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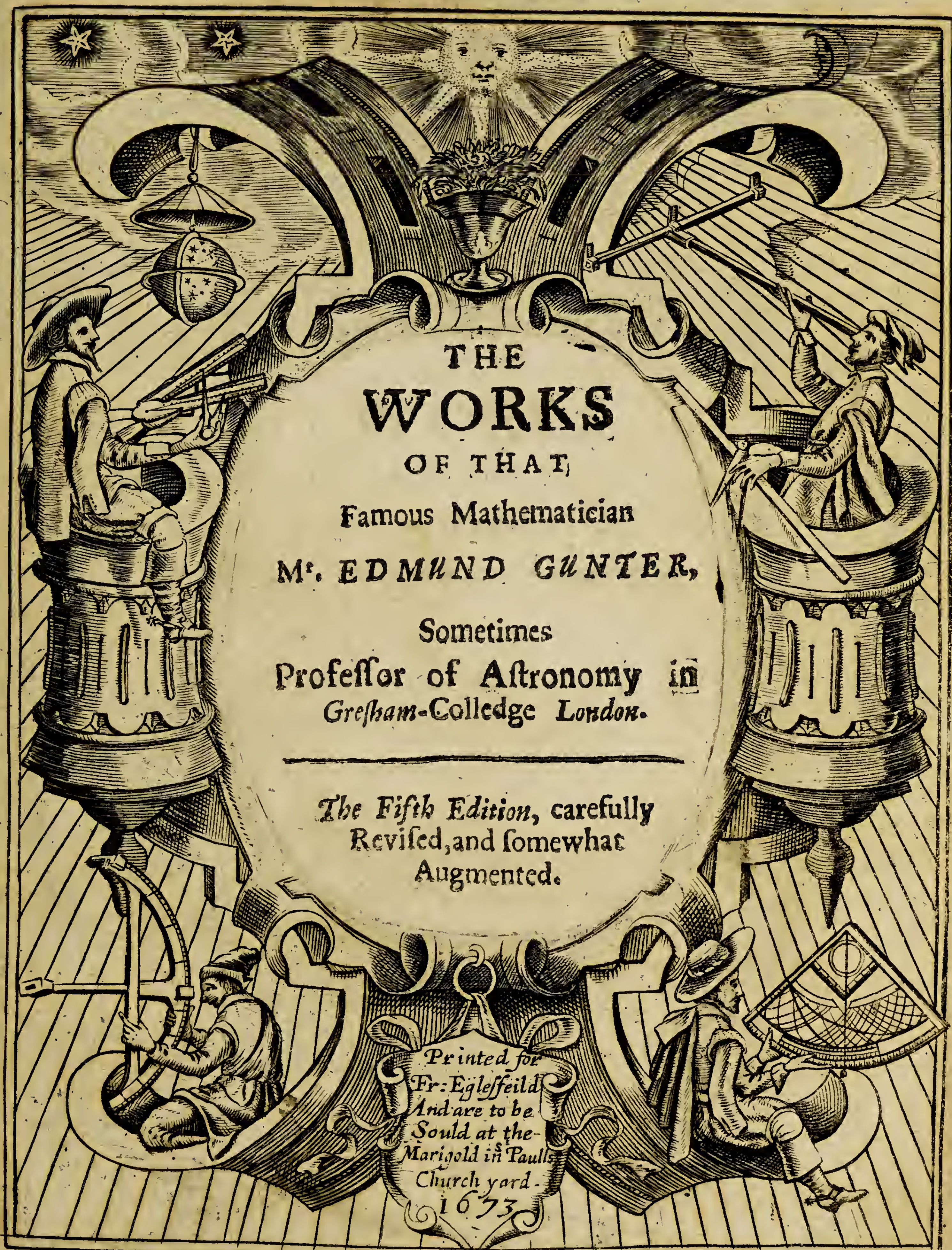
THE

WORKS

OF

Edmund Gunter.


THE
WORKS
OF THAT
Famous Mathematician
M^r. EDMUND GUNTER,
Sometimes
Professor of Astronomy in
Gresham-Colledge London.
The Fifth Edition, carefully Revised, and somewhat Augmented.
Printed for Fr: Eglesfeild And are to be Sould at the Marigold in S^t. Paulls Church yard 1673

THE WORKS OF EDMUND GUNTER:

Containing the *Description* and *Use* of the

Sector, Cross-staff, Bow, Quadrant, And other Instruments.

With a Canon of Artificial Sines and Tangents to a Radius of 10.00000 parts, and the Logarithms from an Unite to 10000:

The Uses whereof are illustrated in the Practice of

Arithmetick, *Geometry*, *Astronomy*, *Navigation*, *Dialling*, and *Fortification*.

And some Questions in Navigation added by Mr. *Henry Bond*, Teacher of Mathematicks in *Ratcliff*, near *London*.

To which is added,

The Description and Use of another Sector and Quadrant, both of them invented by Mr. *Sam. Foster*, Late Professor of Astronomy in *Gresham* Colledge, *London*, furnished with more Lines, and differing from those of Mr. *Gunters* both in form and manner of Working.

The Fifth Edition,

Diligently Corrected, and divers necessary Things and Matters (pertinent thereunto) added, throughout the whole work, not before Printed.

By *William Leybourn*, Philomath.

LONDON,
Printed by *A. C.* for *Francis Eglesfield* at the *Marigold* in St. *Pauls* Church-yard. MDCLXXIII.

TO THE RIGHT HONOURABLE, JOHN, Earl of *Bridgwater*, Viscount *Brackley*, Baron *Ellesmore* : One of the Lords of His Majesties most Honourable Privie Council, and Lord Lieutenant of the County of *Buckingham*.

My Lord,

T*Hese Works of the Learned* Gunter, *do naturally and of right, address themselves to your Honours Patronage: having been originally Insignalized with the Titles of your Renowned Ancestors, under whom (near Fifty years since) they received their first Life: So that they seem to be Intailed on your Illustrious Family: Especially, considering the whole World owns your Lordship, no less Heir to your Ancesters resplendent Demeans, then of their love to Noble Arts, amongst which those of the Mathematicks, is none of the meanest you are Master of.*

My

My Lord, The World taking example by so good a Commendum has encouraged this Work to this Fifth Edition. But it having met with the ill usage in former Impressions to have contracted several Typographical Sphalmata, which did somewhat disfigure its beauty, I thought it requisite to bestow some pains in their removal; as also to furnish it with the attendance of some compleat Tracts of Mr. Samuel Foster *our Learned Authors Successor in his Astronomical Profession in* Gresham *Colledge: What other Additions I have made to the Work, in several places, which are not only pertinent, but necessary, the Reader is acquainted with in the Preface. My Lord, This Work being arrived to this state of Perfection, pleads for a bolder access to your Honours hands; and makes it humbly confident to find your Honour no less favourable to it, now grown up, than your Predecessors have been to its infancy. My Lord, I have derived, likewise, hence some share of that humble confidence, that your Honour will pardon this presumption, of my subscribing my self,*

My Lord,

Your Lordships most obliged,

and obsequious Servant,

William Leybourn.

WILLIAM LEYBOURN TO THE READER.

I Am far from the vanity of desiring to have it thought, that I prefix my *Name* as a *Bush* or *Garland* to invite any to to the *Purchasing* of this *Book*; The Learned *Authors* Authority is more than *I* or any other can say for it, and the number of *Impressions* that have been so welcomed by the Publick is a sufficient *Testimony* of its good *acceptance* in the *World*, for indeed, of all the *Mathematical Books* yet *extant*, I know not one more full of *Variety* of matter, nor more *Practical* than this is.

All that I design in this *Preface* is an *Apology* for my self, to ask *pardon* of the more knowing *Mathematician*, for my confidence in presuming to *shelter* any of my *mean* and *weak Performances* under the *Canopy* of so profound a *Master* of *Mathematical Learning* as this our *Author* was. But to such as shall be offended therewith (as, I hope, none justly can) let me say thus much for my self:

To the Reader.

1. I am not the firſt that (with good ſucceſs) have attempted the like.

2. In what I have done in this *Work*, I have not diminiſhed or expunged one *Syllable* of the learned *Authors*, but retained his own *Method*, and the ſeveral *Examples* throughout the *Book* I have carefully *examined*, and where I found any *Typographical* Error, I made bold to *correct* it, for which, I preſume, I deſerve rather *Thanks* than *Blame*.

3. That whatſoever herein I have attempted to *inſert*, is nothing but what is abſolutely *pertinent* to our *Authors Works*, and renders his *Inſtruments* to young Tyroes in theſe Sciences more uſeful than they could otherwiſe imagine.

4. In what part of this *Book* ſoever I have *added* any thing, I have done the *Author* this *right*, for in the Contents before the Book, relating to the Page wherein any Inſerſion of mine is, I have before it placed the figure of a *hand* pointing thus ☞: So that if I have done any thing, misbecoming an *Artiſt*, the Author may not be *charged* with it, but my ſelf juſtly *blamed*.

And although, there are here and there ſome hints of things in ſeveral places of the *Book* of mine *inſerted*, yet the principal are theſe, *viz*.

1. In the SECTOR, where (after our *Author*

hath

hath treated of *Projecting* of the *Sphere* in *Plano* upon all the principal *Spherical Circles*) I have added one other *Projection* upon an *Oblique Circle,* wherein (if I deceive not my ſelf) I have given more light to *Projection* in *Plano*, than is yet extant in our *Mother Tongue*: for out of this *Oblique Projection* may be demonſtrated the whole *Art* of *Dialling*, and in ſome meaſure it is there effected.

2. In the CROSS-STAFF (after our Author hath treated of the *Menſuration* of *Plain Regular Superficies*) I have inſerted the *Menſuration* of ſuch as are not *Uniform*, as alſo of *Multangulars*, *Regular Poligons*, &c. And (after his Menſuration of *Regular ſquared Solids*) I have added the *Menſuration* of *Priſmes*, *Pyramids*, and *Cones*, both *whole* and *diſſected*. And with theſe and ſuch like neceſſary *matters*, I have in ſeveral other places ſupplied a *Vacancy*.

To the ſecond *Appendix*, which is the uſe of a *Quadrant*, of Mr. *Samuel Foſters* Invention, Printed with the former Edition of theſe our *Authors Works*, I have altered nothing, but have added the *Conſtruction* of the ſame *Quadrant* formerly wholly omitted. And in his *Alteration* of the SECTOR, I have corrected ſome Overſights, and miſtakes, which were in the former Edition (that being Printed by a Copy leſs Correct) by the help of Mr. *Foſters* own *Manu-*

ſcript, which I was accommodated with from the worthy Dr. *John Twiſden*, a moſt induſtrious *Mathematician*, and a worthy honourer of the *Learned* Mr *Foſter*, to whom (not only my ſelf, but) the whole World in general is engaged for his care and pains in the Publication of divers of Mr. *Foſters Works* with ſeveral of *his own* both in *Latine* and *Engliſh* in a Book Entituled *Miſcellanies*, or *Mathematical Lucubrations* of Mr. *Samuel Foſter*.

Having thus far declared my ſelf, and endeavoured to take off ſuch aſperſions as might poſſibly have been thrown upon me; Give me leave (for the Dead cannot plead for themſelves) to take notice of ſome *Plagiaries* and *Purloiners* of other mens Labours and Ingenuities, who out of *Lucre* to *themſelves*, and *Emulation* to *others* of better parts, have lately thrown into the *World* (to the grand abuſe thereof) ſeveral trivial *Tractates*, extracted (or rather tranſcribed) both from our *Author*, and alſo from the *Works* and *Manuſcripts* of the fore-mentioned Mr. *Foſter*, our *Authors* Succeſſor in the *Aſtronomical Profeſſion* in *Greſham Colledge, London*, Publiſhing them to the World in their own names, without taking the leaſt notice of the learned Authors, whence they originally filcht thoſe ornaments wherewith they pride themſelves in their ſeveral Pamphlets, not ſo much as mentioning their names with any due reſpect. I need not tell thee who they be, Their own Impertinencies having made them notorious enough; for ſome of them (rather than they will want applauſe) become their own *Encomiaſters*, ſounding their own *Trumpets* before their *Books*, both in *Engliſh*, *Greek*, and *Latine*. But leaving theſe to the juſt cenſure of all that ſhall take due notice of them, give me leave to commend thee to the peruſal of theſe *Works* of our Judicious *Authors*, in the *Uſe* and *Practice* whereof (as in all other thy honeſt *Attempts* and *Endeavours*) I wiſh thee good *ſucceſs*, and ſo for this time bid thee

April 18.
1673.

Farewel.

Arts and Sciences MATHEMATICAL
Professed and Taught by *William Leybourn.*

Arithmetick,
- In Whole Numbers, and Fractions.
- In Decimals, and by Logarithms.
- Instrumentally, by Decimal Scales, Napiers Bones: and to extract the Square and Cube Roots by Inspection.

Geometry:
- The Principles thereof with the
  - Practice, and
  - Demonstration.

Astronomy:
- The Description of the Circles of the Sphere.
- The Use of the Globes,
  - Celestial, and
  - Terrestrial.
- To project the Sphere *in Plano* upon any Circle,
  - Right, or
  - Oblique.

And upon these Foundations the following Superstructures.

The Use of Geometrical Instruments, in the Practice of
- *Longimetria*, or the Mensuration of
  - Heights, as of Trees, Towers, &c.
  - Depths, as of Mines, Wells, Descents, &c.
  - Distances, as of Churches, Towers, &c.
- *Planometria*, or the Mensuration of
  - Board,
  - Glass,
  - Pavement,
  - Tiling, &c.
  - Or any other Superficies.
- *Stereometria*, or the Mensuration of
  - Timber, growing or squared.
  - Stone, regular or irregular.
  - Cask, commonly called Gauging.
- *Geodæsia*, or the Measuring of Land divers ways, and by several Instruments; to draw the Plot of a whole Mannor or Lordship; to cast up the Content thereof; and to beautifie the same with all necessary Ornaments thereunto belonging.

Trigonometria:
- Or, the Mensuration of Triangles, both
  - Plain, and
  - Spherical.
- The Application thereof, in the solution of Problems in
  - Geometry.
  - Astronomy.
  - Geography.
  - Navigation.
  - Fortification.
  - Dialling, &c.

Navigation:
- The Principles thereof, and the manner of Sailing by
  - The Plain Sea-Chart.
  - *Mercator*'s Chart.
  - The Arch of a great Circle.

Horologiographia, Or Dialling:
- *Arithmetically*, by the Tables of
  - Sines.
  - Tangents.
  - Logarithms.
- *Geometrically*, by
  - Scale, and
  - Compasses.
- *Instrumentally*, by the Sector, Quadrants, Scales, and other Instruments, accommodated with Lines for that purpose,

You may hear of him at Mr. *Hayes*'s at the *Cross-daggers* in *Moor-fields*.

THE CONTENTS OF The First Book of the Sector.

CHAP. I.

To

The Contents of the second Book of the *Sector.*

Having

The Contents of the third Book of the *Sector.*

The Contents of the First Book of the *Croſs-Staff*.

CHAP. I.

The Contents of the ſecond Book of the *Croſs-Staff*.

The

The Contents of the third Book of the *Cross-Staff.*

Chap. 12.

4. The

The Contents of the general use of the *Canon*, and Table of *Logarithms*.

THE

Advertisement.

WHereas the whole Subject of the following Treatises do contain the use of Instruments, and that the true and exact making of them is principally to be minded and enquired into, I thought good to give notice, That if any Gentlemen studious in the Mathematicks have or shall have occasion for any Instrument belonging to this Book, as also with all others useful both for Sea or Land, they may be furnished either in Silver, Brass, or Wood, by Walter Hayes, *at the* Cross-daggers *in* Moor-fields, *next door to the* Pope's-head *Tavern*; *where they may have all sorts of Maps, Globes, Sea-plats, Carpenters Rules, Post and Pocket-Dials for any Latitude,* &c.

THE FIRST BOOK OF THE SECTOR.

CHAP. I.

The Description, the Making, and the General Use of the SECTOR.

A SECTOR in Geometry, is a Figure comprehended of two right Lines containing an Angle at the Center, and of the Circumference assumed by them. This Geometrical Instrument having two Legs, conteining all variety of Angles, and the distance of the Feet, representing the Subtenses of the Circumference, is therefore called by the same name.

It conteineth 12 several Lines or Scales, of which 7 are general, the other 5 more particular. The first is the Scale of Line, divided into 100 equal parts, and numbred by 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

The second, the Lines of Superficies, divided into 100 unequal parts, and numbred by 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

3. The third, the Lines of Solids, divided into 1000 unequal parts, and numbred by 1, 1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

4. The fourth, the Lines of Sines and Chords, divided into 90 degrees, and numbred with 10, 20, 30 unto 90.

Theſe four Lines, of *Lines*, of *Superficies*, of *Solids*, and of *Sines*, are all drawn from the Center of the *Sector* almoſt to the end of the Legs. They are drawn on both the Legs, that every Line may have his fellow. All of them are of one Length, that they may anſwer one to the other: And every one hath his Parallels, that the eye may the better diſtinguiſh the Diviſions. But of the Parallels thoſe only which are inwardmoſt contein the true Diviſions.

There are three other general Lines, which becauſe they are infinite are placed on the Side of the Sector.

5. The firſt a Line of *Tangents*, numbred with 10, 20, 30, 40, 50, 60, ſignifying ſo many degrees from the beginning of the Line, of which 45 are equal to the whole Line of Sines, the reſt follow as the length of the Sector will bear.

6. The ſecond, a Line of Secants, divided by Pricks into 60 degrees, is the ſame with that of the Line of Tangents to which it is joyned.

7. The third is the Meridian Line, or Line of Rumbs, divided unequally into degrees, of which the firſt 70 are almoſt equal to the whole Line of Sines, the reſt follow unto 85, according to the Length of the Sector.

Of the particular Lines inſerted among the general, becauſe there was void ſpace.

8. The firſt are the Lines of Quadrature placed between the Lines of Sines, and noted with 10, 9, 8, 7, S, 6, 5, 90, Q.

9. The ſecond, the Lines of Segments, placed between the Lines of Sines and Superficies, divided into 50 parts, and numbred with 5, 6, 7, 8, 9, 10.

10. The third, the Lines of inſcribed bodies in the ſame Sphere, placed between the Scales of Lines, and noted with *D. S. I. C. O. T.*

11. The fourth, the Lines of Equated Bodies, placed between the Lines of Lines and Solids, and noted with *D. I. C. S. O. T.*

12. The fifth, are the Lines of Metals, inſerted with the Lines of Equated Bodies, (there being room ſufficient) and noted with the Characters ☉ ☿ ♄ ☽ ♀ ♂ ♃.

There remain the Edges of the Sector, and on the one I have ſet a Line of Inches, which are the twelfth parts of a Foot *Engliſh*: on the other a leſſer Line of Tangents to which the Gnomon is Radius.

2. *Of the making of the* SECTOR.

LEt a Ruler be first made either of Brass or of Wood like unto the Figure before the Book annexed. Which may open and shut upon his Center: the Head of it may be about the twelfth part of the whole Length, that it may bear the moveable Foot, and yet the most part of the Divisions may fall without it. Then let a moveable Gnomon be set at the end of the moveable Foot, and there turn upon an Axis, so as it may sometimes stand at a right Angle with the Feet; and sometimes be inclosed within the Feet. But this is well known to the Work-man.

For drawing of the Lines. Upon the Center of the Sector, and Semidiameter somewhat shorter than one of the Feet, draw an occult Ark of a Circle, crossing the Closure of the inward Edge of the Sector, about the Letter *T*.

In this Ark, at one degree on either Side from the Edge, draw right Lines from the Center, fitting them with Parallels, and divide them into an hundred equal parts, with Subdivisions into 2, 5, or 10, as the Line will bear, but let the Numbers set to them, be only 1, 2, 3, 4, *&c.* unto 10, as in the Example. These Lines so divided, I call the Lines or Scales of Lines; and they are the ground of all the rest.

In this Ark, at 5 degrees on either Side, from the Edge near *T*, draw other right Lines from the Center, and fit them with Parallels: these shall serve for the Lines of Solids.

Then on the other Side of the Sector, in like manner, upon the Center, and equal Semidiameter, draw another like Ark of a Circle: and here again at one degree near on either Side from the Edge, near the letter Q, draw right Lines from the Center, and fit them with Parallels: These shall serve for the Lines of Sines.

At 5 degrees, on either Side from the Edge near Q, draw other right Lines from the Center, and fit them with Parallels: these shall serve for the Lines of Superficies.

These four principal Lines being drawn, and fitted with Parallels, we may draw other Lines in the middle between the Edges and the Lines of Lines, which shall serve for the Lines of Inscribed Bodies, and others between the Edges and the Sines for the Lines of Quadrature. And so the rest as in the Example.

3. *To divide the Lines of Superficies.*

SEeing the Superficies do hold in the Proportion of their *homologal* Sides duplicated by the 29. *Prop.* 6. *Lib. Euclid.* If you shall find mean Proportionals between the whole Side, and each hundred part of the like Side, by the 13. *Prop.* 6. *Lib. Euclid.* all of them cutting the same Line, that Line so cut shall contain the Divisions required; wherefore upon the Center A, and Semidiameter equal to the Line of Lines, describe a Semicircle A C B D, with A B perpendicular to the Diameter C D. And let the Semidiameter A D be divided as the Line of Lines into an hundred parts, and A E the one half of A C divided also into an hundred parts, so shall the Divisions in A E be the Centers from whence you shall describe the Semicircles C 10. C 20. C 30. *&c.* dividing the Line A B into an hundred unequal parts: and this Line A B so divided shall be the Line of Superficies, and must be transferred into the Sector. But let the numbers set to them be only 1. 1. 2. 3. unto 10. as in the Example.

Or these Lines of Superficies may otherwise be transferred into the Sector, out of the Line of Lines, by a Table of Square Roots; For the Root taken out of the Line of Lines, shall give the Square in the Lines of Superficies.

As, to inscribe the Division of 25 in the Lines of Superficies; put six Ciphers to 25, and make it 25000000, then find the Square Root of this Number, which will be 5000.

Take therefore 5000 out of the Line of Lines (supposing the whole Line to be 10000) and it will give the true Distance between the Center, and the points of 25, in the Lines of Superficies.

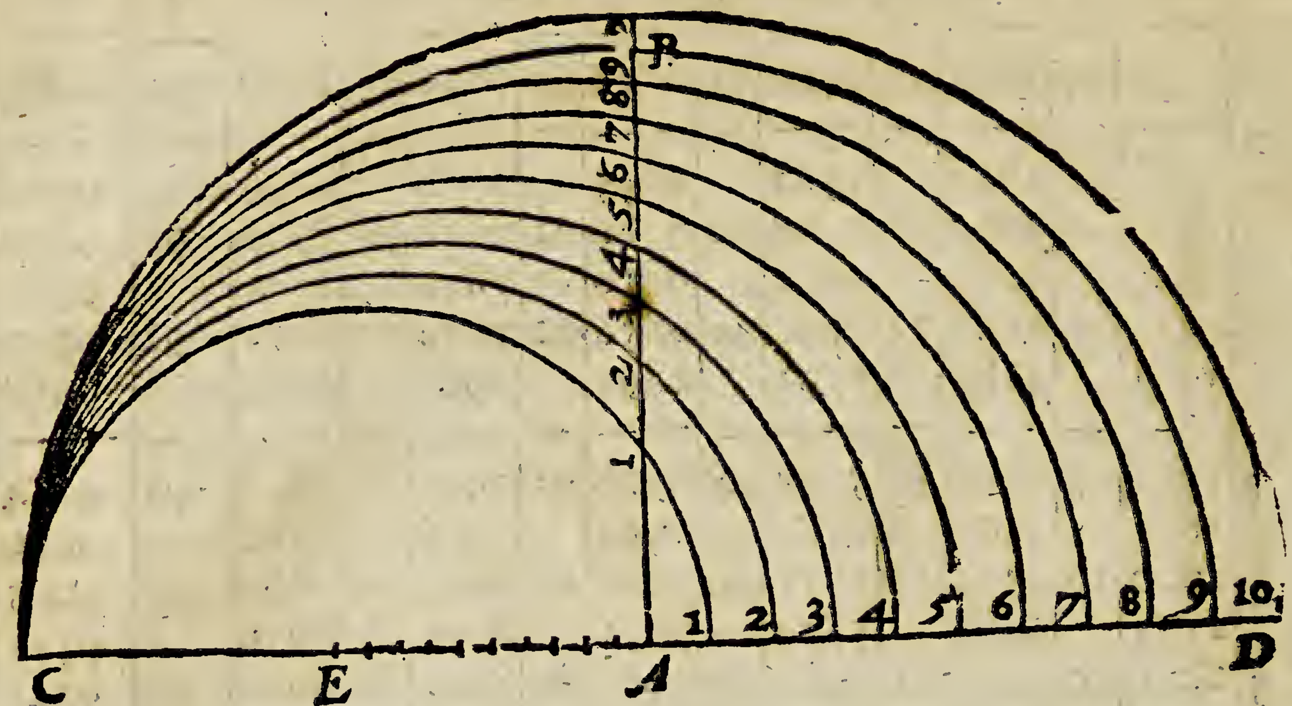
So, for the Division of 30, put to 30 six Ciphers, and make it 30000000, whose Square Root is 5477. This (taken out of the Line of Lines) shall give the place for the Points of 30, in the Lines of Superficies. And the like reason holdeth for all the rest, according to this following Table.

If any please to make use of a Diagonal Scale, equal to the Line of Lines, he may put eight Ciphers to the Number proposed, and make the Table of Roots to five Places: So, his work will be more exact.

A Table of Square Roots *for the Division of the Lines of* Superficies.

A Table of Square Roots for Division of the Line of Superficies.

Sq	Root.	Sq.	Root.	Sq	Root	Sq	Root.	Sq.	Root.	Sq	Root.	Sq.	Root.
0		15	3873	30	5477	45	6708	60	7746	75	8660	90	9487
	707		3937		5523		6745		7778		8689		9513
1	1000	16	4000	31	5568	46	6782	61	7810	76	8718	91	9539
	1225		4062		5612		6819		7842		8746		9565
2	1414	17	4123	32	5657	47	6856	62	7874	77	8775	92	9592
	1581		4183		5701		6892		7906		8803		9618
3	1732	18	4243	33	5744	48	6928	63	7937	78	8832	93	9644
	1871		4301		5788		6964		7969		8860		9670
4	2000	19	4359	34	5831	49	7000	64	8000	79	8888	94	9695
	2121		4416		5874		7036		8031		8916		9721
5	2236	20	4472	35	5916	50	7071	65	8062	80	8944	95	9747
	2345		4528		5958		7106		8093		8972		9772
6	2449	21	4582	36	6000	51	7141	66	8124	81	9000	96	9798
	2550		4637		6042		7176		8155		9028		9823
7	2646	22	4690	37	6083	52	7211	67	8185	82	9055	97	9849
	2739		4743		6124		7246		8216		9083		9874
8	2828	23	4796	38	6164	53	7280	68	8246	83	9110	98	9899
	2915		4848		6205		7314		8276		9138		9925
9	3000	24	4899	39	6245	54	7348	69	8307	84	9165	99	9950
	3082		4950		6285		7382		8337		9192		9975
10	3162	25	5000	40	6325	55	7416	70	8367	85	9219	100	10000
	3240		5050		6364		7450		8396		9247		
11	3317	26	5099	41	6403	56	7483	71	8426	86	9274		
	3391		5148		6442		7517		8556		9300		
12	3464	27	5196	42	6481	57	7550	72	8485	87	9327		
	3536		5244		6519		7583		8515		9354		
13	3606	28	5291	43	6557	58	7616	73	8544	88	9381		
	3674		5338		6595		7648		8573		9407		
14	3742	29	5385	44	6633	59	7681	74	8602	89	9434		
	3808		5431		6671		7714		8631		9460		
15	3873	30	5477	45	6708	60	7746	75	8660	90	9487		



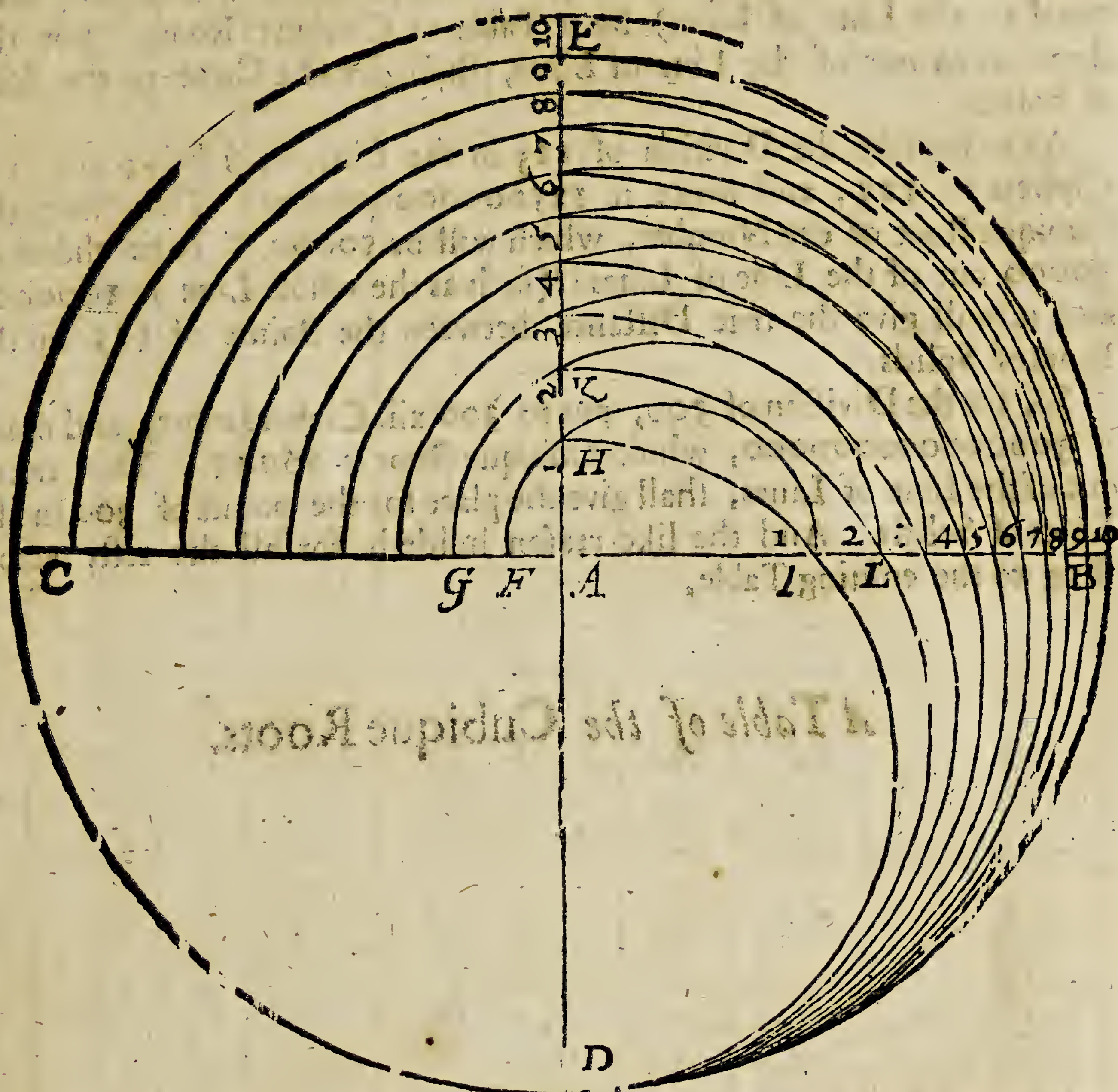
4. *To divide the Lines of Solids.*

SEeing like Solids do hold in the Proportion of their homologal Sides triplicated, if you ſhall find two mean Proportionals between the whole Side and each thouſandth part of the like Side: all of them cutting the ſame two right Lines, the former of thoſe Lines ſo cut, ſhall contain the Diviſions required.

Wherefore upon the Center A, and Semidiameter equal to the Line of Lines, deſcribe a Circle and divide it into 4 equal parts C E B D, drawing the croſs-Diameters C B E D. Then divide the Semidiameter A C, firſt into 10 equal parts, and between the whole Line A D and A F, the tenth part of A C, ſeek out two mean Proportional Lines A I and A H: again between A C and A G (being two Tenths of A B) ſeek out two mean Proportionals A L and A K, and ſo forward in the reſt. So ſhall the Line A B, be divided into 10 unequal parts.

Secondly, divide each tenth part of the Line A C into 10 more, and between the whole Line A D, and each of them, ſeek out two mean Proportionals as before: So ſhall the Line A B be divided now into an hundred unequal parts.

Thirdly,



Thirdly, if the Length will bear it, ſubdivide the Line A C once again, each part in ten more, and between the whole Line A D, and each Subdiviſion, ſeek two mean Proportionals as before. So ſhall the Line A B be now divided into 1000 parts. But the Ruler being ſhort, it ſhall ſuffice, if thoſe 10 which are neareſt the Center be expreſſed, the reſt be underſtood to be ſo divided, though actually they be divided into no more than 5 or 2, and this Line A B ſo divided ſhall be the Line of Solids, and muſt be transferred into the Sector: But let the Numbers ſet to them be only 1. 1. 1. 2. 3. *&c.* unto 10, as in the Example.

On

Or theſe Lines of Solids may otherwiſe be transferred into the Sector, out of the Line of Lines (or rather, out of a Diagonal Scale equal to the Line of Lines) by a Table of Cubique Roots. For the Root taken out of the Line of Lines, ſhall give the Cube in the Line of Solids.

As to inſcribe the Diviſion of 125 in the Lines of Solids; put xii. Ciphers to 125, and make it 125000000000000: Then find the Cubique Root of the Number, which will be 50000. Take therefore 50000 out of the Line of Lines; (ſuch as the whole Line is 100000) and it will give the true Diſtance between the Points of 125 in the Lines of Solids.

So, for the Diviſion of 300, put to 300 xii. Ciphers more, and make it 300000000000000, whoſe Cubique Root is *66943*. This, taken out of the Line of Lines, ſhall give the place for the points of 300 in the Lines of Solids. And the like reaſon holdeth for all the reſt, according to the enſuing Table.

A Table of the Cubique Roots.

A Table

A Table of Cubique Roots.

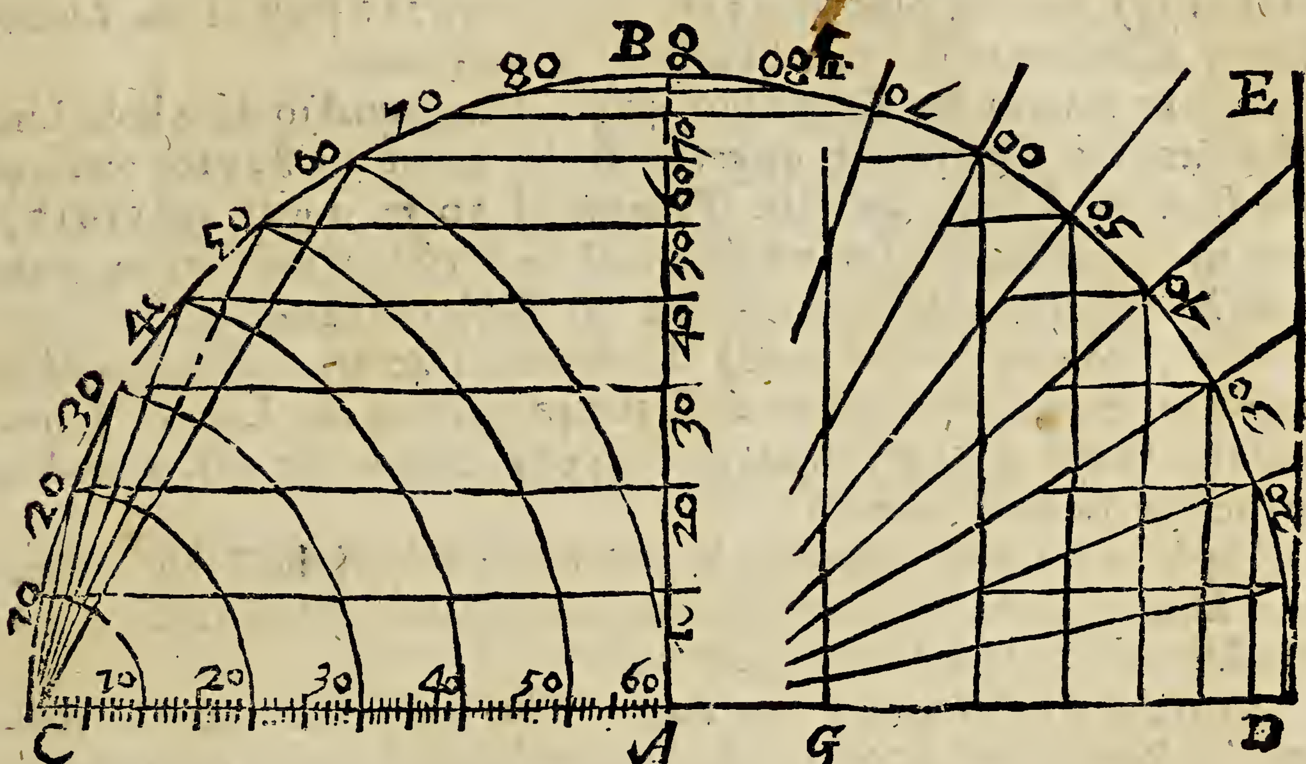
Cub	Root.	Cub	Root.	Cub.	Root.	Cub.	Root.	Cub.	Root.
0	0	20	2714	50	3684	125	5000	275	6502
	794	21	2758	52	3732	130	5065	280	6542
1	1000	22	2802	54	3779	135	5129	285	6580
	1144	23	2843	56	3825	140	5192	290	6619
2	1259	24	2884	58	3870	145	5253	295	6656
	1357	25	2924	60	3914	150	5313	300	6694
3	1442	26	2962	62	3957	155	5371	305	6731
	1518	27	3000	64	4000	160	5428	310	6767
4	1587	28	3036	66	4041	165	5484	315	6804
	1650	29	3072	68	4081	170	5539	320	6839
5	1709	30	3107	70	4121	175	5593	325	6875
	1765	31	3141	72	4160	180	5646	330	6910
6	1817	32	3174	74	4198	185	5698	335	6945
	1866	33	3207	76	4235	190	5748	340	6979
7	1912	34	3239	78	4272	195	5798	345	6913
	1957	35	3271	80	4308	200	5848	350	7047
8	2000	36	3301	82	4344	205	5896	355	7080
	2040	37	3332	84	4379	210	5943	360	7119
9	2080	38	3361	86	4414	215	5990	365	7146
	2117	39	3391	88	4447	220	6036	370	7179
10	2154	40	3419	90	4481	225	6089	375	7211
11	2223	41	3448	92	4515	230	6126	380	7243
12	2289	42	3476	94	4546	235	6171	385	7274
13	2351	43	3503	96	4578	240	6214	390	7306
14	2410	44	3530	98	4610	245	6257	395	7337
15	2466	45	3556	100	4641	250	6299	400	7368
16	2519	46	3583	105	4717	255	6341	405	7398
17	2571	47	3608	110	4791	260	6382	410	7428
18	2620	48	3634	115	4862	265	6423	415	7459
19	2668	49	3659	120	4931	270	6463	420	7488
20	2714	50	3684	125	5000	275	6502	425	7518

The Division of the Lines of Solids.

Cub.	Root.	Cub.	Root.	Cub.	Root.	Cub.	Root.
425	7518	575	8315	725	8983	875	9564
430	7547	580	8339	730	9004	880	9580
435	7576	585	8363	735	9024	885	9600
440	7605	590	8387	740	9045	890	9619
445	7634	595	8410	745	9065	895	9636
450	7663	600	8434	750	9085	900	9654
455	7691	605	8457	755	9105	905	9672
460	7719	610	8480	760	9125	910	9690
465	7747	615	8504	765	9145	915	9708
470	7774	620	8527	770	9165	920	9725
475	7802	625	8549	775	9185	925	9743
480	7829	630	8572	780	9205	930	9761
485	7856	635	8595	785	9224	935	9778
490	7883	640	8617	790	9244	940	9795
495	7910	645	8640	795	9263	945	9813
500	7937	650	8662	800	9283	950	9830
505	7963	655	8684	805	9302	955	9847
510	7989	660	8706	810	9321	960	9864
515	8015	665	8728	815	9340	965	9881
520	8041	670	8750	820	9359	970	9898
525	8067	675	8772	825	9378	975	9915
530	8092	680	8793	830	9397	980	9932
535	8118	685	8815	835	9416	985	9949
540	8143	690	8836	840	9435	990	9966
545	8168	695	8857	845	9454	995	9983
550	8193	700	8879	850	9472	1000	10000
555	8217	705	8900	855	9491		
560	8242	710	8921	860	9509		
565	8267	715	8942	865	9529		
570	8291	720	8962	870	9546		
575	8315	725	8983	875	9564		

5. *To divide the Lines of Sines and Tangents on the Side of the Sector.*

UPon the Center A, and Semidiameter equal to the Line of Lines, describe a Semicircle A B C D, with A B, perpendicular to the Diameter C D. Then divide the Quadrant C B, B D, each of them into 90, and subdivide each degree into two parts: For so if streight Lines be drawn parallel to the Diameter C D, through these 90, and their Subdivisions, they shall divide the Perpendicular A B unequally into 90.



And this A B (so divided) shall be the Line of Sines, and must be transferred into the Sector. The Number set to them are to be 10, 20, 30, *&c.* unto 90, as in the Example.

If now in the point D, unto the Diameter C D, we shall raise a Perpendicular D E, and to it draw streight Lines from the Center A, through each Degree of the Quadrant D B, these streight Lines shall be Secants, and this Perpendicular so divided by them shall be the Line of Tangents, and must be transferred unto the Side of the Sector. The Number set to them, are so be 10, 20, 30, *&c.* as in the Example.

If between A and D, another streight Line G F be drawn parallel to D E, it will be divided by those Lines from the Center in like sort as

DE is divided, and it may serve for a lesser Line of Tangents, to be set on the Edge of the Sector.

If the Compasses shall be extended from C to each degree of the Quadrant C B, and those Extents transferred into one Line (C A) this Line C A so divided into 60 (or rather into 90 *gr.*) shall be a Line of Chords, and may be set on some void place of the Sector.

These Lines of Sines and Tangents, may yet otherwise be transferred into the Sector out of the Line of Lines (or rather out of a Diagonal Scale equal to the Line of Lines,) by Tables of Natural Sines and Tangents.

For the Sine of 90 *gr.* being equal to the whole Line of Lines of 100000 parts, the Sine of 30 *gr.* will be equal to 50000 (half the Line of Lines;) and the Sine of 45 *gr.* equal to 70710 parts of the Line of Lines, according to the usual Table of natural Sines.

In like manner the Tangent of 45 *gr.* being equal to the whole Line of Lines, the Tangent of 40 *gr.* will be equal to 83910 parts of the Line of Lines: and the Tangent of 50 *gr.* equal to 119175, that is, to one Radius (or whole Line) and 19175 parts more of the same Line of Lines, according to the old Table of Tangents.

And (upon the same ground) the Secant of 40 *gr.* will be equal to 1.30540, that is, one Radius and 30540 parts of the Line of Lines: and the Secant of 50 *gr.* equal to 1.55572, and so the rest, according to the like Table of Secants.

The Line of Chords may also be divided by help of the Table of Sines and Line of Lines. For the double Sine of half the Ark taken out of the Line of Lines will give the Chord.

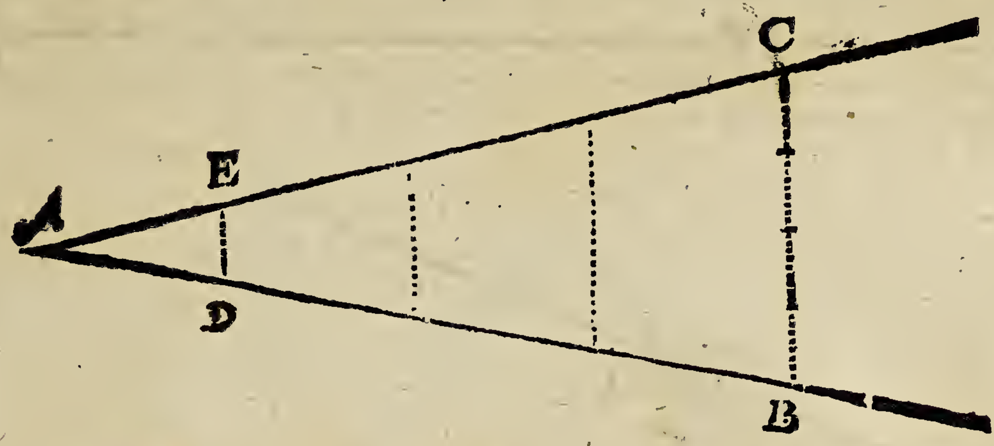
As if the Ark proposed were 60 *gr.* The half of this Ark is 30 *gr.* and the Sine thereof 50000, which being doubled, make 100000, the whole Line of Lines, equal to a Chord of 60 *gr.*

So for the Chord of 90 *gr.* the half Ark is 45 degrees, and the Sine thereof 70710, which being doubled, make 1414240, that is, one Radius, and 41420 parts of the Line of Lines, equal to the Chord of 90 *gr.* required.

6. *To shew the Ground of the Sector.*

LEt A B, A C, represent the Legs of the Sector; then seeing these two A B, A C are equal, and their Sections A D, A E, also equal, they shall be cut proportionally: and if we draw the Lines B C, D E, they will be parallel by *Prop.2. Lib.6. of Euclid,* and so the Triangles A B C

A B C, A D E, ſhall be equiangled, by reaſon of the common Angle at A, and the equal Angles at the Baſe, and therefore ſhall have the Sides proportional about thoſe equal Angles, by *Prop*.4.*Lib*.6. of *Euclid*.



The Side A D ſhall be to the Side A B, as the Baſis, D E, unto the parallel Baſis B C, and by converſion A B ſhall be unto A D, as B C unto D E; and by permutation A D ſhall be unto D E, as A B to B C, *&c*. So that if A D be the fourth part of the Side A B, then D E ſhall alſo be the fourth part of his parallel Baſis B C. The like reaſon holdeth in all other Sections.

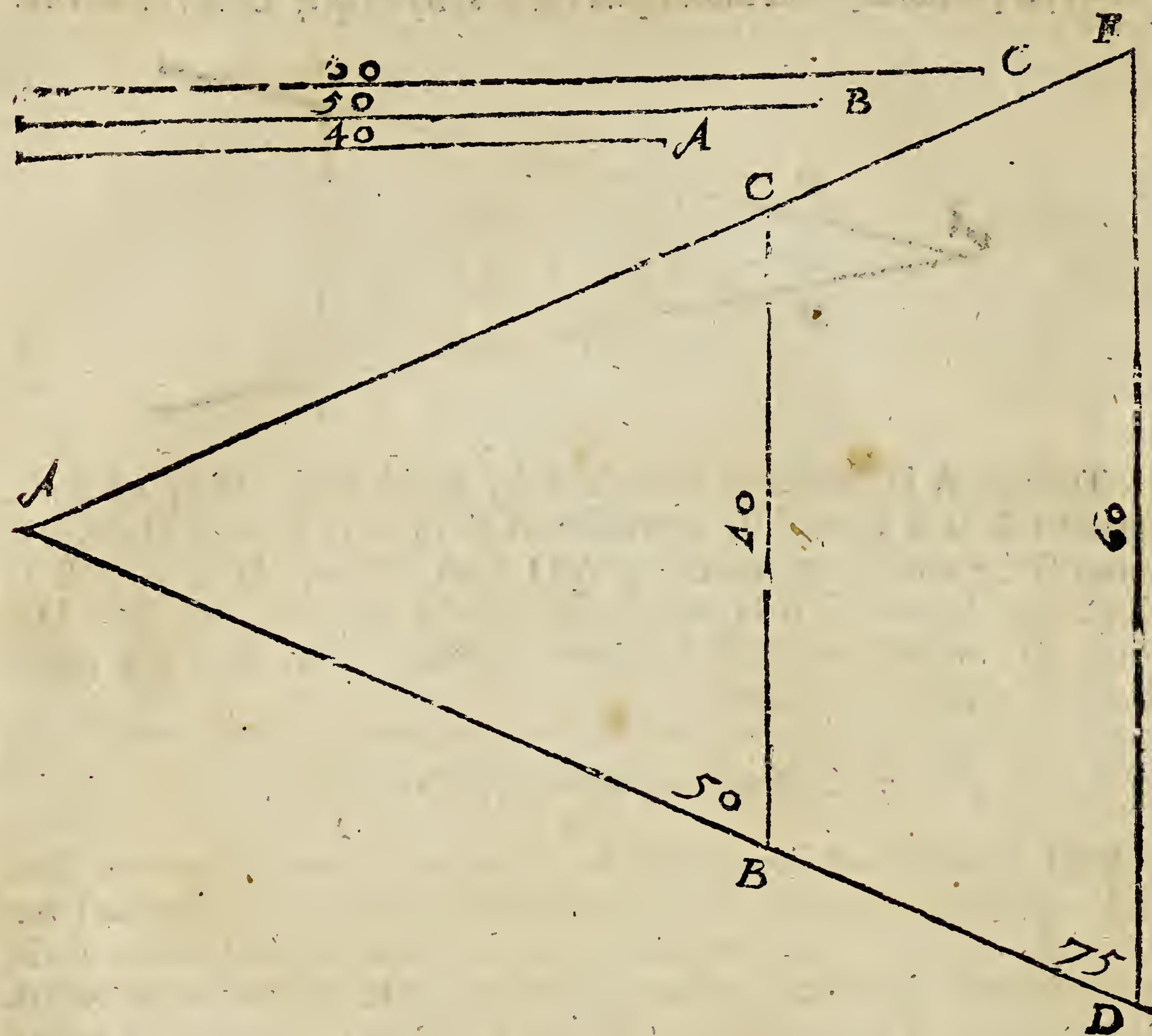
7. *To ſhew the general Uſe of the Sector.*

THere may ſome Concluſions be wrought by the Sector even then when it is ſhut, by reaſon that the Lines are all of one length: but generally the Uſe hereof conſiſts in the ſolution of the Golden Rule, where three Lines being given of a known Denomination, a fourth Proportional is to be found. And this Solution is diverſe in regard both of the Lines and of the Entrance into the Work.

The Solution in regard of the Lines is ſometimes ſimple, as when the Work is begun and ended upon the ſame Lines. Sometimes it is compound, as when it is begun on one kind of Lines and ended on another. It may be begun upon the Lines of Lines, and finiſhed upon the Lines of Superficies. It may begin on the Sines, and end on the Tangents.

The Solution in regard of the Entrance into the work, may be either with a Parallel, or elſe Lateral on the Side of the Sector, I call it Parallel Entrance, or entring with a Parallel, when the two Lines of the firſt Denomination are applied in the Parallels, and the third Line, and that which is ſought for, are on the ſide of the Sector: I call it lateral Entrance, or entring on the ſide of the Sector, when the two Lines of the

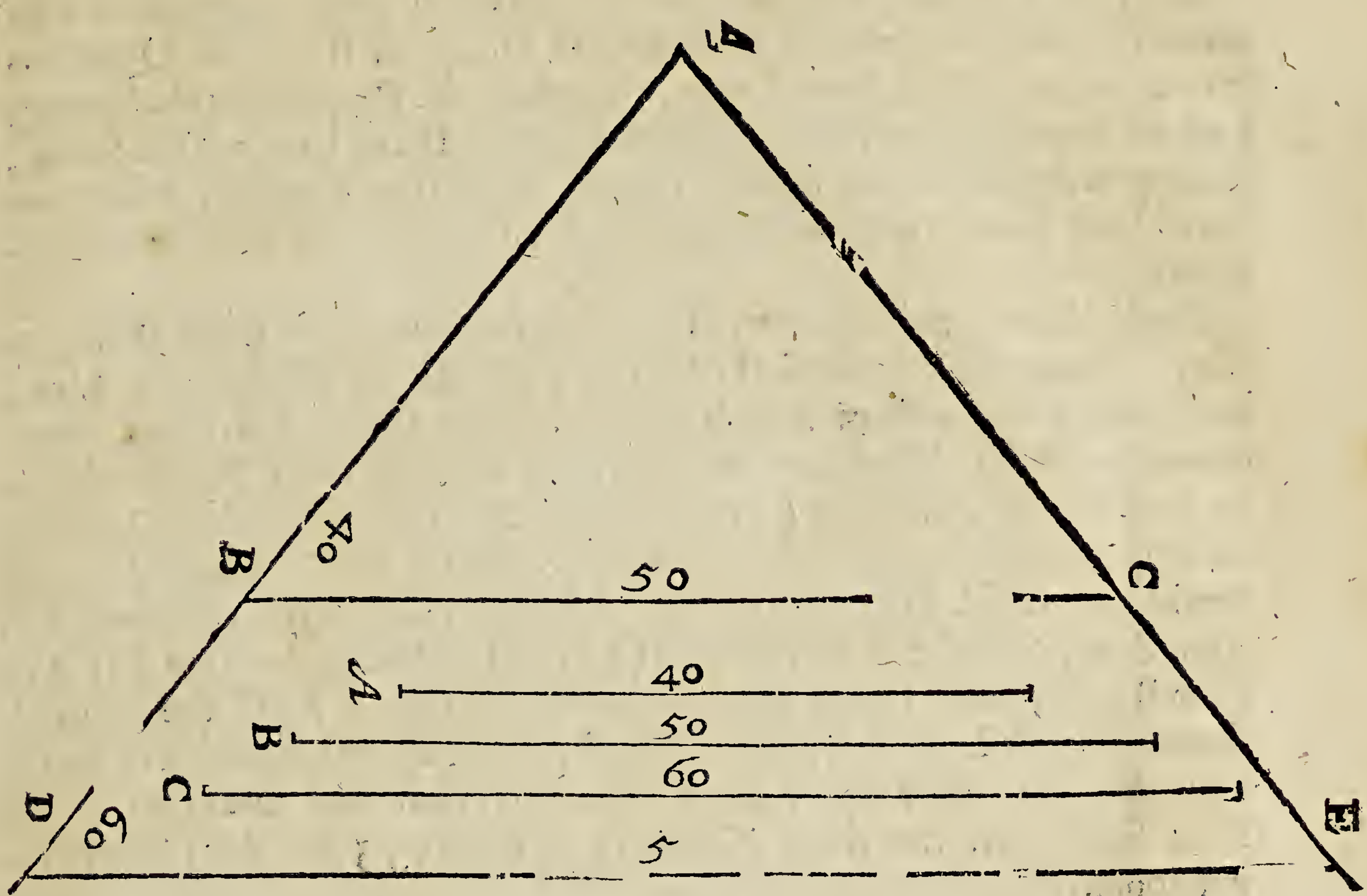
the first Denomination are on the side of the Sector, and the third Line, and that which is to be found out do stand in the Parallels.



As for Example, let there be given three Lines A, B, C, to which I am to find a fourth Proportional, let A measured in the Line of Lines, be 40, B 50, and C 60, and suppose the Question be this: If 40 Months give 50 pounds, what shall 60? Here are Lines of two Denominations, one of Months, another of Pounds, and the first, with which I am to enter, must be that of 40 Months. If then I would enter with a Parallel, first I take A, the Line of 40, and put it over as a Parallel in 50, reckoned in the Line of Lines, on either side of the Sector from the Center, so as it may be the Base of an Isosceles Triangle B A C, whose Sides A B, A C are equal to B, the Line of the second Denomination.

Then

Then the Sector being thus opened, I take C the Line of 60, between the Feet of the Compasses, and carrying them parallel to B C, I find them to cross the Lines A B, A C, on the side of the Sector in D and E, numbred with 75, wherefore I conclude the Line A D or A E is the fourth Proportional and the correspondent Number 75, which was required.



But if I would enter on the Side of the Sector, then would I dispose the Lines of the first Denomination A and C in the Line of Lines, on both sides of the Sector in A B, A C, and in A D, A E, so as they should all meet in the Center A, and then taking B the Line of the second Denomination, put it over as a Parallel in B C, that it may be the Basis of the Isoscheles Triangle B A C (whose Sides A B, A C, are equal to A the first Line of the first Denomination) for so the Sector being thus opened, the other Parallel from D to E, shall be the fourth Proportional which was required, and if it be measured with the other Lines, it shall be 75, as before.

In both these manners of Operations, the two first Lines do serve to open the Sector to his due Angle, the Difference between them is especially this,

this, that in Parallel Entrance, the two Lines of the first Denomination, are placed in the Parallels B C, D E, and in Lateral Entrance they are placed on both Sides of the Sector, in A B, A D, and in A C, A E.

Now in simple solution which is begun and ended upon the same kind of Lines, it is all one which of the two latter Lines be put in the second or third places. As in our Example we may say, *As* 40 *are to* 50, *so* 60 *unto* 75, or else, *As* 40 *are to* 60, *so* 50 *unto* 75. And hence it cometh, that we may enter both with a Parallel, and on the Sides two manner of ways at either Entrance, and so the most part of Questions may be wrought four several ways, though in the Propositions following, I mention only that which is most convenient. If any have not the Sector, he may make use of the former Figure, as in our Example, where we have three Numbers given (40. 50. 60.) to find the fourth Proportional.

First, draw a right line (A D) to represent one of the Lines of the Sector. Then take out the first Number (48) out of the Line of Lines, and prick it down from A to B; and on the Center (A,) and Semidiameter (A B) describe an occult Ark of a Circle from B towards C. In like manner, take out (60) the other Number of the first Denomination, and prick it down from A to D. And on the Center (A) and Semidiameter (A D) describe a second Ark of a Circle, from D toward E. That done, take the third Number (50) and inscribe it into the first Ark from B to C; and laying the Ruler to the Center (A) and the Point C, draw the right Line A C, out in length, till it cut the second Ark in the point E. So the Distance from D to E (taken and measured in the same Scale with the third Number) will give 75 for the fourth Proportional.

Thus much for the general Use of the Sector, which being considered, and well understood, there is nothing hard in that which followeth.

CHAP.

CHAP. II.

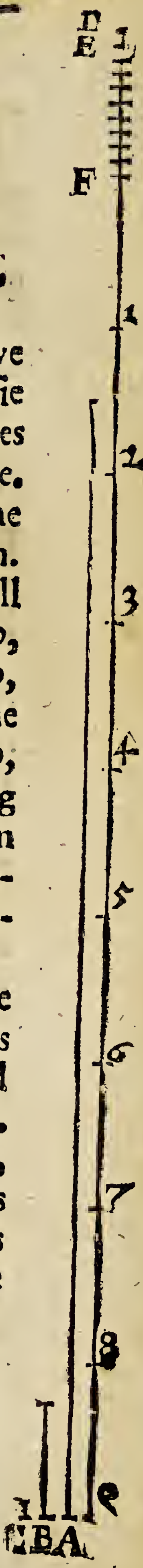
The Use of the Scale of Lines.

1. *To set down a Line, resembling any given Parts or Fraction of Parts.*

THe Lines of Lines are divided actually into 100 parts, but we have put only 10 Numbers in them. These we would have to signifie either themselves alone, or ten times themselves, or an hundred times themselves, or a thousand times themselves, as the matter shall require. As if the Numbers given be no more than 10, then we may think the Lines only divided into ten parts according to the number set to them. If they be more than 10, and not more than 100, then either Line shall contain 100 parts, and the Numbers set by them shall be in value 10,20, 30, *&c.* as they are divided actually. If yet they be more than 100, then every part must be thought to be divided into 10, and either Line shall be 1000 parts, and the Numbers set to them shall be in value 100, 200, 300, and so forward still increasing themselves by 10. This being presupposed, we may number the Parts and Fraction of Parts given in the Line of Lines; and taking out the Distance with a Pair of Compasses, set it by, for the Line so taken shall resemble the Number given.

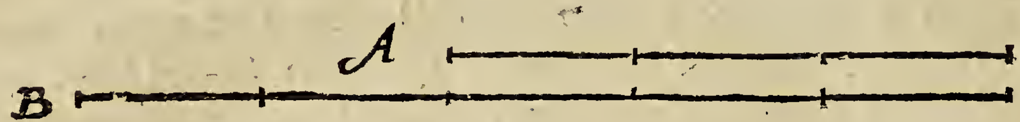
In this manner may we set down a Line resembling 75, if either we take 75 out of the hundred parts, into which one of the Line of Lines is actually divided, and note it in A, or $7\frac{1}{2}$ of the first 10 parts, and note it in B, or only $\frac{3}{4}$ of one of those hundred Parts, and note it in C. Or if this be either too great or too small, we may run a Scale at pleasure, by opening the Compass to some small distance, and running it ten times over, then opening the Compass to these ten, run them over nine times more, and set Figures to them as in this Example, and out of this we may take what parts we will as before.

To this end I have divided the Line of Inches on the Edge of the Sector, so as one Inch conteineth 8 parts, another 9, another 10, *&c.* according as they are figured, and as they are distant from the other end of the Sector, that so we might have the better Estimate.



2. To increase a Line in a given Proportion.
3. To diminish a Line in a given Proportion.

TAke the Line given with a pair of Compaſſes, and open the Sector, ſo as the Feet of the Compaſſes may ſtand in the point of the Number given, then keeping the Sector at this Angle, the Parallel Diſtance of the points of the Number required, ſhall give the Line required.



Let A be a Line given to be increaſed in the Proportion of 3 to 5. Firſt, I take the Line A with the Compaſſes, and open the Sector till I may put it over in the Points of 3 and 3, ſo the Parallel between the Points of 5 and 5, doth give me the Line B, which was required.

In like manner, if B be a Line given to be diminiſhed in the Proportion of 5 to 3, I take the Line B: and to it open the Sector in the points of 5 and 5, ſo the Parallel between the Points of 3 and 3, doth give me the Line A, which was required.

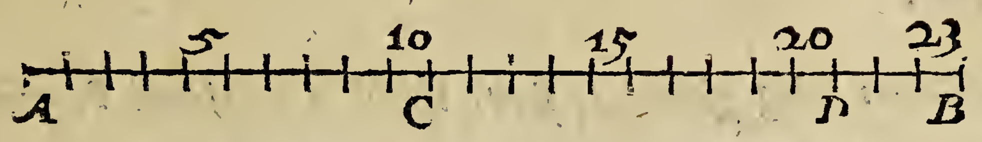
If this manner of work doth not ſuffice, we may multiply or divide the Numbers given by 2, or 3, or 4, *&c.* And ſo work by their Numbers *æqui-multiplices*, as for 3 and 5, we may open the Sector in 6 and 10, or elſe in 9 and 15, or elſe in 12 and 20, or in 15 and 25, or in 18 and 30, *&c.*

4. To divide a Line into any number of Parts given.

TAke the Line given, and open the Sector according to the length of the ſaid Line in the points of the parts, whereinto the Line ſhould be divided, then keeping the Sector at this Angle, the Parallel Diſtance between the points of 1 and 1 ſhall divide the Line given into the Parts required.

Let

Let A B be the Line given, to be divided into five parts, first I take this Line A B, and to it open the Sector in the point of 5 and 5, the Parallel between the points of 1 and 1, doth give me the Line A C, which doth divide it into the parts required.



Or let the like Line A B be to be divided into twenty three parts. First, I take out the Line and put it upon the Sector in the points of 23, then may I by the former Proposition diminish it in A C, C D, in the Proportion of 23 to 10, and after that divide the Line A C into 10, *&c.* as before.

5. *To find a Proportion between two or more right Lines given.*

Take the greater Line given, and according to it open the Sector in the points of 100 and 100, then take the lesser Lines severally, and carry them parallel to the greater, till they stay in like points, so the Number of points wherein they stay, shall shew their Proportion unto 100.

Let the Lines given be A B, C D, first I take the Line C D, and to it open the Sector in the points of 100 and 100, then keeping the Sector at this Angle, I enter the lesser Line A B, parallel to the former, and find it to cross the Lines of Lines in the points of 60. Wherefore the Proportion of A B to C D, is as 60 to 100.

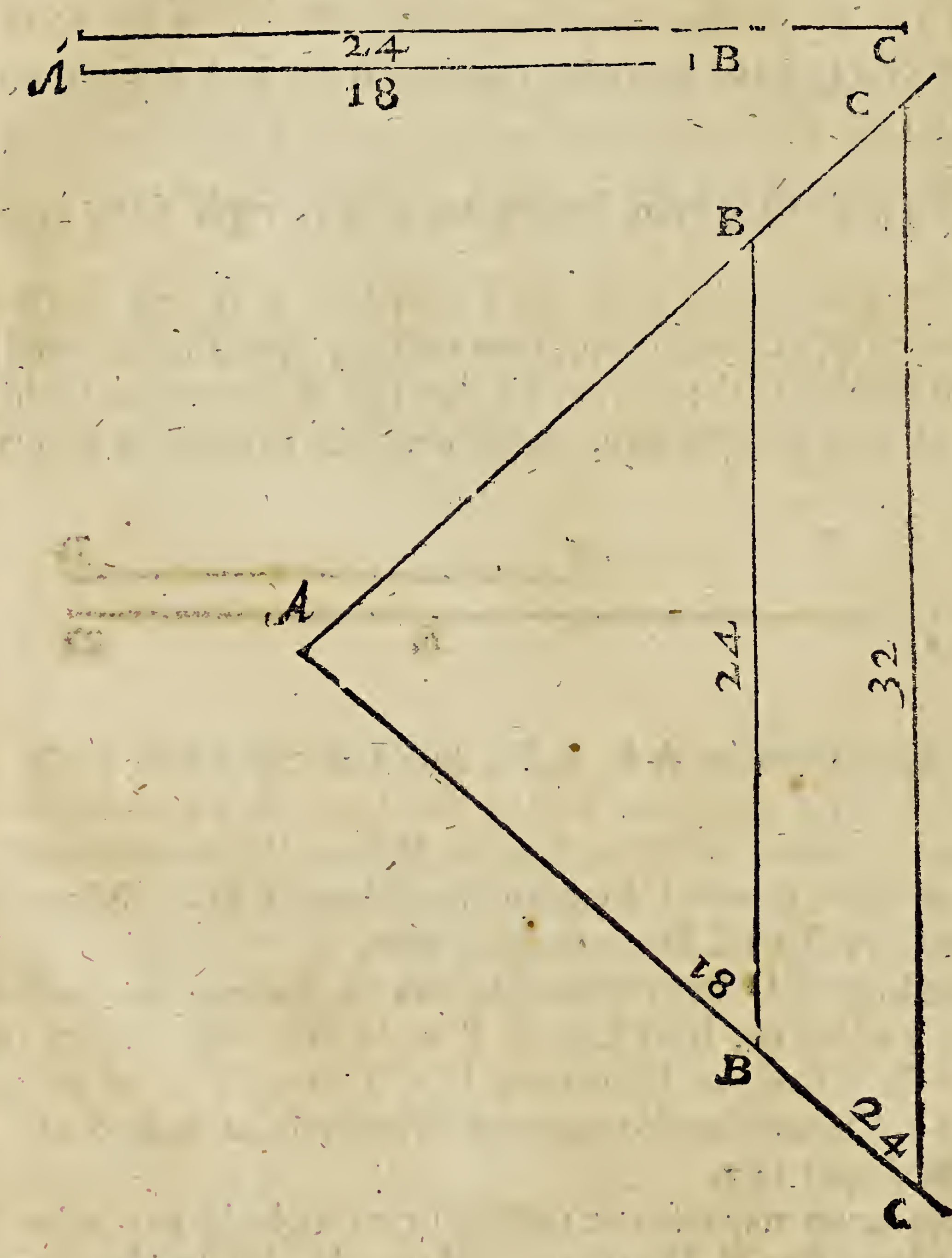
Or if the Line C D be greater than can be put over in the Points of 100, then I admit the lesser Line A B to be 100, and cutting off C E equal to A B, I find the Proportion of C E unto E D to be as 100, almost to 67; wherefore this way the Proportion of A B unto C D, is as 100 unto almost 167.

This Proportion may also not unfitly be wrought by any other Number, that admits several Divisions, and namely, by the Numbers of 60.

And ſo the leſſer Line will be found to be 36, which is as before in leſſer Numbers, as 3 unto 5. It may alſo be wrought without opening the Sector. For if the Lines between which we ſeek a proportion, be applied to the Lines of Lines (or any other Scale of equal parts) there will be ſuch Proportion found between them, as between the Lines to which they are equal.

6. *Two Lines being given, to find a third in continual Proportion.*

First place both the Lines given, on both ſides of the Sector from the Center, and mark the terms of their Extenſion, then take out the



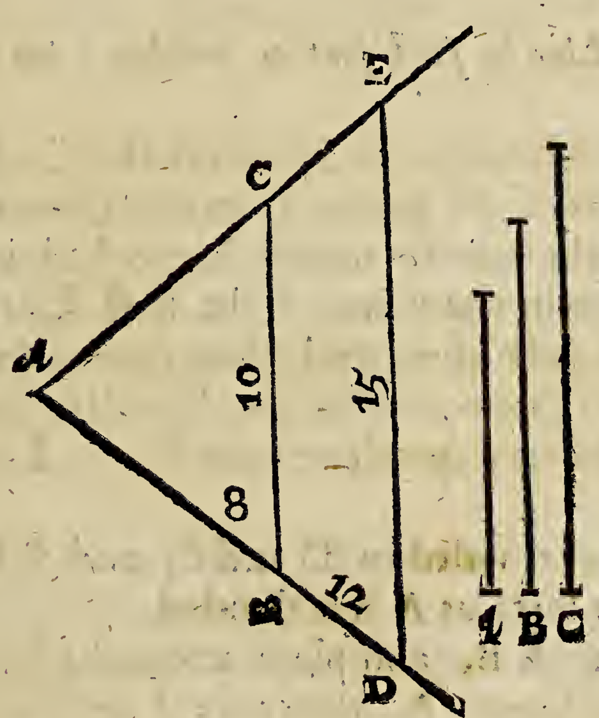
ſecond

ſecond Line again, and to it open the Sector, in the terms of the firſt Line, ſo keeping the Sector at this Angle, the parallel Diſtance between the terms of the ſecond Line, ſhall be the third Proportional.

Let the two Lines given be A B, A C, which I take out and place on both ſides of the Sector, ſo as they all meet in the Center A, let the terms of the firſt Line be B and B, the terms of the ſecond C and C. Then do I take out A G the ſecond Line again, and to it open the Sector in the terms B B. So the Parallel between C and C doth give me the third Line in continual Proportion. For as A B is unto A C, ſo B B equal to A C, is unto C C.

7. *Three Lines being given, to find the fourth in diſcontinual Proportion.*

HEre the firſt Line and the third are to be placed on both ſides of the Sector from the Center, then take out the ſecond Line, and to it open the Sector in the terms of the firſt Line. For ſo keeping the Sector at this Angle, the parallel Diſtance between the terms of the third Line, ſhall be the fourth Proportional. Let the three Lines given be A, B, C.



Firſt, I take out A and C, and place them on both ſides of the Sector, in A B, A C, and A D, A E, laying the beginning of both Lines acat the Center A, then do I take out B the ſecond Line, according to it I open the Sector in B and C, the terms of the firſt Line: ſo the Parallel between D and E, doth give me the fourth Proportional which was required.

As in *Arithmetick*, it ſufficeth if the firſt and third Number given be of one Denomination, the ſecond and the fourth which is required be of another. For one and the ſame Denomination is not required neceſſarily in them all. So in *Geometry*, it ſufficeth if the Sides A B, A D, reſembling the firſt and third Lines given be meaſured in one Scale, and the Parallels B C, D E be meaſured in another. Wherefore knowing the Proportion of A the firſt Line, and C the third Line by the fifth Propoſition before. Which is here as 8 to 12, and deſcending in leſſer Numbers, is as 4 to 6, or as 2 to 3, or aſcending into greater Numbers, as 16 unto 24, or 18 to 27, or 20 to 30, or 30 to 45, or 40 to 60, *&c.* If the Sector be opened in the points of 8 and 8, to the quantity of B, the ſecond Line given, then a Parallel between 12 and 12, ſhall give D E, the fourth Line required. So likewiſe if it be opened in 4 and 4, then a Parallel between 6 and 6; or if in 16 and 16, then a Parallel between 24 and 24 ſhall give the ſame D E: and ſo in the reſt.

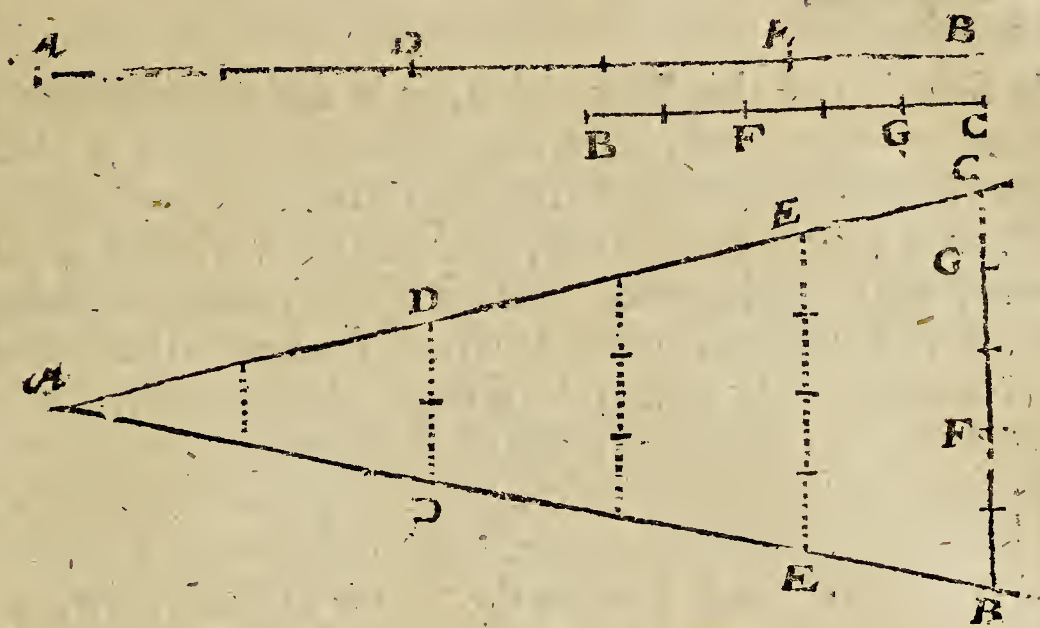
8. *To divide a Line in ſuch ſort as another Line is before divided.*

FIrſt, take out the Line given, which is already divided, and laying it on both ſides of the Sector from the Center; mark how far it extendeth. Then take out the ſecond Line which is to be divided, and to it open the Sector in the terms of the firſt Line. This done, take out the parts of the firſt Line, and place them alſo on the ſame ſide of the Sector from the Center. For the Parallels taken in the terms of theſe parts ſhall be the correſpondent parts in the Line which is to be divided.

Let A B, be a Line divided in D and E, and B C the Line which I am to divide in ſuch ſort, as A B is divided.

Firſt, I take Line A B, and place it on the Line of Lines in A B, A C, both from the Center A, then take I out the ſecond B C, and to it open the Sector in B and C, the terms of the firſt Line. The Sector thus opened to his due Angle, I take out A D and A E, the parts of the firſt Line A B, and place them alſo on both the ſides of the Sector A D, A E, ſo the Parallel D D giveth me B F, and the Parallel E E giveth

giveth B G, and the Line B C is divided in F and G, as is the other Line A B in D and E, which was required.



If the Line A B were longer than one of the Sides of the Ruler, then ſhould I find what proportion it hath to his parts A D, A E, and that known, I may work as before in the former Propoſition.

9. Two Numbers being given, to find a third in continual Proportion.

First reckon the two numbers given on both ſides of the Lines of Lines from the Center, and mark the terms to which either of them extendeth, then take out a Line reſembling the ſecond number again, and to it open the Sector in the terms of the firſt number, for ſo keeping the Sector at this Angle, the Parallel Diſtance between the terms of the ſecond lateral Number, being meaſured in the ſame Scale, from whence his Parallel was taken, ſhall give the third Number Proportional.

Let the two Numbers given be 18, 24. Theſe being reſembled in Lines, the work will be in a manner all one with that in *Prop.6.* and ſo the third Proportional number will be found to be 32.

10. *Three Numbers being given, to find a fourth in discontinual Proportion.*

THe Solution of this Propofition, is in a manner all one with that before in *Prop.*7. only there may be fome difficulty in placing of the numbers. To avoid this, we muft remember that three numbers being given, the queftion is annexed but to one, and this muft always be placed in the third place, that which agrees with this third number in denomination, fhall be the firft number, and that which remaineth the fecond number. This being confidered, reckon the firft and third numbers, which are of the firft Denomination on both fides of the Lines of Lines from the Center, and mark the terms to which either of them extendeth, then take out a Line refembling the fecond number, and to it open the Sector in the terms of the firft number, for fo keeping the Sector at this Angle, the parallel Diftance between the terms of the third lateral Number, being meafured in the fame Scale from whence his Parallel was taken, fhall give the fourth number Proportional.

As if a queftion were propofed in this manner, 10 yards coft 8 *l.* how many yards may we buy for 12 *l.* here the queftion is annexed to 12; and therefore it fhall be the third number, and becaufe 8 is of the fame denomination, it fhall be the firft number, then 10 remaining, it muft be the fecond number, fo will they ftand in this order, 8, 10, 12. Thefe being refembled in Lines, the work will be in a manner the fame with that in *Prop.*7. and the fourth Proportional number will be found to be 15: for *as* 8 *are to* 10, *fo* 12 *unto* 15.

And this holdeth in direct Proportion; where as the firft number is to the fecond, fo the third to the fourth. So that if the third number be greater than the firft, the fourth will be greater than the fecond; or if the third number be lefs than the firft, the fourth will be lefs than the fecond, but in reciprocal Proportion, commonly called the *Back Rule*, where, by how much the firft number is greater than the third, fo much the fecond will be lefs than the fourth, or by how much the firft number is lefs than the third, fo much the fecond will be greater than the fourth; the manner of working muft be contrary, that is, the Sector is to be opened in the terms of the third number: and the Parallel refembling the number required, is to be found between the terms of the firft number, the reft may be obferved as before, as for example.

If

If twelve men would raise a Frame in ten days, in how many days would eight men raise the same Frame? Here, because the fewer men would require longer time, though the numbers be 12, 10, 8, *yet the fourth Proportional will be found to be* 15.

So if 60 *Yards of three quarters of a Yard in bredth would hang round about a room, and it were required to know how many Yards of half a Yard in bredth would serve for the same room. The fourth Proportional would be found to be* 90.

So if to make a Foot superficial 12 *inches in bredth do require* 12 *inches in length, and the bredth being* 16 *inches, it were required to know the length. Here, because the more bredth, the less length, the fourth Proportional will be found to be* 9.

So if to make a solid Foot, a Base of 144 *inches, require* 12 *inches in height, and a Base given being* 216 *inches, it were required to know how many inches it shall have in height. The fourth Proportional would be found to be* 8.

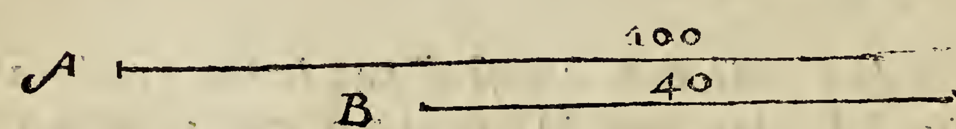
This last Proposition of finding a fourth Proportional Number may be wrought also by the Lines of Superficies, and by the Lines of Solids.

CHAP. III.

The Uſe of the Lines of Superficies.

1. *To find a Proportion between two or more like Superficies.*

TAke one of the ſides of the greater Superficies given, and according to it open the Sector in the points of 100 and 100 in the Lines of Superficies, then take the like ſides of the leſſer Superficies ſeverally, and carry them parallel to the former, till they ſtay in like points, ſo the number of points wherein they ſtay, ſhall ſhew their Proportion unto 100.



Let A and B, be the ſides of like Superficies, as the ſides of two Squares, or the Diameters of two Circles, firſt I take the ſide A, and to it open the Sector in the points of 100, then keeping the Sector to this Angle, I enter the leſſer ſide B, parallel to the former, and find it to croſs the Lines of Superficies in the points of 40, wherefore the Proportion of the Superficies, whoſe ſide is A, to that whoſe ſide is B, is as 100 unto 40, which is in leſſer number as 5 unto 2.

This Propoſition might have been wrought by 60, or any other Number that admits ſeveral Diviſions. It may alſo be wrought without opening the Sector, for if the ſides of the Superficies given be applied to the Lines of Superficies, beginning always at the Center of the Sector, there will be ſuch Proportion found between them, as between the number of parts whereon they fall.

2. *To augment a Superficies in a given Proportion.*
3. *To diminiſh a Superficies in a given Proportion.*

TAke the ſide of the Superficies, and to it open the Sector in the points of the numbers given; then keeping the Sector at that Angle, the parallel diſtance between the points of the number required, ſhall give the like ſide of the Superficies required.

Let

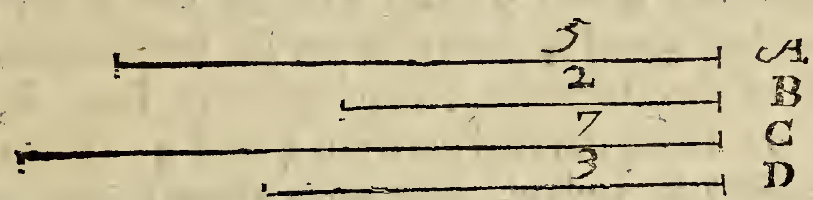
Let A be the ſide of a Square, to be augmented in the Proportion of 2 to 5. Firſt, I take the ſide A, and put it over in the Lines of Superficies in 2 and 2; ſo the Parallel between 5 and 5, doth give me the ſide B, on which if I ſhould make a Square, it would have ſuch Proportion to the Square of A, as 5 unto 2.

In like manner, if B were the Semidiameter of a Circle to be diminiſhed in the Proportion of 5 unto 2, I would take out B, and put it over in the Lines of Superficies in 5 and 5; ſo the Parallel between 2 and 2 would give me A; on which Semidiameter if I ſhould make a Circle, it would be leſs than the Circle made upon the Semidiameter B, in ſuch Proportion as 2 is leſs than 5.

For variety of work, the like caution may be here obſerved to that which we gave in the third Propoſition of Lines.

4. *To add one like Superficies to another.*
5. *To ſubtract one like Superficies from another.*

First, the Proportion between like ſides of the Superficies given, is to be found by the firſt Propoſition of Superficies, then add or ſubtract the numbers of thoſe Proportions, and accordingly augment or diminiſh by the former Propoſition.



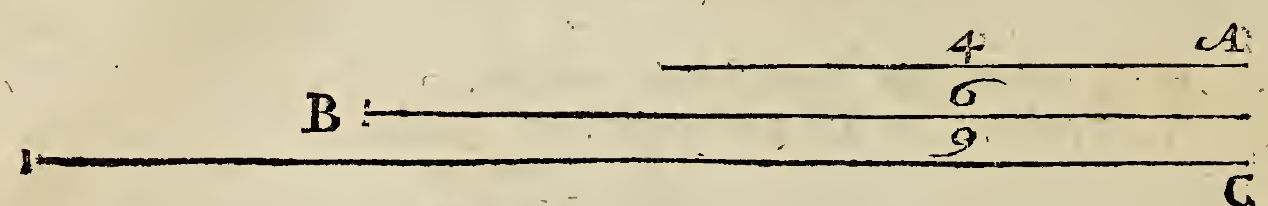
As if A and B were the ſide of two Squares, and it were required to make a third Square equal to them both. Firſt the Proportion between the Squares of A and B, would be found to be as 100 unto 40, or in the leſſer numbers as 5 to 2; then becauſe 5 and 2 added do make 7, I augment the ſide A in the Proportion of 5 to 7, and it will produce the ſide C, on which if I make a Square, it will be equal to both the Squares of A and B, which was required.

In like manner A and B being the ſides of two Squares, if it were required to ſubtract the Square of B, out of the Square of A, and to make a Square equal to the Remainder, here the Proportion being as 5 to 2, becauſe 2 taken out of 5, the Remainder is 3, I would diminiſh the ſide A in the Proportion of 5 to 3, and ſo I ſhould produce the ſide D, on

D, on which if I make a Square, it will be equal to the Remainder, when the Square of B is taken out of the Square of A, that is, the two Squares made upon B and D, ſhall be equal to the firſt Square made upon the ſide A.

6. To find a mean Proportional between two Lines given.

FIrſt find what Proportion is between the Lines given, as they are Lines, by the fifth Propoſition of Lines, then open the Sector in the Lines of Superficies, according to his Number, to the quantity of the one, and a Parallel taken between the points of the Number belonging to the other Line ſhall be the mean Proportional.



Let the Lines given be A and C. The Proportion between them (as they are Lines) will be found, by the fifth Propoſition of Lines, to be, as 4 to 9. Wherefore, I take the Line C, and put it over to the Lines of Superficies between 9 and 9, and keeping the Sector at this Angle, his Parallel between 4 and 4 doth give me B, for the mean Proportional. Then for proof of the Operation I may take this Line B, and put over between 9 and 9: ſo his Parallel between 4 and 4, ſhall give me the firſt Line A. Whereby it is plain, that theſe three Lines do hold in continual Proportion; and therefore B is a mean Proportional between A and C, the extremes given.

Upon the finding out of this mean Proportion, depend many Corollaries, as

To make a Square equal to a Superficies given.

IF the Superficies given be a rectangle Parallelogram, a mean Proportional between the two unequal Sides ſhall be the Side of his equal Square:

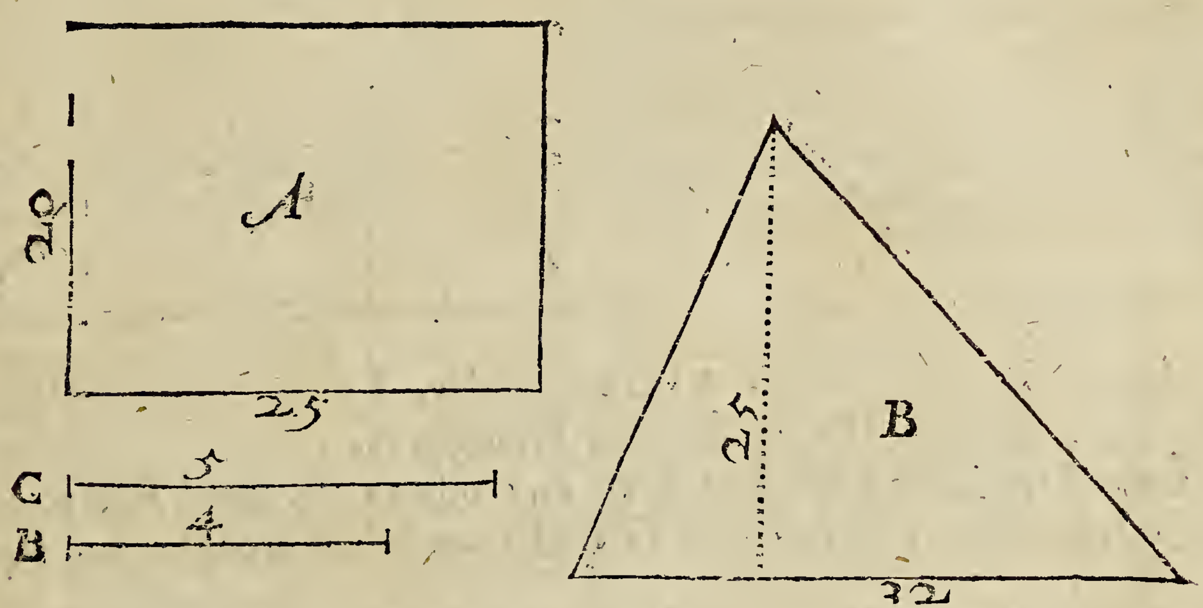
If it ſhall be a Triangle, a mean Proportion between the Perpendicular and half the Baſe ſhall be the Side of his equal Square. If it ſhall be any other right-lined Figure, it may be reſolved into Triangles, and ſo

a

a Side of a Square found equal to every Triangle, and theſe being reduced into one equal Square, it ſhall be equal to the whole right-lined Figure given.

To find a Proportion between Superficies, though they be unlike one to the other.

IF to every Superficies we find the ſide of his equal Square, the Proportion between theſe Squares ſhall be the Proportion between the Superficies given.



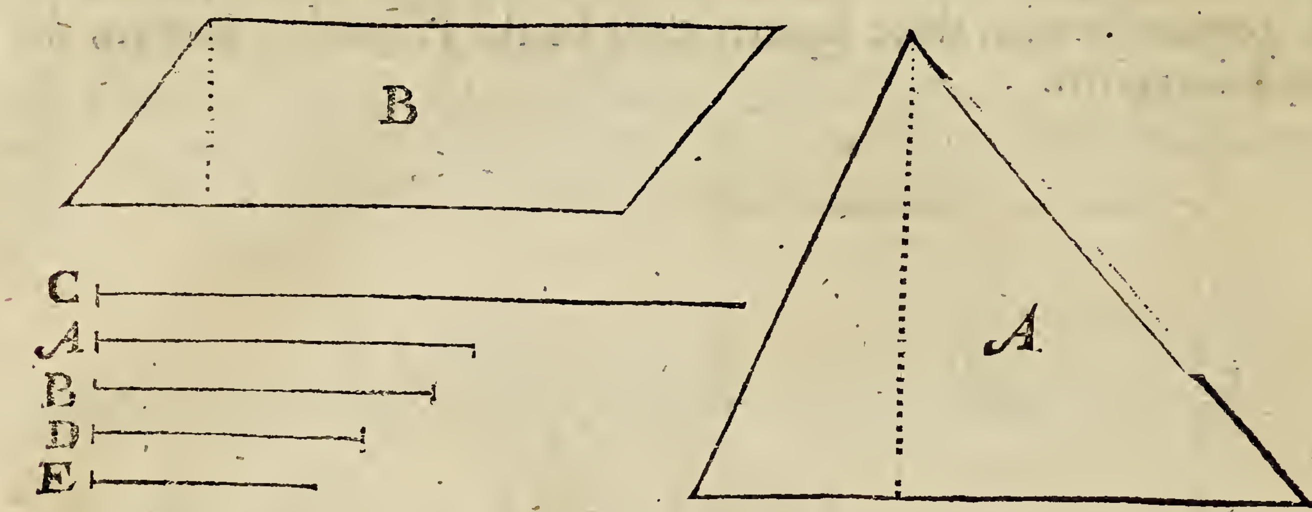
Let the Superficies given be the oblong A, and the Triangle B. Firſt between the unequal Sides of A, I find a mean Proportional, and note it in C: This is the ſide of a Square equal unto A. Then between the Perpendicular of B, and half his Baſe, I find a mean Proportional, and note it in B: this is the ſide of a Spuare equal to B: but the Proportion between the Squares of C and B, will be found, by the firſt Propoſition of Superficies to be as 5 to 4: and therefore this is the Proportion between thoſe given Superficies.

To make a Superficies, like to one Superficies, and equal to another.

LEt the one Superficies given be the Triangle A, and the other the Rhomboides B; and let it be required to make another Rhomboides like to B, and equal to the Triangle A.

Firſt,

First, between the Perpendicular and the Base of B, I find a mean Proportional, and note it in B, as the side of his equal Square, then between the Perpendicular of the Triangle A, and half his Base, I find a mean Proportional, and note it in A, as the side of his equal Square. Wherefore now as the side B is to the side A, so shall the sides of the Rhomboides given be to C and D, the sides of the Rhomboides required, and his Perpendicular also to E, the Perpendicular required.



Having the Sides and the Perpendicular, I may frame the Rhomboides up, and it will be equal to the Triangle A.

If the Superficies given had been any other right-lined Figures, they might have been resolved into Triangles and then brought into Squares as before.

Many such Corollaries might have been annexed, but the means of finding a mean Proportional being known, they all follow of themselves.

7. *To find a mean Proportional between two Numbers given.*

First, reckon the two Numbers given on both sides of the Lines of Superficies, from the Center, and mark the terms whereunto they extend; then take a Line out of the Line of Lines, or any other Scale of equal parts resembling one of those Numbers given, and put it over in the terms of his like Number in the Lines of Superficies; for so keeping the Sector at this Angle, the Parallel taken from the terms of the other Number and measured in the same Scale from which the other Parallel was taken, shall here shew the Mean Proportional which was required.

Let the Numbers given be 4 and 9. If I shall take the Line A in the

the Diagram of the sixth Proposition resembling 4, in a Scale of equal parts, and to it open the Sector in the terms of 4 and 4, in the Lines of Superficies, his Parallel between 9 and 9 doth give me B for the Mean Proportional. And this measured in the Scale of equal parts doth extend to 6, which is the Mean Proportional Number between 4 and 9: For as 4 to 6, so 6 to 9.

In like manner, if I take the Line C, resembling 9, in a Scale of equal parts, and to it open the Sector in the terms of 9 and 9, in the Lines of Superficies, his Parallel between 4 and 4 doth give me the same Line B, which will prove to be 6, as before, if it be measured in the same Scale whence C was taken.

For the Figures 1, 2, 3, 4, *&c.* here set down upon the Line, do sometime signifie themselves alone: sometime 10, 20, 30, 40, *&c.* sometime 100, 200, 300, 400, *&c.* and so forward, as the matter shall require. The first Figure of every Number is alway that which is here set down: the rest must be supplied according to the nature of the Question.

If you suppose Pricks under the Number given (as in Arithmetical Extraction) and the last Prick to the left hand shall fall under the last Figure (which will be as oft as there be odd Figures) the unite will be best placed at 1, in the middle of the Line; so the Root and the Square will both fall forward, toward the end of the Line. But, if the last Prick shall fall under the last Figure but one (which will be as oft as there be even Figures) then the unite may be placed at 1 in the beginning of the Line, and the Square in the second length: or the unite may be placed at 10, in the end of the Line, so the Root and the Square will both fall backward, toward the middle of the Line.

8. *To find the Square Root of a Number.*
9. *The Root being given, to find the Square Number of that Root.*

IN the Extraction of a Square Root it is usual to set Pricks under the first Figure, the third, the fifth, the seventh, and so forward, beginning from the right hand toward the left, and as many Pricks as fall to be under the Square Number given, so many Figures shall be in the Root: so that if the Number given be less than 100, the root shall be only of one Figure; if less than 10000, it shall be but two Figures; if less than 100000, it shall be three Figures, *&c.*

Thereupon the Lines of Superficies are divided first into an hundred parts,

parts, and if the Number given be greater than 100, the firſt Diviſion (which before did ſignifie only one) muſt ſignifie 100, and the whole Line ſhall be 10000 parts: if yet the number given be greater than 10000, the firſt Diviſion muſt now ſignifie 10000; and the whole Line be eſteemed at 1000000 parts: and if this be too little to expreſs the Number given, as oft as we have recourſe to the beginning, the whole Line ſhall increaſe itſelf an hundred times.

By theſe means if the laſt Prick to the left hand ſhall fall under the laſt Figure, which will be as oft as there be odd Figures, the Number given ſhall fall out between the Center of the Sector and the tenth Diviſion: but if the laſt Prick ſhall fall under the laſt Figure but one, which will be as oft as there be even Figures, then the Number given ſhall fall out between the tenth Diviſion and the end of the Sector.

This being conſidered, when a Number is given, and the Square Root is required, take a pair of Compaſſes, and ſetting one Foot in the Center, extend the other to the term of the number given in one of the Lines of Superficies; for this Diſtance applied to one of the Lines of Lines, ſhall ſhew what the Square Root is, without opening the Sector.

Thus 36 doth give a Root of 6; and 360, a Root of almoſt 19: and 3600, a Root of 60; and 36000, a Root of 189, *&c.*

In like manner, the neareſt Root of 725 is here found to be (about) 27, the neareſt Root of 7250, about 85: the neareſt of 72500, about 269: and the neareſt Root of 725000, about 851: And ſo in the reſt.

On the contrary, a Number given may be ſquared, if firſt we extend the Compaſſes to the Number given in the Lines of Lines, and then apply that Diſtance to the Lines of Superficies, as may appear by the former Examples.

10. *Three Numbers being given, to find the fourth in a duplicated Proportion.*

It is plain (by *Euclid. Lib.6. Prop.*19 & 20.) that like Superficies doth hold in a duplicated Proportion of their homologal Sides, whereupon a queſtion being moved concerning Superficies and their Sides: It is uſual in Arithmetick, that the Proportion be firſt duplicated before the Queſtion be reſolved, which is not neceſſary in the Uſe of the Sector, only

only the Numbers which do signifie Superficies, must be reckoned in the Lines of Superficies, and they which signifie the Sides of Superficies, in the Lines of Lines, after this manner.

If a Question be made concerning a Superficies, the two Numbers of the first Denomination must be reckoned in the Lines of Lines: and the Sector opened in the terms of the first Number to the quantity of a Line out of the Scale of Superficies resembling the second Number; so his Parallels taken between the terms of the third Number, being measured in the same Scale of Superficies, shall give the Superficial Number which was required.

As if a Square, whose side is 40 Perches in length, shall contain 10 Acres in the Superficies, and it be required to know how many Acres the Square should contain, whose side is 60 Perches.

Here if I took 10 out of the Line of Superficies, and put it over in 40, in the Lines of Lines, his Parallel between 60 and 60, measured in the Line of Superficies, would be 22 ½, and such is the number of Acres required. For Squares do hold in a duplicated Proportion of their sides; wherefore when the Proportion of their sides is as 4 to 6, and 4 multiplied into 4 become 16, and 6 multiplied into 6 become 36, the Proportion of their Squares shall be as 16 to 36, and such is the Proportion of 10 to 22 ½.

If a Field measured with a Statute Perch of 16 ½ foot, shall contain 288 Acres, and it be required to know how many Acres it would contain if it were measured with a Woodland Perch of 18 foot.

Here because the Proportional is reciprocal, if I took 288 out of the Line of Superficies, and put it over in 18 in the Lines of Lines, his Parallel between 16 ½ and 16 ½ measured in the Line of Superficies, would be 242; and such is the Number of Acres required.

For seeing the Proportion of the Sides is as 16 ½ to 18, or in lesser Numbers as 11 to 12, and that 11 multiplied into 11 become 121, and 12 into 12 become 144, the Proportion of these Superficies shall be as 121 to 144, and so have 288 to 242, in reciprocal Proportion.

On the contrary, if a question be proposed concerning the Side of a Superficies, the two Numbers of the first Denomination must be reckoned in the Lines of Superficies, and the Sector opened in the terms of the first Number to the quantity of a Line, out of the Line of Lines or some Scale of equal parts, resembling the second Number; so his Parallel taken between the terms of the third Number being measured in the ſame Scale with the ſecond Number, ſhall give the fourth Number required.

As if a Field contained 288 Acres when it was meaſured with a Statute Perch of 16 ½, and being meaſured with another Perch, was found to contain 242 Acres, it were required to know what was the length of the Perch with which it was ſo meaſured.

Here becauſe the Proportion is reciprocal, if I took 16 ½ out of the the Line of Lines, and put it over in 242 in the Lines of Superficies, his Parallel between 288 and 288, being meaſured in the Line of Lines, would be 18, and ſuch is the length of the Perch (in Feet) wherewith the Field was laſt meaſured.

For ſeeing the Proportion of the Acres is as 288 unto 242, or in the leaſt Numbers, as 144 to 121, and that the Root of 144 is 12, and the Root of 121 is 11, the Proportion of Roots, and conſequently of the Perches, ſhall be as 12 to 11, and ſo are 16 ½ to 18 in reciprocal Proportion.

If 360 men were to be ſet in form of a long Square, whoſe Sides ſhall have the Proportion of 5 to 8; and it were required to know the Number of men to be placed in front and file: If the Sides were only 5 and 8, there ſhould be but 40 men; but there are 360: therefore, working as before, I find that,

As 40 *to the Square of* 5 :
So 360 *to the Square of* 15.

As 40 *to the Square of* 8 :
So 360 *to the Square of* 24.

and ſo 15 and 24 are the Sides required.

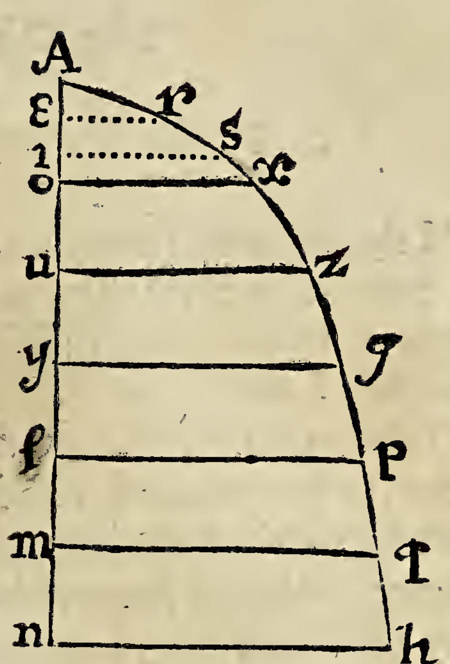
If 1000 men were lodged in a ſquare ground whoſe Side were 60 paces, and it were required to know the Side of the Square wherein 5000 might be ſo lodged, here working as before, I ſhould find that,

As 1000, *are to the Square of* 60 :
So 5000 *to the Square of* 134.

And ſuch, very near, is the Number of paces required.

11. *How*

11. *How to describe a Parabola, by help of the Line of Lines and Superficies.*



UPon A n as the Diameter, prick down, by the Line of Lines, the equal Parts A o, A u, A y, A l, A m, A n, *&c.* and from these Points raise the Perpendiculars o x, u z, y g, l p, m q, n h, *&c.* And upon the Perpendicular o x, assume the Point x, and open the Sector in the Line of Superficies, so that o x (being the first Perpendicular) may fall in with the Points 1... 1 (the first of the Line of Superficies:) Then if you take off from the same Line 2...2, you shall prick down u z, and 3 ... 3 gives y g; and 4 ... 4, l p; 5 ... 5, m q; 6 ... 6, n h; *&c.*

Or, you may begin your work from n h, which (because it is the sixth Perpendicular) take from n to h, the Point assumed, and set that length in the Line of Superficies from 6 to 6, so may you prick down the other Points correspondently.

Through these Points h, q, p, g, with an even hand draw the Parabola.

And here note, that Parabola's may be described of infinite Varieties, according to the Cones from whence they are taken, yet keeping all one and the same length.

CHAP. IV.

The Uſe of the Lines of Solids.

1. *To find a Proportion between two or more like Solids.*

IN the Sphere, in regular, parallel, and other like bodies, whoſe Sides next the equal Angles are proportional, the work is in a manner the ſame, with that in the firſt Propoſition of Superficies, but that it is wrought on other Lines.

Take one of the ſides of the greater Solid, and according to it open the Sector in the points of 1000 and 1000, in the Lines of Solids, then take the like Sides of the leſſer Solids ſeverally, and carry them parallel to the former, till they ſtay in like points, ſo the number of points wherein they ſtay ſhall ſhew their proportion to 1000.

Let A and B be the like Sides of like Solids, either the Diameters or Semidiameters of two Spheres, or the ſides of two Cubes or other like. Firſt I take the ſide A, and to it open the Sector in the points of 1000, then keeping the Sector at this Angle, I enter the leſſer Side B parallel to the former, and find it to croſs the Line of Solids in the points of 400, and ſuch is the Proportion between the Solids required, which in leſſer Number is, as 5 to 2.

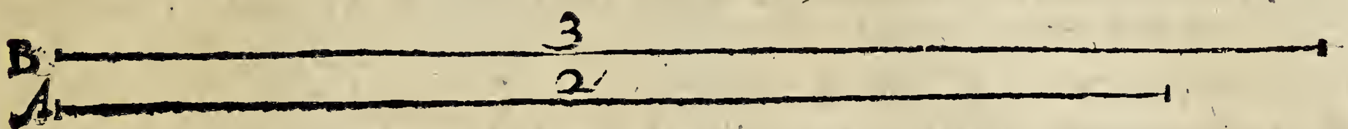
This Propoſition might have been wrought by 60, or any other Number that admits ſeveral Diviſions.

It may alſo be wrought without opening the Sector, for if the ſides of the Solids given be applied to the Lines of Solids, beginning always at the Center of the Sector, there will be ſuch Proportion between them, as between the Numbers of parts whereon they fall.

2. *To augment a Solid in a given Proportion.*
3. *To diminiſh a Solid in a given Proportion.*

TAke the ſide of the Solid given, and to it open the Sector, in the points of the Number given: then keeping the Sector at that Angle, the parallel Diſtance between the points of the Number required, ſhall give the like Side of the Solid required.

If it be a parallelopipedon or ſome irregular Solid, the other like Sides may be found out in the ſame manner, and with them the Solids required, may be made up with the ſame Angles.



Let A be the ſide of a Cube, to be augmented in the Proportion of 2 to 3. Firſt, I take the ſide A, and put it over in the Lines of Solids in 2 and 2; ſo the Parallel between 3 and 3, doth give me the ſide B, on which if I make a Cube, it will have ſuch Proportion to the Cube of A, as 3 to 2.

In like manner, if B were the Diameter of a Sphere, to be diminiſhed in the proportion of 3 to 2. I would take out B, and put it over in the Lines of Solids, in 3 and 3, ſo the Parallel between 2 and 2, would give me A: to which Diameter if I ſhould make a Sphere, it would be leſs than the Sphere, whoſe Diameter is B, in ſuch proportion as 2 is leſs than 3.

Here alſo for variety of work, may the like caution be obſerved to that which we gave in the third Propoſition of Lines.

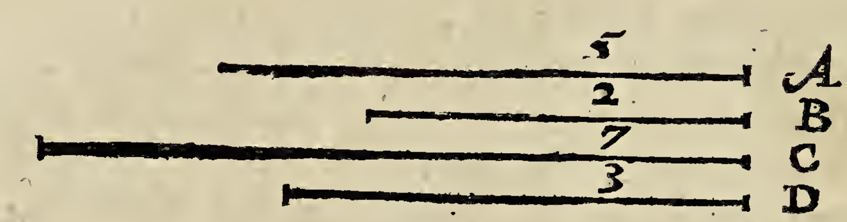
4. *To add one like Solid to another.*
5. *To ſubtract one like Solid from another.*

FIrſt the Proportion between the ſides of the like Solids given, is to be found by the firſt Propoſition of Solids: then add or ſubtract thoſe Proportions, and accordingly augment or diminiſh by the former Propoſition.

As if A and B were the ſides of two Cubes, and it were required to make a third Cube equal to them both: firſt the Proportion between the ſides A and B, would be found to be as 100 to 40, or in leſſer terms as

5 to

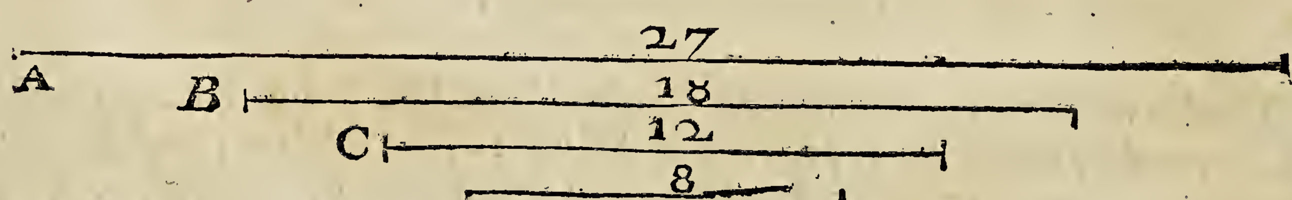
5 to 2: then becauſe 5 and 2 being added do make 7, I augment the ſide A, in the proportion of 5 to 7, and produce the ſide C, on which if I make a Cube, it will be equal to both the Cubes of A and B, which was required.



In like manner A and B being the ſides of two Cubes, if it were required to ſubtract the Cube of B out of the Cube of A, and to make a Cube equal to the Remainder. Here the Proportion being as 5 to 2, becauſe 2 taken out of 5, the Remainder is 3, I ſhould diminiſh the ſide A in the proportion of 5 to 2, and ſo I ſhould have the ſide D, on which if I make a Cube, it will be equal to the Remainder, when the Cube of B is taken out of the Cube of A, that is, the two Cubes made upon B and D ſhall be equal to the firſt Cube made upon the ſide A.

6. To find two mean proportional Lines between two extreme Lines given.

FIrſt I find what Proportion is between the two extreme Lines given, as they are Lines, by the fifth Propoſition of Lines, then open the Sector in the Lines of Solids, to the quantity of the former Extreme, and a Parallel between the points of the number belonging to the other Extreme, ſhall be that mean Proportional, which is next the former Extreme. This done, open the Sector again to this mean Proportional in the points of the former Extreme, and the parallel Diſtance between the points of the latter Extreme, ſhall be the other mean Proportional required.



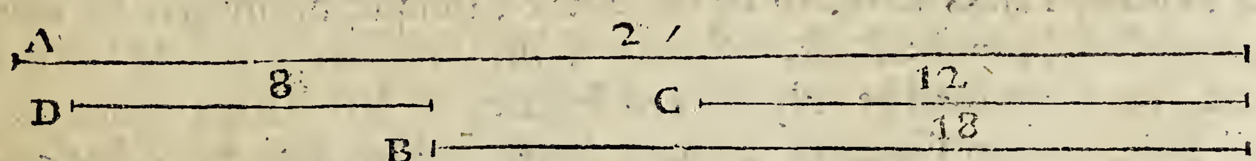
Let the two extreme Lines given be A and D, the Proportion between them, as they are Lines, will be found to be as 27 to 8. Wherefore I take the Line A, and put it over in the Lines of Solids between 27 and

27, and keeping the Sector at this Angle, his Parallel between 8 and 8, doth give me B the mean Proportional next unto A. Then put I over this Line B, between the aforesaid 27 and 27, and this Parallel between 8 and 8 doth give me the Line C, the other mean Proportional which was required.

Again, for proof of the operation I put over this Line C in the aforesaid 27 and 27, and his Parallel between 8 and 8 doth give me the very Line D: whereby it is plain that these four Lines do hold in continual Proportion; and so B and C are found to be the Mean Proportionals between A and D the Extreme given.

7. *To find two mean proportional Numbers between two extreme Numbers given.*

FIrst reckon the Numbers given on both sides of the Lines of Solids, beginning from the Center, and marking the terms whereto they extend: then take a Line out of the Line of Lines, or any other Scale of equal parts resembling the former of those Numbers, and put it over in the Lines of Solids, between the points of his like Number, and a Parallel between the points belonging to the other Extreme, measured in the Scale from whence the other Parallel was taken, shall give that mean proportional Number which is next the former Extreme. This done open the Sector again to this mean Proportional in the Points of the former Extreme, and the parallel Distance between the points of the latter Extreme, measured in the same Scale as before, shall there shew the other mean Proportional required.



Let the two extreme Numbers given be 27 and 8; if I shall take the Line A, resembling 27 in a Scale of equal parts, and to it open the Sector in 27 and 27, in the Line of Solids, his Parallel between 8 and 8 doth give me B, for his next mean Proportional, and this measured in the former Scale doth extend to 18. Then put I over this Line B, between the foresaid 27 and 27, and his Parallel between 8 and 8 doth give me C, for the other mean Proportional, and this measured in the former Scale

Scale doth extend to 12. Again, for proof of my work, I put over this Line C, between 27 and 27, as before, and his Parallel between 8 and 8 doth give me D, which meaſured in the former Scale doth extend to 8, which was the latter extreme Number given; whereby it is plain, that theſe four Numbers do hold in continual Proportion: and therefore 18 and 12 are Mean Proportionals between 27 and 8, which was required.

If you ſuppoſe Pricks under the Number given as in Arithmetical Extraction, and that laſt Prick to the left hand ſhall fall under the laſt figure, as in 1728, the unite will be left placed at 1, in the middle of the Line, and the Root, Square and Cube will all fall forward toward the end of the Line.

If the laſt Prick ſhall fall under the laſt Figure but one, as in 17280; the unite may be placed at 1, in the beginning of the Line, and the Cube in the ſecond length: or the unite may be placed at 10, in the end of the Line, and the Cube in the firſt length.

But if the laſt Prick ſhall fall on the laſt Figure but two, as in 172800; then, place the unite always at 10 in the end of the Line: ſo the Root, Square and Cube will all fall backward and be found in the ſecond length.

8. *To find the Cubique Root of a Number.*
9. *The Root being given to find the Cube Number of that Root.*

IN the Extraction of a Cubique Root, it is uſual to ſet Pricks under the firſt Figure, the fourth, the ſeventh and tenth, and ſo forward omitting two, and pricking the third from the right hand toward the left; and as many Pricks as fall to be under the Cubique Numbers, ſo many Figures ſhall be in the Root. So that if the Number given be leſs than 1000, the Root ſhall be only of one Figure; if leſs than 1000000, it ſhall be but of two Figures; if above theſe, and leſs than 1000000000, it ſhall be but three Figures, *&c.* whereupon the Lines of Solids are divided, firſt into 1000 parts, and if the Numbers given be greater than 1000 the firſt Diviſion (which before did ſignifie only one) muſt ſignifie 1000, and the whole Line ſhall be 1000000: if yet the Number given be greater than 1000000, the firſt Diviſion muſt now ſignifie 1000000, and the whole Line be eſteemed at 1000000000 parts, and if theſe be too little to expreſs the Numbers given, as oft as we have recourſe to the beginning, the whole Line ſhall increaſe itſelf a thouſand times.

By

By these means, if the last Prick, to the left hand, shall fall under the last Figure, the Number given shall be reckoned at the beginning of the Lines of Solids from 1 to 10, and the first Figure of the Root shall be always either 1 or 2. If the last Prick shall fall under the last Figure but one, then the Number given shall be reckoned in the middle of the Line of Solids, between 10 and 100, and the first Figure of the Root shall be always either 2, or 3, or 4. But if the last Prick shall fall under the last Figure but two, then the Number given shall be reckoned at the end of the Line of Solids, between 100 and 1000.

This being considered, when a Number is given, and the Cubique Root required, set one Foot of the Compasses in the Center of the Sector, extend the other in the Line of Solids to the Points of the Number given: For this Distance applied to one of the Lines of Lines, shall shew what the Cubique Root is, without opening the Sector.

So the nearest Root of 8490000, is about 204.
The nearest Root of 84900000, is about 439.
The nearest Root of 849000000, is about 947.

On the contrary, a Number may be cubed, if first we extend the Compasses to the Number given, in the Line of Lines, and then apply the Distance to the Lines of Solids, as may appear by the former Examples.

10. *Three Numbers being given, to find a fourth in a triplicated Proportion.*

As like Superficies do hold in a duplicated Proportion, so like Solids in a triplicated Proportion of their homologal Sides: and therefore the same Work is to be observed here on the Lines of Solids, as before in the Lines of Superficies, as may appear by these two Examples.

If a Cube whose side is 4 inches, shall be 7 pound weight, and if it be required to know the weight of a Cube whose sides is 7 inches; here the Proportion would be,

As 4 *are to a Cube of* 7 :
So 7 *to a Cube of* 37 $\frac{1}{2}$.

And if I took 7 out of the Lines of Solids, and put it over in 4 and 4, in the Lines of Lines, his Parallel between 7 and 7, measured in the Lines of Solids, would be 37 ½; and such is the weight required.

If a Bullet of 27 pound weight, have a Diameter of 6 inches, and it be required to know the Diameter of the like Bullet, whose weight is 125 pounds; here the Proportion would be,

As the Cubique Root of 27, *is unto* 6,
So the Cubique Root of 125, *is unto* 10.

And if I took 6 out of the Line of Lines, and put it over in 27, and 27 of the Lines of Solids, his Parallel between 125 and 125 measured in the Line of Lines, would be 10; and such is the length of the Diameter required.

The End of the first Book.

THE

THE

SECOND BOOK

OF THE

SECTOR,

Containing the Uſe of the Circular Lines.

CHAP. I.

Of the Nature of Sines, Chords, Tangents, and Secants, fit to be known before-hand; in reference to right-lined Triangles.

IN the Canon of Triangles, a Circle is commonly divided into 360 Degrees, each Degree into 60 Minutes, each Minute into 60 Seconds.

A Semicircle therefore is an Ark of 180 *gr.*

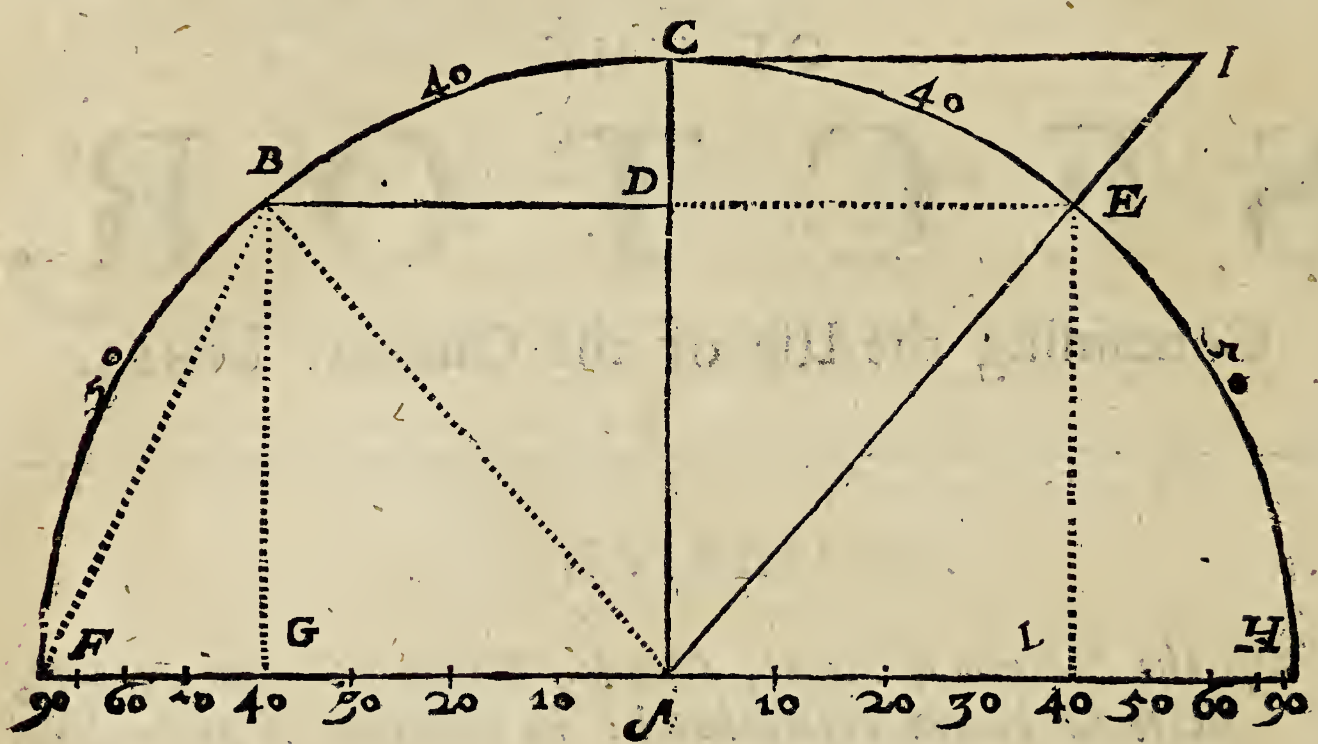
A Quadrant is an Ark of 90 *gr.*

The meaſure of an Angle is the Ark of a Circle deſcribed out of the angular point, intercepted between the Sides ſufficiently produced.

So the meaſure of a right Angle is always an Ark of 90 *gr.* and in this Example the meaſure of the Angle B A D in the following Figure, is the Ark B C of 40 *gr.* the meaſure of the Angle B A G, is the Ark B F of 50 *gr.*

The Complement of an Ark or of an Angle doth commonly ſignifie the Ark which the given Ark doth want of 90 *gr.* and ſo the Ark B F is the Complement of the Ark B C, and the Angle B A F, whoſe meaſure is B F, is the Complement of the Angle B A C; and on the contrary,

The Complement of an Ark or Angle in regard of a Semicircle, is that Ark which the given Ark wanted to make up 180 *gr.* and ſo the Angle E A H is the Complement of the Angle E A F, as the Ark E H is the Complement of the Ark F E, in which the Ark C E is the exceſs above the Quadrant.



The Proportions which theſe Arks (being the meaſures of Angles) have to the Sides of a Triangle, cannot be certain, unleſs that which is crooked be brought to a ſtreight Line, and that may be done by the application of *Chords*, *Right Sines*, *Verſed Sines*, *Tangents* and *Secants* to the Semidiameter of a Circle.

A Chord is a right Line ſubtending an Ark: ſo B E is the Chord of the Ark B C E, and B F a Chord of the Ark B 40 F.

A right Sine is half the Chord of the double Ark, *viz.* the right Line which falleth perpendicularly from the one Extreme of the given Ark, upon the Diameter drawn to the other Extreme of the ſaid Ark.

So if the given Ark be B C, or the given Angle be B A C, let the Diameter be drawn through the Center A unto C, and a Perpendicular B D be let down from the Extreme B upon A C, this Perpendicular B D ſhall be the right Sine both of the Ark B C, and alſo of the Angle

B A C:

B A C: and it is alſo the half of the Chord B E, ſubtending the Ark B C E, which is double to the given Ark B C. In like manner, the Semidiameter F A, is the right Sine of the Ark F C, and of the right Angle F A C; for it falleth perpendicularly upon A C, and it is the half of the Chord F H.

This whole Sine of *90 gr.* is hereafter called *Radius*, but the other Sines take their Denomination from the Degrees and Minutes of their Arks.

Sinus verſus, the Verſed Sine is a Segment of the Diameter, intercepted between the right Sine of the ſame Ark, and the Circumference of the Circle. So D C is the Verſed Sine of the Ark C B, and G F the Verſed Sine of the Ark B F, and G H the Verſed Sine of the Ark B H.

A Tangent is a right Line perpendicular to the Diameter drawn by the one Extreme of the given Ark, and terminated by the Secant drawn from the Center, through the other Extreme of the ſaid Ark.

A Secant is a right Line drawn from the Center, through one Extreme of the given Ark, till it meet with the Tangent raiſed from the Diameter at the other Extreme of the ſaid Ark.

So if the given Ark be C E, or the given Angle be C A E, let the Diameter be drawn through the Center A to C, and in C to A C, be raiſed a Perpendicular C I. Then let another Line be drawn from the Center A through E, till it meet with the Perpendicular C I in I; the Line C I is a Tangent, and A I is the Secant both of the Ark C E, and of the Angle C A E.

CHAP.

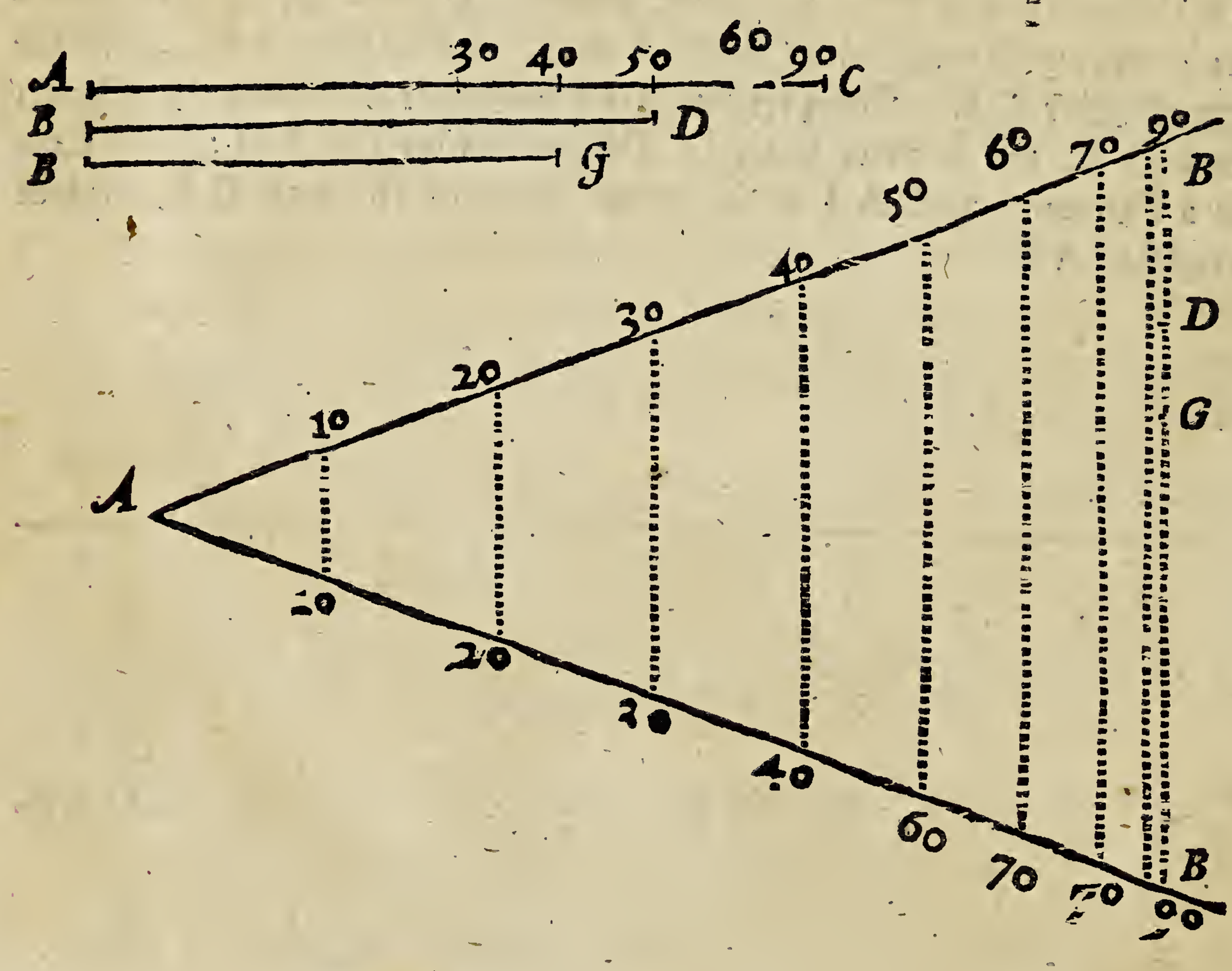
CHAP. II.

Of the general Use of Sines and Tangents.

1. *The Radius being known, to find the right Sine of any Ark or Angle.*

IF the Radius of the Circle given be equal to the lateral Radius, that is to the whole Line of Sines on the Sector, there needs no farther work, but to take the other Sines alſo out of the Side of the Sector. But if it be either greater or leſſer, then let it be made a parallel Radius, by applying it over in the Lines of Sines, between 90 and 90, ſo the Parallel taken from the like lateral Sines, ſhall be the Sine required.

As if the given Radius be A C, and it were required to find the Sine of 56 *gr.* and his Complement agreeable to that Radius.



Let A B, A B represent the Lines of Sines on the Sector, and let B B the Distance between 90 and 90, be equal to the given Radius A C. Here the Lines A 40, A 50, A 90 may be called the lateral Sines of 40, 50, and 90, in regard of their place on the side of the Sector. The Lines between 40 and 40, between 50 and 50, between 90 and 90, may be called the parallel Sines of 40, 50, and 90, in regard they are parallel one to the other. The whole Sine of 90 *gr.* here standing for the Semidiameter of the Circle, may be called the Radius. And therefore if A C be put over in the Line of Sines in 90 and 90, and so made a Parallel Radius, his parallel Sine between 50 and 50 shall be B D, the Sine of 50 required. And because 50 taken out of 90, the Complement is 40, his parallel Sines between 40 and 40 shall be B G, the Sine of the Complement which was required.

2. *The right Sine of any Ark being given to find the Radius.*

TUrn the Sine given into a parallel Sine, and his parallel Radius shall be the Radius required.

As if B D were the given Sine of 50 *gr.* and it were required to find the Radius, let B D be made a parallel Sine of 50 *gr.* by applying it over in the Lines of Sines between 50 and 50, so his parallel Radius between 90 and 90 shall be A C, the Radius required.

3. *The Radius of a Circle, or the right Sine of any Ark, being given, and a streight Line resembling a Sine, to find the quantity of that unknown Sine.*

LEt the Radius or right Sine given be turned into his Parallel, then take the right Line given, and carry it parallel to the former, till it stay in like Sines, so the number of Degrees and minutes where it stayeth, shall give the quantity of the Sine required.

As if B D were the given Sine of 50 *gr.* and B G the streight Line given, first I make B D a parallel Sine of 50 *gr.* then keeping the Sector at this Angle, I carry the Line B G parallel, and find it to stay in no other but 40 and 40, and therefore 40 *gr.* is his Quantity required.

4. The

4. *The Radius or any right Sine being given, to find the Versed Sine of any Ark.*

IF the Ark, whose Versed Sine is required, be less than the Quadrant, take the Sine of the Complement out of the Radius, and the Remainder shall be the *Sinus Versus*, the Versed Sine of that Ark.

As if A B being the lateral Radius, it were required to find the Versed Sine of 40*gr.* here the Sine of the Complement is A 50, and therefore B 50 is the Versed Sine required. Or if I reckon from B at the end of the Sector, toward the Center, the Distance from 90 to 80 is the Versed Sine of 10*gr.* from 90 to 70, the Versed Sine of 20*gr.* from 90 to 60 is the Versed Sine of 30*gr.* and so in the rest.

If A D be the given Sine of 50*gr.* and it be required to find the Versed Sine of 50*gr.* here because A D is unequal to the lateral Sine of 50*gr.* I make it a Parallel. And first I find the Radius A C, then the Sine of the Complement A 40, which being taken out of A C, leaveth C 40, for the Versed Sine of 50*gr.* which was required.

But if the Ark whose Versed Sine is required, be greater than the Quadrant, his Versed Sine also is greater than the Radius, by the right Line of his excess above 90*gr.*

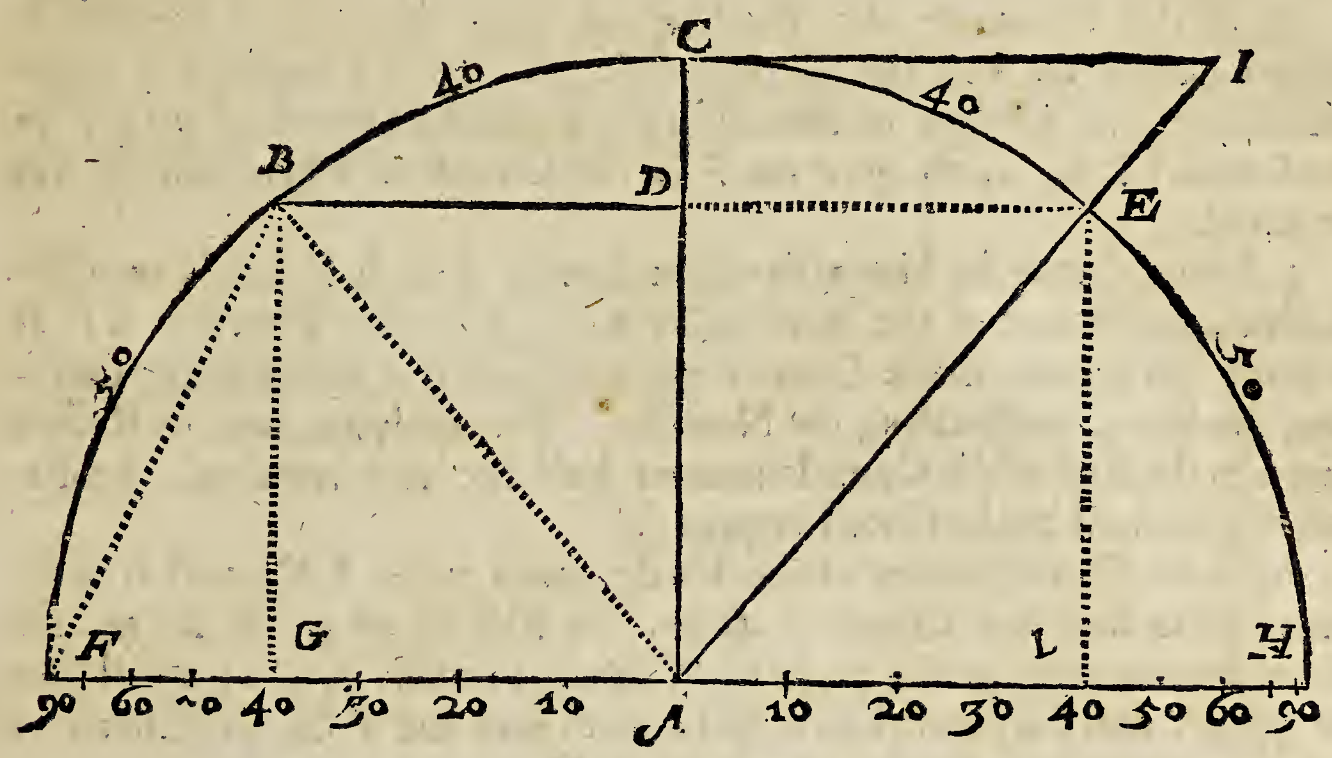
As if A C being the Radius given, it were required to find the Versed Sine of 130*gr.* here the excess above 90*gr.* is 40*gr.* and therefore the Versed Sine required is equal to the Radius A C and A 40, both being set together.

5. *The Diameter or Radius being given, to find the Chords of every Ark.*

THe Sines may be fitted many ways to serve for Chords, 1. A Sine being the half of the double Ark, if the Sine be doubled; it giveth the Chord of the double Ark, a Sine of 10*gr.* doubled giveth a Chord of 20*gr.* and a Sine of 25*gr.* being doubled giveth a Chord of 50*gr.* and so in the rest; As here B D, the Sine of B C, an Ark of 40*gr.* being doubled, giveth B E the Chord of B C E, which is an Ark of 80*gr.* Wherefore if the Radius of the Circle given be equal to the lateral Radius, let the Sector be opened near unto his length, so that both the Lines of Sines may make but one direct Line: so the Distance on the Sines between 10 and 10 shall be a Chord of 20, the Distance between 20and

20 and 20 ſhall be a Chord of 40, and the Diſtance between 30 and 30, ſhall be a Chord of 60, and ſo in the reſt.

2. Becauſe a Sine is the half of the Chord of the double Ark, the Proportion holdeth.



As the Diameter F H unto the Radius A H, ſo the Chord B E unto the Sine D E, or the Chord G L unto the Sine A L, and then if the Radius A H be put for the Diameter, which is a Chord of 180 *gr.* the Sine D E or A L, ſhall ſerve for a Chord of 80 *gr.* and the Semiradius which is the Sine of 30 *gr.* ſhall ſerve for a Chord of 60 *gr.* and go for the Semidiameter of a Circle, and ſo in the reſt. So that by theſe means we ſhall not need to double the Lines of Sines as before, but only to double the Numbers. And to this purpoſe I have ſubdivided each degree of the Sines into two, that ſo they might ſhew how far the half degrees do reach in the Sines, and yet ſtand for whole degrees when they are uſed as Chords.

Wherefore if the Radius of the Circle given be equal to the lateral Semiradius (the Sine of 30 *gr.* and Chord of 60 *gr.*) there needs no farther work, then to take the Sine of 10 *gr.* for a Chord of 20 *gr.* and a Sine of 15 *gr.* for a Chord of 30 *gr.* &c.

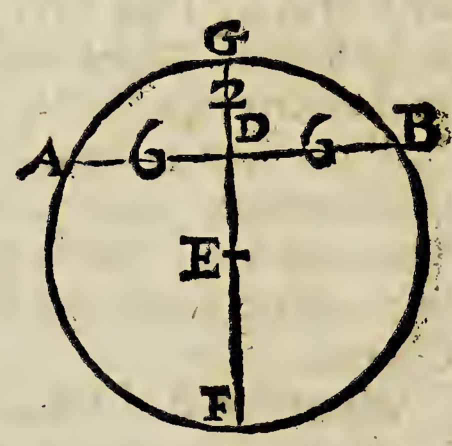
But if the Radius of the Circle given be either greater or leſſer than the lateral Semiradius, take the Diameter of it, and make it a Parallel Chord of 180 *gr.* by applying it over the Lines of Sines between 90 and 90, or take the Radius or Semidiameter, which is equal to the Chord of 60*gr.* and make it a parallel Radius of 60*gr.* by applying it over in the Sines of 30 and 30, and keep the Sector at this Angle. The Parallels taken from the lateral Chords ſhall be the Chords required.

As if the Diameter of a Circle given were the Line A B, and it were required to find the Chord of 80*gr.* Firſt I make A B a parallel Chord of 180*gr.* or the half of it a parallel Chord of 90*gr.* ſo his Parallel L G, doth give me F G the Chord of 80*gr.* which was required.

3. Seeing that as the Sine of the Complement of the half Ark is unto the Radius, ſo the Sine of the ſame whole Ark is unto the Chord of it: If we ſeek but for one ſingle Chord, we may find it without either doubling the Sines, or doubling the Number. For applying over the Radius given in the Sine of the Complement of half the Ark required, his Parallel Sine ſhall be the Chord required.

As if the Semidiameter of the Circle given were A C, and it were required to find the Chord of 40*gr.* the half of 40*gr.* is 20*gr.* the Complement of 20*gr.* is 70*gr.* Wherefore I make A C a parallel Sine of 70*gr.* and his parallel Sine G L, doth give me F G, the Chord of 40*gr.* agreeable to the Semidiameter A C.

Having two right Lines reſembling the Chord and Verſed Sine, to find the Diameter and Radius.



Let the two right Lines given be A B, reſembling the Chord, G D the Verſed Sine of a Circle, whoſe Arch A G B is unknown, and and let it be required to find the Diameter G F.

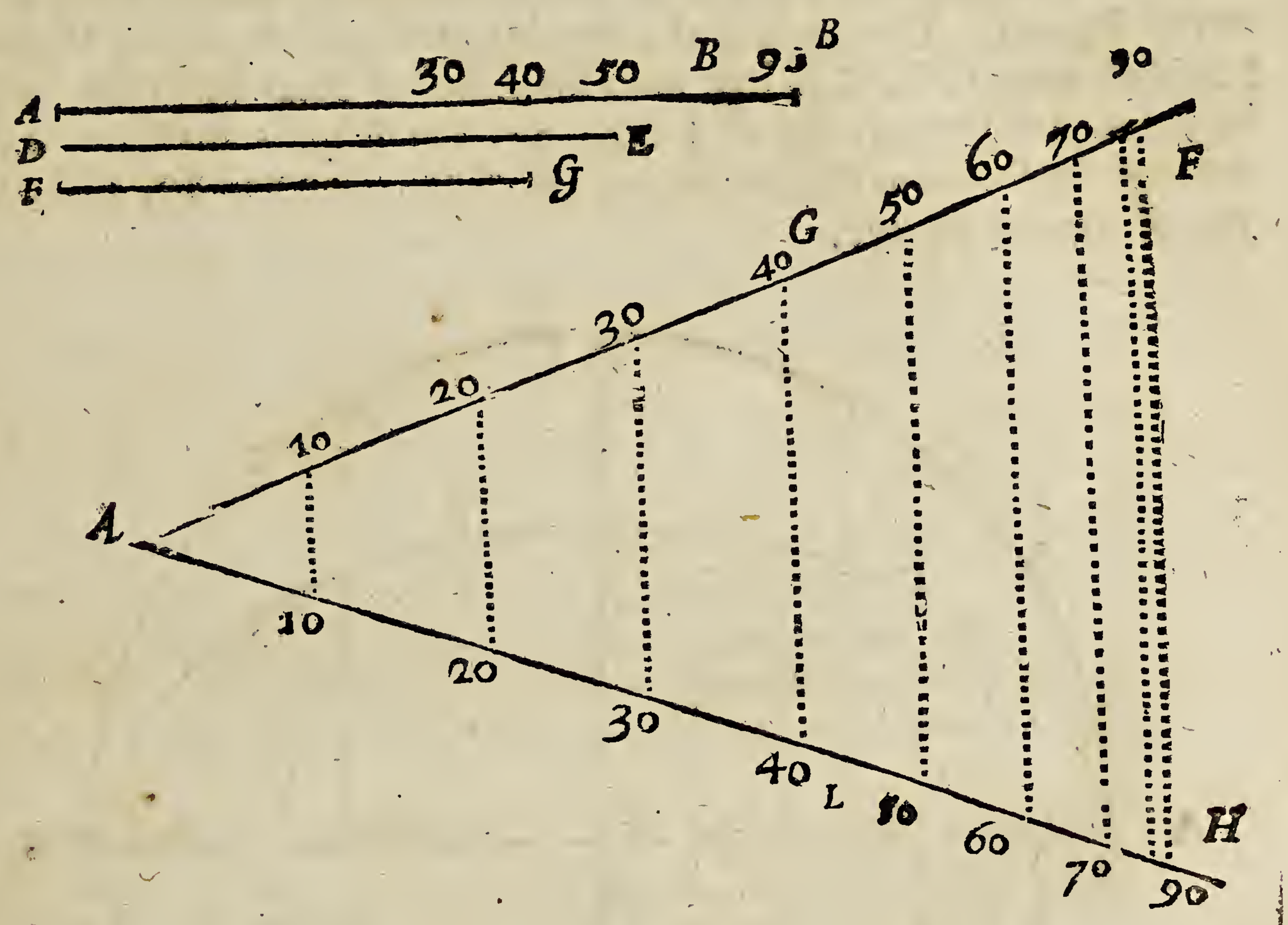
Having two Lines given, the firſt G D, the ſecond A D, the half of A B, we may find a third in continual Proportion (by the ſixth or nineth Propoſition of the Lines) and that ſhall be the Line D F (18) the Sum whereof and of G D gives the Diameter G F (20) and the half thereof is the Radius (E G).

6. The

6. The Chord of any Ark being given, to find the Diameter and Radius.

TUrn the Chord given unto a parallel Chord, and his parallel Semiradius shall be the Semidiameter, and the parallel Radius shall be the Diameter.

As if F G be the Chord of 80 *gr.* I put this over in G and L, the Sine of 40, and Chord of 80 *gr.* and the parallel Chord of 180 *gr.* giveth me A B the Diameter required.



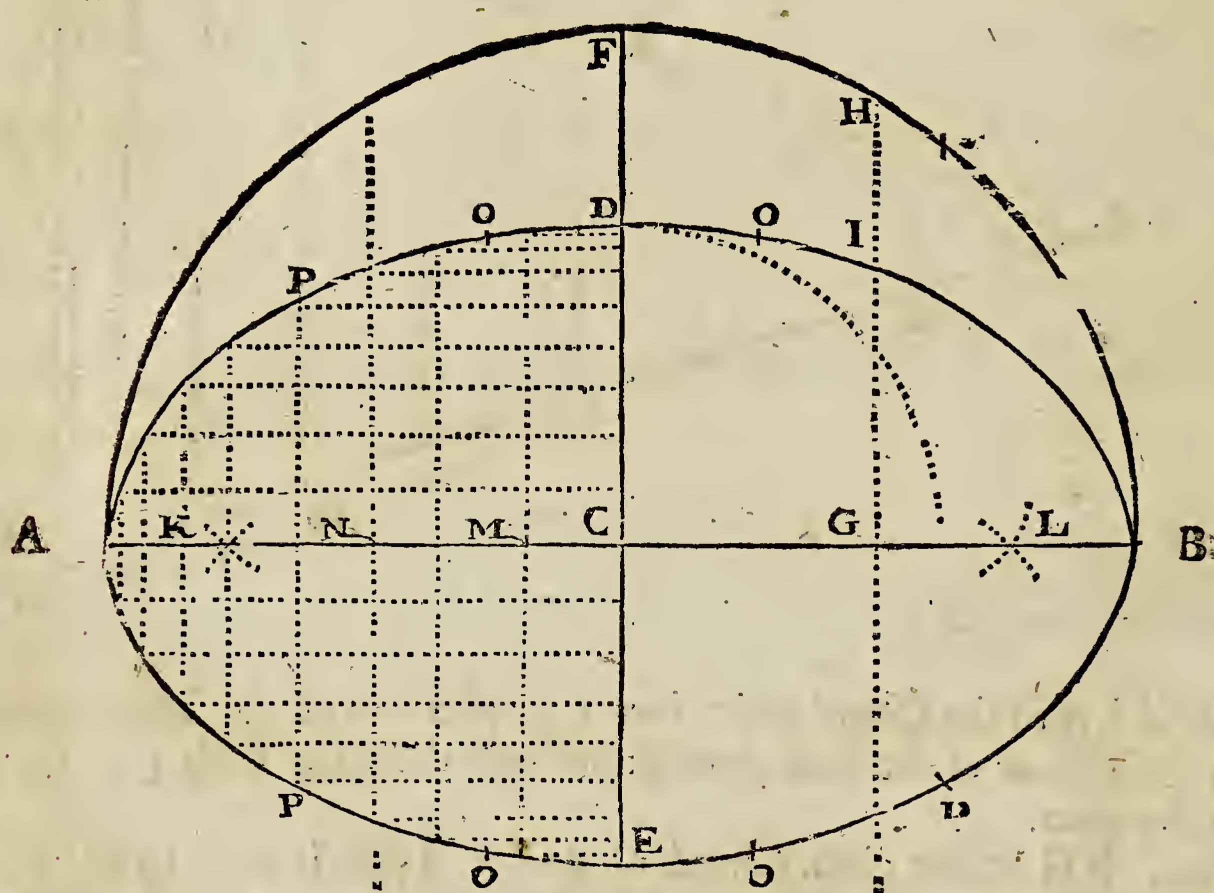
Or if I turn the Chord given into a parallel Sine of the same quantity, his parallel Sine of the Complement of half the Ark, doth give me the Semidiameter.

As if F G be the given Chord of 40 *gr.* I put it over in G and L, the Sines of 40 *gr.* then because the half of 40 *gr.* is 20 *gr.* and the Complement of 20 *gr.* is 70 *gr.* I take out the parallel Sine of 70 *gr.* and it giveth me A B for the Semidiameter, agreable to that Chord of 40 *gr.*

Having the Diameter of an Ellipsis to describe the same upon a plain.

If each Semidiameter be divided, in such sort, as the Line of *Sines* is divided upon the *Sector*, and right Lines drawn through each division Perpendicular to those Semidiameters like unto Sines; The Points where the Sines drawn through the one Semidiameter do meet the Sines of the Complement drawn through the other Semidiameter, shall be the Points through which the Ellipsis is to be drawn.

Let the Diameters be A B, D E, one crossing the middle of the other in the Point C. Divide first the Semidiameters C A, C B; then the Semidiameters C D, C E, like unto the Lines of *Sines* upon the *Sector*, by the eighth Proposition of Lines: So, the Ellipsis shall be drawn through the Points at the meeting of the Sines of 10 and 80, of 20 and 70, of 30 and 60, *&c.*



Or (without the help of the Line of Sines) we may draw the Circle A F B upon the Center C, and Semidiameter A C, for so, crossing the Diameter A B with several Perpendicular Lines continued unto the Circumference of the Circle, if we divide these Perpendiculars on

either

either side of the Diameter in such sort as the greater Semidiameter C F is divided by the lesser, in the Point D, and draw a Line winding through all those Points, the Line so drawn shall be the Ellipsis.

Or (without the help of the *Sector*) we may with the Radius A C, upon the Centers D and E, describe two occult Arches meeting in the Points K and L. Then taking between C and K, any Number of of Points M N, we may from the Centers K and L, with the Semidiameter M B describe four occult Arches; and with the Radius A M, and the same Centers K and L, cross them again with other four Arches in the Points at O. In like manner, from the same Centers K and L, with the Radius N B, we may describe other four occult Arches; and with the Radius A N, and the former Centers cross them again, with four Arches in the Points at P, and so draw the Ellipsis through the Points O P, *&c.*

This is (in effect) as we should tye a thread about A and L, and then draw it easily from the Point A round about the two former Centers K and L, until it were brought to the Point *A* again: which is also an easie way to describe an Ellipsis.

The distance of these former Points from either Semidiameter may be set down in Numbers. For supposing the lesser Semidiameter C D, to be 10, the greater (C B) to be 16, (or otherwise divided into any Number of known Points,) If we have the proportion between C G and C B, we may find the length of the Perpendicular G I.

If the Proportion be as 1 to 2, the Perpendicular will be 8, *66.*

If the Proportion be as 2 to 3, the Perpendicular will be about 7. 45.

As the greater Semidiameter	C B
to the part given	C G
So 100000, the Radius	C B
to the Sine of	C G
whose Complement is	G H
As the Radius	C F
to the Sine of the Complement	G H
So the lesser Semidiameter	C D
to the Perpendicular.	G I

The same may also be found without knowing the Sines. For the Perpendicular G H is a mean Proportional between A G and G B: which being known

As C F unto E D, so is G H unto G I.

7. *To open the* Sector *to the quantity of any Angle given.*
8. *The* Sector *being opened, to find the quantity of the Angle.*

IT is one thing to open the Edges of the Sector to an Angle, and another thing to open the Lines on the Sector to the same Angle. For the Lines of *Lines* on the one side, and the Lines of *Sines* on the other side, do make an Angle of 2 *gr.* when the Sector is close shut, and the Edges do make no Angle at all. So likewise the Lines of *Superficies* and the Lines of *Solids* do make an Angle of 10 *gr.* which are to be allowed to the Edges.

The Lines of *Lines* may be opened to a right Angle, if the whole Line of 100 parts be applied over in 80 and 60.

The Line of *Sines* may be opened to a right Angle, if the large Secant of 45 *gr.* be applied over in the Sines of 90 *gr.* or if the Sine of 90 *gr.* be applied over in the Sines of 45 *gr.* or if the Sine of 45 *gr.* be applied over in the Sines of 30 *gr.*

If it be required to open those Lines to any other Angle, take out the Chord thereof, and apply it over in the *Semiradius*, and those Lines shall be opened to that Angle.

As if it were required to open the Sector in the Lines of *Sines* to an Angle of 40 *gr.* take out the Chord of 40 *gr.* and to it open the Sector in the Chord of 60 *gr.* so shall the Lines of *Sines* be opened to the Angle required. Or if the same Chord of 40 *gr.* be applied over between 50, and 50, in the Line of *Lines,* they shall also be opened to the same Angle. If it be applied over in 25 of the Lines of *Superficies,* or 125 in the Lines of *Solids,* they also shall be opened to the same Angle: because the Chord of 60 *gr.* or Sine of 30 *gr.* and 50 in the Lines of *Lines,* and 25 in the Lines of *Superficies,* and 125 in the *Solids,* are all of the same length with the *Semiradius.*

Or if the *Semiradius* by applied over between the Sine of 30 *gr.* and the Sine of the Complement of the Angle required, it will open the Lines of *Sines* to that Angle.

As if the *Semiradius* be applied over in the Sines of 30 *gr.* and the Sine of 50 *gr.* it shall open the Lines of *Sines* to an Angle of 40 *gr.*

On the contrary, if the *Sector* be opened to an Angle, and it be required to know the quantity thereof, open the Compasses to the *Semiradius,* and setting one foot in the Sine of 30 *gr.* turn the other toward the other Line of *Sines,* and it shall fall there in the Complement

of

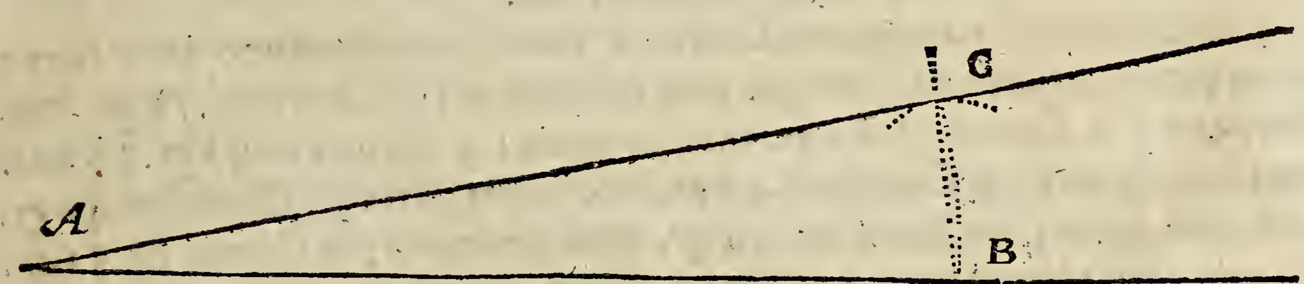
of the Angle; if it fall on 50 *gr.* the Angle is 40 *gr.* if on 60 *gr.* the Angle is 30 *gr.* &c.

Or take over the parallel Chord of 60 *gr.* and meaſure it in the lateral Chord, and it ſhall there ſhew the quantity of the Angle. As if the *Sector* being opened to an Angle, I ſhould take over the Parallel of 30 *gr.* of the Sines, and 60 *gr.* of the Chords, and meaſure it in the lateral Chords, find it to be 40 *gr.* the Angle comprehended between the Lines of *Sines* is 40 *gr.* but the Angle between the Edges of the Sector is 2 *gr.* leſs, and therefore but 38 *gr.*

9. *To find the quantity of any Angle given.*

If out of the Angular Point, to the quantity of the *Semiradius*, be deſcribed an occult Ark that may cut both ſides of the Angle, the Chord of this Ark meaſured in the lateral Chord, ſhall give the quantity of the Angle.

Let the Angle given be B A C: firſt I take the *Semiradius* with the Compaſſes, and ſetting one foot in A, I cut the ſides of the Angle in B and C; then I take the Chord B C, and meaſure it in the lateral Chord, and I find it to be 11 *gr.* and 15 *min.* and ſuch is the quantity of he Angle given.



Or if the Ark be deſcribed out of the Angular Point at any other diſtance, let the Semidiameter be turned into a parallel Chord of 60 *gr.* then take the Chord of this Ark, and carry it Parallel, till it croſs in like Chords: ſo the place where it ſtayeth ſhall give the quantity of the Angle.

As in the former example, if I make the Semidiameter A B a parallel Chord of 60 *gr.* and then keeping the *Sector* at that Angle, carry the Chord B C parallel, till it ſtay in like Chords; I ſhall find it to ſtay in no other but 11 *gr.* 15 *min.* and ſuch is the Angle B A C.

10. *Upon a right Line, and a Point given in it, to make an Angle equal to any Angle given.*

FIrst out of the Point given describe an Ark, cutting the same Line: then by the 5 *Prop.* afore, find the Chord of the Angle given agreeable to the Semidiameter, and inscribe it into this Ark: so a right Line drawn through the Point given, and the end of this Chord, shall be the side that makes up the Angle.

Let the right Line given be A B, and the Point given in it be *A*, and let the Angle given be 11 *gr.* 15 *min.* Here I open the Compasses to any Semidiameter A B, (but as oft as I may conveniently to the lateral Semiradius) and setting one foot in *A*, I describe an occult Ark B C; then I seek out the Chord of 11 *gr.* 15 *min.* and taking it with the Compasses, I set one foot in B, the other crosseth the Ark in C, by which I draw the Line A C, and it makes up the Angle required.

11. *To divide the Circumference of a Circle into any parts required.*

IF 360, the measure of the whole Circumference, be divided by the Number of parts required, the Quotient giveth the Chord, which being found will divide the Circumference.

So a Chord of 120 *gr.* will divide the Circumference into three equal parts; a Chord of 90 *gr.* into four parts; a Chord of 72 *gr.* into five parts; a Chord of 60 *gr.* into six parts; a Chord of 51 *gr.* 26 *min.* into seven parts; a Chord of 45 *gr.* into eight parts; a Chord of 40 *gr.* into nine parts; a Chord of 36 *gr.* into ten parts; a Chord of 32 *gr.* 44 *min.* into eleven parts; a Chord of 30 *gr.* into twelve parts.

In like manner if it be required to divide the Circumference of the Circle whose Semidiameter is A B, into 32; first I take the Semidiameter A B, and make it a parallel Chord of 60 *gr*, then because 360 *gr.* being divided by 32 the Quotient will be 11 *gr.* 15 *min.* I find the parallel Chord of 11 *gr.* 15 *min.* and this will divide the Circumference into 32.

But here the parts being many, it were better to divide it first into fewer, and after to come over it again. As first to divide the Circumference into 4, and then each 4 parts into 8, or otherwise, as the parts may be divided.

12. *To divide a right Line by extreme and mean proportion.*

THe Line to be divided by extreme and mean proportion, hath the ſame proportion to his greater Segment, as in Figures inſcribed in the ſame Circle, the ſide of an *Hexagon* a figure of ſix Angles, hath to a ſide of a *Decagon* a figure of ten Angles: but the ſide of a *Hexagon* is a Chord of 60 *gr.* and the ſide of a *Decagon* is a Chord of 36 *gr.*

Let A B be the Line to be divided: if I make A B a parallel Chord of 60 *gr.* and to this Semidiameter find A C a Chord of 36 *gr.* this A C ſhall be the greater Segment, dividing the whole Line in C, by extreme and mean proportion. So that,

As A B the whole, is unto A C the greater Segment: ſo A C the greater Segment, unto C B the leſſer Segment.

Or let A C be the greater Segment given: if I make this a parallel Chord of 36 *gr.* the correſpondent Semidiameter ſhall be the whole Line A B, and the difference C B the leſſer Segment.

Or let C B be the leſſer Segment given: if I make this a parallel Chord of 36 *gr.* the correſpondent Semidiameter ſhall be the greater Segment A C, which added to C B, gives the whole Line A B.

To avoid doubling of Lines or Numbers, you may put over the whole Line in the *Sines* of 72 *gr.* and the parallel Sine of 36 *gr.* ſhall be the greater Segment.

Or if you put over the whole Line in the Sines of 54 *gr.* the parallel Sine of 30 *gr.* ſhall be the greater Segment, and the parallel Sine of 18 *gr.* ſhall be the leſſer Segment.

CHAP. III.

Of the projection of the Sphere *in Plano.*

SECT. I.

To Project the Sphere in Plano, by streight Lines.

1. THe Sphere may be projected in *Plano* in streight Lines, as in the *Analemma*, if the Semidiameter of the Circles given be divided in such sort as the Line of *Sines* on the Sector.

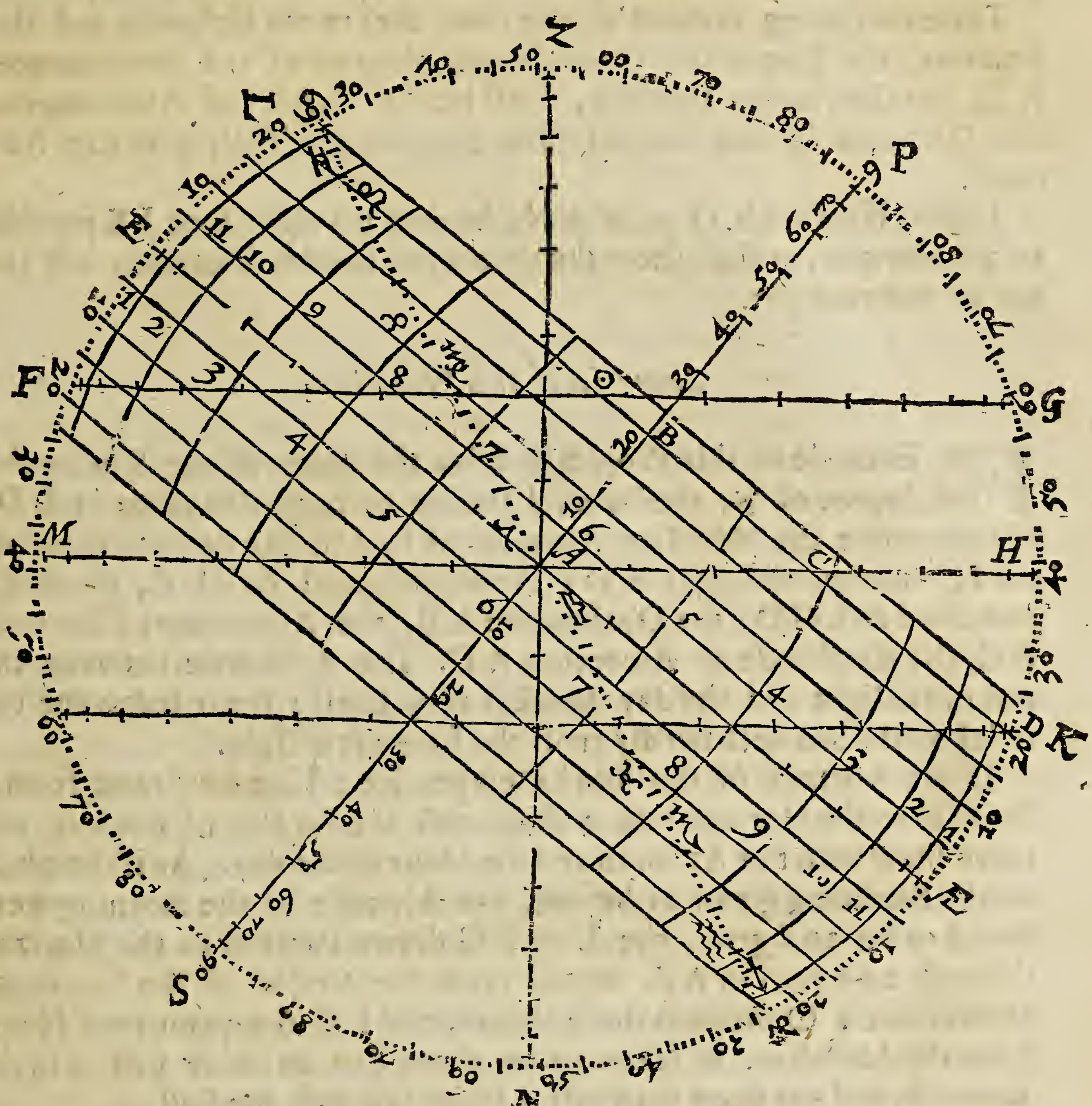
As if the Radius of the Circle given were A E, the Circle thereon described may represent the Plane of the general Meridian, which divided into four equal parts in E, P, Æ, S, and crossed at right Angles with E Æ and P S, the Diameter E Æ, shall represent the Equator, and P S, the Circle of the hour of *6*. And it is also the Axis of the World, wherein P stands for the North Pole, and S for the South Pole. Then may each quarter of the Meridian be divided into *90 degrees* from the Equator towards the Poles. In which we number 23 *degrees*, 30 *min.* the greatest declination of the Sun from E to ♋ Northwards, from Æ to ♑ Southwards, the Line drawn from ♋ to ♑ shall be the Eclipttick, and the Lines drawn parallel to the Equator through ♋ and ♑ shall be the Tropick.

Having these common Sections with the Plane of the Meridian, if we shall divide each Semidiameter of the Ecliptick into *90 degrees*, in such sort as the *Sines* are divided on the *Sector*. The first 30 *degr.* from A towards ♋ shall stand for the Sign of ♈. The *30 degr.* next following for ♉. The rest of ♊ ♋ ♌, *&c.* in their order. So that by these means we have the place of the *Sun* for all times of the year.

If again we divide A P, A S, in the like sort, and set thereto the Numbers 10, 20, 30, *&c.* unto *90 degrees*, the Lines drawn through each of these degrees parallel to the Equator shall shew the declination of the Sun, and represent the Parallels of Latitude.

If farther, we divide A E, A Æ, and each of his Parallels equally in the like sort, and then carefully draw a Line through each 15 *degr.* so as it makes no Angles, the Lines so drawn shall be Eliptical, and represent

represent the Hour-circles. The Meridian P E S, the hour of 12 at noon; that next unto it drawn through 75 *degrees* from the Center, the hours of 11 and 1, that which is drawn through 60 *degrees* from the Center, the hours of 10 and 2, *&c.*



To these we may add the months of the year, and the days of each month, placing *January* about F, *March* about E, *June* about J, *July* about K, *September* about E Æ, *December* about the Tropick of ♑: and so the rest according to their Declination from the Equator.

Then having respect unto the Latitude, we may number it from

E Northward unto Z, and there place the Zenith: by which, and the Center, the Line drawn Z A N, ſhall the Vertical Circle, paſſing through the Zenith and Nadir, and through the Center at A, in the Points of Eaſt and Weſt, and the Line M A H croſſing it at right Angles, ſhall repreſent the Horizon.

Theſe two being divided in the ſame ſort as the Ecliptick and the Equator, the Line drawn through each degree of the Semidiameter A Z, parallel to the Horizon, ſhall be the Circles of Altitude, and the Diviſions in the Horizon and his parallels ſhall give the Azimuth.

Laſtly, If through 18 *gr.* in A N, be drawn a right Line I K parallel to the Horizon, it ſhall ſhew the time when the day breaketh, and the end of the twilight.

Some Uſes of this Projection.

FOr Example of this Projection, let the place of the Sun be the laſt degree of ♉, the Parallel paſſing through this place is L D, and therefore the Meridian Altitude M L, and the depreſſion below the Horizon at midnight H D: the Semidiurnal Ark L C, the Seminocturnal Ark C D; the Declination A B, the Aſcentional difference B C, the Amplitude of Aſcention A C. The difference between the end of twilight and the day break is very ſmall; for it ſeems the Parallel of the Sun doth hardly croſs the Line of twilight.

If the Altitude of the Sun be given, let a Line be drawn from it Parallel to the Horizon: ſo it ſhall croſs the Parallel of the Sun, and there ſhew both the Azimuth and the Hour of the day. As if the place of the Sun being given as before, the Altitude in the morning were found to be 20 *degrees*, the Line F G drawn Parallel to the Horizon through 20 *degrees* in A Z, would croſs the Parallel of the Sun in ☉. Wherefore F ☉ ſheweth the Azimuth, and L ☉ the quantity of Hours from the Meridian. It ſeems to be about half an hour paſt 6 in the morning, and yet more than half a Point ſhort of the Eaſt.

The diſtance of two places may be alſo ſhewed by this Projection, their Latitudes being known, and their difference of Longitude.

For ſuppoſe a place in the Eaſt of *Arabia*, having 20 *degrees* of North Latitude, whoſe difference of Longitude from *London*, is found by an Eclipſe to be 5 Hours, ½. Let Z be the Zenith of *London*, the Parallel of Latitude for that other place muſt be L D, in which the difference

difference of Longitude is L ⊙. Wherefore ⊙ representing the site of that place, I draw through ⊙ a Parallel to the Horizon M H, crossing the Vertical A Z near about 70 *degrees* from the Zenith, which multiplied by 20, sheweth the distance of *London*, and that place to be 1400 Leagues. Or multiplied by 60, to be 4200 miles.

Sect. II.

To project the Sphere in Plano, by Circular Lines.

2. THe Sphere may be projected in *Plano* by Circular Lines, as in the general Astrolabe of *Gemma Frisius*, by the help of the Tangent on the side of the *Sector*.

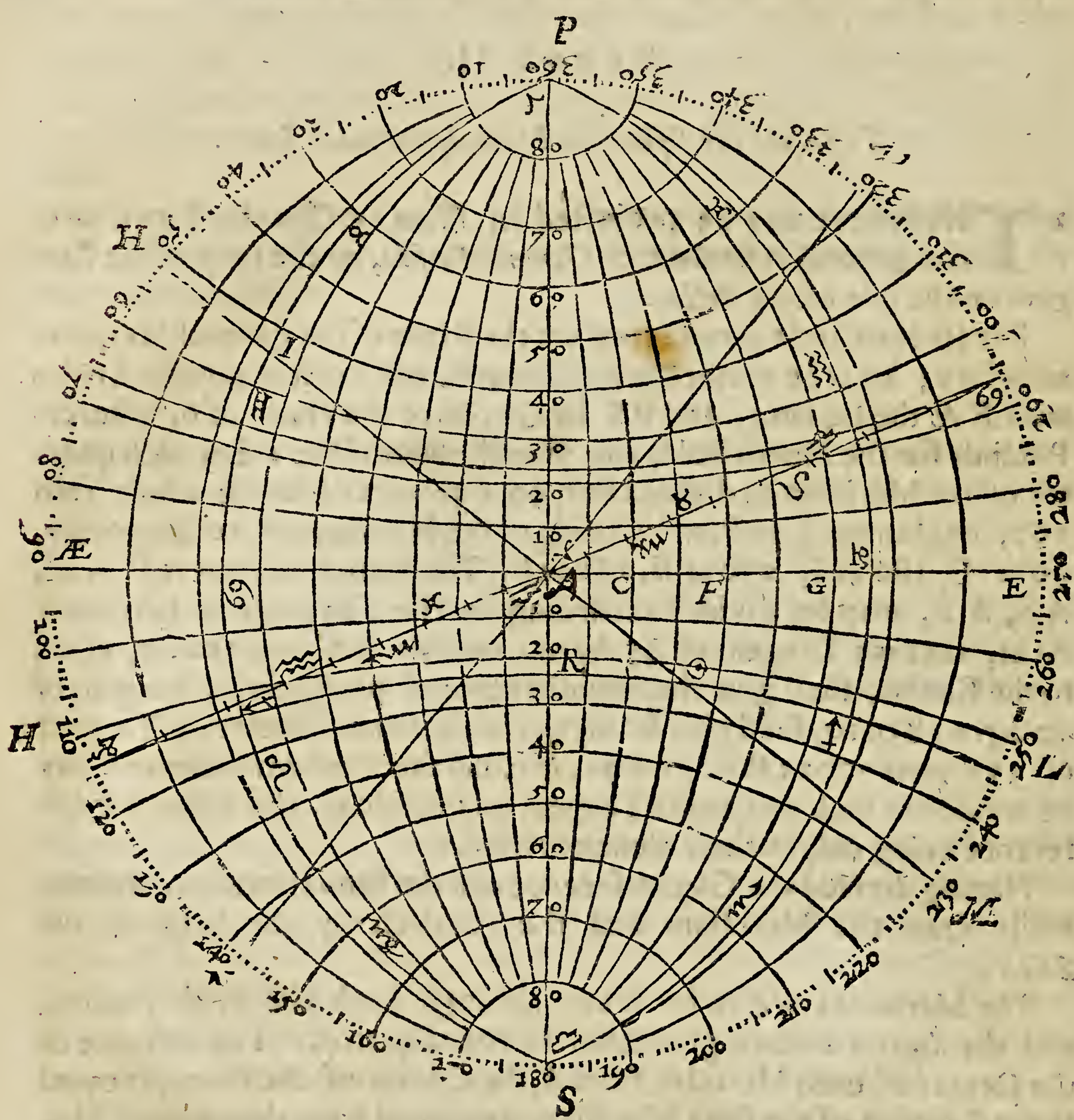
For let the Circle given represent the Plane of the general Meridian as before; let it be divided into four parts, and crossed at right Angles with E Æ the Equator, and P S the Circle of the Hour of 6, wherein P stands for the North Pole, and S for the South Pole: Let each quarter of the Meridian be divided into 90 *degrees*, and so the whole into 360, beginning from P, and setting to the Numbers of 10, 20, 30, *&c.* 90 at Æ, 180 at S, 270 at E, 360 at P. The Semidiameters A P, A Æ, A S, A E, may be divided according to the Tangents of half their Arks, that is a Tangent of 45 *degrees*, which is always 10000, equal to the Radius, shall give the Semidiameter of 90 *degrees*, a Tangent of 40 *degrees* 83910, shall give 80 *degrees* in the Semidiameter: a Tangent of 35 *degrees* 70021 shall give 70, *&c.* So that the Semidiameters may be divided in such sort as the Tangent on the side of the *Sector*, the difference being only in their denomination.

Having divided the Circumference and the Semidiameters, we may easily draw the Meridians and the Parallels by the help of the *Sector*.

The Meridians are to be drawn through both the Poles P and S, and the *degrees* before graduated in the Equator. The distance of the Center of each Meridian from A, the Center of the Plane, is equal to the Tangent of the same Meridian, reckoned from the general Meridian P Æ S E, and the Semidiameter equal to the Secant of the same *degree*.

As for example, If I should draw the Meridian P B S, which is the tenth from P Æ S, the Tangent of 10 *gr.* 17633, giveth me A C, and the Secant of 10 *gr.* 101543, giveth me S C, wherefore C is the

Center of the Meridian, P B S, and C S his Semidiameter: ſo A F a Tangent of 20 *gr.* 36397 ſheweth F to be the Center of P D S, the twentieth Meridian from P Æ S, and A G a Tangent of 23 *gr.* 30 *m.* 43481, ſheweth G to be the Center of P ♋ S, *&c.*



The Parallels are to be drawn through the degrees, in A P, A S, and their correſpondent degrees in the general Meridian. The diſtance of the Center of each Parallel from A the Center of the Plane, is equal to the Secant of the ſame Parallel from the Pole, and the Semidiameter equal

equal to the Tangent of the same *degree.* As if I should draw the Parallel of 80 *degrees*, which is the tenth from the Pole S, first I open the Compasses unto A C the Tangent of 10 *degrees* 17633, and this giveth me the Semidiameter of this Parallel, whose Center is a little from S, in such distance as 101543 the Secants S C is longer than 10000, the Radius S A.

The Meridians and Parallels being drawn, if we number the 23 *degr.* 30 *min.* from E to ♋ Northwards, from Æ to ♑ Southward, the Line drawn from ♋ to ♑ shall be the Ecliptick: which being divided in such sort as the Semidiameter A P, the first 30 *degrees* from A to ♋ shall stand for the Sine of ♈; the 30 *degr.* next following for ♉; the rest for ♊, ♋, ♌, *&c.* in their order.

If farther we have respect unto the Latitude, we may number it from E Northward unto Z, and there place the Zenith, by which and the Center, the Line drawn Z A N, shall represent the verticle Circle, and the Line M A H, crossing it at right Angles, shall represent the Horizon, and these divided in the same sort as A P, the Circles drawn through each *degree* of the Semidiameter A Z, Parallel to the Horizon, shall be the Circles of Altitude: and the Circles drawn through the Horizon and his Poles, shall give the Azimuths.

Some Uses of this Projection.

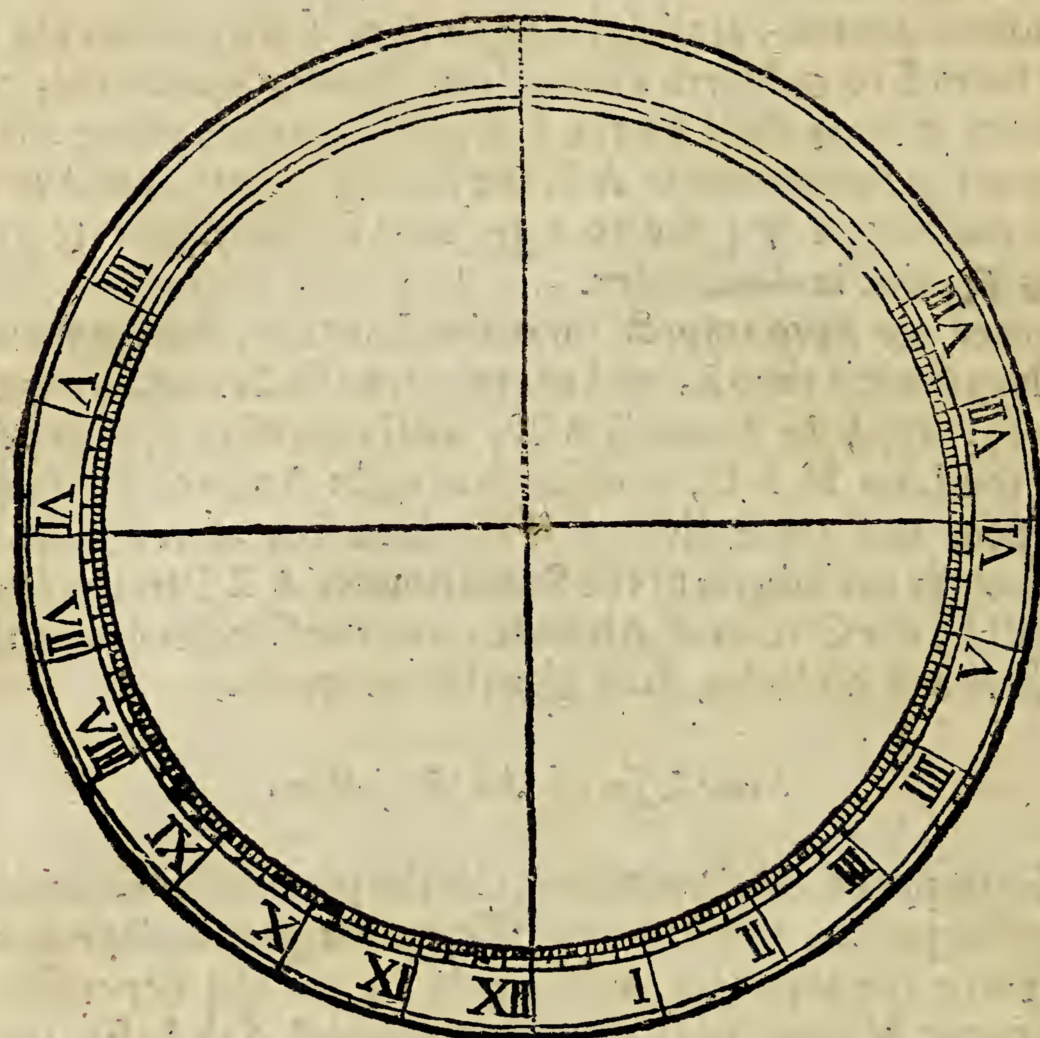
FOr Example of this Projection, let the place of the Sun be in the beginning of ♒, the Parallel passing through this place is ♒ ☉ L, and therefore the Meridian Altitude M L, and the depression below the Horizon at Midnight H O, the Semidiurnal Ark L ☉, the Seminocturnal Ark O ☉, the Declination A R, the Ascensional difference R ☉, the Amplitude of the Ascension A ☉.

Or if A be put to represent the Pole of the World, then shall P Æ S E stand for the Equator, and P ♋ S ♑ for the Ecliptick, and the rest which before stood for Meridians, may now serve for particular Horizons, according to their several Elevations. Then suppose the place of the Sun given to be 24 *degrees* of ♉, his Longitude shall be P I, his right Ascension P H, his Declination H I. And if the place given be 19 *degrees* of ♌, his Longitude shall be P K, his right Ascension P N, his Declination N K. Again, the Declination brought to the Horizon of the place, shall there shew the Ascensional difference; Amplitude of Ascension, and the like conclusions of the Globe. But I

intend

intend not here to ſhew the Uſe of the Aſtrolabe, but the Uſe of the *Sector* in Projection.

And after this manner may a Nocturnal be projected to ſhew the Hour of the Night, whereof I will ſet down a Type for the uſe of Seamen.



It conſiſts, as you ſee, of two parts, the one is a Plane divided equally according to the 24 hours of the day, and each hour into quarters or minutes, as the Plane will bear: the Line from the Center to XII, ſtands for the Meridian, and XII ſtands for the hour of 12 at midnight. The other part is a rundle for ſuch ſtars as are near the North-pole, together with the twelve months, and the days of each month fitted to the right Aſcenſion of the ſtars. Thoſe that have occaſion to ſee the South-pole, may do the like for the Southern Conſtellations, and put them in a Rundle on the back of this Plane, and ſo it may ſerve for all the World.

The

The Use of this Nocturnal.

The Uſe of this Nocturnal is eaſie and ready. For look up to the

ſhall be given by the Tangent of 11 *degrees* 45 *min.* The Center of the Circle of Longitude paſſing through this Pole E ♈ and ♎, ſhall intend not here to ſhew the Uſe of the Aſtrolabe, but the Uſe of the *Sector* in Projection.

And after this manner may a Nocturnal be projected to ſhew the Hour of the Night, whereof I will ſet down a Type for the uſe of Seamen.

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The

The Use of this Nocturnal.

The Use of this Nocturnal is easie and ready. For look up to the Pole, and see what Stars are near the Meridian: then place the Rundle to the like situation, so the day of the month will shew the hour of the Night.

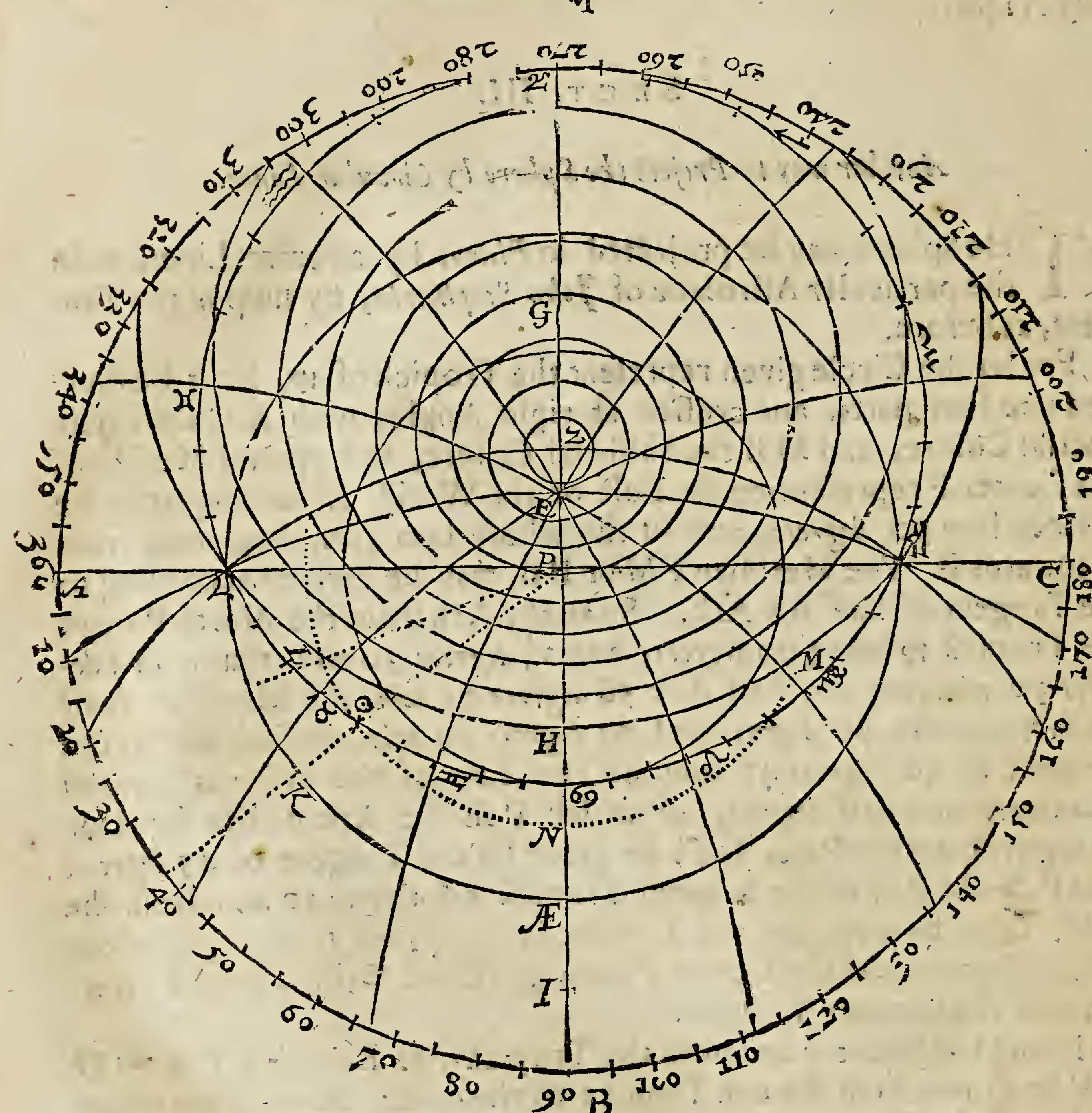
Sect. III.

Another way to Project the Sphere by Circular Lines.

3. THe Sphere may be projected in *Plano*, by circular Lines, as in the particular Astrolabe of *John Stophlerin*, by help of the Tangent, as before.

For let the Circle given represent the Tropick of ♑, let it be divided into four parts, and crossed at right Angles with A C the Equinoctial Colure, and M B the Solstitial Colure, and general Meridian, the Center P representing the Pole of the World. Let each quarter be divided into 90 *degrees*, and so the whole into 360, beginning from A towards B. The Meridian P M or P B, may be divided according to the Tangent of half his Ark. So as the Ark from the North Pole to the Tropick ♑ being 90 *degrees*, and 23 *degrees* 30 *min.* that is 113 *degrees* 30 *min.* and the half Ark 56 *degrees* 45 *min.* the Meridian shall be divided into 90 *degrees* and 23 *degrees* 30 *min.* in such sort as the Tangent of 56 *degrees* 45 *min.* on the side of the *Sector* is divided into *degrees* and half *degrees*, of which P Æ the Ark of the Equator 90 *degrees* from the Pole, shall be given by the Tangent of 45 *degrees*. And P ♋ the Ark of the Summer Tropick 66 *degrees* 30 *min.* from the Pole, shall be given by the Tangent of 33 *degrees* 15 *min.* And the Circles drawn upon the Center P through Æ and ♋, shall be the Equator, and the Summer Tropick.

Having the Equator and both the Tropicks, the Ecliptick ♈ ♋ ♎ ♑ shall be drawn from the one Tropick to the other, through the intersection of the Equator and the Equinoctial Colure. And it may be divided first into twelve Signs after this manner: P E the Ark of the Pole of the Ecliptick 23 *degrees* 30 *min.* from the Pole of the World, shall be given by the Tangent of 11 *degrees* 45 *min.* The Center of the Circle of Longitude passing through this Pole E ♈ and ♎, shall be found at D (somewhat below B) by the Tangent of 66 *degrees* 30 *min.* Then through D draw an occult Line parallel to A C, and divide it on each side from D, in such sort as the Tangent is divided on the side of the Sector, allowing 45 *degrees* to be equall to D E, so the thirtieth *degree* from D toward the right hand, shall be the Center of the Circle of Longitude passing through E ♉ and ♍. The



sixtieth *degree*, the Center of ♊ E ♐. The thirtieth *gr.* from D towards the left hand, the Center of ♓ E ♍. The sixtieth, the Center of ♒ E ♌. And the other intermediate *degrees* shall be the Centers to divide each sign into 30 *gr.*

If

If farther we have respect unto the Latitude, we may (the Meridian being before divided) number it from P Northward unto H, and there place the North Interfection of the Meridian and Horizon: then the Complement of the Latitude being numbred from P Southward unto Z, shall there give the Zenith; and 90 *degr.* from Z Southward unto F, shall there give the South interfection of the Meridian and Horizon. The middle between F and H shall be G the Center of the Horizon ♈ H ♎ F, passing through the beginning of ♈ and ♎, unless there be some former errrour.

All Parallels to the Horizon may be found in like sort by their Interfections with the Meridian, and the middle between those Interfections is always the Center.

The Azimuths may be drawn as the Circles of Longitude were before. For the Circle of the first Verticle ♈ Z ♎, will be found at I (somewhat near unto B) by the Tangent of the Latitude. And if through I we draw an occult Line parallel to A C, and divide it on each side from I, in such sort as the Tangent is divided on the side of the *Sector*, allowing 45 *degrees* to be equal to I Z, these Divisions shall be the Centers, and the distance from these Divisions unto Z, shall be the Semidiameters whereon to describe the rest of the Azimuths.

Some Uses of this Projection.

For example of this Projection, let ☉ the place of the Sun given be 10 *degr.* of ♉: a right Line drawn from P through this place unto the Equator, shall there shew his right Ascension ♈ K, and his Declination K ☉. Then may we on the Center P and Semidiameter ☉ P draw an occult Parallel of Declination, crossing the Horizon in L, M. the Meridian in G and N. So the right Lines P L and P M produced, shall shew the time of the Suns rising and setting, ♈ Q the difference of Ascension, R the difference of Descension, ♈ L the Amplitude of rising, and ♎ M the Amplitude of his setting, L N M sheweth the length of the night, Z G sheweth his distance from the Zenith at noon, H N his depression below the Horizon at midnight. And then having the Altitude of the Sun at any time of the day, the Interfection of the parallel of Altitude with the parallel of Declination, sheweth the Azimuth, and a right Line drawn from P through this Interfection, giveth the hour of the day.

Sect. IV.

A third way to Project the Sphere in Plano, by Circular Lines.

4. THe Sphere may be Projected in *Plano* by Circular Lines, after the manner of the old concave Hemisphere, by the help of the Tangent on the side of the *Sector.*

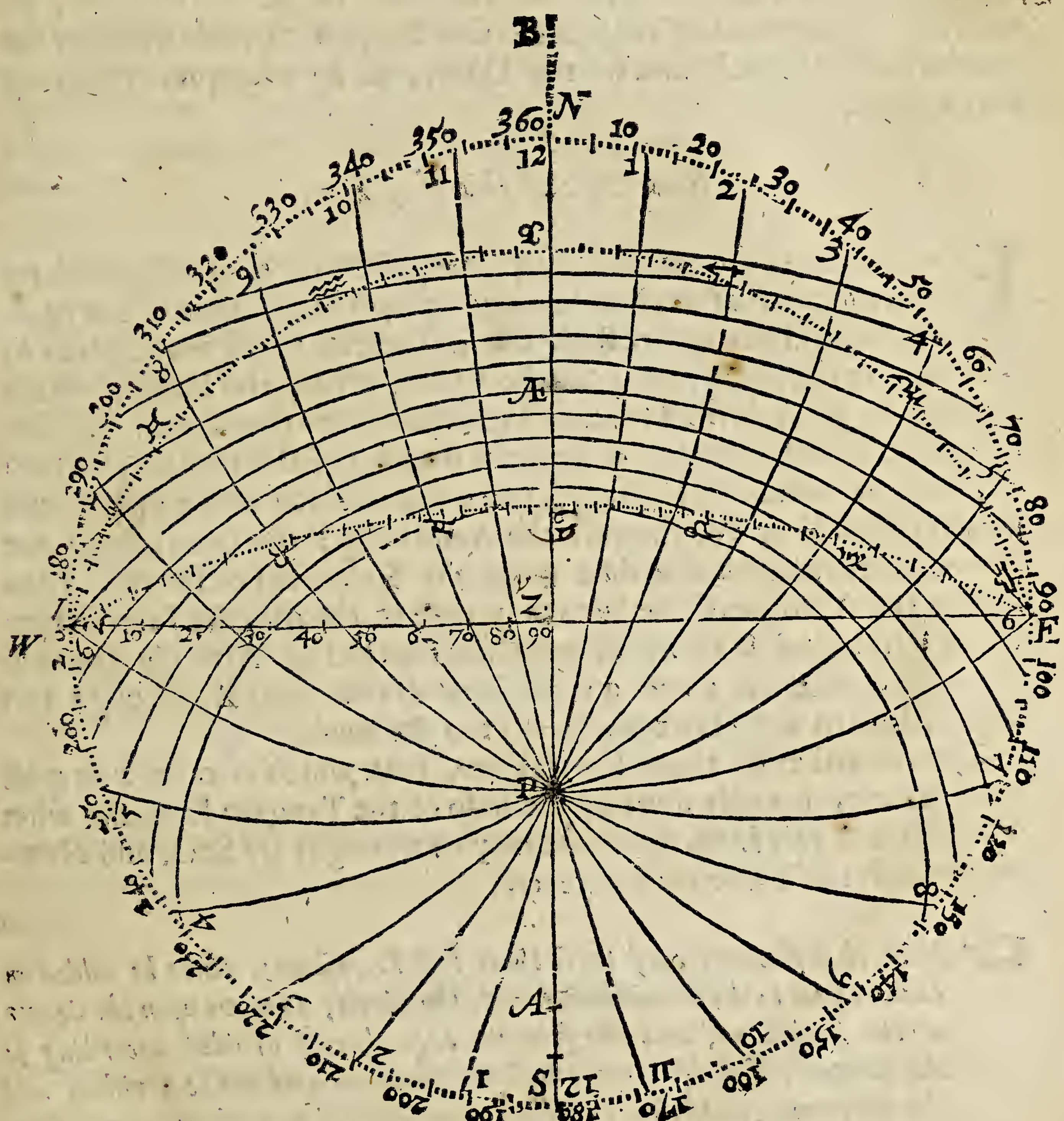
For let the Circle given represent the Plane of the Horizon, let it be divided into four parts, and crossed at right Angles with S N the Meridian, and E V the Verticle; so as S may stand for the South, *N* for the North, E for the East, W the West part of the Horizon, and the Center Z represent also the Zenith. Let each quarter of the Horizon be divided into 90 *degrees*, and so the whole into 360 *degrees*, beginning from N, and setting to the numbers of 10,20,30,*&c.* 90 at E, 180 at S, 270 at W, 360 at N.

The Semidiameters Z N, Z S, may be divided according to the Tangent of half their Arks: so as the Ark from the Zenith to the Horizon being 90 *gr.* and the half Ark 45 *gr.* the Semidiameters are to be divideed in such sort as the Tangent of 45 *gr.* as was shewed before in the second Projection. And if from Z we draw Circles through each of these Divisions, they shall be Parallels of Altitude.

Then having respect unto the Altitude, we may (the Meridian being before divided) number it from Z to Æ, and there place the Intersection of the Meridian and Equator. The Complement of the Latitude from Z unto P, shall there give the Pole of the World, and 90 further from P, shall there give the other intersection of the Meridian and Equator.

The middle between these intersections shall be A the Center of the Equator, passing through E and W, unless there be some former errour. The intersections of the Tropicks depend on the Equator. From Æ 23 *degrees* 30 *min.* farther shall be ♑ the intersection of the Meridian and the Southern Tropick. From Æ 23 *degrees* 30 *min.* nearer shall be ♋, the Intersection of the Meridian and the Northern Tropick. The Intersections of the other intermediate Parallels, shall be given in like sort, by their *degrees* of distance from the Equator, and the middle between those Intersections is always the Center.

The Hour Circles may be here drawn as the Azimuths in the third Projection. For the Center of E P W, the hour of 6, will be found at B, (ſomewhat near unto N) by the Tangent of the Latitude. And if through B we draw an occult Line parallel unto E W, and divide it on each ſide from B, in ſuch ſort as the Tangent is divided on the ſide of the *Sector*, allowing 45 *degrees* to be equal to B P, and 15 *degrees*, for every hour, thoſe Diviſions ſhall be the Centers, and the diſtance from the Diviſions unto P, ſhall be the Semidiameters, whereon to deſcribe the reſt of the hour Circles.



The

The Ecliptick may be drawn as the Equator. For the Center of that half which hath Southern Declination, ſhall be given by the Tangent of the Altitude, which the Sun hath in his entrance into ♑. And the Center of the other half by the Tangent of his Altitude, at his entrance into ♋, and it may be divided, as in the former Projection, or elſe by Tables calculated to that purpoſe.

To theſe Circles thus drawn, if we ſhall add the months of the year, and the days of each month, as we may well do, at the Horizon, on either ſide, between the Tropicks: this Projection ſhall be fitted for the moſt uſeful Concluſions of the Globe, as by examples following may appear.

Some Uſes of this Projection.

FOr the day of the month being given, the Parallel that ſhooteth on it, doth ſhew what declination the Sun hath at that time of the year. And where this Parallel croſſeth the Ecliptick, there is the place of the Sun. Or the place of the Sun being firſt given, the Parallel which croſſeth it, ſhall at the Horizon ſhew the day of the month. Either of theſe then being given, or only the parallel of Declination, we may follow it, firſt unto the Horizon, there the diſtance of the end of the parallel from E or W, ſheweth the Amplitude; the ſame among the hour-circles ſheweth the time when the Sun riſeth or ſetteth. Then having the Altitude of the Sun at any time of the day, the Interſection of the Parallel of Declination with the Parallel of Altitude ſheweth the hour of the day; and a right Line drawn from Z, through this Interſection to the Horizon, giveth the Azimuth.

Thus in either of theſe Projections, that which is otherwiſe moſt troubleſome, is eaſily done by the help of the Tangent Line, and what I have ſaid of this Line, the ſame may be wrought by Scale and Numbers out of the Table of Tangents.

☞ *Note, that if unto any of theſe three laſt Projections, there be added an Index equal to the Semidiameter of the Circle, to move upon the Center of the Projection, and the fiducial edge thereof divided according to the Tangents of half Arks, the Semidiameters need not be divided, and the Inſtruments will then be fitly accommodated to perform many Concluſions of the Sphere.* ☜

☞By

☞By the former ways of Projecting of the Sphere, the whole Art of Dialling may be performed upon any of them, but especially upon this last, which may be fitted to the Horizon of any place, the manner whereof in this place I shall briefly deliver.

1. *For an Horizontal Dial.*

If streight Lines be drawn from the Center of the Projection through the intersections of the hour Circles with the outermost Circle or Horizon, those Lines so drawn shall be the true hour Lines of an Horizontal Dial in that Latitude for which the Projection was made, for the hour Circles cut the Horizon at those degrees of distance.

2. *For an Erect direct North or South Dial.*

If an Index be divided as the Semidiameter of the Projection ZW is, on both sides, and laid upon the same Diameter WE, the hour Lines of the Projection will cut the same Index in such degrees from the Meridian on either side, as the hour Lines on such a North or South Dial ought to have upon the Plane: As,

					deg.	*min.*	
The hour Lines of	11	and	1	do cut the Index at	9	28	From the Meridian.
	10		2		19	54	
	9		3		31	54	
	8		4		47	09	
	7		5		66	42	

And these are the true hour distances for a North or South Plane in this Latitude of 51 *degrees* 30 *min.* for which this Projection was made.

3. *For a Vertical Declining Dial.*

Suppose an upright Plane to decline from the South Westward 24 *degr.* 20 *min.* Such a Plane is described in the third Book of this Treatise *Chap.*7. If you lay your Index to 24 *degr.* 20 *min.* counted from E towards S, and there keep it fixed, the hour Lines of the Projection

Projection will cut the Index in these degrees from the Meridian, on either side thereof, at which they are to be drawn upon the Dial Plane.

4. *For direct Incliners.*

Let the Inclining Plane be projected upon the Scheme, a Ruler laid to the Pole of the inclining Plane, and to the several Points where the hours cross the Plane, the Ruler will cut the outermost Circle in the degrees that the hour Lines ought to have upon such an inclining Plane.

Thus let the Circle W Æ E, which is the Equinoctial Circle, represent a Plane inclining to the Horizon, a Ruler laid to the Pole of the World (which is the Pole of the Equinoctial Circle) and the several intersections of the hour Circles with this Circle, shall cut the outermost Circle in every fifteenth degree, and such distance ought each hour have from other upon the Plane.

5. *For Declining inclining Planes.*

Let a Plane decline from the South Westward 24 *degrees* 20 *min.* and incline to the Horizon Northward 36 *degrees*, such a Plane is repesented in the Diagram of the seventh Chapter of the third Book of this Treatise, by the Circle B M D. Now a Ruler laid upon the Pole of this Plane, (which is in the Line Q H, 90 *degrees* distant from M) and the intersections of the hour Circles with the Plane, shall cut the primitive or horizontal Circle in the degrees of distance that the respective hour Lines of such a declining inclining Plane ought to have upon the Dial Plane. ☜

☞ Sect. 5.

Of Projecting of the Sphere upon Oblique Circles.

IN the four first Sections of this Chapter, Mr *Gunter* hath shewed how to Project the Sphere in *Plano* upon the principal Great Circles of the Sphere, *viz.* Twice upon the Plane of the Meridian, once upon the Tropick of ♑, or the Equinoctial, (parallel thereto) and lastly upon the Horizon.

To

To these Projections I think it will not be impertinent (but very beneficial and satisfactory to the Reader) to shew how the Sphere may be Projected in *Plano* upon any Oblique Circle, as upon any Plane whatsoever and howsoever situate, for all or most Diall Planes are Oblique Circles, and are Horizons in some part of the World or other. As for Example, A Dial Plane declining from the South Westward 24 *degrees* 20 *min.* and inclining Northward 36 *degrees* (such is the Diall Plane in the tenth Chapter of the third Book of this Treatise of Dialling) will in some place or other be an Horizontal Plane: And by projecting of the Circles of the Sphere in their true positions upon this Oblique Plane, you shall not only discover in what Longitude and Latitude this will be an Horizon, but will also delineate out unto you the places of the Hour-lines proper for this declining inclining Plane, in a quite different manner and form than that which Mr *Gunter* hath shewed how to make the Diall in the forementioned tenth Chapter of the third Book, by drawing the Plane upon the Horizontal Projection for this Latitude. And seeing the difference of the two ways of working are so various, and the variety that will appear in the placing of the Circles of the Sphere in their true positions upon such an Oblique Plane cannot but be both beneficial and delightful, I shall here insert the manner how the same may be effected, not only upon this, but upon any other Oblique Plane whatsoever.

To proceed then, Let the Circle H X O D, represent a Dial Plane declining from the South Westward 24 *degrees* 20 *min.* and inclining Northward 36 *degrees.*

1. Draw the Diameter H O, and cross it at right Angles with the Line C F meeting in the Center Q.

2. Take the half Tangent of 36 *degrees*, the Planes inclination, and set it from Q to Z, so shall Z be the Zenith of the Place.

3. Take the half Tangent of 54 *degrees*, the Complement of the Planes inclination, and set that from Q to B, so shall B be the Point through which the Horizon of the place must pass.

4. Take the Tangent of 36 *degrees* the Plains Inclination, and set it from Q to C. Or take the Secant of 36 *degrees*, and set it from B to C, so shall C be the Center of the Horizon H B O.

5. Take the Tangent of 54 *degrees*, the Complement of the Inclination, and set it from Q to F.

6. Take 24 *degrees* 20 *min.* out of your Line of Chords and set that distance from H to c, from D to d, and from O to c.

7. A Ruler laid from Z to c, d, and c, will give the Points W, S and E, for the Weſt South and Eaſt Points thereof.

8. Draw F G perpendicular to Q F, or parallel to H O, and extend it as far as you ſhall ſee requiſite.

9. Draw a Line through the Points E and W extending it till it croſs the Line F G laſt drawn, at G, ſo ſhall G be the Center of the Meridian of the place repreſented by P Z S.

10. Lay a Ruler from W to Z and it will cut the Circle in *a*, from which Point *a* ſet 38 *degrees* 30 *min*. the Co-latitude to *b*, and a Ruler laid from W to *b* will give the Point P in the Meridian for the Pole of the World.

11. Set 90 *degrees* of your Chords from *b* to *f*, and from *f* to *g*. A Ruler laid from W to *f* gives Æ in the Meridian Circle, for the Equinoctial Point, and from W to *g* gives M for the South Pole, and a right Line drawn through P Q and M ſhall be the Axis of the World.

12. Through the Points W Æ E draw the Equinoctial Circle, to find the Center whereof,

13. Divide W E into two equal parts in R, and raiſe the Perpendicular R T, drawing it forth till it meet with Q P being extended, here repreſented by the two Lines R T and Q V, whoſe meeting ſhall be the Center of the Equinoctial, which Q T extended would be equal to the Secant of the height of the Pole above the Plane. Or if from T you draw a Line through C it will interſect Q V in the Center of the Equinoctial alſo.

14. Divide M P in two equall parts in D, and draw D G at right Angles to P M, and extend D G infinitely.

15. Upon P, at the diſtance P D (or any other) deſcribe the Semicircle L D N, and laying a Ruler from P to G the Center of the Meridian, it will cut the Semicircle L D N in L, at which Point L begin to divide L D N into twelve equal parts, and a Ruler laid from P through each of thoſe equal parts ſhall give the Tangents of 15, 30, 45, *&c.* upon the Tangent G D and ſhall be the Centers of the ſeveral Meridians, G being the Center of twelve a clock, or the Meridian of the Place.

15. From the Center Q through the Points where the ſeveral Meridians do cut the Primitive Circle draw right Lines, and thoſe Lines ſhall be the true hour Lines of a South Plane declining Weſtward 24 *degrees* 20 *min.* and inclining Northward 36 *degrees* in the Latitude of 51 *degrees* 30 *min.*

And

A brief Synopsis of this Oblique Projection.

HXOD, the declining inclining Plain.

QZ = ½ Tang. 36 *d.* = Plains inclination.

QB = ½ Tang. 54 *d.* = Co-Plains inclination.

QC = Tang. 36 *d* Or BC = Secant. of 36 *d* = Plains inclin. and is the Center of the Horizon.

QF = Tan. 54 *d.* = Co-Plains inc.

Hc = Dd = Oe = 24 *d.* 20 *m.* = to the Plains Declination.

A Ruler laid from Z to c, d, e, will give W, S and E in the Horizon: GF ⊥ to QF or ∥ to HO.

EW extended, gives G the Center of the Meridian.

A Ruler laid from W to Z gives a.

ab = Chord 38 *d.* 30 *m.* = Co-Lat.

Wb gives P for the Pole of the world

bf = fg = Chord 90 *deg.*

A Ruler laid from W to f gives Æ the Equinoct. Wg, gives M the South Pole, and PQM is the Axis of the World.

Through WÆE, draw the Equinoctial Circle.

WR = RE = ½ Tang. WE.

RT ⊥ to WE. Or, draw RT through the Center of the Horizon, and its intersection with QV will be the Center of the Equinoctial.

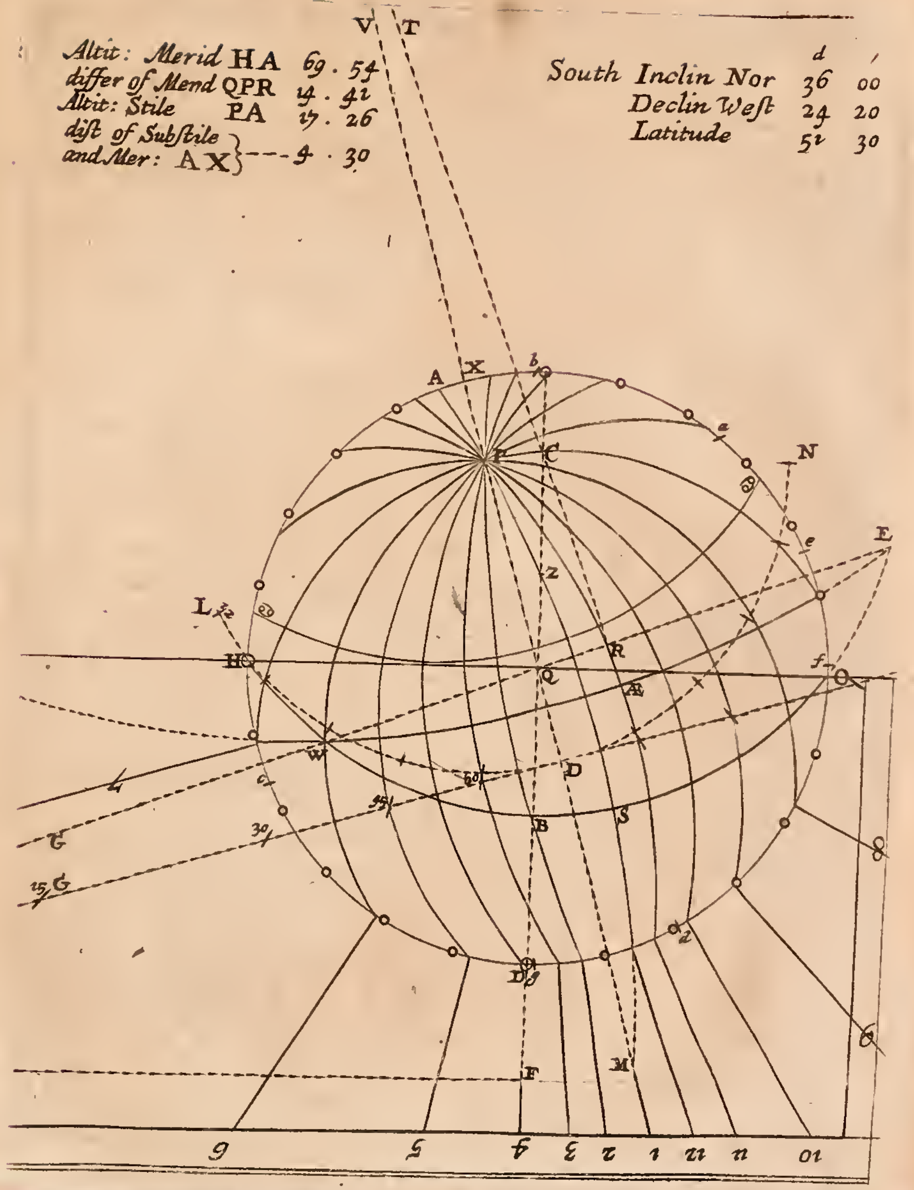
QV and RT extended, and intersecting, gives the Center of the Equinoctial = Sec. of the Poles height above the Plain.

Divide MP in 2 = parts in D and draw DG infinitely.

Upon P, at any distance, describe the Semicircle LDN, lay a Ruler from P to G, it will cut LDN in L, at L begin to divide LDN into 12 = parts. A Ruler laid from P to those = parts, will give the Tangents of 15, 30, 45, *&c.* upon GD, and be the Centers of the Meridians, G being the Center of 12.

Lines drawn from Q through the intersections of the Meridians with the Primitive Circle (representing the declining inclining Plain) shall be the Hour-Lines.

Place this *Synopsis* against Page 74 of the Sector, so that it, together with the Scheme, may lie out when the Book is shut.



24 *degrees* 10 *min.* and reaching Northward 30 *degrees* in the Latitude of 51 *degrees* 30 *min.*

And

And from this Scheme may further be found that,

	degr.	min.
1. The Elevation of the Meridian H A is	69	54
2. The difference of Meridians Q P Æ is	14	41
3. The height of the Stile P A is	17	26
4. The diſtance of the Subſtile and Meridian A X is	4	30

Note, In like manner might be inſerted in this Projection, the Tropicks and other Parallels of the Suns place or Declination; The Azimuths, Almicanthar's, the Ecliptick, and other (either ſmall or great) Circles; as is inſtanced in the Scheme by the Tropick of Cancer, which is thereon deſcribed; but of this Oblique Projection I have ſaid enough in this place. ☞

CHAP. IV.

Of the Resolution of right-lined Triangles.

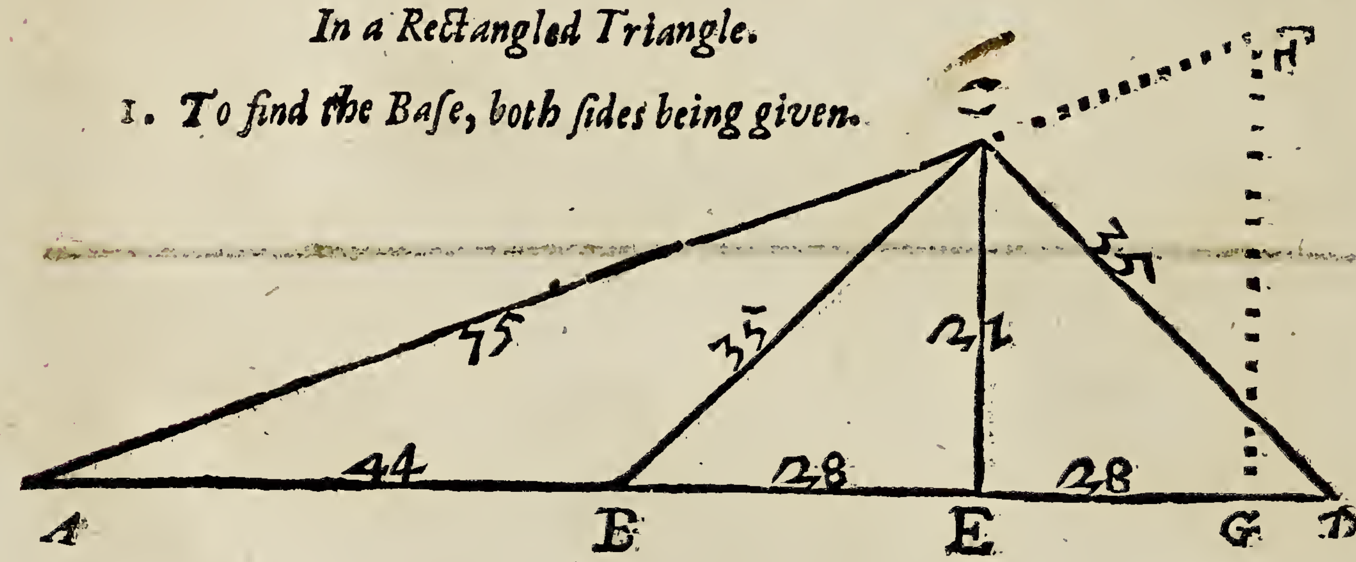
IN all Triangles there being six parts, *viz.* three Angles, and three sides, any three of them being given, the rest may be found by the *Sector*.

As may appear by the *Prop.* following, wherein for our practise we may use these Triangles CEA, CEB, CED, all which are Rectangled in E. And AGF Rectangled in G. All the rest consist of Oblique Angles.

Ang.	*Gr.*	*M.*	*S.*	*Lin.*	*Parts.*
E	90	0	0	*A C*	75
G	90	0	0	*A F*	100
A	16	15	36	*F G*	28
D	36	52	12	*C E*	21
B	36	52	12	*C D*	35
B	143	7	48	*B B*	35
A F G	73	44	12	*A G*	96
A C E	73	44	12	*A E*	72
A C B	20	36	36	*A B*	44
B C E	53	7	48	*B D*	28
E C D	53	7	48	*A D*	28
B C D	106	15	36	*B E*	56
A C D	126	52	12	*E D*	100

In a Rectangled Triangle.

1. *To find the Base, both sides being given.*



Let the *Sector* be opened in the Line of *Lines* to a right Angle (as before was shewed *Cap.* 2, *Prop.* 7.) then take out the sides of the Triangle, and lay them, one on one *Line*, the other on the other *Line*, so as they meet in the Center, and mark how far they extend. For the *Line* taken

taken from the terms of their extension, shall be the Base required, *viz.* the side opposite to the right Angle.

Or add the squares of the two sides (as in *Prop.* 4. Superfic.) and the side of the compound Square shall be the * Base.

* *Note that I call the longest side of the Triangle the Base.*

As if the Lines A E, C E, should be the sides about the right Angle, and it were required to find the Base subtending the right Angle.

First, I set the Line of *Lines* to a right Angle by applying the whole Line of 10 from 6 in the one Line, to 8 in the other. Then if the greater of the two Lines given be less than the Line of *Lines*, I take the greater of them A E, and transfer it with the Compasses into one of the Lines of *Lines*, and find, that, in my *Sector* (which is 14 Inches long, and so, the Line of *Lines*, almost 7 Inches) it reacheth from the Center to 518.

Again, I take the lesser Line C E, and transfer it into the other Line of *Lines*, and find, that it reacheth from the Center unto 151, wherefore I take the distance from 151 unto 518, and such is the length of the Base A C required.

If either of the Lines given be too large for the *Sector*, then I may measure them by Feet or Inches, as suppose I find the length of A E to be about 720, and of C E 210. Then in the Line of *Lines* (being set, one Perpendicular to the other, as before) I extend the Compasses from 210 unto 720; and measuring this extent in the Line of *Lines*, find it to be 750 parts, wherefore I prick down 750 parts in the Line A C, from the same Scale by which I measured A E, and C E. So, this Line A C shall be the Base required.

In working by the Line of *Superficies*, I need no opening of the *Sector*. For, taking the Line C E with my Compasses, and measuring it in the Line of *Superficies* upon my *Sector*, I find it near 13 parts.

Then taking the Line A E, I find it to be about 269. These two being added together make 292: and this extent is the length of the Base A C required.

2. To find the Base, by having the Angles, and one of the sides given.

Take the side given, and turn it into the parallel Sine of his opposite Angle; so the parallel Radius shall be the Base.

As if the Line A E were the side of a Rectangle Triangle opposite

opposite to an Angle of 73 *gr.* 45, and it were required to find the Base.

First, I take the side A E with my Compasses, and set it over in the Sines of 73 *gr.* 45. So, the Parallel Radius taken from between 90 and 90, will give the Base A C required.

If the side given be such as cannot well be fitted over in the Sines of his opposite Angle, I may measure it by feet or inches, and suppose I find the length of A E to be 720, then would I take 720 parts out of the Line of Lines, and make it a Parallel Sine of 73 *gr.* 45. So, the Parallel Radius taken from between 90 and 90, and measured in the Line of *Lines* will be found to be about 750 parts: Wherefore, I prick down 750 in the Line A C, by the same Scale, whereby I measured A E: and this Line A C shall be the Base required.

3. To find a side by having the Base and the other side given.

Let the Sector be opened in the Lines of *Lines* to a right Angle, and the side given laid on one of those Lines from the Center: then take the Base with a pair of Compasses, and setting one foot in the term of the given side, turn the other to the other Line of the Sector, and it shall there shew the side required.

Or take the Square of the side out of the Square of the base (as in *Prop. 4. Superfic.*) and the side of the remaining Square shall be the side required.

Thus having A C for the Base, and C E, for the side of a Rectangle Triangle, the other side will be found to be A E.

Or, if A C, being measured, be 750, and C E, 210, the other side A E will be found to be 720.

To find a side having the Base, and the Angles given.

Take the Base given, and make it a Parallel Radius, so the parallel Sines of the Angles, shall be the opposite sides required.

Thus in the Rectangle A E C, if A C be made a Parallel Radius, the Parallel Sine of 73 *gr.* 45, will give the side A E; and the Parallel Sine of 16 *gr.* 15. will give the other side C E.

5. To

5. *To find a side by having the other side and the Angles given.*

Take the side given, and turn it into his Parallel Sine of his opposite Angle: so the Parallel Sine of the Complement shall be the side required.

Thus in the Rectangle D E C, if C E be made a Parallel Sine of 53*gr.* 8 *m.* the parallel Sine of 36*gr.* 52 *m.* will give the side E D. and the Parallel Sine of 90*gr.* will give the Base C D.

6. *To find the Angles by having the Base and one of the sides given.*

First, take out the Base given, and laying it on both sides of the Sector, so as they may meet in the Center, and mark how far it extendeth. Then take out the lateral Radius, and to it open the Sector in terms of the Base. This done, take out the side given, and place it also on the same Lines of the Sector from the Center. For the Parallel taken in the terms of this side, shall be the Sine of his opposite Angle.

Or take the base given, and make it a Parallel Radius; then take the side given, and carry it parallel to the Base, till it stay in like Sines: so they shall give the quantity of the opposite Angle.

Thus in the Rectangle A E C, having the Base A C, and the side A E, you may find the Angle C A E, to be 16*gr.* 15 *m.*

7. *To find the Angles by having both the sides given.*

Take out the greater side, and lay it on both sides of the Sector, so as they meet in the Center, and mark how far it extendeth. Then take the other side, and to it open the Sector in the terms of the greater side; so the Parallel Radius shall be the Tangent of the lesser Angle. The third Angle is always known by the Complement.

Thus in the Rectangle D E C, having the sides C E, and E D, you may find the lesser Angle E C D to be 36 *gr.* 52 *m.* and therefore the other Angle E D C to be 53 *d.* 8 *m.*

8. *The Radius being given, to find the Tangent and Secant of any Ark.*

9. The Tangent of any Ark being given, to find the Secant thereof, and the Radius.

10. The Secant of any Ark being given, to find the Tangent thereof, and the Radius.

The Tangent, and the Secant, together with the Radius of every Ark, do make a right Angle Triangle; whose sides are the Radius and Tangent, and the Base always the Secant; and the Angles always known by reason of the given Arks. As in the Rectangle A E C, if on the Center A, and Semidiameter A E, you describe a Circle, then make A E, to be the Radius, and E C, a Tangent of *16.* 15 and A C a Secant of 16 *gr.* 15 *m.*

If you describe a Circle on the Center C, and Semidiameter C E, then is C E the Radius and E A, a Tangent of 73 *gr.* 45 *m.* and C A a Secant of 73. 45.

Wherefore the Solution is the same with those before.

In any right-lined Triangle whatsoever,

11. To find a side by knowing the other two sides, and the Angle contained by them.

Let the *Sector* be opened in the Lines of *Lines* to the Angle given as I shewed before, *Cap.* 2. *Prop.* 7. Then take out the sides of the Triangle, and laying them the one on the one Line, the other on the other, so as they meet in the Center, mark how far they extend. For the Line taken between the terms of their Extension, shall be the third side required.

As if A C and A D were two sides of a right lined Triangle containing an Angle of 16 *gr.* 16 *m.* and it were required, to find the third side subtending this Angle.

First, I set the Lines to an Angle of 16. 16 *m.* by applying the Sine of 8 *gr.* 8 *m.* over in the Points of 50 and 50, in the Line of Lines. That done, I take the longer Line A D, and transfer it with my Compasses into one of the Lines of *Lines*, and find it to reach from the Center to 720.

Again, I take the lesser Line A C, and transfer it into the other Line of *Lines*, where it reacheth from the Center to 540, wherefore, I take the distance from 540 to 720, and such is the length of the third side C D required.

Or (if the Lines be given in measure) A D 100, and A C 75. I extend the Compasses from 100 to 75, and measuring this extent in the Line of Lines, find to be 35. Whereupon I take 35 parts out of the Scale, by which A C, and A D were measured, and prick them down in the Line C D. So, this Line C D shall be the third side required.

12. *To find a side by having the other two sides, and one of the adjacent Angles, so it be known which of the other Angles is Acute or Oblique.*

Let the *Sector* be opened in the Line of *Lines* to the Angle given, and the adjacent side laid on one of those Lines from the Center; then take the other side with a pair of Compasses, and setting one foot in the term of the former given side, turn the other to the other Line of the *Sector* which here representeth the side required, and it shall cross it in two places; but with which of them is the term of the side required, must be judged by the Angle.

As if in the Triangle following, the side A C being given, and the side C D and the Angle C A D 16 *gr.* 16 *m.* it were required to find the side A D.

First, I open the Sector in the Line of *Lines* to an Angle of 16 *gr.* 16 *m.* and laying the adjacent side from the Center A, find where it extendeth in C. Then I take the other side C D with the Compasses, and setting one foot in C, and turning the other to the other Line of the *Sector*, I find that it doth cross it both in B and D.

Or, (if the Lines be given in measure) A C 75, and C D 35; I may take 35 out of the Line of *Lines*, and setting one foot in 75, I shall find the other foot to cross the other Line of the *Sector*, both at 44 (answerable to A B) and at 100 (answerable to A D.)

So that it is uncertain whether the side required be A B or A D, only it may be judged by the Angle. For if the inward Angle where they cross be Obtuse, the side required is the lesser; if it be Acute, it is the greater.

13. *To find a side by having the Angles, and one of the other sides given.*

Take the side given, and turn it into the Parallel Sine of his opposite Angle; so the Parallel Sines of the other Angle shall be the opposite sides required.

As if in the Triangle A B C, having the side A D, and knowing the Angle C A B to be 16 *gr.* 16 *m.* and the Angle A B C to be 143 *deg.* 8 *m.* it were required to find the other two sides, A C, and B C.

The three Angles of a right-lined Triangle, are always equal to 180 *gr.* wherefore I add 16 *gr.* 16 *m.* unto 143 *gr.* 8 *m.* and by the remainder to 180 *gr.* find the third Angle A C B opposite to the known side A B, to be 20 *gr.* 36 *m.* Then I take the side A B, and make it a Parallel Sine of 20 *gr.* 36 *m.*

So, his Parallel Sine of 16 *degr.* 16 *m.* will be the the side B C; and the Parallel Sine of 143 *degr.* 8 *m.* will be the side A C.

Or if measuring the side A B, I find it to be 44; I may take 44 parts, either out of the Line of Lines, or o t of any other Scale of equal parts, and make it a Parallel Sine of 20 *gr.* 36 *m.* So his Parallel Sine of 16 *gr.* 16 *m.* measured in the same Scale, will give 35 for the length of the side B C: and the Parallel Sine of 36 *gr.* 52 *m.* will give 75, for the length of the other side A C.

When the Angle comes to be above 90 *gr.* the Sine of 80 *gr.* doth stand for a Sine of 100 *gr.* and the Sine of 70 *gr.* for a Sine of 110 *gr.* and so the rest; for those, which are their Complements to 180 *gr.*

14. *To find the proportion of the sides by having the three Angles.*

Take the lateral Sines of the Angles, and measure them in the Line of Lines. For the numbers belonging to those Lines do give the proportion of the sides. pa 76

Thus, in the two equi-angle Triangles A E C, A G F, if you take the lateral Sine of 90 *gr.* for the right Angle at E and G, and measure it in the Line of *Lines*, you shall find it to be 100. Then take the lateral Sine of 16 *gr.* 16 *m.* for the common Angle at A, you shall find it to be 28. Take the lateral Sine of 73 *gr.* 44 *m.* For the third Angle at C and F, you shall find it to be 96. Such therefore is the proportion of the sides.

75

As 100 : 96. 28 :: So are 75 : 72. 21.

15. *To find an Angle, by knowing the Three sides.*

Let the two containing sides be laid on the Lines of the *Sector*, from the Center, one on one Line, and the other on the other; and let the third side, which is opposite to the Angle required, be fitted over in their terms, so shall the *Sector* be opened in those Lines to the quantity of the Angle required.

The quantity of this Angle is found as in *Cap.2. Prop* 8. Thus having the three ſides of the Triangle A C D, to find the Angle at A, I take the two containing ſides A D, A C, and transfer them with my Compaſſes into the Line of *Lines:* where I find the one to reach from the Center, to 72; the other, to 54.

Then I take C D, (the ſide oppoſite to the Angle at A) and ſet that over between 72 and 54.

Or if the three ſides be given in meaſure A D 103; A C 75; C D 35: I might take 35 for the ſide C D out of the Line of *Lines*, and ſet that over from 100 to 75. This done I take the diſtance between 50 and 50 and meaſuring it in the Line of *Sines* I find it to be about 8 *gr.* 8 *m.* the double whereof is 16 *gr.* 16 the Angle required.

16. *To find an Angle, by having two ſides, and one adjacent Angle.*

Firſt take out the ſide oppoſite to the Angle given, and laying it on both ſides of the Sector, ſo as they meet in the Center, mark how far it extendeth; then take out the lateral Sine of the Angle, and to it open the Sector in the terms of the firſt ſide: this done take out the other ſide given, and place it alſo on the ſame Lines of the Sector from the Center, for the Parallels taken in the terms of this ſide; ſhall be the Sine of the Angle oppoſite to the ſecond ſide.

Or take out the ſide oppoſite to the Angle given, and make it a Parallel Sine of that Angle: then take the other ſide given and carry it Parallel to the former, till it ſtay in like Sines: ſo they ſhall give the quantity of the Angle oppoſite to the ſecond ſide.

Thus in the Triangle A C D, knowing two ſides A C, C D, with the Angle C A D oppoſite to the ſide C D, you may find the Angle A D C oppoſite to the other known ſide A C, to be about 36 *gr.* 52 *m.*

17. *To find an Angle by having two ſides, and the Angle contained by them.*

Firſt find the third ſide by the 11 *Prop.* and then the Angles may be found by the 15 or 16 *Prop.*

For obſervation of Angles, the *Sector* may have ſights ſet on the movable foot: ſo that by looking through them, the edges of the Sector may be applied to the ſides of the Angle.

For meaſuring of the ſide of leſſer Triangles, any Scale may ſuffice, either of feet, or inches, or leſſer parts. But for greater Triangles, eſpecially for plotting of grounds. I hold it fit to uſe a chain of four Perches in length, each Perch divided into 25, and the whole Chain an hundred Links, wherein, if the whole Chain be (according to 16 $\frac{1}{2}$ foot in a Perch) 66 foot, (that is 792 inches) each ſeveral Link will be ſeven inches and $\frac{92}{100}$.

If (according to 18 in the Perch) the whole Chain be 72 feet in length (that is, 864 inches) then each ſeveral Link will be eight inches and $\frac{64}{100}$.

For ſo the length being multiplied into the breadth, the five laſt Figures give the content in Roods and Perches by this Table; the other Figures towards the left hand do ſhew the number of Acres directly.

As in a long Square, where the length is 24 Chains $\frac{1}{2}$ the breadth 13 Chains $\frac{1}{2}$ the uſual way is, to reſolve the Chains into Perches: So the length is 97 Perches, and the breadth 54 Perches. Theſe multiplied one into the other make 5238 ſquare Perches, and thoſe (divided by 160) give 32 Acres, 2 Roods, and 38 Perches for the content required.

But, reckoning by Chains and Links, the length is 24 Chains 25 Links, the breadth 13 Chains 50 Links. Theſe multiplied one into the other make 32.73750 ſquare Links. Then, cutting off the five laſt Figures, I find 32 Acres 73750 *lin.* ſuch as an 100000 do make an Acre, Of which 70000 are equal to two Roods 32 Perches: and the reſt 3750 equal to 6 Perches more (as appeareth by this Table.) So, the whole content is 32 Acres, 2 Roods, 38 Perches, as before.

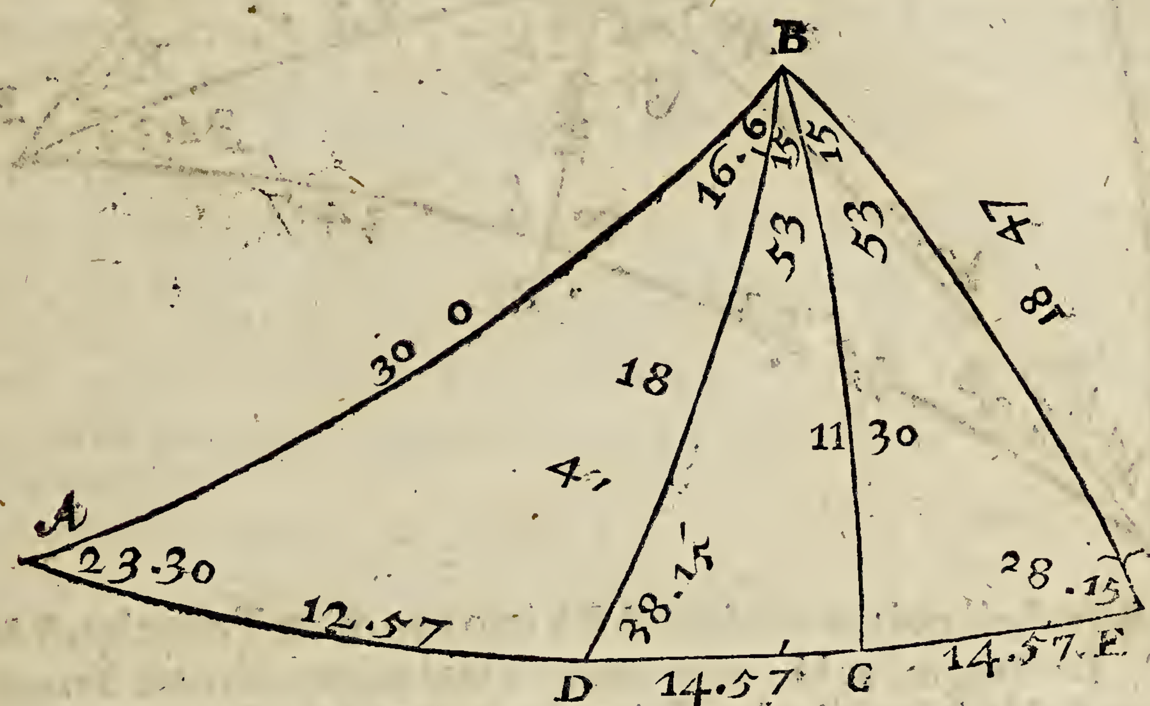
Link.	*R.*	*P.*
100000	4	0
90000	3	24
80000	3	8
70000	2	32
60000	2	16
50000	2	0
40000	1	24
30000	1	8
20000	0	32
10000	0	16
9375	0	15
8750	0	14
8125	0	13
7500	0	12
6875	0	11
6250	0	10
5625	0	9
5000	0	8
4375	0	7
3750	0	6
3125	0	5
2500	0	4
1875	0	3
1250	0	2
624	0	1

CHAP.

CHAP. V.

Of the Resolution of Spherical Triangles.

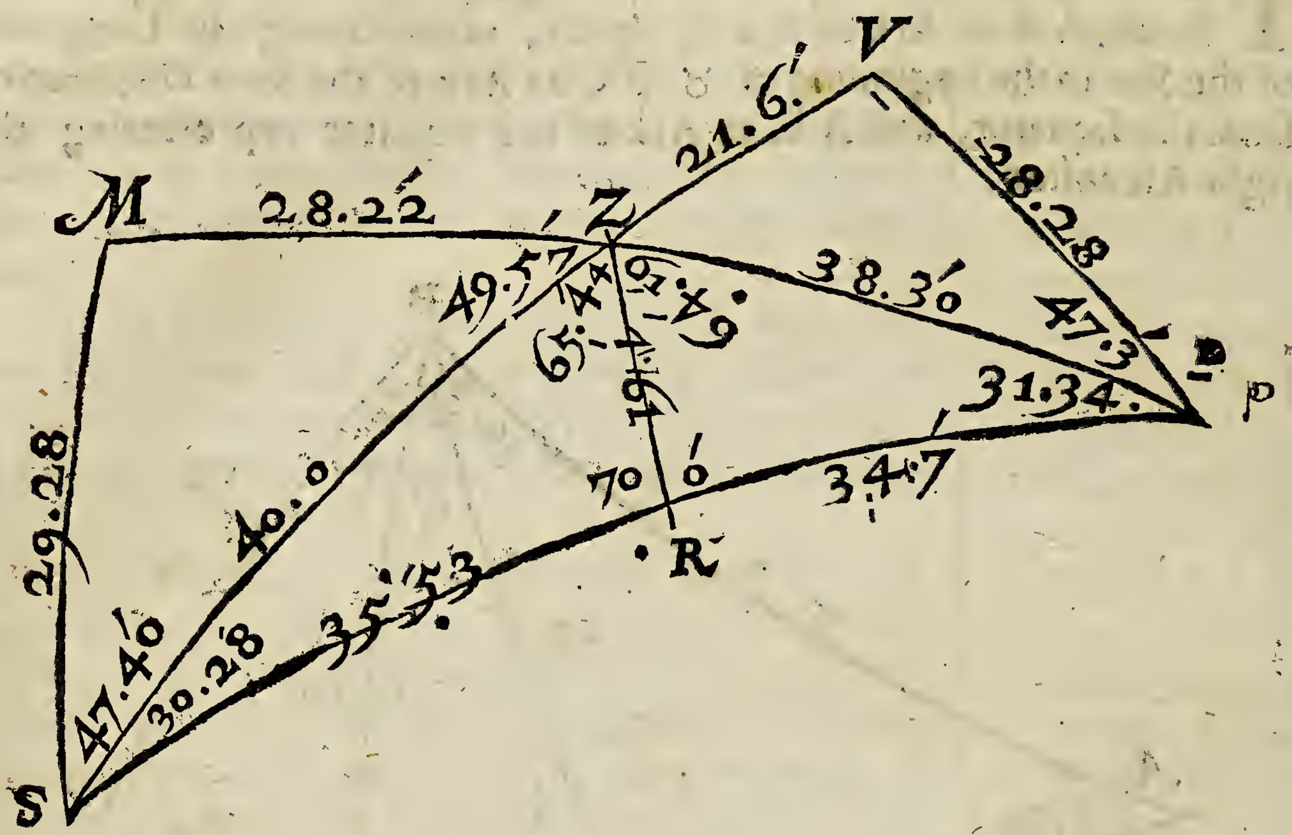
FOr our practise in Spherical Triangles, let A be the Equinoctial Point, A B an Ark of the Ecliptick, representing the Longitude of the Sun in the beginning of ♉, B C an Ark of the Suns Declination from the Equator, and A C an Ark of the Equator representing the right Ascension.



Let B D be an Ark of the Horizon, representing the Amplitude of the Suns rising from the East, and B E an Ark of the Horizon for his setting from the West: so D C shall be the difference of Ascension, and C E the difference of Descension; A D the Oblique Ascension, and A E the Oblique Descension of the same place of the Sun in our Latitude at *Oxford* of 51 *gr.* 45 *m.* whose Complement 38 *gr.* 15 *m.* is the Angle at E and D. The Triangles A C B, D C B, E C B, are Rectangled in C: the other A D B, A E B, consist every way of Oblique Angles.

Or, to fit an Example nearer to the Latitude of *London*. Let Z P S represent the Zenith, Pole, and Sun, Z P being 38 *gr.* 30 *m.* the Complement

ment of the Latitude, P S 70 *gr.* the Complement of the Declination, and Z S 40 *gr.* the Complement of the Suns Altitude. The Angle at Z ſhall ſhew the Azimuth, and the Angle at P, the Hour of the Day from the Meridian. Then if from Z to P S we let down a Perpendicular Z R, we ſhall reduce the Oblique Triangle into two Rectangle Triangles Z R P, Z R S. Or if from S to Z P we let down a Perpendicu-



lar S M, we ſhall reduce the ſame Z P S into two other Triangles, S M Z, S M P, Rectangled at M: whatſoever is ſaid of any of theſe Triangles, the ſame holdeth for all other Triangles in the like caſes.

For the Reſolution of each of theſe, there be ſeveral ways. I only chuſe thoſe which are fitteſt for the *Sector*, wherein if that be remembred which before is ſhewed in the general uſe of the *Sector* concerning Lateral and Parallel entrance, it may ſuffice only to ſet down the Propoſition of the three parts given, to the fourth required, and ſo ſhew firſt by the *Sines* alone.

In

In a Rectangled Triangle.

1. *To find a side, by knowing the Base, and the Angle opposite to the required side.*

As the Radius
is to the Sine of the Base:
So the Sine of the opposite Angle
to the Sine of the side required.

* As in the Rectangle A C B, having the Base A B, the place of the Sun 30 *gr.* from the Equinoctial Point, and the Angle B A C of 23 *gr.* 30 *m.* the greatest Declination, if it were required to find the side B C the Declination of the Sun.

**In Rectangled Triangles, the side opposite to the Rectangle is called the Base.*

Take either the lateral Sine of 20 *gr.* 30 *m.* and make it a Parallel Radius; so the Parallel Sine of 30 *gr.* taken and measured in the side of the *Sector*, shall give the side required 11 *gr.* 30 *m.* Or take the Sine of 30 *gr.* and make it a Parallel Radius; so the Parallel Sine of 23 *gr.* 30 *m.* taken and measured in the lateral Sines, shall be 11 *gr.* 30 *m.* as before.

So in the Triangle Z P S, having Z P 38 *gr.* 30 *m.* and the Angle P 31 *gr.* 34 *m.* given, we shall find the Perpendicular Z R to be 19 *gr.* 1 *m.* or having P S 70 *gr.* and the said Angle P 31 *gr.* 34 *m.* given, we may find the Perpendicular S M to be 29 *gr.* 28 *m.*

2. *To find the side by knowing the Base and the other side.*

As the Sine of the Complement of the side given
is to the Radius:
So is the Sine of the Complement of the Base
to the Sine of the Complement of the side required.

So in the Rectangle A C B, having A B 30 *gr.* and B C 11 *gr.* 30 *m.* given, the side A C will be found 27 *gr.* 54 *m.*

Or in the Rectangle Z R P, having Z P 38 *gr.* 30 *m.* and Z R 29 *gr.* 1 *m.* given, the side R P will be found 34 *gr.* 7 *m.*

3. To

3. To find a side, by knowing the two Oblique Angles.

As the Sine of either Angle
to the Sine of the Complement of the other Angle ;
So is the Radius
to the Sine of the Complement of the side opposite to the second Angle.

So in the Rectangle A C B having C A B for the first Angle 23 *gr.* 30 *m.* and A C B, for the second 69 *gr.* 22 *m.* the side A C will be found 27 *gr.* 54 *m.* Or making A B C the first Angle, and C A B the second, the side B C will be found 11 *gr.* 30 *m.*

4. To find the Base, by knowing both the sides.

As the Radius
to the Sine of the Complement of the one side :
So the Sine of the Complement of the other side,
to the Sine of the Complement of the Base required.

So in the Rectangle A C B having A C 27 *gr.* 54 *m.* and B C 11 *gr.* 30 *m.* the Base A B will be found 30 *gr.*

5. To find the Base by knowing the one side, and the Angle opposite to that side.

As the Sine of the Angle given,
to the Sine of the side given :
So is the Radius
to the Sine of the Base required.

So in the Rectangle B C D, knowing the Latitude and the Declination, we may find the Amplitude ; as having B C the side of the Declination 11 *gr.* 30 *m.* and B D C the Angle of the Complement of the Latitude 38 *gr.* 15 *m.* the Base B D, which is the Amplitude, will be found to be 18 *gr.* 47 *m.*

6. *To*

6. *To find an Angle, by the other Oblique Angle, and the side opposite to the inquired Angle.*

As the Radius,
to the Sine of the Complement of the side:
So the Sine of the Angle given,
to the Sine of the Complement of the Angle required.

So in the Rectangle A C B, having the Angle B A C 23 *gr:* 30 *m.* and the side A C 27 *gr.* 54 *m.* the Angle A B C will be found 69 *gr.* 21 *m.*

7. *To find an Angle, by the other Oblique Angle, and the side opposite to the Angle given.*

As the Sine of the Complement of the side
to the Sine of the Complement of the Angle given:
So is the Radius,
to the Sine of the Angle required.

So in the Rectangle A C B, having B A C 23 *gr.* 30 *m.* and B C 11 *gr.* 30 *m.* the Angle A B C will be found 69 *gr.* 21 *m.*

8. *To find an Angle, by the Base, and the side opposite to the inquired Angle.*

As the Sine of the Base
is to the Radius:
So the Sine of the side
to the Sine of the Angle required.

So in the Rectangle B C D, having B D 18 *gr.* 47 *m.* and B C 11 *gr.* 30 *m.* the Angle B D C will be found 38 *gr.* 15 *m.*

These eight Propositions have been wrought by the Sines alone; those which follow require joynt help of the Tangent.

And forasmuch as the Tangent could not well be extended beyond 63 *gr.* 30 *m.* I shall set down two ways for the resolution of each Proposition; so that if the one will not hold, the other may.

9. *To find a side, by having the other side, and the Angle opposite to the inquired side.*

1. As the Radius
   to the Sine of the side given:
So the Tangent of the Angle,
   to the Tangent of the side required.

2. As the Sine of the side given,
   is to the Radius:
So the Tangent of the Complement of the Angle
   to the Tangent of the Complement of the side required.

So in the Rectangle A C B, having the side A C 27 *gr.* 54 *m.* and the Angle B A C 23 *gr.* 30 *m.* the side B C will be found to be 11 *gr.* 30 *m.*

10. *To find a side, by having the other side, and the Angle next the inquired side.*

As the Tangent of the Angle,
   to the Tangent of the side given:
So is the Radius
   to the Sine of the side required.

2. As the Tangent of the Complement of the side,
   to the Tangent of the Complement of the Angle:
So is the Radius
   to the Sine of the side required.

This and the like, where the Tangent standeth in the first place, are best wrought by Parallel entrance. And so in the Rectangle B C D, having B C the side of Declination 11 *gr.* 30 *m.* and B D C the Angle of the Complement of the Latitude 38 *gr.* 15 *m.* the side D C, which is the Ascensional difference, will be found 14 *gr.* 57 *m.*

By the Ascensional difference is given the time of the Suns rising and setting, and length of the day: allowing an hour for each 15 *gr.* and four minutes of time for each several degree. As in the example, the difference between the Suns Ascension in a right Sphere, which is always

always at six of the clock, and his Ascension in our Latitude being 14 *gr.* 57 *m.* it sheweth that the Sun riseth very near an hour before six, because of the Northern Declination; or after six, if the Sun be declining to the Southward.

11. *To find a side by knowing the Base, and the Angle adjacent next to the inquired side.*

1. As the Radius,
To the Sine of the Complement of the Angle:
So the Tangent of the Base,
to the Tangent of the side required,

2. As the Sine of the Complement of the Angle,
is to the Radius:
So is the Tangent of the Complement of the Base,
to the Tangent of the Complement of the side required.

So in the Rectangle A C B, knowing the place of the Sun from the next Equinoctial Point, and the Angle of his greatest Declination, we may find his right Ascension: *viz.* the Base A B 30 *gr.* and the Angle B A C 23 *gr.* 30 *m.* being given, the right Ascension A C will be found 27 *gr.* 54 *m.*

12. *To find the Base by knowing the Oblique Angles.*

As the Tangent of the one Angle,
to the Tangent of the Complement of the other Angle:
So is the Radius,
to the Sine of the Complement of the Base.

So in the Rectangle A C B, having B A C 23 *gr.* 30 *m.* and A B C 69 *gr.* 21 *m.* the Base A B will be fonnd 30 *gr.*

13. *To find the Base, by one of the sides, and the Angle adjacent next that side.*

1. As the Radius,
is to the Sine of the Complement of the Angle:

So the Tangent of the Complement of the side,
to the Tangent of the Complement of the Base.

2. As the Sine of the Complement of the Angle,
is to the Radius :
So the Tangent of the side given,
to the Tangent of the Base required.

So in the Rectangle A C B, having A C 27 *gr.* 54 *m.* and B A C 23 *gr.* 30 *m.* the Base A B will be found 30 *gr.* 0 *m.*

14. *To find an Angle, by knowing both the sides.*

1. As the Radius,
is to the Sine of the side next the inquired Angle :
So the Tangent of the Complement of the opposite side,
to the Tangent of the Complement of the Angle required.

2. As the Sine of the side next the inquired Angle,
is to the Radius :
So the Tangent of the opposite side,
to the Tangent of the Angle required.

So in the Rectangle A C B, having A C 27 *gr.* 54 *m.* and B C 11 *gr.* 30 *m.* the Angle at A will be found 23 *gr.* 30 *m.* and the Angle at B 69 *gr.* 21 *m.*

15. *To find the Angle, by the Base, and the side adjacent to the required Angle.*

1, As the Tangent of the Complement of the side,
to the Tangent of the Complement of the Base :
So is the Radius,
to the Sine of the Complement of the Angle required.

2. As the Tangent of the Base,
to the Tangent of the side :
So is the Radius,
to the sign of the Complement of the Angle required.

So

So in the Rectangle B C D, having the Base B D 16 *gr.* 47 *m.* and the side B C 11 *gr.* 30 *m.* the Angle D B C between them will be found 53 *gr.* 15 *m.*

16. *To find an Angle, by knowing the other Oblique Angle, and the base.*

1. As the Radius,
 to the Sine of the Complement of the Base:
So the Tangent of the Angle given,
 To the Tangent of the Complement of the Angle required.

2. As the Sine of the Complement of the Base,
 is to the Radius:
So the Tangent of the Complement of the Angle given,
 to the Tangent of the Angle required.

So in the Rectangle A C B, having the Angle at A 23 *gr.* 30 *m.* and the Base A B 30 *gr.* the Angle A B C will be found 69 *gr.* 12 *m.*

These sixteen Cases are all that can fall out in a Rectangle Triangle: those which follow do hold

In any Spherical Triangle whatsoever.

17. *To find a side, opposite to an Angle given, by knowing one side, and two Angles, whereof one is opposite to the given side, the other to the side required.*

As the Sine of the Angle opposite to the side given,
 is to the Sine of the side given:
So the Sine of the Angle opposite to the side required,
 to the Sine of the side required.

So in the Triangle A B E, having the place of the Sun, the Latitude, and the greatest Declination, we may find the Amplitude. As having A B 30 *gr.* B A E 23 *gr.* 30 *m.* and A B E 38 *gr.* 15 *m.* the side B E which is the Amplitude, will be found 18 *gr.* 47 *m.*

18. *To find an Angle opposite to a side given, by having one Angle and two sides, the one opposite to the given Angle, the other to the Angle required.*

As the Sine of the side opposite to the Angle given,
is to the Sine of that Angle given:
So the Sine of the side opposite to the Angle required,
to the Sine of the Angle required.

So in the Triangle Z P S, having the Azimuth, and Latitude, and Declination, we may find the hour of the day. As having P Z S 130 *gr.* 3 *m.* P S 70 *gr.* and Z S 40 *gr.* the Angle Z P S, which sheweth the hour from the Meridian, shall be found 31 *gr.* 34 *m.*

19. *To find an Angle by knowing the three sides.*

This Proposition is most useful, but most difficult of all others: as in Arithmetick, so by the *Sector*, yet may it be performed several ways.

1. According to *Regiomontanus* and others.
As the Sine of the lesser side, next the Angle required,
to the difference of the versed Sines of the Base, and difference of (the sides:
So is the Radius,
to a fourth proportional.

Then as the Sine of the greater side next the Angle required,
is to that fourth proportional:
So is the Radius,
to the versed Sine of the Angle required.

So in the Triangle Z P S, having the side P S, the Complement of the Declination 70 *gr.* 0 *m.* the side Z P the Complement of the Latitude 38 *gr.* 30 *m.* and the Base Z S the Complement of the Altitude 40 *gr.* the Angle of the hour of the day Z P S will be found 31 *gr.* 34 *m.* which is 2 *h.* 6 *m.* from the Meridian.

For the Base being 40 *gr.* 0 *m.* and the difference of the sides 38 *gr.* 30 *m.* and 70 *gr.* 0 *m.* being 31 *gr.* 30 *m.* the difference of their versed Sines

Sines will be the same with the distance between the right Sine of 50 *gr.* and 58 *gr.* 30 *m.* This difference I take out, and make it a Parallel Sine of the lesser side 38 *gr.* 30 *m.* so the Parallel Radius will be the fourth proportional. Then coming to the second operation, I make this fourth proportional a Parallel Sine of the greater side of 70 *gr.* 0 *m.* and take out this Parallel Radius. For this measured from 90 *gr.* toward the Center, will be the versed Sine of 31 *gr.* 34 *m.*

In the like sort in the same Triangle Z P S, having the same Complements given, the Angle P Z S which is the Azimuth from the North part of the Meridian, will be found 130 *gr.* 3 *m.* For here the Base opposite to the Angle required being 70 *gr.* and the difference of the sides 38 *gr.* 30 *m.* and 40 *gr.* being 1 *gr.* 30 *m.* the difference of their versed Sines will be the same with the distance between the right Lines of 20 *gr.* and 88 *gr.* 30 *m.* This difference I take and make it a Parallel Sine of the lesser side 38 *gr.* 30 *m.* so the Parallel Radius will be the fourth proportional. Then coming to the second operation, I make this fourth proportional a Parallel Sine of the greater side 40 *gr.* and take out this Parallel Radius; for this measured from 90 *gr.* beyond the Center, in the Lines of *Sines*, stretched forth at their full length, will be the versed Sine of 130 *gr.* 3 *m.*

2. I may find an Angle by knowing three sides, by that which I have elsewhere demonstrated upon *Barth. Pitiscus*, and that at one operation in this manner.

As the Sine of the greater side,
is to the Secant of the Complement of the other side:
So the differerence of Sines of the Complement of the Base,
and the Ark compounded of the lesser side with the
Complement of the greater,
to the versed Sine of the Angle required.

So in the same Triangle Z P S, having the same Complements given, the Angle at P, which sheweth the hour from the Meridian, will be found as before, 31 *gr.* 34 *m.*

For the sides being 38 *gr.* 30 *m.* and 70 *gr.* 0 *m.* I take the Secant of the Complement of 38 *gr.* 30 *m.* and make it a Parallel Sine of 70 *gr.* then keeping the Sector at this Angle, I consider that the Complement of 70 *gr.* being 20 *gr.* added unto 38 *gr.* 30 *m.* the compounded side (which is here the Meridian Altitude) will be 58 *gr.* 30 *m.*

30 *m.* and that the Base being 40 *gr.* the difference of Sines of the compounded side, and the Complement of the Base will be (as before) the distance between the Sines of 50 *gr.* and 58 *gr.* 30 *m.* Wherefore I take out this difference, and lay it on both the Lines of *Sines* from the Center: so the Parallel taken in the terms of this difference, and measured from 90 *gr.* towards the Center, doth give the versed Sine of 31 *gr*, 34 *m.*

This example of finding the hour of the day might otherwise have been pro posed in these terms,

As the Sine of the Complement of the Declination,
is to the Secant of the Latitude:
So the differerence between the Sine of the Altitude proposed,
and the Sine of the Meridian Altitude,
to the versed Sine of the hour from the Meridian:

Then the Latitude being 51 *gr.* 30 *m.* the Declination 20 *gr.* Northward, and the Altitude 50 *gr.* the work would be the same as before.

The other Angles P Z S, P S Z, may be found in the same sort; but having the sides and one Angle, it will be sooner done by that which we shewed before in the 18 *Prop.*

20. *To find a side, by knowing the three Angles.*

If for the greater Angle, we take his Complement to 180 *gr.* the Angles shall be turned into sides, and the sides into Angles, and the operation shall be the same, as in the former *Prop.*

As in the Triangle Z P S, having the Angle Z P S 31 *gr.* 34 *m.* Z S P 30 *gr.* 28 *m.* and P Z S 130 *gr.* 3 *m.* I would take the greater Angle of 130 *gr.* 3 *m.* out of 180 *gr.* and there remains 49 *gr.* 57 *m.* Then as I had a Triangle of three known sides, one of 31 *gr.* 34 *m.* another of 30 *gr.* 20 *m.* and a third of 49 *gr.* 57 *m.* I would seek the Angle oppoiite to one of these sides, by the last *Prop.* So the Angle which is thus found, would be the side, which is here required.

21. *To find a side, by having the other two sides, and the Angle comprehended.*

This Proposition being the converse of the nineteenth, may be wrought

wrought accordingly: but the best way both for it and those which follow, is to resolve them into two Rectangles, by letting down a Perpendicular, as was shewed in the first *Prop.*

So in the Triangle Z P S, having Z P the Complement of the Latitude, and P S the Complement of the Declination, with Z P S the Angle of the hour from the Meridian, we may find Z S the Complement of the Altitude of the Sun.

For having let down the Perpendicular Z R, by the first *Prop.* we have two Triangles, Z R P, Z R S, both rectangled at R. Then may we find the side P R, either by the second, or tenth, or eleventh *Prop.* which taken out of P S, leaveth the side R S: with this R S and Z R we may find the Base Z S by the fourth *Prop.*

Or having let down the Perpendicular S M, we have two Rectangle Triangles S M Z, S M P. Then may we find M P, from which if we take Z P, there remaineth M Z: but with M Z and S M, we may find the Base Z S.

22. *To find a side, by having the other two sides, and one of the Angles next the inquired side.*

So in the Triangle Z P S, having Z P, the Complement of the Latitude, and P S the Complement of the Declination, with P Z S the Angle of the Azimuth, we may find Z S the Complement of the Altitude of the Sun.

For having Z P, and the Angle at Z, we may to S Z produced, let down a Perpendicular P V. Then we have two Rectangle Triangles P V Z, P V S, wherein if we find the sides V Z, V S, and take the one out of the other, there will remain the side required Z S.

23. *To find a side, by having one side, and the two Angles next the inquired side.*

So in the Triangle A B D, having A B the place of the Sun, and B A D the Angle of the greatest Declination, and A D B the Angle of the Equator with the Horizon, we may find A D the Oblique Ascension.

For having let down B C the Perpendicular of Declination, we have two Rectangled Triangles, A C B, D C B. Then may we find A C the right Ascension, and D C the ascensioral difference; and comparing the one with the other, there remaineth A D.

24 *To find a side, by having two Angles, and the side inclosed by them.*

So in the Triangle Z P S, having the Angles at Z and P, with the side intercepted Z P, we may find the side P S. For having let down the Perpendicular P V, we have two Rectangles P V Z, P V S, Then may we find the Angle V P Z, either by the seventh, or fifteenth, or sixteenth *Prop.* which added to Z P S, maketh the Angle V P S, with this V P S, and P V, we may find the Base P S, according to the 13 *Prop.*

25. *To find an Angle by having the other two Angles and the side inclosed by them.*

So in the Triangle Z P S, having the Angles at Z and P, with the side intercepted Z P, we may find the other Angle Z S P. For having let down the Perpendicular Z R, we have two Rectangles Z R P, Z R S. Then may we find the Angle P Z R by the sixteenth *Prop.* and that compared with P Z S, leaveth the Angle R Z S: with this R Z S, and Z R, we may find the Angle required Z S R, according to the sixth *Proposition.*

26. *To find an Angle, by having the other two Angles, and one of the sides next the inquired Angle.*

So in the Triangle A B D, having the Angles at A and D, with the side A B, we may find the Angle A B D. For having let down the Perpendicular B C, we have two Rectangles, A C B, D C B. Then may we find the Angles A B C, D B C, and take D B C out of A B C, for so there remaineth the Angle required A B D.

27. *To find an Angle, by knowing two sides, and the Angle contained by them.*

So in the Triangle Z P S, having the sides Z P, P S, with the Angle comprehended Z P S, we may find the Angle P Z S. For having let down the Perpendicular S M, we have two Rectangles S M Z, S M P

The

Then may we find the side M P, and taking Z P out of M P, there remaineth M Z: with this M Z and the Perpendicular M S, we may find the Angle M Z S, by the fourteenth *Prop.* This Angle M Z S, taken out of 180 *gr.* there remaineth P Z S.

28. *To find an Angle, by knowing the two sides next it and one of the other Angles.*

So in the Triangle Z P S, having the sides Z P, and P S, with the Angle P Z S, we may find the Angle Z P S; For having let down the Perpendicular P V, we have two Rectangles P V Z, P V S. Then may we find the Angles V P Z, V P S: and taking V P Z out of V P S, there remaineth Z P S, which was required.

These 28 Cases are all that can fall out in any Spherical Triangle: if any do not presently understand them, let them once more read over the use of the Globes, and they shall soon become easie unto them.

CHAP. VI.

Of the Use of the Meridian Line in Navigation.

THe Meridian Line is here set on the side of the Sector stretched forth at full length, on the same Plane with the Line of *Lines* and *Solids*, and is divided unequally toward 87 *gr.* (whereof 70 *gr.* are about one half) in such sort as the Meridian in the Chart of *Mercators* Projection. The Use of it may be,

1. *To divide a Sea-chart according to* Mercators *Projection.*

If a degree of the Equator on the Sea-chart, be equal to the hundred part of the Line of *Lines* in the *Sector*, the degrees of the Meridian upon the *Sector*, shall give the like degrees upon the Sea-chart: if otherwise they be unequal, then may the Meridians of the Sea-chart be divided in such sort as the Line of Meridians is divided on the *Sector*, by that which we shewed before in the 8 *Prop.* of the Line of Lines.

But to avoid error, I have here set down a Table, whereby the Meridian Line may be divided out of the degrees of the Equator supposing each degree in the Equator, to be subdivided into a thousand parts. By which Table, and the usual Table of Sines, Tangents, and Secants, the Proportions following may be also resolved Arithmetically. For the manner of division, let the Equator be drawn, and divided, and crossed with Parallel Meridians, as in the common Sea-chart: then look into the Table, and let the distance between the Equator and 40 *gr.* in the Meridian, from the Equator, be equal to 43 *gr.* 711 *parts* of the Equator, as in the Table: let 50 *gr.* in the Meridian from the Equator, be equal to 57 *gr.* 909 *parts* of the Equator, and so in the rest.

The making of this Table is, by addition of Secants. For the Parallels of Latitudes being less than the Equator or Meridian in such proportion as the Radius is to the Secant of the Parallel. For example, the Parallel of 60 *degrees* of Latitude is less than the Equator (and consequently, each degree of this Parallel of 60 degrees less than a degree of the Equator, or Meridian) in such proportion as 100000 the Radius, hath unto 200000 the Secant of 60 *degrees.*

M	*Gr. Par.*	*M*	*Gr. Par.*	*M*	*Gr. Par.*	*M*	*Gr. Par.*	*M*	*Gr. Par.*
0	0. 0	3	3.001	6	6.011	9	9.037	12	22.088
	.100		3.101		6.111		9.138		12.190
	.200		3.201		6.212		9.239		12.293
	.300		3.301		6.312		9.341		12.395
	.400		3.402		6.413		9.442		12.497
	.500		3.502		6.514		9.543		12.600
	.600		3.602		6.614		9.645		12.702
	.700		3.702		6.715		9.740		12.805
	.800		3.803		6.816		9.848		12.907
	.900		3.903		6.916		9.949		13.010
1	1.000	4	4.003	7	7.017	10	10.051	13	13.112
	1.100		4.103		7.118		10.152		13.215
	1.200		4.204		7.219		10.254		13.318
	1.300		4.304		7.319		10.355		13.421
	1.400		4.404		7.420		10.457		13.523
	1.500		4.504		7.521		10.559		13.626
	1.600		4.605		7.622		10.661		13.729
	1.700		4.705		7.723		10.762		13.832
	1.800		4.805		7.824		10.864		13.935
	1.900		4.906		7.925		10.966		14.038
2	2.000	5	5.006	8	8.026	11	11.068	14	14.141
	2.100		5.106		8.127		11.170		14.244
	2.200		5.207		8.228		11.272		14.347
	2.300		5.307		8.329		11.374		14.450
	2.400		5.408		8.430		11.476		14.553
	2.500		5.508		8.531		11.578		14.656
	2.601		5.609		8.632		11.680		14.760
	2.701		5.709		8.733		11.782		14.863
	2.801		5.810		8.834		11.884		14.967
	2.901		5.910		8.936		11.986		15.070
3	3.001	6	6.011	9	9.037	12	12.088	15	15.174

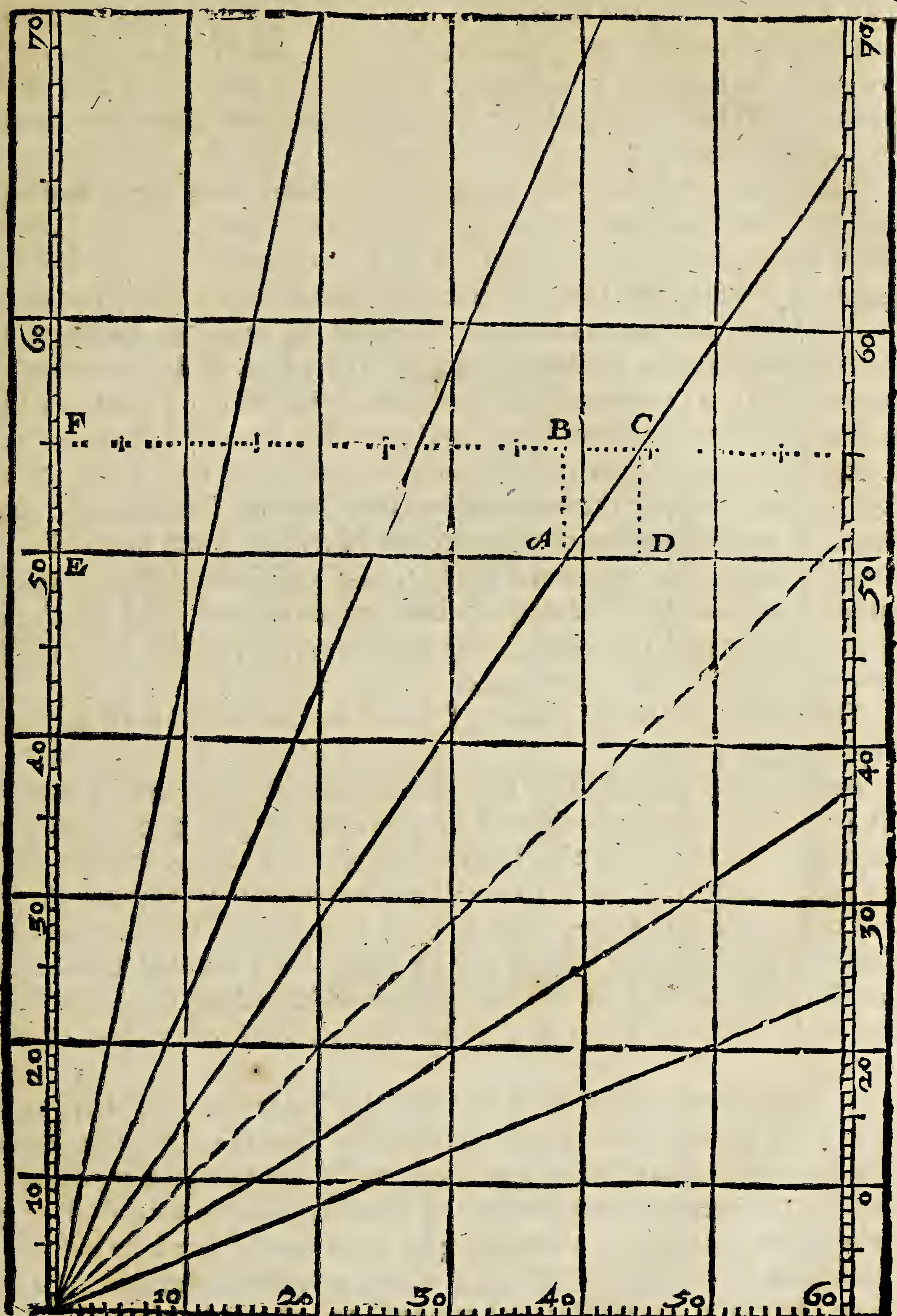
M	Gr. Par.	M	Gr. Par.	M	Gr. Par	M	Gr. Par.	M	Gr. Par.
15	15.174	18	18.303	21	21 486	24	24.734	27	28.058
	15.277		18.408		21.503		24.844		28.171
	15.381		18 513		21.70		24.953		28.283
	15.485		18.619		21.808		25.063		28.396
	15.588		18.724		21.915		25.173		28.508
	15.692		18.830		21.023		25 282		28.621
	15.796		18 935		22.130		25.392		28.734
	15 900		19.041		22.238		25.502		28.847
	16.004		19.146		22.345		25.613		28.959
	16.107		19.251		22.453		25.723		29.072
16	16.211	19	19.356	22	22.561	25	25.833	28	29.186
	16.316		19.463		22.669		25.943		29.299
	16.420		19.569		22.777		26.054		29.313
	16.524		19.675		22.885		26.164		29.526
	16.628		19.781		22.993		26.275		29.640
	16.732		19.887		23.101		26.386		29.753
	16.836		19.693		23.210		26.497		29.867
	16.941		20.100		23.318		26.608		29.981
	17.045		20.206		23.427		26.719		30.095
	17.150		20.312		23.535		26.830		30.209
17	17.255	20	20.419	23	23.643	26	26.941	29	30.324
	17.359		20.525		23.752		27.052		30.438
	17.464		20.632		23.861		27.164		30.553
	17.568		20 738		23.970		27.275		30.667
	17.673		20.845		24.079		27.387		30.782
	17.778		20.952		24.188		27.499		30.897
	17.883		21.059		24.297		27.610		31.012
	17.988		21.165		24.406		27.722		31.127
	18.093		21.292		24.515		27.834		31.242
	18.198		21.379		24.624		27.946		31.357
18	18.303	21	21.486	24	24.734	27	28.058	30	31.473

M	Gr. Par	M	Gr. Par.	M	Gr. Par	M	Gr. Par.	M	Gr. Par.
30	31.473	33	34.992	36	38 633	9	42.415	42	46 362
	31.885		35.111		38.757		42.544		40.496
	31.704		35.231		38 880		42.673		46.631
	31.820		35.350		39 004		42.802		46 766
	31.936		35.470		39 129		42.931		46.702
	32.052		35.590		39.853		43.061		47.037
	32.168		35.710		39.377		43 191		47.173
	32.284		35.830		39.502		43.320		47.309
	32.409		35.950		39.627		43.451		47.440
	32.516		36.071		39.752		43.581		47.581
31	32.633	34	36.191	37	39.877	40	43.711	23	47.718
	32.750		36.312		40.002		43.842		47.855
	32.867		36.433		40.128		43.973		47 992
	32.984		36.554		40.255		44.104		48.129
	33.101		36.675		40.379		44.235		48.267
	33.218		36.796		40 505		44.366		48.404
	33.336		36.917		40.631		44.498		48.542
	33.453		37.039		40.757		44 630		48.681
	33.571		37.161		40 884		44.762		48.819
	33.688		37.282		41.014		44 894		48.958
32	33.806	35	37.405	38	41.137	41	45.026	44	49.079
	33.924		37.527		41.264		45.159		49.236
	34.042		37.643		41.322		45.292		49.375
	34.161		37.771		41.519		45.425		49.515
	34.279		37.894		41.646		45.558		49.655
	34.397		38.017		41.774		45.691		49.795
	34.516		38.140		41.902		45.825		49.935
	34.635		38 263		42.030		45.959		50.076
	34.754		38.386		42.158		46.093		50.217
	34.873		38.509		42.287		46.227		50.358
33	34.992	36	38.633	39	42.415	42	46.362	45	50.499

M	Gr. Par.	M	Gr. Par.	M	Gr. Par.	M	Gr. Par.	M	Gr. Par.
45	50.499	48	54.860	51	59.481	54	64.412	57	69.711
	50.641		55.010		59.640		64.582		69.895
	50.783		55.160		59.800		64.753		70.080
	50.925		55.310		59.960		64.924		70.265
	51.068		55.460		60.120		65.096		70.449
	51.210		55.611		60.280		65.268		70.635
	51.353		55.762		60.441		65.440		70.821
	51.496		55.913		60.601		65.613		71.008
	51.639		56.065		60.763		65.786		71.195
	51.783		56.217		60.925		65.960		71.483
46	51.927	49	56.369	52	61.088	55	66.134	58	71.572
	52.071		56.522		61.250		66.308		71.701
	52.215		56.675		61.413		66.483		71.950
	52.360		56.828		61.577		66.659		72.140
	52.505		56.981		61.741		66.835		72.331
	52.650		57.135		61.904		87.011		72.522
	52.795		57.289		62.069		67.188		72.714
	52.941		57.444		62.234		67.365		72.906
	53.087		57.598		62.399		67.543		73.099
	53.233		57.704		62.564		67.721		73.292
47	53.380	50	57.909	53	62.730	56	67.900	59	73.486
	53. 26		58.065		62.897		68.079		73.680
	53.673		58.221		63.063		68.258		73.875
	53.821		58.377		63.231		68.438		74.071
	53.968		58.534		63.398		68.618		74.267
	54.116		58.691		63.566		68.799		74.464
	54.264		58.848		63.734		68.981		74.661
	54.413		59.006		63.903		69.163		74.859
	54.562		59.164		64.072		69.345		75.057
	54.711		59.322		64.042		69.528		75.256
48	54.860	51	59.481	54	64.412	57	69.712	60	75.456

M	Gr. Par.	M	Gr. Par.	M	Gr. Par.	M	Gr. Par.	M	Gr. Par.
60	75.451	63	81.749	66	88.725	69	96.575	72	105.579
	75.656		81.970		88.971		96.854		105.904
	75.853		82.191		89.219		97.135		106.230
	76.057		82.413		89.467		97.418		106.558
	76.261		82.635		89.716		97.701		106.888
	76.464		82.860		89.967		97.986		107.220
	76.667		83.084		90.218		98.272		107.5[illegible]3
	76.871		83.313		90.470		98.560		107.888
	77.076		83.536		90.723		98.849		108.226
	77.281		83.763		90.978		99.139		108.665
61	77.487	64	83.990	67	91.232	70	99.431	73	108.906
	77.694		84.219		91.489		99.724		109.249
	77.901		84.448		91.746		100.018		109.594
	78.109		84.678		92.005		100.314		109.941
	78.367		84.909		92.264		100.612		110.290
	78.526		85.141		92.525		100.910		110.641
	78.736		85.374		92.787		101.211		110.994
	78.947		85.607		93.050		101.513		111.349
	79.158		85.842		93.314		101.816		111.707
	79.370		86.077		93.579		102.121		112.066
62	79.583	65	86.313	68	93.846	71	102.427	74	112.428
	79.796		86.550		94.213		102.735		112.792
	80.010		86.788		94.382		103.044		113.158
	80.225		87.027		94.652		103.356		113.526
	80.441		87.267		94.923		103.668		113.897
	80.657		87.508		95.195		103.983		114.270
	80.874		87.749		95.468		104.299		114.645
	81.091		87.992		95.743		104.616		115.023
	81.310		88.235		96.019		104.936		115.403
	81.529		88.480		96.296		105.257		115.786
63	81.749	66	88.725	69	96.575	72	105.579	75	116.171

M	Gr. Par	M	Gr. Par.	M	Gr. Par.	M	Gr. Par.	M	Gr. Par.
75	116.171	78	129.075	81	145.650	84	168.947	87	208 705
	116.559		129.558		146.292		169.912		210.649
	116.949		130.065		146 942		170.893		212.668
	117.342		130.536		147.600		171.891		214.745
	117.737		131.031		148.265		172.907		216.909
	118.135		131.530		148.937		173.941		219.158
	118.536		132.034		149.618		174 994		221.498
	118.939		132.542		150.307		176.067		223.938
	119.345		133.055		151.003		177.160		226.486
	119.755		133.572		151.709		178.275		229.153
76	120.166	79	134.094	82	152 423	85	179.411	88	231.950
	121.581		134.620		153.147		180.569		234.891
	121.000		135.151		153.878		181.752		237.991
	121.420		135.687		154.620		182.960		241.268
	121.843		136.228		155.372		184.194		244.744
	122.270		136.775		156.132		185.454		248.445
	122.700		137.326		156.903		186.743		252.402
	123.133		137.883		157.685		188.062		256.652
	123.570		138.445		158.478		189.411		261.243
	124.009		139.012		159.281		190.793		266.235
77	124.452	80	139.585	83	160.096	86	192 210	89	271.705
	124.898		140.164		160.922		193.661		277.753
	125.348		140.748		161.761		195 151		284.517
	125.801		141.339		162.612		196.680		292.191
	126.258		141.936		163.474		198.251		301.058
	126.718		142.538		164.352		199.867		311.563
	127.182		143.147		165.243		201.529		324 455
	127.649		143.763		166.146		203.240		341.166
	128.121		144.385		167.065		205.005		365.039
	128.596		145.014		167.999		206.825		408.011
78	129.075	81	145 650	84	168.947	87	208.705	90	Infinite.


70
60
F
B
C
A
D
50
E
40
30
20
10
0
10
20
30
40
50
60

If it be a particular Chart, I would first draw the Line A B serving for the first Meridian, and cross it with two Perpendiculars B C and A D, the one at the upper end, the other at the lower end of the Chart, which may serve for the extreme Parallels of Latitude that you are to make use of.

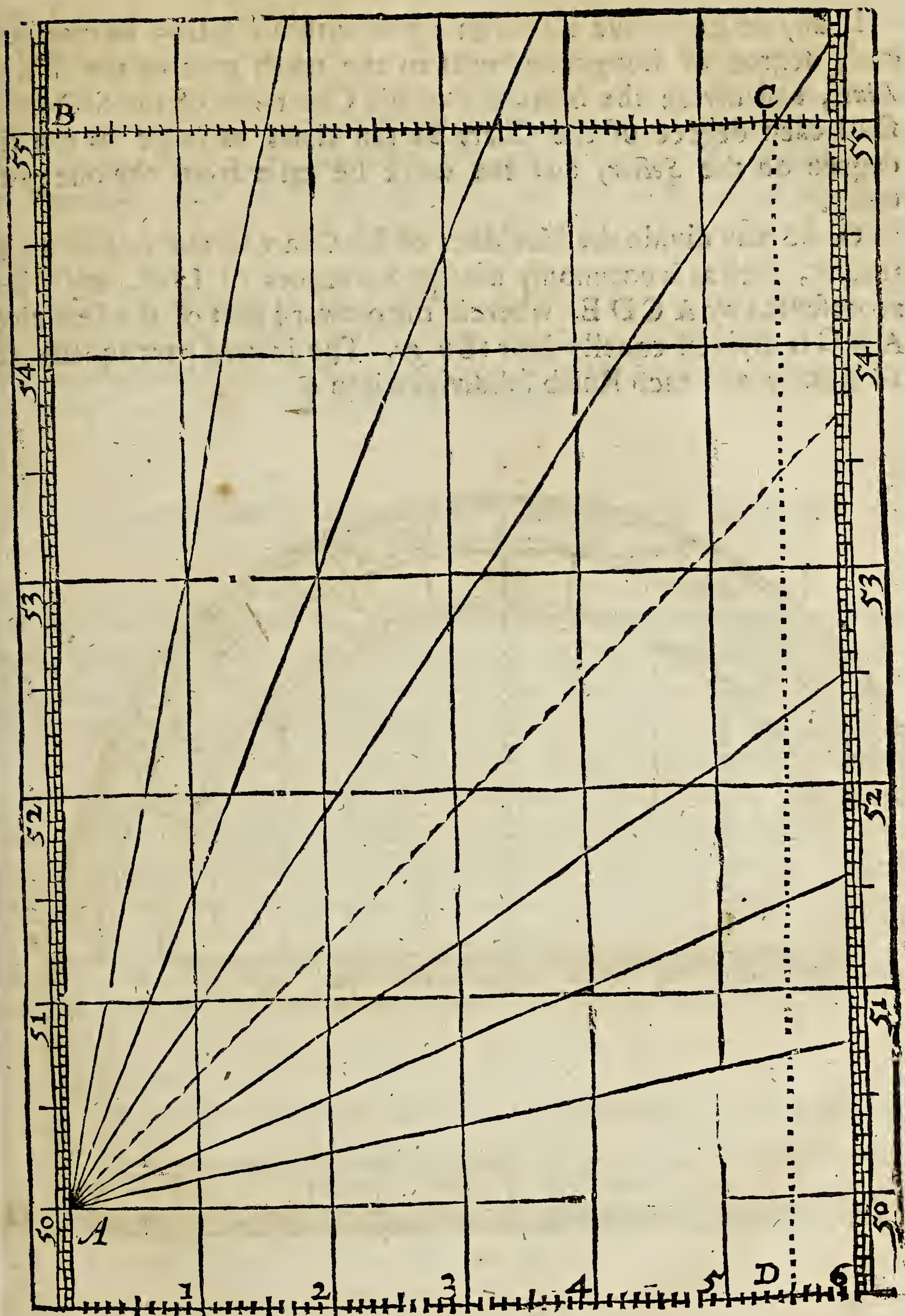
Then considering at what Latitude the Chart is to begin and end, and that this Chart intended for the Latitude of these parts, is to begin at 50 *gr.* and so end at 55 *gr.* I look into the Table, and find that 50 *gr.* of Latitude must be drawn at 57 *gr.* 909 parts; and 55 *gr.* of Latitude at 66 *gr.* 134 parts from the Equator; and that the Meridian distance between the Parallel of 50 *gr.* and 55 *gr.* of Latitude must be equal to 8 *gr.* 225 parts of the Equator. Whereupon I take the Line A B out of the Meridian Line, and diminish it in such proportion as 8. 225 hath unto 1000 per 3 *Prop. Lin.* and with that extent of the Compasses, I divide the two extreme Parallels of Latitude into equal degrees, and through each degree draw Meridian Lines parallel to the first Meridian, noting them with 1, 2, 3, 4, *&c.* and then, I subdivide either one or all of those degrees into ten parts, and (if I may) each tenth part into ten parts more, but howsoever, I suppose each degree to be subdivided into 1000 parts.

66.134
57.909
8.225

The Meridians being drawn, I come to the Parallels of Latitude, beginning at 50 *gr.*

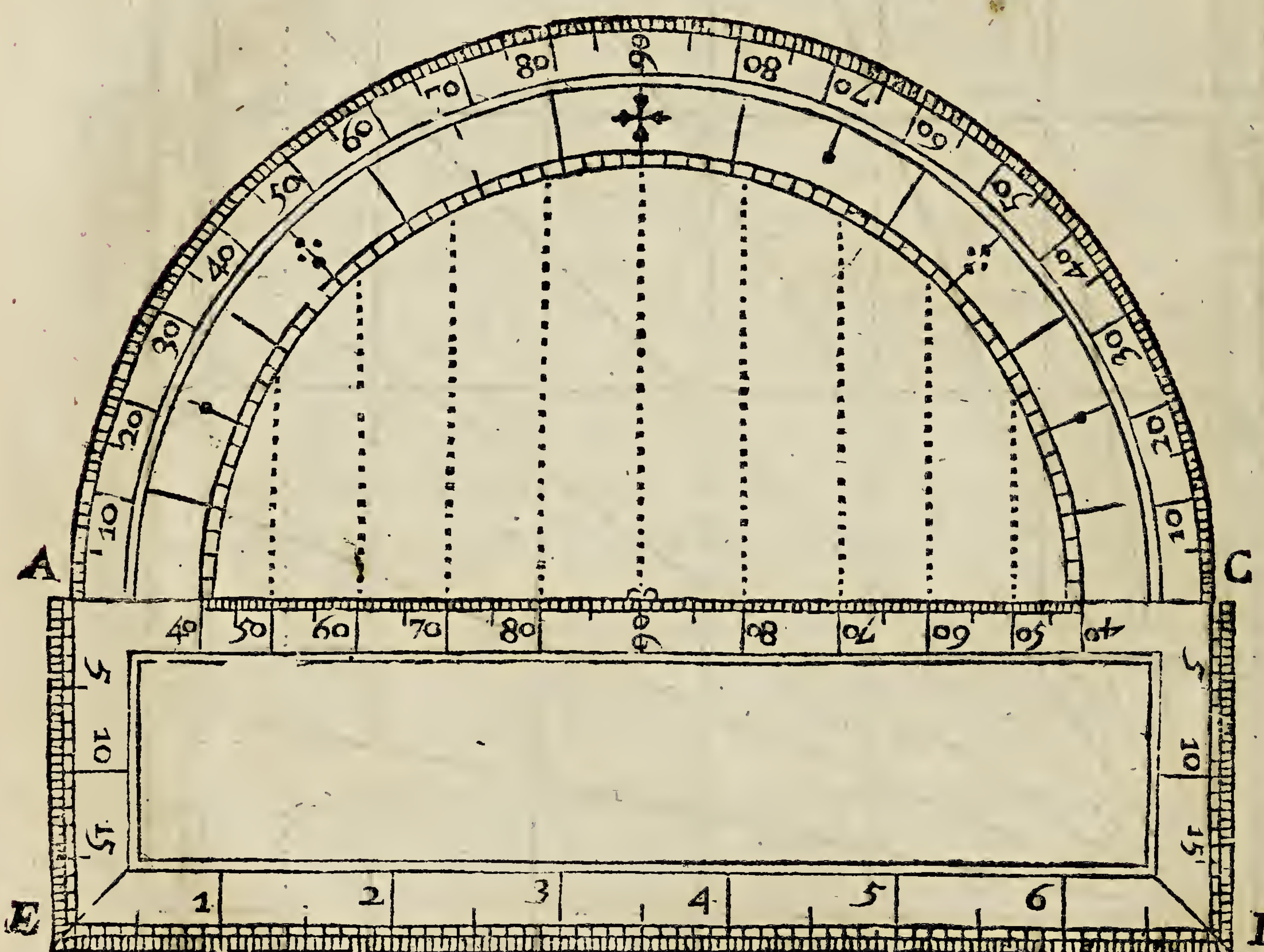
And finding in the Table, that the distance between the Equator and 50 *gr.* in the Meridian should be equal to 57 *gr.* 909 parts in the Equator and his Parallels, I may suppose the lowest Parallel to be 57 *gr.* from the Eq ator: so the distance between this lowest Parallel and the Parallel of 50 *gr.* will be only 909 parts. Wherefore I take these 909 odd parts, out of the degrees that I divided before, and prick them down into the two uttermost Meridians from the lowest Parallel upwards, and there draw the Parallel of 50 *gr.* of Latitude.

In like manner, because I find by the Table that the distance between the Equator and 51 *gr.* in the Meridian is 59 *gr.* 881 parts of the Equator, I abate the former 57 *gr.* and there remain 2 *gr.* 481 parts for the distance between the lowest Parallel, and this Parallel of 51: wherefore I take these 2 degrees 481 parts out of the Line before divided, and prick them down in the two uttermost Meridians (as before) from the lowest Parallel upward, and there draw the Parallel of 51 degrees of Latitude.


B
C
55
54
53
52
51
50
A
1
2
3
4
5
D
6

If any desire to have his Chart agree with his *Sector*, he may make each degree of Longitude equal to the tenth part of the Line of *Lines*, and divide the Meridian of his Chart out of the Sector: so shall each degree of the Chart be ten times as large as the like degree on the *Sector*, and the work be easie from the one to the other.

Or he may divide the Meridian of his Chart by the side of a Protractor, such as is commonly used by Surveyors of Land, and is here represented by A C D E, wherein the outward part of the Semicircle A B C is divided equally into 180 *gr.* The inward part equally into 16 Rumbs, and each Rumb subdivided into 4.



The

The Lines C D, D E, E A, divided equally according to the Line of Lines upon the *Sector*, or the Parallels upon the Chart. Onely the Diameter A C would be divided unequally, by letting down occult perpendicular Lines upon it from each Degree in the Semicircle, which being done, the intermediate part between the Rumbs and the Diameter may be all cut forth: And the back side of the long Square may be filled with 6 Lines of Chords, or Scales of several parts in the Inch.

So may the Meridian be divided by the parts of the Side E D, the Angles of each Rumb may readily be pricked down by the Degrees in the Semicircle, and the Line of Chords and the other Scales may serve to do the like with more variety.

2. *To find how many Leagues answer to one Degree of Longitude in every several Latitude.*

In sailing by the Compass, the Course holds sometime upon a Great Circle, sometime upon a Parallel to the Equator; but most commonly upon crooked Lines, winding towards one of the Poles, which Lines are well known by the Name of *Rumbs*.

If the Course hold upon a Great Circle, it is either North or South under some Meridian, or East or West under the Equator. And in these Cases, every Degree requires an allowance of twenty Leagues; every twenty Leagues will make a Degree difference in the sailing: so that here needs no further Precept than the Rule of Proportion in the Chapter of Lines.

But if the Course hold East or West, or any of the Parallels to the Equator,

As the Radius,
is to twenty Leagues, the Measure of one Degree at the Equator:
So the Sine of the Complement of the Latitude,
to the Measure of Leagues answering to one Degree in that Latitude.

Where-

Gr.	'	Leag.
0	0	20
18	12	19
25	15	18
31	48	17
36	52	16
41	25	15
45	34	14
49	28	13
53	8	12
56	38	11
60	0	10
63	15	9
66	25	8
69	30	7
72	32	6
75	31	5
78	28	4
81	23	3
84	15	2
87	8	1

Wherefore I take 20 Leagues out of the Line of Lines, and make it a parallel Radius, by fitting it over in the Sines of 90 and 90: so his parallel Sine taken out of the Complement of the Latitude, and measured in the Line of Lines, shall shew the number of Leagues required.

Thus in the Latitude of 18 *gr.* 12 *m.* we shall find 19 Leagues answering to one Degree of Longitude, and 18 Leagues in the Latitude of 25 *gr.* 15 *m.* as in this Table.

This may be done more readily without opening the Sector, by doubling the Sine of the Complement of the Latitude, as may appear in the same Example.

It may also be done by the Line of Meridians, either upon the Sector, or upon the Chart: For if we open a pair of Compasses to the quantity of one Degree of Longitude in the Equator, or one of his Parallels, and measure it in the Meridian-line, setting one Foot as much above the Latitude given, as the other falleth beneath it, so that the Latitude may be in the middle between the Feet of the Compasses, the number of Leagues intercepted shall be that which was required.

But if the Course hold upon any of the *Rumbs*, between a Parallel of the Equator and the Meridian, we are to consider (besides the Equator of the World to which we tend, which must be always known),

1. The difference of Longitude, at least in general.
2. The difference of Latitude, and that in particular.
3. The Rumb whereon the Course holds.
4. The distance upon the *Rumb*, which is the distance which we are here to consider, and is always somewhat greater than the like distance upon a greater Circle. And for these, first, I shew in general this third Proposition.

3. *To find how many Leagues do answer to one Degree of Latitude in every several Rumb.*

The Seamans Compaſs is commonly divided into 32 Points; the half, into 16; the quarter, into 8; which have their names of *N, NbE, NNE, &c.* according to thoſe parts of the World to which they point. Anſwerable to theſe Points, are the Rumbs upon their Chart; each quarter divided into 8, each Rumb 11 *gr.* 15 *m.* diſtant one from the other: The firſt Rumb being that which is 11 *gr.* 15 *m.* diſtant from the Meridian; the ſecond, 22 *gr.* 30 *m.* the third, 33 *gr.* 45 *m.* and ſo the reſt. And (if they have need of ſmaller parts) they ſubdivide each Rumb into quarters, allowing 2 *gr.* 48 *m.* to the firſt quarter, 5 *gr.* 37 to the half Rumb, *&c.* as in the Table following.

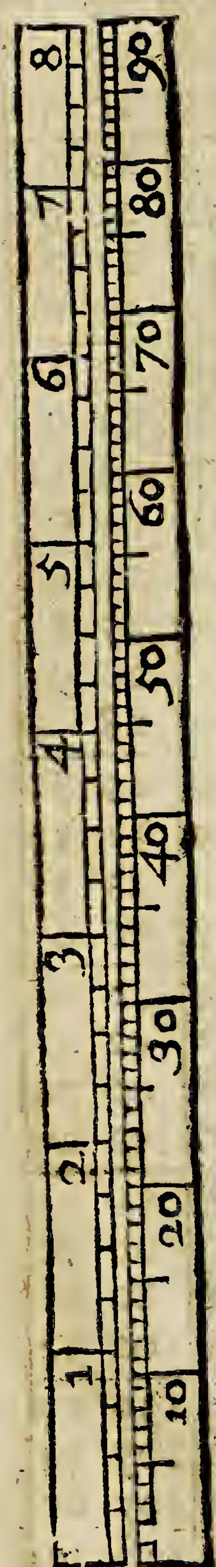
As the Sine of the Complement of the Rumb from the Meridian,
is to 20 Leagues, the Meaſure of one Degree of the Meridian:
So is the Radius,
to the Leagues anſwering to one Degree upon the Rumb.

As if in ſailing *NE b N*, from 50 *gr.* of North Latitude, it were required how many Leagues the Ship ſhould run before it could come to 51 *gr.* of Latitude, becauſe this is the third Rumb, and the Inclination thereof 33 *gr.* 45 *m.* I would take 20 Leagues, *&c.*

Wherefore I take 20 Leagues out of the Line of Lines, and make it a parallel Sine of 56 *gr.* 15 *m.* the Complement of the Rumb from the Meridian; ſo his parallel Radius taken and meaſured in the Line of Lines, ſhall ſhew me 24 for the number of Leagues required.

Rumbs.	*Inclinat. to the Merid.* Gr.	M	*Number of Leagues* Lgs.	Pa.
	2	49	20	02
	5	37	20	10
	8	26	20	22
1	11	15	20	39
	14	4	20	62
	16	52	20	90
	19	41	21	24
2	22	30	21	65
	25	19	22	12
	28	7	22	68
	30	56	23	32
3	33	45	24	05
	36	34	24	90
	39	22	25	87
	42	11	26	99
4	45	0	28	8
	47	49	29	78
	50	37	31	52
	53	26	33	57
5	56	15	36	00
	52	4	38	90
	61	52	42	43
	64	41	46	78
6	67	30	52	26
	70	19	59	37
	73	7	68	90
	75	56	82	31
7	78	45	102	52
	81	34	136	30
	84	22	205	24
	87	11	407	60
8	90	0	Infinita.	

And thus in the firſt Rumb from the Meridian we ſhall find 20 Leagues 39 parts anſwering to one Degree of Latitude, and 21 Leagues 65 parts in the ſecond Rumb, *&c.* as in this Table, where we ſubdivide each League into a hundred parts, and ſhew beſides what Inclination the Rumb hath to the Meridian.



This may be done more readily without opening the Sector, by doubling the Secant of the Rumb, as may appear in the ſame Example.

It may alſo be done upon the Chart, if firſt we draw the Rumb; then we take the diſtance upon the Rumb between two Parallels, and meaſure it in the Meridian-line, as far above the greater Latitude as beneath the leſſer. For the number of Leagues intercepted ſhall be that which was required.

For Example: In the ſecond Chart, *pag.* 113. I firſt draw the 8 Rumbs, from the Interſection of the Meridian with the Parallel of 50 *gr.* of Latitude, either by that which I have ſhewed before in the general Uſe of Sines, *Chap.* 11. *Prop.* 10. or by help of the Protraction laſt mentioned: For, laying the Center of the Protractor to the Point of Interſection (which is to be the Center of the Rumbs) and turning the Diameter of the Protractor until it be parallel to the Meridians of the Chart (which is then done, when the Meridians and Parallels in the Chart fall under like diviſions in the Protractor) I may make one prick at 11 *gr.* 15 *m.* another at 22 *gr.* 30 *m.* in the outward part of the Semicircle; and ſo the reſt.

Or, having neither Sector nor Protractor, I would have a Line of Chords ſet on the ſide of the Ruler which I am to uſe, from which I may take 60 *gr.* and with that extent ſetting one Foot of the Compaſſes in the former Point of Interſection, draw an occult Ark of a Circle, and therein prick down the former Arks from the Meridian, as in *Chap.* 11. *Prop.* 10. So theſe Arks being pricked down by either of theſe ways, the Right Lines drawn through the Center and thoſe pricks, ſhall be the Rumbs required.

The Rumbs being drawn, I take the diſtance between the Parallels of 50 and 51 *gr.* upon A C, the third Rumb; and

and meaſuring it in the Meridian-line, I find the Compaſſes to reach from above $\frac{1}{10}$ of a Degree below the Parallel of 50, but $\frac{1}{10}$ above the Parallel of 51 *gr.* intercepting 1 *gr.* $\frac{2}{10}$ or 24 Leagues, ſuch as 20 make a degree.

Again, I take the diſtance upon the ſame Rumb between the Parallel of 54 and 55 *gr.* which I find to be ſomewhat longer than the former diſtance between the Parallels of 50 and 51; but meaſuring it in the Meridian Line, according to the Latitude of the Parallel, I find but 1 *gr.* $\frac{2}{10}$ (or 24 Leagues) as before, for the number of Leagues anſwering to 1 Degree of Latitude upon this third Rumb.

And by the ſame reaſon, I may find the number of Leagues anſwering to a Degree of Latitude upon the reſt of the Rumbs agreeable to the Table.

This conſidered in general, I ſhew more particularly in twelve Propoſitions following, how of theſe four any two being given, the other two may be found, both by *Mercator*'s Chart, and by this *Sector.*

1. *By one Latitude, Rumb, and Diſtance, to find the Difference of Latitudes.*

As the Radius,
to the Sine of the Complement of the Rumb from the Meridian:
So the Diſtance upon the Rumb,
to the difference of Latitudes.

Let the Place given be A, in the Latitude of 50 *gr.* C in a greater Latitude, but unknown, the diſtance upon the Rumb being 6 *gr.* between them, and the Rumb the third from the Meridian.

Firſt, I take 6 *gr.* from the diſtance upon the Rumb, out of the Line of Lines, and make it a Parallel Radius, by putting it over in the Sines of 90 and 90: Then keeping the Sector at this Angle, I take out the Parallel Sine of 56 *gr.* 15 *m.* which is the Sine of the Complement of the third Rumb from the Meridian, and meaſuring it in the Line of Lines, I find it to be 5 *gr.* and ſuch is the difference of Latitude required.

Or, I may take out the Sine of 56 *gr.* 15 *m.* for the Complement of the third Rumb from the Meridian, and make it a parallel Radius; then keeping the Sector at this Angle, I take 6 *gr.* for the diſtance, either out of the Line of Lines, or any other Scale of Equal parts, or elſe out of the Meridian Line, and lay it on both ſides of the Sector from the Center, either on the Line of Lines or Sines: ſo the Parallel taken from the Terms of this diſtance, and meaſured in the ſame Scale wherein the diſtance was meaſured, ſhall ſhew the difference of Latitude to be 5 *gr.* as before.

But in ſhorter diſtances, ſuch as fall within the compaſs of a days ſailing, this Work will hold much better; as may appear by comparing the Work with the Table following, where the Numbers in the front do ſignifie the Leagues; thoſe in the ſide, the Rumb; and the reſt in the middle, the difference of Latitude.

In the Chart let a Meridian A B be drawn through A, and in A with A B make an Angle of the Rumb B A C: Then open the Compaſſes, according to the Latitude of the Places, to E F the quantity of 6 *gr.* in the Meridian, transferring them into the Rumb from A to C, and through C draw the Parallel B C, croſſing the Merinian A B in B: ſo the Degrees in the Meridian from A to B ſhall ſhew the difference of Latitude to be 5 *gr.*

2. *By the Rumb and both Latitudes to find the Diſtance upon the Rumb.*

As the Sine of the Complement of the Rumb from the Meridian,
is to the Radius:
So is the Difference of Latitudes,
to the Diſtance upon the Rumb.

As if the Places given were A in the Latitude of 50 *gr.* C in the Latitude of 55 *gr.* and the Rumb the third from the Meridian.

Here I may take 5 *gr.* for the difference of Latitude out of the Line of Lines, and put it over in the Sine of 56 *gr.* 15 *m.* for the

Com-

A Table of Leagues, Rumbs, and Difference of Latitude.

Lg	100	80	60	40	20	19	18	17	16	15
Rum.	G.M.	G.M.	G.M.	G.M	M	M	M	M	M	M
	5 0	4 0	3 0	2 0	60	57	54	51	48	45
	4 59	3 59	2 59	1 59	60	57	54	51	48	45
	4 58	3 58	2 59	1 59	60	57	54	51	48	45
	4 56	3 58	2 58	1 58	59	56	53	50	47	44
1	4 54	3 57	2 50	1 57	59	56	53	50	47	44
	4 51	3 53	2 55	1 56	58	50	52	50	47	43
	4 47	3 50	2 52	1 55	57	55	52	49	46	43
	4 42	3 46	2 49	1 53	56	54	51	48	45	42
2	4 37	3 42	2 46	1 51	55	53	50	47	44	41
	4 31	3 37	2 43	1 48	54	52	49	40	42	40
	4 25	3 32	2 39	1 46	53	50	48	45	42	39
	4 17	3 26	2 34	1 43	51	49	46	44	41	38
3	4 10	3 20	2 30	1 40	50	47	45	42	40	37
	4 1	3 13	2 25	1 36	48	46	43	41	39	30
	3 52	3 5	2 19	1 32	46	44	42	39	37	35
	3 42	2 58	2 13	1 28	44	42	40	38	36	33
4	3 32	2 50	2 7	1 25	42	40	38	36	34	32
	3 22	2 41	2 1	1 21	40	38	36	34	32	30
	3 10	2 32	1 54	1 16	38	36	34	32	30	28
	2 59	2 23	1 47	1 12	36	34	32	30	29	27
5	2 47	2 14	1 40	1 7	33	32	30	28	27	25
	2 34	2 3	1 32	1 2	31	29	28	26	25	23
	2 22	1 53	1 25	0 57	28	27	25	24	23	22
	2 8	1 40	1 17	0 52	26	24	23	22	21	19
6	1 55	1 32	1 8	0 46	23	22	21	20	18	17
	1 41	1 20	1 0	0 40	20	19	18	17	16	15
	1 27	1 9	0 52	0 35	17	16	16	15	14	13
	1 13	0 58	0 44	0 30	15	14	13	12	12	11
7	0 59	0 47	0 35	0 24	12	11	11	10	9	9
	0 44	0 36	0 16	0 18	9	8	8	7	7	7
	0 30	0 24	0 18	0 12	6	6	5	5	5	4
	0 15	0 12	0 9	0 9	3	3	3	3	2	2
8	0 0	0 0	0 0	0 0	0	0	0	0	0	

A Table of Leagues, Rumbs, and Difference of Latitudes.

14	13	12	11	10	9	8	7	6	5	4	3	2	1	Le.
M	M	M	M	M	M	M	M	M	M	M	M	M	M	Rum.
40	39	36	33	30	27	24	21	18	15	12	9	6	3	
42	39	36	33	30	27	24	21	18	15	12	9	6	3	
42	39	36	33	30	27	24	21	18	15	12	9	6	3	
42	39	36	33	29	27	24	21	18	15	12	9	6	3	
41	38	35	32	29	26	24	21	18	15	12	9	6	3	1
41	38	35	32	29	26	23	20	17	14	12	9	6	3	
40	37	34	32	29	26	23	20	17	14	11	9	6	3	
40	37	34	31	28	25	23	20	17	14	11	8	6	3	
39	36	33	31	28	25	22	19	17	14	11	8	6	3	2
38	35	33	30	27	24	22	19	16	14	11	8	5	3	
37	34	32	29	26	24	21	19	16	14	11	8	5	3	
36	33	31	28	26	23	21	18	15	13	10	8	5	3	
35	32	30	27	25	22	20	17	15	12	10	7	5	2	3
34	30	29	26	24	22	19	17	14	12	10	7	5	2	
33	30	28	25	23	21	19	16	14	12	9	7	5	2	
31	29	27	24	22	20	18	16	13	11	9	7	4	2	
30	28	25	23	21	19	17	15	13	11	8	6	4	2	4
28	26	24	22	20	18	16	14	12	10	8	6	4	2	
27	25	23	21	19	17	15	13	11	10	8	6	4	2	
25	23	21	20	18	16	14	13	11	9	7	5	4	2	
23	22	20	18	17	15	13	12	10	8	7	5	3	2	5
22	20	18	17	15	14	12	11	9	8	6	5	3	2	
20	18	17	16	14	13	11	10	8	7	6	4	3	1	
18	17	15	14	13	12	10	9	8	6	5	4	3	1	
16	15	14	13	11	10	9	8	7	6	5	3	2	1	6
14	13	12	11	10	9	8	7	6	5	4	3	2	1	
12	11	10	10	9	8	7	6	5	4	3	3	2	1	
10	9	9	8	7	7	6	5	4	4	3	2	1	1	
8	8	7	6	6	6	5	4	3	3	2	2	1	1	7
6	6	5	5	4	5	4	3	3	2	2	1	1	1	
4	4	4	3	3	3	2	2	2	1	1	1	1	0	
2	2	2	2	1	1	1	1	1	1	1	1	1	0	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	8

Complement of the third Rumb from the Meridian. Then keeping the Sector at this Angle, I take out the Parallel Radius, and measuring it in the Line of *Lines*; I find it to be 6 *gr.* and such is the distance upon the Rumb, which was required.

Or I may take the Lateral Radius, and make it a Parallel Sine of 56 *gr.* 15 *m.* the Complement of the Rumb from the Meridian: then keeping the Sector at this Angle, I take 5 *gr.* for the difference of Latitude, either out of the Line of *Lines*, or out of some other Scale of equal parts, and lay it on both sides of the Sector from the Center, either on the Line of *Lines* or of *Sines*: so the Parallel taken from the terms of his difference, and measured in the same Scale with the the difference, shall shew the distance upon the Rumb to be 6 *gr.* or 120 Leagues.

Or keeping the *Sector* at this Angle, I may take the difference between 50 *gr.* and 55 *gr.* out of the Meridian Line, and measuring it in the Equator, I shall find it to be equal to 8 *gr.* 22 *p.* of the Equator. Wherefore I take the Parallel between 822 and 822 out of the Line of *Lines*, and measuring it in the Line of *Lines*, I shall find it to be 989; which shews that according to this projection, the distance upon this third Rumb, answerable to the former difference of Latitudes, will be equal to 9 *gr.* 89 *p.* of the Equator.

Or the Sector remaining at this Angle, I may take the difference between 50 *gr.* and 55 *gr.* out of the Meridian Line, and lay it from the Center on both sides of the Sector, either on the Line of Lines or of Sines: so the Parallel taken from the terms of this difference, shall be the very Line of distance required, the same with A C or E F upon the Chart; which may serve for the better pricking down of the distance upon the Rumb, without taking it forth of the Meridian Line, as in the former *Proposition*.

Or if the Rumb fall nearer to the Equator, that the lateral Radius cannot be fitted over in it, this Proposition may be wrought by Parallel entrance.

For, if I first take out the Sine of 56 *gr.* 15 *m.* and make it a parallel Radius, by fitting it over in the Sines of 90 and 90, or in the ends of the Line of *Lines*, and then take 5 *gr.* for the difference of Latitudes out of the Line of *Lines*, and carry it parallel to the former, I shall find it to cross both Lines of *Lines* in the Points of 6: and so it gives the same distance as before.

Or if the distance be small, it may be found by the former Table.

For

For the Rumb being found in the ſide of the Table, and the difference of Latitude in the ſame Line; the top of the Column wherein the difference of Latitude was found, ſhall give the number of Leagues in the diſtance required.

Or we may find this diſtance in the Table of Rumbs in the fifth *Propoſition* following. For according to the example, look into the Table of the third Rumb for 5 *gr.* of Latitude, and there we ſhall find 6 *gr.* 10 parts under the title of diſtance.

So if the difference of Latitude upon the ſame Rumb were 50 *gr.* the diſtance would be 60 *gr.* 13 parts. If the difference of Latitude upon the ſame Rumb were only ½ of a degree, the diſtance would be only 60 parts, ſuch as 100 do make a degree.

In the Chart let a Meridian A B be drawn through A, and Parallels of Latitude through A and C; and then in A, with A B, make an Angle of the Rumb B A C: ſo the diſtance take from A to C, and meaſured in the Meridian Line, according to the Latitude of the places, ſhall be found to be 6 *gr.* or 120 Leagues. And ſuch is the diſtance required.

3. *By the diſtance and both Latitudes, to find the Rumb.*

As the diſtance upon the Rumb,
 to the difference of Latitudes:
So is the Radius,
 to the Sine of the Complement of the Rumb from the Meridian.

As if the places given were A, in the Latitude of 50 *gr.* C in the Latitude of 55 *gr.* the diſtance between them being 6 *gr.* upon the Rumb. Firſt I take 6 *gr.* for the diſtance upon the Rumb, and lay it on both ſides of the *Sector* from the Center; then out of the ſame Scale I take 5 *gr.* for the difference of Latitude, and to it open the *Sector* in the terms of the former diſtance: ſo the parallel Radius taken and meaſured in the Sines, doth give 56 *gr.* 15 *m.* the Complement whereof 33 *gr.* 45 *m.* is the Angle of the Rumbs inclination to the Meridian, which was required.

In the Chart let a Meridian A B be drawn through A, and Parallels of Latitude, both through A and C; then open the Compaſſes according to the Latitude of the places to E F, the quantity of 6 *gr.* in the Meridian, and ſetting one foot in A, turn the other till it croſs the

Parallel

Parallel B C in C, and draw the right Line A C: so the Angle B A C shall shew the inclination of the Rumb to the Meridian to be 33 *gr.* 45 *m.* as before.

These three last Propositions depend one on the other, and may be wrought as truly by the Common Sea-Chart as by this of *Mercators* Projection: and therefore in working them by the *Sector*, the distance and the difference of Latitudes may as well or better be taken out of the Line of *Lines* (which here representeth the Equator) or any other Line of equal parts, as out of the inlarged degrees in the Meridian Line. But in the Propositions following, the difference of Longitude must be taken out of the Equator; the difference of Latitudes and distance upon the Rumb must alwaies be taken out of the Meridian Line: which I therefore call the proper difference, and proper distance.

4. *By the Longitude and Latitude of two places to find the Rumb.*

As if the places given were A, in the Latitude of 50 *gr.* C in the Latitude of 55 *gr.* and the difference of Longitude between them were 5 *gr.* 30 *m.*

In the Chart let Meridians and Parallels be drawn through A and C, and a straight Line for the Rumb from A to C; then by that we shewed *Cap.* 2. *Proposition* 9. inquire the quantity of the Angle B A C, and it shall be found to be 33 *gr.* 45 *m.* which is the third Rumb from the Meridian. Wherefore the proportion holds for the Sector,

As A B the proper difference of Latitude:
is to B C the difference of Longitude:
So is A B Radius,
to B C, the Tangent of the Rumb from the Meridian.

According to this I take the proper difference of Latitude from 50 *gr.* to 55 *gr.* out of the Line of Meridians, and lay it on both sides of the *Sector* from the Center; then I take the difference of Longitude 5 *gr.* ½ out of the Line of *Lines*, and to it open the Sector in the terms of the former difference of Latitudes, so the Parallel Radius taken from between 90 and 90; and measured in the greater

Tangent on the side of the *Sector*, doth give 33 *gr.* 45 *m.* for the Rumb required.

But if the Rumb fall nearer to the Equator;

As A D the difference of Longitudes,
is to D C the proper difference of Latitudes:
So A D the Radius,
to D C the Tangent of the Rumb from the Equator.

According to this I take the former difference of Latitudes from 50 *gr.* to 55 *gr.* out of the Line of Meridians, and to it open the *Sector* in the terms of the difference of Longitude reckoned in the Line of *Lines* from the Center, so the Parallel Radius taken and measured in the Tangent, doth give 56 *gr.* 15 *m.* for the Rumb from the Equator: which is the Complement to the former 33 *gr.* 45 *m.* and so both ways it is found to be the third Rumb from the Meridian.

But if this Rumb were to be found in the common Sea-chart, it should seem to be above 47 *gr.* which is more than the fourth Rumb from the Meridian.

5. *By the Rumb and both Latitudes, to find the difference of Longitude.*

As if the places given were A, in the Latitude of 50 *gr.* and C in the Latitude of 55 *gr.* and the Rumb the third from the Meridian.

In the Chart, let a Meridian be drawn through A, and a Parallel of Latitude through C, then in A, with A B, make the Angle of the Rumb from the Meridian B A C, (as was shewed *Cap.* 2. *Prop.* 10.) So the degrees in the Parallel between B and C, shall be found to be 5 *gr.* ½, the difference of Longitude which was required. Wherefore the proportion holds for the *Sector*,

As A B the Radius,
to B C the Tangent of the Rumb from the Meridian:
So A B the proper difference of the Latitudes,
to B C the difference of Longitude.

Accord-

According to this we may take the Tangent of the Rumb which is here 33 *gr.* 45 *m.* from the Meridian, out of the greater Tangent on the ſide of the *Sector*, and putting it over between 90 and 90, make it a Radius: then keeping the *Sector* at this Angle, take the proper difference of Latitudes from 50 *gr.* to 55 *gr.* out of the Line of Meridians, and lay it on both ſides of the Sector from the Center: ſo the Parallel taken from the terms of this difference, and meaſured in the Line of *Lines*, ſhall ſhew the difference of Longitude to be 5 *gr.* ½.

As D C the Tangent of the Rumb from the Equator,
 to A D the Radius:
So C D the proper difference of the Latitudes,
 to A D the difference of Longitude.

According to this, we may beſt work by Parallel entrance, firſt taking 56 *gr.* 15 *m.* for the Angle of the Rumb from the Equator, out of the greater Tangent, and make it a Parallel Radius: then take the proper difference of Latitudes out of the Line of Meridians, and carry it Parallel to the former: ſo we ſhall find it to croſs the Line of Lines in 5 *gr.* ½. And this is the difference of Longitude required, the ſame as before.

But if this difference were to be found by the common Sea-chart, it ſhould ſeem to be only 3 *gr.* 20 *m*, which is more than two degrees leſs than the truth. And yet this error would be greater, if either the Latitude be greater, or the Rumb fall nearer the Equator, as may appear by comparing the common Sea-chart with the Table following.

The first Rumb, from the Meridian.			*North and by East, South and by East,*			*North and by West. South and by West.*		
La	Long	Dist.	La	Long.	Dist.	La	Long.	Dist.
Gr	Gr Par	Gr. Par.	Gr	Gr. Par	Gr. Par	Gr	Gr. Par.	Gr. Par.
0	0	0	30	6 20	30 59	60	15 10	61 18
1	20	1 02	31	6 49	31 61	61	15 41	62 20
2	40	2 04	32	6 72	32 63	62	15 83	63 21
3	60	3 06	33	6 96	33 65	63	16 26	64 23
4	80	4 08	34	7 20	34 67	64	16 71	65 25
5	1 00	5 10	35	7 44	35 69	65	17 17	66 27
6	1 20	6 12	36	7 68	36 71	66	17 65	67 29
7	1 40	7 14	37	7 92	37 73	67	18 15	68 31
8	1 60	8 16	38	8 17	38 75	68	18 77	69 33
9	1 80	9 18	39	8 43	39 77	69	19 21	70 35
10	2 00	10 20	40	8 70	40 78	70	19 78	71 37
11	2 20	11 22	41	8 96	41 80	71	20 35	72 36
12	2 40	12 24	42	9 22	42 82	72	21 07	73 41
13	2 61	13 25	43	9 50	43 84	73	21 60	74 43
14	2 81	14 27	44	9 76	44 86	74	22 36	75 45
15	3 02	15 29	45	10 04	45 88	75	23 16	76 47
16	3 22	16 31	46	10 33	46 90	76	23 90	77 49
17	3 43	17 33	47	10 62	47 92	77	24 75	78 51
18	3 64	18 35	48	10 91	48 94	78	25 67	79 53
19	3 85	19 37	49	11 21	49 96	79	26 67	80 55
20	4 06	20 39	50	11 52	50 98	80	27 76	81 57
21	4 27	21 41	51	11 83	52 0	81	28 97	82 59
22	4 49	22 43	52	12 15	53 2	82	30 32	83 61
23	4 70	23 45	53	12 47	54 4	83	31 84	84 63
24	4 92	24 47	54	12 81	55 6	84	33 61	85 65
25	5 14	25 49	55	13 16	56 8	85	35 69	86 67
26	5 36	26 51	56	13 50	57 10	86	38 24	87 69
27	5 58	27 53	57	13 86	58 12	87	41 52	88 71
28	5 80	28 55	58	14 23	59 14	88	46 15	89 73
29	6 03	29 57	59	14 62	60 16	89	54 06	90 75
30	6 26	30 59	60	15 01	61 18	90		

The second Rumb from the Meridian.			*North, North-east, South, South east,*			*North, North-west, South, South west.*		
La	*Long.*	*Dist.*	*La*	*Long.*	*Dist.*	*La*	*Long.*	*Dist.*
Gr	*Gr.Par.*	*Gr Par.*	*Gr*	*Gr.Par.*	*Gr.Par.*	*Gr*	*Gr.Par.*	*Gr.Par.*
0	0	0	30	13 03	32 47	60	31 29	64 94
1	0 42	1 08	31	13 51	33 54	61	32 09	66 03
2	0 83	2 16	32	14 00	34 54	62	32 96	67 11
3	1 24	3 25	33	14 49	35 72	63	33 86	68 19
4	1 65	4 33	34	15 00	36 80	64	34 79	69 27
5	2 07	5 41	35	15 50	37 88	65	35 75	70 35
6	2 49	6 49	36	16 00	38 97	66	36 75	71 44
7	2 91	7 57	37	16 51	40 05	67	37 80	72 52
8	3 32	8 66	38	17 03	41 13	68	38 88	73 60
9	3 74	9 74	39	17 56	42 21	69	40 00	74 68
10	4 16	10 82	40	18 10	43 30	70	41 19	75 77
11	4 59	11 90	41	18 65	44 38	71	42 43	76 85
12	5 01	12 99	42	19 20	45 46	72	43 74	77 99
13	5 43	14 07	43	19 76	46 54	73	45 11	79 01
14	5 85	15 15	44	20 33	47 62	74	46 57	80 10
15	6 28	16 23	45	20 92	48 71	75	48 12	81 18
16	6 71	17 32	46	21 50	49 79	76	49 78	82 26
17	7 14	18 40	47	22 11	50 87	77	51 55	83 34
18	7 58	19 48	48	22 72	52 95	78	53 46	84 42
19	8 91	20 56	49	23 35	53 03	79	55 54	85 52
20	8 45	21 65	50	23 98	54 12	80	57 82	86 59
21	8 90	22 73	51	24 63	55 20	81	60 33	87 67
22	9 34	23 81	52	25 30	56 28	82	63 13	88 76
23	9 79	24 89	53	25 98	57 37	83	66 32	89 84
24	10 24	25 98	54	26 69	58 45	84	69 99	90 92
25	10 70	27 06	55	27 39	59 53	85	74 32	92 00
26	11 16	28 14	56	28 12	60 61	86	79 63	93 09
27	11 62	29 22	57	28 87	61 79	87	86 46	94 17
28	12 08	30 31	58	29 64	62 78	88	96 10	95 25
29	12 55	31 39	59	30 44	63 86	89	112 57	96 38
30	13 03	32 47	60	31 25	64 94	90		

The third Rumb, from the Meridian.			*North-east by North, South-east by South,*			*North-west by North, South-west by South,*		
La	Long	Dist.	La	Long.	Dist.	La	Long.	Dist.
Gr	Gr. Par	Gr. Par.	Gr	Gr. Par	Gr. Par	Gr	Gr. Par.	Gr. Par.
0	0	0	30	21 03	36 08	60	50 42	72 16
1	0 66	1 20	31	21 80	37 28	61	51 78	73 36
2	1 33	2 40	32	22 58	38 49	62	53 18	74 56
3	2 00	3 61	33	23 38	39 69	63	54 63	75 77
4	2 67	4 81	34	24 18	40 89	64	56 12	76 97
5	3 34	6 01	35	25 00	42 09	65	57 68	78 17
6	4 01	7 22	36	25 82	43 30	66	59 29	79 37
7	4 68	8 42	37	26 64	44 50	67	69 99	80 58
8	5 36	9 62	38	27 48	45 70	68	62 71	81 78
9	6 03	10 82	39	28 39	46 90	69	64 53	82 98
10	6 71	12 03	40	29 21	48 11	70	66 44	84 19
11	7 39	13 23	41	30 09	49 31	71	68 45	85 39
12	8 07	14 43	42	30 98	50 51	72	70 55	86 59
13	8 76	15 64	43	31 88	51 71	73	72 77	87 79
14	9 44	16 84	44	32 80	52 92	74	75 12	89 00
15	10 13	18 04	45	33 74	54 12	75	77 62	90 20
16	11 83	19 24	46	34 69	55 32	76	80 30	91 40
17	12 53	20 45	47	35 67	56 52	77	83 15	92 61
18	12 23	21 65	48	36 66	57 73	78	86 25	93 81
19	12 93	22 85	49	37 67	58 93	79	89 60	95 01
20	13 64	24 05	50	38 69	60 13	80	93 27	96 22
21	14 35	25 26	51	39 74	61 33	81	97 32	97 42
22	15 07	26 46	52	40 82	62 54	82	101 85	98 62
23	15 80	27 66	53	41 91	63 74	83	106 97	96 82
24	16 53	28 86	54	43 03	64 94	84	112.90	101 03
25	17 26	30 07	55	44 19	66 15	85	119 90	102 23
26	18 00	31 27	56	45 37	67 45	86	128 45	103 43
27	18 75	32 47	57	46 58	68 55	87	139 47	104 64
28	19 50	33 67	58	47 82	69 75	88	155 00	105 84
29	20 26	34 88	59	49 11	70 96	89	181 58	107 04
30	21 03	36 08	60	50 42	72 16	90		

The fourth Rumb, from the Meridian.			*North-east, South east,*			*North-west, South-west.*		
La	*Long.*	*Dist.*	*La*	*Long.*	*Dist.*	*La*	*Long.*	*Dist.*
Gr	*Gr.Par.*	*Gr.Par.*	*Gr*	*Gr.Par.*	*Gr.Par.*	*Gr*	*Gr.Par.*	*Gr.Par.*
0	0	0	30	31 47	42 42	60	75 46	84 85
1	1 00	1 41	31	32 63	43 84	61	77 49	85 27
2	2 00	2 83	32	33 81	45 25	62	79 58	87 68
3	3 00	4 24	33	34 99	46 67	63	81 75	89 09
4	4 00	5 66	34	36 19	48 07	64	83 99	90 51
5	5 01	7 07	35	37 41	49 50	65	86 31	91 92
6	6 01	8 49	36	38 63	50 91	66	88 73	93 34
7	7 02	9 90	37	39 88	52 33	67	91 23	94 75
8	8 03	11 31	38	41 14	53 74	68	93 85	96 17
9	9 04	12 73	39	42 42	55 15	69	96 58	97 58
10	10 05	14 14	40	43 71	56 65	70	99 43	98 99
11	11 07	15 56	41	45 03	57 98	71	102 43	100 41
12	12 09	16 97	42	46 36	59 40	72	105 58	101 82
13	13 11	18 38	43	47 72	60 81	73	108 91	103 24
14	14 14	19 80	44	49 10	62 22	74	112 43	104 65
15	15 17	21 21	45	50 50	63 64	75	116 17	106 06
16	16 21	22 63	46	51 93	65 05	76	120 17	107 48
17	17 35	24 04	47	53 38	66 46	77	124 45	108 89
18	18 30	25 45	48	54 86	67 88	78	129 08	110 31
19	19 36	26 87	49	56 37	69 29	79	134 10	111 72
20	20 42	28 28	50	57 91	70 71	80	139 59	113 14
21	21 49	29 70	51	59 48	72 12	81	145 65	114 55
22	22 56	31 11	52	61 09	73 54	82	152 42	115 96
23	23 64	32 54	53	62 73	74 95	83	160 10	117 38
24	24 73	33 94	54	64 41	76 37	84	168 9	118 79
25	25 83	35 35	55	66 13	77 78	85	179 41	120 21
26	26 94	36 77	56	67 90	79 20	86	192 21	121 62
27	28 06	38 18	57	69 71	80 61	87	208 71	123 04
28	29 18	39 60	58	71 57	82 02	88	231 95	124 45
29	30 32	41 01	59	73 49	83 44	89	271 71	125 86
30	31 47	42 43	60	75 49	84 85	90		

The fifth Rumb from the Meridian. *North east and by East. South-east and by East.* *North west and by West. South-west and by West.*

La	Long.	Dist.	La	Long.	Dist.	La	Long.	Dist.
Gr	Gr. Par.	Gr. Par.	Gr	Gr. Par.	Gr. Par	G	Gr Par.	Gr. Par.
0	0	0	30	47 10	54 00	60	112 9	108 01
1	1 49	1 80	31	48 84	55 80	61	115 97	109 80
2	2 99	3 60	3	50 60	57 60	62	119 10	111 60
3	4 49	5 40	33	52 37	59 40	6	122 34	113 40
4	6 00	7 20	34	54 16	61 20	64	125 70	115 20
5	7 50	9 00	35	55 98	63 00	65	129 18	117 00
6	9 00	10 80	36	57 82	64 80	66	132 78	118 80
7	10 50	12 60	37	59 68	66 60	67	136 54	120 60
8	12 01	14 40	38	61 57	68 40	68	140 45	122 40
9	13 52	16 20	39	63 48	70 20	69	144 53	124 20
10	15 04	18 00	40	65 42	72 00	70	148 81	126 00
11	16 56	19 80	41	66 39	73 80	71	153 30	127 80
12	18 09	21 60	42	69 39	75 60	72	158 00	129 60
13	19 62	23 40	43	61 43	77 40	73	163 00	131 40
14	21 16	25 20	44	73 48	79 20	74	168 26	133 20
15	22 70	27 00	45	75 58	81 00	75	173 86	135 00
16	24 26	28 80	46	77 72	82 80	76	179 84	136 80
17	25 82	30 60	47	79 89	84 60	77	186 26	138 60
18	27 39	32 40	48	82 10	86 40	78	193 17	140 40
19	28 97	34 20	49	84 36	88 10	79	200 69	142 20
20	30 55	36 00	50	86 67	90 00	80	208 91	144 00
21	32 15	37 80	51	89 03	91 80	81	217 98	145 80
22	33 76	39 60	52	91 43	93 60	82	228 13	147 60
23	35 38	41 40	53	92 88	95 40	83	239 61	149 40
24	37 01	43 20	54	96 40	97 20	84	252 85	151 20
25	38 66	45 00	55	98 98	99 00	85	268 51	153 00
26	40 32	46 80	56	101 62	100 80	86	287 67	154 80
27	42 00	48 60	57	104 33	102 60	87	312 36	156 60
28	43 67	50 40	58	107 12	104 40	88	345 15	158 40
29	45 38	52 20	59	109 98	106 20	89	406 72	160 20
30	47 10	54 00	60	112 93	108 00	90		

The sixth Rumb from the Meridian.			*East North east, West North west,*			*East South-east. West South-west.*		
La	Long.	Dist.	La	Long.	Dist.	La	Long.	Dist.
Gr	Gr. Par.	Gr. Par.	Gr	Gr. Par.	Gr. Par	Gr	Gr. Par.	Gr. Par.
0	0	0	30	75 98	78 39	60	182 18	156 78
1	2 41	2 61	31	78 78	81 00	61	187 07	159 40
2	4 83	5 23	32	81 61	83 62	62	192 13	162 01
3	7 25	7 84	33	84 48	86 23	63	197 36	164 62
4	9 66	10 45	34	87 37	88 84	64	202 77	167 24
5	12 08	13 06	35	90 30	91 46	65	208 38	169 85
6	14 51	15 68	36	93 27	94 07	66	214 20	172 46
7	16 94	18 29	37	96 27	96 68	67	220 25	175 08
8	19 37	20 90	38	99 31	99 30	68	226 57	177 69
9	21 81	23 52	39	102 40	101 91	69	233 15	180 30
10	24 26	26 13	40	105 53	104 52	70	240 06	182 92
11	26 71	28 74	41	108 71	107 14	71	247 27	185 53
12	29 17	31 36	42	111 93	109 75	72	254 90	188 14
13	31 65	33 97	43	115 20	112 36	73	262 92	190 75
14	34 14	36 58	44	118 53	114 97	74	271 43	193 37
15	36 63	39 20	45	121 92	117 59	75	280 46	195 98
16	39 13	41 81	46	125 36	120 20	76	290 11	198 59
17	41 65	44 42	47	128 87	122 81	77	300 46	201 21
18	44 18	47 03	48	132 44	125 43	78	311 62	203 82
19	46 75	49 65	49	136 09	128 04	79	323 73	206 43
20	49 29	52 26	50	139 81	130 65	80	337 00	209 05
21	51 87	54 87	51	143 60	133 27	81	351 64	211 66
22	54 47	57 49	52	147 47	135 88	82	368 00	214 27
23	57 08	60 10	53	151 44	138 46	83	386 51	216 89
24	59 71	62 71	54	155 50	141 10	84	407 89	219 50
25	62 36	65 33	55	159 66	143 72	85	433 13	222 11
26	65 04	67 94	56	163 93	146 33	86	464 05	224 73
27	67 74	70 55	57	168 31	148 95	87	503 88	227 34
28	70 46	73 17	58	172 80	151 56	88	560 00	229 95
29	73 20	65 78	59	177 42	154 17	89	656 08	232 56
30	75 98	78 39	60	182 18	156 78	90		

The seventh Rumb from the Meridian. — *East and by North, West and by North,* — *East and by South, West and by South.*

La Gr	Long Gr. Par.	Dist. Gr Par.
0	0	0
1	05 02	5 12
2	10 05	10 25
3	15 08	15 38
4	20 12	20 50
5	25 16	25 63
6	30 21	30 75
7	35 27	35 88
8	40 34	41 00
9	45 42	46 13
10	50 52	51 26
11	55 63	56 38
12	60 77	61 51
13	65 92	66 63
14	71 09	71 76
15	76 28	76 88
16	81 50	82 01
17	86 75	87 14
18	92 02	92 26
19	97 31	97 39
20	102 64	102 51
21	108 01	107 64
22	113 42	112 77
23	118 87	117 89
24	124 35	123 02
25	129 87	128 14
26	135 44	133 27
27	141 05	138 40
28	146 71	143 52
29	152 44	148 65
30	158 23	158 23

La Gr	Long. Gr. Par.	Dist. Gr. Par.
30	158 23	153 77
31	64 00	158 90
32	169 96	164 02
33	175 92	169 15
34	181 95	174 28
35	188 04	179 40
36	194 22	184 53
37	200 48	189 65
38	206 82	194 78
39	213 24	199 90
40	219 76	205 03
41	226 37	210 16
42	233 08	215 28
43	239 90	220 41
44	246 84	225 53
45	253 89	230 66
46	261 05	235 79
47	268 36	240 91
48	275 80	246 04
49	283 40	251 16
50	291 13	256 29
51	299 03	261 41
52	307 11	266 54
53	315 37	271 66
54	323 82	276 79
55	332 48	281 92
56	341 36	287 04
57	50 47	292 17
58	359 81	297 30
59	369 45	302 43
60	379 35	307 55

La Gr	Long. Gr. Par.	Dist. Gr. Par.
60	379 35	307 55
61	389 56	312 67
62	400 10	317 80
63	410 98	322 93
64	422 26	328 05
65	433 94	333 18
66	446 03	338 30
67	458 66	343 43
68	471 80	348 55
69	485 52	353 68
70	499 89	358 81
71	514 94	363 93
72	530 79	369 06
73	547 52	374 18
74	565 22	379 31
75	584 03	384 43
76	604 13	389 56
77	625 67	394 69
78	648 91	399 81
79	674 15	404 94
80	701 75	410 06
81	732 25	415 19
82	766 30	420 32
83	804 86	425 44
84	849 38	430 57
85	901 98	435 69
86	966 31	440 82
87	1049 26	445 94
88	1166 11	451 07
89	1366 23	456 20
90	[illegible]	[illegible]

The eighth Rumb of East and West, with the Longitude answering to one deg. of distance, and the distance belonging to one degree of Longitude.

La	Long.	Dist.	La	Long.	Dist.	La	Long.	Dist.
Gr	Gr. Par.	Parts.	Gr	Gr. Par	Parts.	Gr	Gr. Par.	Parts.
0	0	10000	30	1 25	86 60	00	2 00	50 00
1	1 00	99 98	31	1 17	85 71	61	2 06	48 48
2	1 00	99 94	32	1 18	84 80	62	2 13	46 94
3	1 00	99 86	33	1 19	83 86	63	2 20	45 40
4	1 00	99 75	34	1 21	82 90	64	2 28	43 83
5	1 00	99 62	35	1 22	81 91	65	2 37	42 26
6	1 01	99 45	36	1 24	80 90	66	2 46	40 67
7	1 01	99 25	37	1 25	79 86	67	2 56	39 07
8	1 01	99 02	38	1 27	78 80	68	2 67	37 46
9	1 01	98 76	39	1 29	77 71	69	2 79	35 83
10	1 02	98 48	40	1 31	76 60	70	2 92	34 20
11	1 02	98 16	41	1 33	75 47	71	3 07	32 55
12	1 02	97 81	42	1 35	74 31	72	3 24	30 90
13	1 03	97 43	43	1 37	73 13	73	3 42	29 23
14	1 03	97 03	44	1 39	71 93	74	3 63	27 56
15	1 03	96 59	45	1 41	70 71	75	3 86	25 88
16	1 04	96 12	46	1 44	69 46	76	4 13	24 19
17	1 04	95 63	47	1 47	68 20	77	4 44	22 49
18	1 05	95 10	48	1 49	66 91	78	4 81	20 79
19	1 06	94 55	49	1 52	65 60	79	5 24	19 08
20	1 06	93 97	50	1 55	64 28	80	5 76	17 36
21	1 07	93 35	51	1 59	62 63	81	6 39	15 64
22	1 08	92 72	52	1 62	61 56	82	7 18	13 91
23	1 09	92 05	53	1 66	60 18	83	8 20	12 18
24	1 09	91 35	54	1 70	58 77	84	9 57	10 45
25	1 10	90 63	55	1 74	57 35	85	11 47	8 71
26	1 11	89 88	56	1 79	55 92	86	14 33	6 97
27	1 12	89 10	57	1 84	54 46	87	19 11	5 23
28	1 13	88 29	58	1 89	52 99	88	28 65	3 49
29	1 14	87 46	59	1 94	51 50	89	57 30	1 74
30	1 15	86 60	60	2 00	50 00	90		0

These Tables are calculated for each of the Rumbs.

The first seven have three Columns, and of them the first containeth the degrees of Latitude from the Equinoctial to the Pole: the second doth give the difference of Longitude; and the third the distance, both of them belonging to that Rumb and Latitude.

As in the Table of the third Rumb; at the Latitude of 50 *gr.* I find under the title of Longitude 38 *gr.* 69 *parts*, and under the title of Distance 60 *gr.* 13 *parts.* This shews that if the course held constantly on the third Rumb from the Equinoctial to the Latitude of 50 *gr.* the difference of Longitude would be 38 *gr.* 69 *parts* of 100, and the distance upon the Rumb 60 *gr.* 13 *parts.* For here I reckon the distance by degrees, rather than by Leagues or Miles, and subdivided each degree into 100 *parts*, rather than into 60 minutes, for the more ease in Calculation, and withal to make the Calculation to agree the better, both with his, and my *Cross-staff* and other Instruments.

The use of these Tables, for the finding of the difference of Longitude, is this. Turn to the Table of the Rumb, and there see what Longitude belongeth to either Latitude, then take the one Longitude out of the other, the Remainder will be the difference of Longitude required.

As in the former Example, where the places given were A, in the Latitude of 50 *gr.* C in the Latitude of 55 *gr.* and the Rumb the third from the Meridian: I look into the Table of the third Rumb and and there find,

Latitude 50 *gr.*	Longitude 38 *gr.* 69 *parts.*
Latitude 55 *gr.*	Longitude 44 *gr.* 19.
Therefore the difference of	Longitude 5 *gr.* 50.

There is another Use of these Tables, for the describing of the Rumbs both on the Globe, and all sorts of Charts. For having drawn the Circles of Longitude and Latitude, and finding by the Tables, the the difference of Longitude belonging to each Rumb and Latitude: If we make a prick in the Chart, at every degree of Latitude, according to that difference of Longitude, and draw Lines through those Pricks, so as they make no Angles, the Lines so drawn shall be the Rumbs required.

The Use of the Eighth Rumb is something different from the rest.

For

For there being here no change of Latitude, I have set to each Latitude, the difference of Longitude, belonging to one degree of distance, and the distance belonging to one degree of Longitude.

As if two places shall be 20 Leagues, or one degree distant one from the other, in the Latitude of 50 *gr.* the difference of Longitude between them will be 1 *gr.* 55 *parts*. But if they differ one degree in Longitude, the distance between them will be only 64 parts, which fall short of 13 Leagues, or at the most 64 *gr.* 28 parts, such as 10000 do make a degree.

6. *By the difference of Longitude, Rumb, and one Latitude, to find the other Latitude.*

As if the places given were A, in the Latitude of 50 *gr.* C in a greater Latitude, but unknown, the difference of Longitude 5 *gr.* ½, and the Rumb the third from the Meridian.

In the Chart let A B, D C, Meridians, be drawn through A and C, according to the difference of Longitude, one 5 *gr.* ½ from the other; and a Parallel of Latitude through A, crossing the Meridian C D in D: then in A, with A B, make an Angle of the Rumb B A E: so the degrees in the Meridian between D and C, shall be found to be 5 *gr.* the proper difference of Latitude which was required. Wherefore the proportion holds for the *Sector*,

As A D the Radius,
  to D C the Tangent of the Rumb from the Equator;
So A D the difference of Longitude,
  to D C the proper difference of Latitude.

According to this, I take 56 *gr.* 15 *m.* for the Angle of the Rumb from the Equator, out of the greater *Tangent*, and make it a Parallel Radius. Then I reckon 5 *gr.* ½ in the Line of *Lines* from the Center, for the difference of Longitude. So the Parallel taken from the terms of this difference, and measured in the Line of Meridians, shall reach from 50 *gr.* the Latitude given, to 55 *gr.* which is the Latitude required.

Or if the Rumb fall nearer to the Meridian,

As

As B C the Tangent of the Rumb from the Meridian,
is to A B the Radius:
So B C the difference of Longitude,
to A D the proper difference of Latitude.

According to this we may best work by Parallel entrance; first take 33 *gr.* 45 *m.* for the Angle of the Rumb from the Meridian, out of the greater Tangent, and make it a Parallel Radius; then take 5 *gr.* ½ for the difference of Longitude out of the Line of *Lines*, and carry it Parallel to the former, till the feet of the Compasses stay in like Points: so the Line between the Center and the place of this stay, being taken and measured in the Line of Meridians from 50 *gr.* forward, shall shew the Latitude required to be 55 *gr.* as in the former way.

The like may be found by the Tables of Rumbs. For in the Table of the third Rumb, at the Latitude of 50 *gr.* I find the Longitude of 38 *gr.* 69 *p.* To this if I add 5 *gr.* 50 *p.* for the difference of Longitude given, the compound Longitude will be 44 *gr.* 19 *p.* and this answers to the Latitude of 55 *gr.*

But if this difference of Latitude were to be found by the common Sea-chart, it should seem to be 8 *gr.* 13 *m.* and so the second Latitude should be 58 *gr.* 13 *m.* which is above 3 *gr.* more than the truth.

7. *By one Latitude, Rumb, and distance, to find the difference of Longitude.*

As if the places given were A in the Latitude of 50 *gr.* C in a greater Latitude but unknown, the distance upon the Rumb be 6 *gr.* between them, and the Rumb the third from the Meridian.

In the Chart, let a Meridian A B, and a Parallel A D, be drawn through A, and in A, with A B, make an Angel B A C, for the Rumb from the Meridian; then open the Compasses according to the Latitude of the places to E F, the quantity of 6 *gr.* in the Meridian, transferring them into the Rumb from A to C, and through C draw another Meridian D C, crossing the Parallel drawn through A in D; so the degrees intercepted in the Parallel from A to D, shall shew the difference of Longitude required to be about 5 *gr.* ½. Wherefore the proportion holds for the Sector.

As

As A C the Radius,
is to A D, equal to B C, the Sine of the Rumb from the Meri- dian:
So A C the proper diſtance upon the Rumb,
to A D the difference of Longitude.

According to this I take the Sine of 33 *gr.* 45 *m.* for the Angle of the Rumb from the Meridian, and make it a Parallel Radius; then keeping the *Sector* at this Angle, I take 6 *gr.* for the diſtance, out of the Meridian Line, according to the eſtimated Latitudes of both places, and lay it on both ſides of the *Sector* from the Center: ſo the Parallel taken from the terms of this diſtance, and meaſured in the Lines of *Lines*, ſhall ſhew the difference of Longitude to be about 5 *gr.* ½.

In this and ſome of the *Prop.* following, where there is but one Latitude known, there may be ſometimes an error of a minute or two, in the eſtimation of the proper diſtance, yet it may be rectified at a ſecond operation.

This Propoſition may alſo be wrought by the Tables of Rumbs. For according to the Example, in the Table of the third Rumb, at the Latitude of 50 *gr.* I find the Longitude of 38 *gr.* 69 *p.* and the diſtance of 60 *gr.* 13 *p.* to this I add 6 *gr.* for the diſtance given; ſo the compound diſtance will be 66 *gr.* 13 *p.* and this anſwers to the Longitude of 44 *gr.* 19 *p.* then if I take the one Longitude out of the other, the difference will be 5 *gr.* 50 *p.* as before.

But if this difference were to be found by the common Sea-chart, it ſhould ſeem to be only 3 *gr.* 20 *m.* which is more than 2 *gr.* leſs than the truth.

8. *By one Latitude, Rumb, and difference of Longitudes, to find the diſtance.*

As if the places given were A, in the Latitude of 50 *gr.* C in a greater Latitude but unknown, the difference of Longitude between them being 5 *gr.* ½, and the Rumb the third from the Meridian.

In the Chart let A B, D C, Meridians be drawn through A and C, according to the difference of Longitude, and a Parallel of Latitude through A, croſſing the Meridian D C in D; then in A, with A B, make an Angle of the Rumb B A C: ſo the diſtance on the Rumb from A to C taken and meaſured in the Meridian, according to the eſtimated Latitude of the places, ſhall be found to be *6 gr.* Wherefore the proportion holds for the *Sector*.

As A D, equal to B C, the Sine of the Rumb from the Meridian,
is to A C the Radius:
So A D the difference of Longitudes,
to A C the proper diſtance upon the Rumb:

According to this, I take the lateral Radius, and make it a Parallel Sine of 33 *gr.* 45 *m.* which is here the Angle of the Rumb from the Meridian; then I reckon 5 *gr.* ½ in the Lines of *Lines* from the Center, for the difference of Longitude: ſo the Parallel taken from the terms of this difference, and meaſured in the Line of Meridians, according to the Latitudes of the places, ſhall there ſhew the diſtance required to be about 6 *gr.* which are 120 Leagues.

Or if the Rumb fall nearer to the Meridian, that the lateral Radius cannot be fitted over in his Sine, this *Prop.* muſt be wrought by Parallel entrance, and ſo alſo it gives the ſame diſtance as before.

Or we may find this diſtance by the Table of Rumbs. For in the Table of the third Rumb, at the Latitude of 50 *gr.* I find the Longitude of 38 *gr.* 69 *p.* and the diſtance of 60 *gr.* 13 *p.* To this Longitude here found, I add 5 *gr.* 50 *p.* for the difference of Longitude given: ſo the compound Longitude will be 44 *gr.* 19 *p.* and this anſwers to the diſtance of 66 *gr.* 15 *p.* Then if I take the one diſtance out of the other, the remainder will be 6 *gr.* 2 *p.* for the diſtance required.

But if this diſtance were to be meaſured on the common Sea-chart, it ſhould ſeem to be almoſt 10 *gr.* or at the leaſt 197 Leagues, above 77 Leagues more than the truth.

9. *By one Latitude, diſtance, and difference of Longitude, to find the Rumb.*

As if the places given were A, in the Latitude of 50 *gr:* C in a greater Latitude, but unknown, the difference of Longitude between them being 5 *gr.* ½, and the diſtance 6 *gr.* upon the Rumb.

In the Chart let A B, D C, Meridians, be drawn through A and C, and

and a Parallel of Latitude through A; then open the Compaſſes according to the Latitudes of the places, to E F the quantity of *6 gr.* in the Meridian, and ſetting the one foot in A, the other foot ſhall croſs the other Meridian in C: and if we draw the right Line A C, the Angle B A C ſhall ſhew the inclination of the Rumb to the Meridian, to be about *33 gr.* 45 *m.* Wherefore the proportion holds for the *Sector*,

As A C the proper diſtance upon the Rumb,
is to A D the difference of Longitude:
So A C Radius,
to A D, equal to B C, the Sine of the Rumb from the Meridian.

According to this, I take the proper diſtance *6 gr.* out of the Line of Meridians, and lay it on both ſides of the *Sector* from the Center; then I take the difference of Longitude 5 *gr.* ½ out of the Line of *Lines*, and to it open the *Sector* in the terms of the former diſtance: ſo the Parallel Radius taken from between 90 and 90, and meaſured in the Sines, doth give about 33 *gr.* 45 *m*, for the Rumb required.

But if this Rumb were to be found by the common Sea-chart, it ſhould ſeem to be above 66 *gr.* and ſo almoſt the ſixth Rumb from the Meridian.

10. *By the Longitude and Latitude of two places, to find their diſtance upon the Rumb.*

Let the *Sector* be opened in the Lines of *Lines* unto a right Angle (as was ſhewed before *Cap.* 2. *Prop.* 7.) then take out the proper difference of Latitude, and lay it on the one Line, and the difference of Longitude, and lay it on the other line, ſo as they may both meet in the Center, marking how far they extend. For the Line taken from the terms of their extenſion, and meaſured in the Meridian, according to their Latitudes, ſhall ſhew the diſtance required.

So if the places given were A and C, A in the Latitude of 50 *gr.* C in the Latitude of 55 *gr.* the proper difference of Latitude ſhall be the Line A B, and let B C the difference of Longitude be 5 *gr.* ½. we ſhall find that A C the diſtance upon the Rumb is about *6 gr.* which make 120 Leagues.

For in the Chart, let an occult Meridian be drawn through A, and a Parallel of Latitude through C, crossing the former Meridian in B, and a right Line for the Rumb, from A to C, so have we a Rectangle Triangle A B C, whose Base A C, taken and measured in the Meridian from E below 50 *gr.* to F, as much above 55 *gr.* doth contain the quantity of 6 *gr.*

In the same manner the *Sector* being opened to a right Angle, in the Lines of *Lines*; if we take the difference of Latitude out of the Line of Meridians, in his proper place from 50 *gr.* to 55 *gr.* and place it on one of the sides from the Center, to resemble A B, then reckon the difference of Longitude on the other Perpendicular Line from the Center to 5 *gr.* ½, instead of B C, we shall have the like Rectangle Triangle on the *Sector*, to that which we had before on the Chart; and if we take out the Base of it, and measure it in the Line of Meridians from below 5 *gr.* to as much above 55 *gr.* we shall find as before, that it containeth about 6 *gr.* or 120 Leagues.

But if this distance were to be measured on the common Sea-chart, it should seem to be almost 7 *gr.* ¼, or 245 Leagues; which is 25 *leagues* more than the truth.

11. *By the Latitude of two places, and the distance upon the Rumb, to find the difference of Longitude.*

Let the *Sector* be opened in the Lines of *Lines* to a right Angle, then take out the proper difference of Latitudes, and lay it on one of the Lines from the Center, then take the proper distance with a pair of Compasses, and setting one foot in the terms of the difference, turn the other foot to the other Line of the *Sector*, and it shall there shew the difference of Longitude required.

So if the place given were A, in the Latitude of 50 *gr.* C in the Latitude of 55 *gr.* with 6 *gr.* of distance one from another, we shall find their difference of Longitude to be about 5 *gr.* ½.

For in the Chart let a Meridian A B be drawn for the one, and A C, A D, Parallels of Latitude for them both: Then open the Compasses according to the Latitude of the places, to E F the quantity of 6 *gr.* in the Meridians, and setting one foot in A, having Latitude of 50 *gr* turn the other to the Parallel of 55 *gr*, and it shall there cut off the required difference of Longitude B C 5 *gr.* ½.

In

In the same manner, the *Sector* being opened to a right Angle, in the Lines of *Lines* : if we take the difference of Longitude out of the Line of Meridians in his proper place from 50 *gr.* unto 55 *gr.* and place it on one of the Lines from the Center; then take 6 *gr.* the distance upon the Rumb out of the same Line of Meridians, according to the Latitudes of the places, and set the one foot in the term of the former difference, turning the other foot to the other Perpendicular Line, we shall find that it will cross it about 5 *gr.* ½ from the Center, which is the difference of *L*ongitude required.

But if this difference of *L*ongitude were to be found by the common Sea-chart, it would seem to be only 3 *gr.* 20 *m.* which is more than 2 *gr.* 10 *m.* less than the truth.

12. *By one Latitude, distance and difference of Longitudes, to find the difference of Latitudes.*

Let the *Sector* be opened in the *L*ine of *Lines* to a right Angle, and let the difference of *L*ongitude be reckoned in one of those *L*ines from the Center; then take the proper distance with a pair of Compasses, and setting the one foot in the term of the former difference, turn the other foot to the other *L*ine of the *Sector*, and it shall thence cut off a *L*ine, equal to the proper difference of *L*atitude required.

So if the places given were A and C, A in the Latitude of 50 *gr.* C in a greater *L*atitude but unknown, the difference of *L*ongitude between them 5 *gr.* ½, and the distance upon the Rumb 6 *gr.* or 120 *L*eagues, we shall find the difference of *L*atitude to be 5 *gr.*

For in the Chart, let occult Meridians be drawn through A and C, and a Parallel of *L*atitude through A, then open the Compasses according to the estimated *L*atitudes of the places to E F the quantity of 6 *gr.* in the Meridian, and setting the one foot in A, turn the other to the Meridian drawn through C, and it shall there cut off the *L*ine D C, which is the difference of *L*atitude required.

In the same manner, the *Sector* being opened to a right Angle in the *L*ine of *Lines*, if in the one *L*ine we reckon the difference of *L*ongitude from the Center to 5 *gr.* ½, then taking 6 *gr.* for the distance out of the *L*ine of Meridians, according to the *L*atitude of the places, we set the one foot in the term of the given difference, and turn the other foot to the other Perpendicular *L*ine, we shall find that it cuts a *Line* from it, which taken and meafured in the *Line* of Meridians, from 50 *gr.* on forward, doth fhew the difference of *Latitude* to be as before 5 *gr.*

But if this difference of *Latitude* were to be found by the common Sea-chart, it would feem to be only 2 *gr.* 25 *m.* which is 2 *gr.* 35 *m.* lefs than the truth. Such is the difference between both thefe Charts.

THE

THE THIRD BOOK OF THE SECTOR,

Containing the Uſe of the particular Lines.

THe Lines of *Lines*, of Superficies, of Solids, of Sines, with the lateral *Lines* of Tangents and Meridians, whereof I have hereunto ſpoken, are thoſe which I principally intended; that little room on the *Sector* which remaineth, may be filled up with ſuch particular *Lines*, as each one ſhall think convenient for his purpoſe. I have made choice of ſuch as I thought might be beſt prickt on without hindring the ſight of the former, *viz.* Lines of *Quadrature*, of *Segments*, of *Inſcribed bodies*, of *Equated bodies*, and of *Metals*.

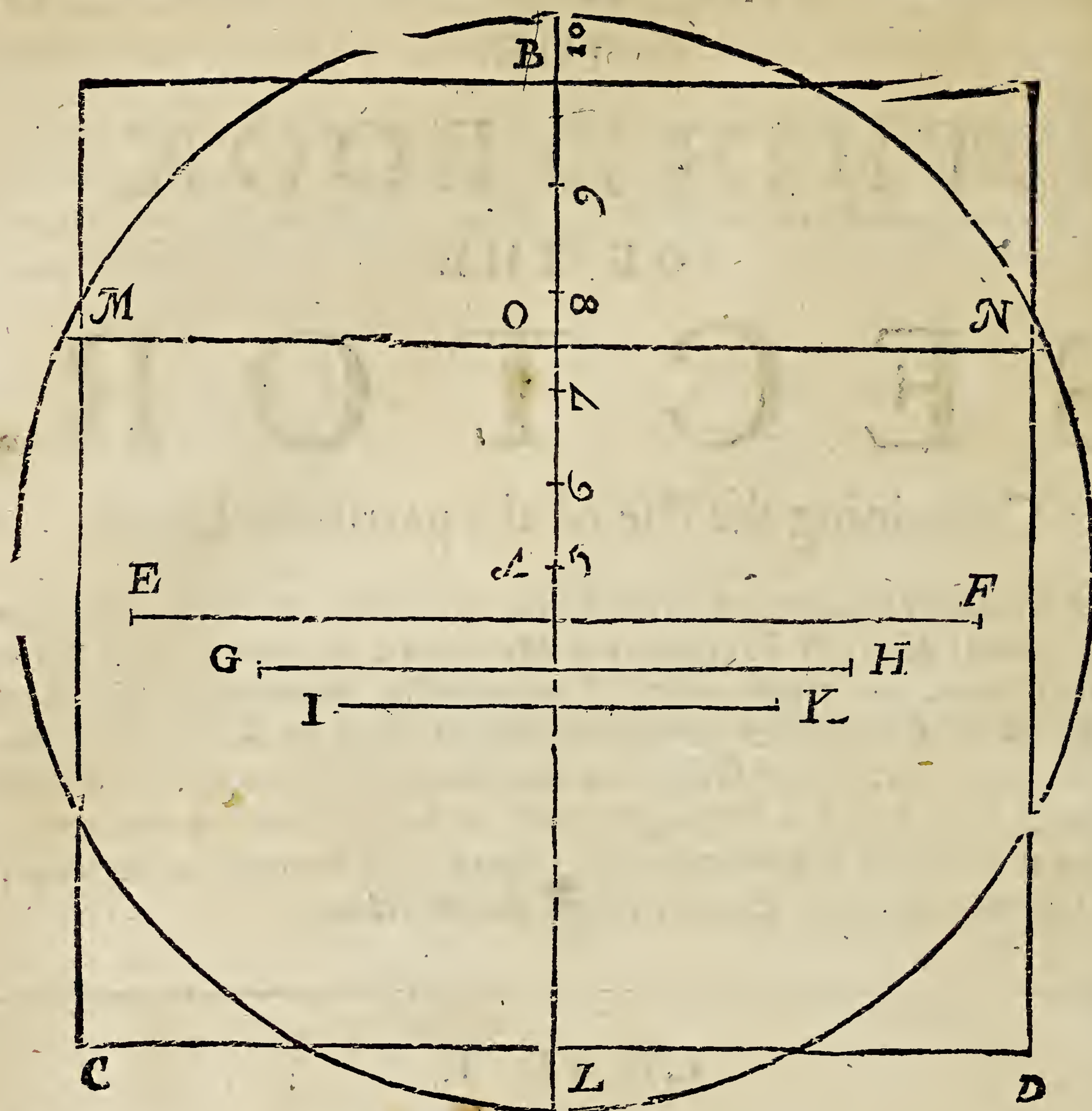
CHAP. I.

Of the Lines of Quadrature.

THe Lines of Quadrature may be known by the letter Q, and by their place between the Lines of Sines. Q ſignifieth the ſide of a Square; 5 the ſide of a Pentagon with five equal ſides, 6 of a Hexagon with ſix equal ſides, and ſo 7, 8, 9, and 10. S ſtands for the Semidiameter of a Circle, and 90 for a Line equal to 90 *gr.* in the Circumference. The uſe of them may be:

1. *To make a ſquare equal to a Circle given:*
2. *To make a Circle equal to a Square given.*

If



If the Circle be firſt given, take his Semidiameter; and to it open the *Sector* in the Points at S: ſo the Parallel taken from between the Points at Q, ſhall be the ſide of the Square required.

If the Square be given, take his ſide, and to it open the *Sector*, in the Points at Q: So the Parallel taken from between the Points at S, ſhall be the Semidiameter of the Circle required.

Let the Semidiameter of the Circle given be A B, the ſide of the Square equal unto it ſhall be found to be C D.

3. *To reduce a Circle given, or a Square into an equal Pentagon, or other like ſided and like angled Figure.*

Take the ſide of the Figure given, and fit it over in his due Points: ſo ſo the Parallels taken from between the Points of the other Figures, ſhall be the ſides of thoſe Figures: which being made up with Equal Angles, ſhall be all equal one to the other.

Let the Semidiameter of the Circle given be A B, the ſide of an *Hexagon* equall to this Circle, ſhall by theſe means be found to be G H; and the ſides of an Octagon to be I K. Other Planes not here ſet down, may firſt be reduced into a Square, by the ſixth *Prop. Superf.* and then into a Circle or other of theſe equal Figures, as before.

4. *To find a right Line, equal to the Circumference of a Circle, or other part thereof.*

Take the Semidiameter of the Circle given, and to it open the *Sector* in the Points at ⁶; ſo the Parallel taken from between the Points at 90 in this Line, ſ[illegible] the fourth part of the Circumference: which being known, the other parts may be found out by the ſecond and third *Prop.* of Lines.

Thus if the Semidiameter of the Circle given be A B, the right Line E F ſhall be found to be the fourth part of the Circumference. Therefore the double of E F ſhall be equal to the Circumference of 180 *gr.* and the half of E F be the Circumference of 45 *gr.* and ſo in the reſt.

CHAP. II.

Of the Lines of Segments.

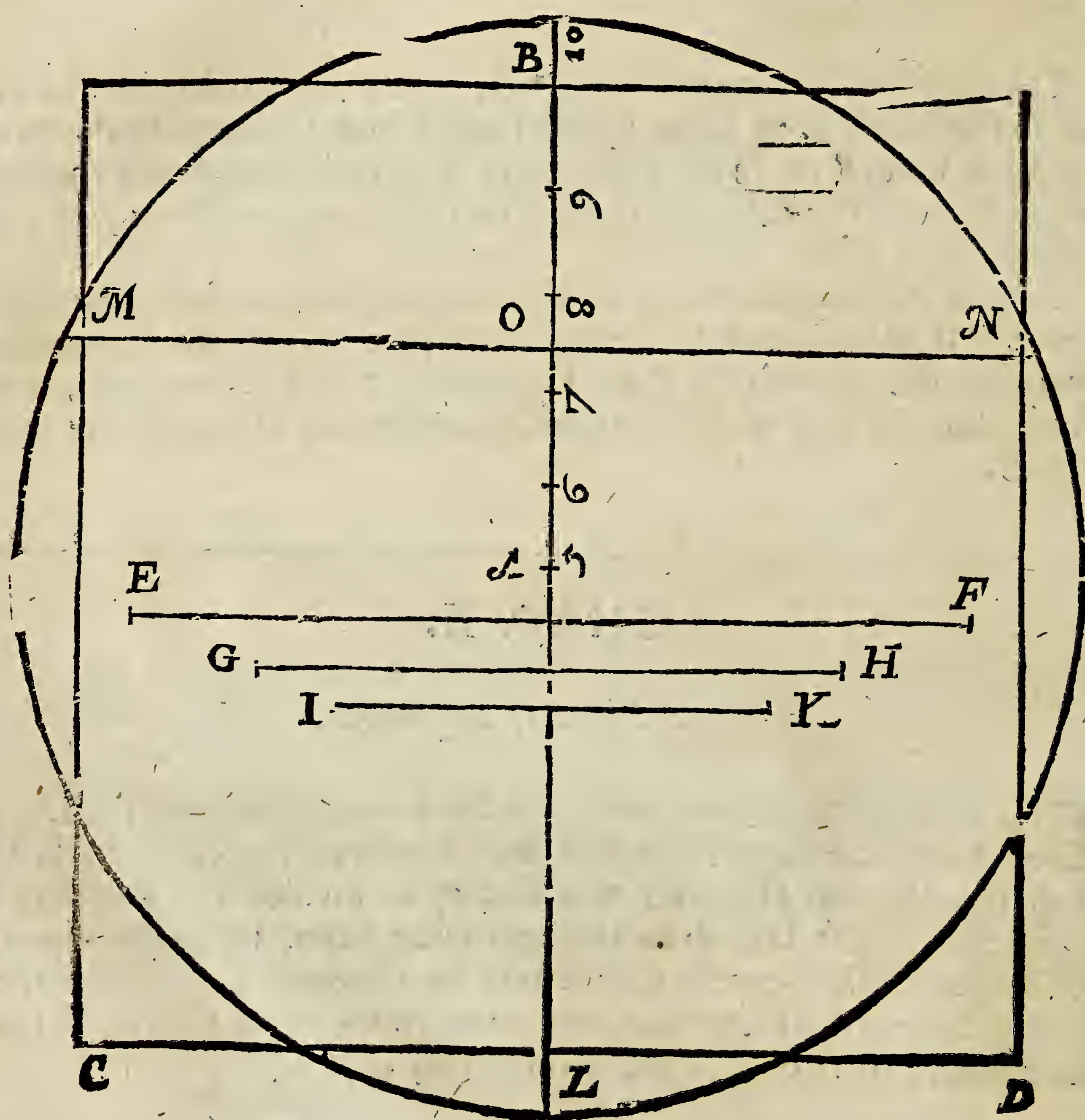
THe Lines of Segments which are here placed between the Lines of Sines and Superficies and are numbred by 5, 6, 7, 8, 9, 10, do repreſent the Diameter of a Circle, ſo divided into a hundred parts, as that a right Line draw through theſe parts, Perpendicular to the Diameter, ſhall cut the Circle into two Segments, of which the greater Segment ſhall have that proportion to the whole Circle, as the parts cut have to 100. The uſe of them may be,

1. *To divide a Circle given into two Segments according to a Proportion given.*

2. *To find a Proportion between a Circle and his Segments given.*

Let the *Sector* be opened in the Points of 100 to the Diameter of the Circle given: so a Parallel taken from the Points proportionable to the great Segment required, shall give the depth of that greater Segment.

Or if the Segments be given, let the *Sector* be opened as before; then take the depth of the greater Segment, and carry it Parallel to the Diameter: so the number of Points wherein they stay, shall shew the proportion to 100.



As if the Diameter of the Circle given were B L, the depth of the greater Segment L O being 75, doth shew the proportion of the Segment O M L N to the Circle, to be as 75, to 100; *viz.* three parts of four.

Hence I might shew, if there were any use of it,

2. *To find the side of a Square, equal to any known Segment of a Circle.*

The side of a Square equal to the whole Circle, may be found by the former *Chap.* and then having the proportion of the Segment to the Circle, we may diminish the Square in such proportion by that which hath been shewed *Lib.*1. *Cap.*3. *Prop.*3.

CHAP. III.

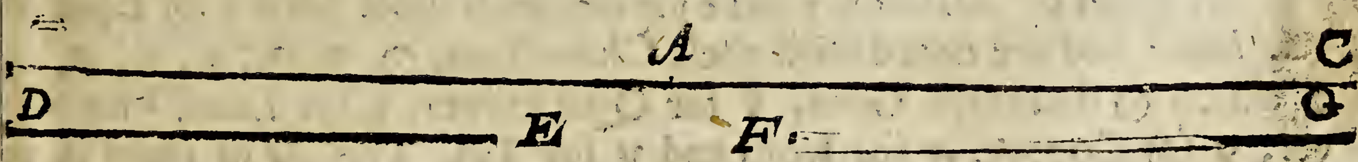
Of the Lines of Inscribed Bodies.

THe Lines of *Inscribed Bodies* are here placed between the Lines of *Lines*, and may be known by the letters D,S,I,C,O,T, of which D signifieth the side of a *Dodecahedron*, I of an *Iscosahedron*, C of a *Cube*, O of an *Octahedron*, and T of a *Tetrahedron*, all inscribed into the same Sphere, whose Semidiameter is here signified by the letter S.

The use of these Lines may be,

1. *The Semidiameter of a Sphere being given, to find the sides of the five regular Bodies, which may be inscribed in the said Sphere.*
2. *The side of any of the five regular Bodies being given, to find the Semidiameter of a Sphere, that will circumscribe the said Body.*

If the Sphere be first given, take his Semidiameter, and to it open the *Sector* in the Points at S: if any of the other bodies be first given, take the side of it, and fit it over in his due Points: so the Parallel taken from between the Points of the other bodies, shall be the sides of those bodies, and may be inscribed into the same Sphere.



So if the Semidiameter of the Sphere be A C, the ſide of the *Dodecahedron* inſcribed ſhall be D E.

CHAP. IV.

Of the Lines of Equated bodies.

THe Lines of *Equated bodies*, are here placed between the Lines of *Lines* and *Solids*, noted with theſe letters D, I, C, S, O, T, of which D ſtands for the ſide of a *Dodecahedron*, I for the ſide of an *Icoſahedron*, C for the ſide of a *Cube*, S for the Diameter of a *Sphere*, O for the ſide of an *Octahedron*, and T for the ſide of a *Tetrahedron*, all equal one to the other. The uſe of theſe Lines may be,

1. *The Diameter of a Sphere being given, to find the ſides of the five regular bodies, equal to that Sphere.*
2. *The ſide of any of the five regular bodies being given, to find the Diameter of a Sphere, and the ſides of the other bodies, equal to the firſt body given.*

If the Sphere be firſt given, take its Diameter, and to it open the *Sector* in the Points at S: if any of the other bodies be firſt given, take the ſide of it, and fit it over in his due Points, ſo the Parallels taken from between the Points of the other bodies, ſhall be the ſides of thoſe bodies equal to the firſt body given.

Thus in the laſt Diagram, if the Diameter of a Sphere given be B C, the ſide of the *Dodecahedron* equal to this Sphere, would be found to be F G.

CHAP. V.

Of the Lines of Metals.

THe Lines of *Metals* are here joyned with thoſe before of *Equated bodies*, and are noted with theſe Characters, ☉, ☿, ♄, ☽, ♀, ♂, ♃, of which ☉ ſtands for Gold, ☿ for Quickſilver, ♄ for Lead, ☽ for Silver, ♀ for Copper, ♂ for Iron, and ♃ for Tin. The uſe of them is to give

give a proportion beween these several Metals, in their magnitude and weight, according to the experiments of *Marinus Ghetaldus*, in his book called *Premotus Archimedes.*

1. *In like bodies of several Metals, and equal weight, having the magnitude of the one, to find the magnitude of the rest.*

Take the magnitude given out of the Lines of Solids, and to it open the *Sector* in the Points belonging to the Metal given: so the Parallels taken from between the Points of the other Metals, and measured in the Lines of Solids, shall give the magnitude of their bodies.

Thus, having Cubes or Spheres of equal weight, but several Metals, we shall find, that if those of Tin contain 10000 D, the others of Iron will contain 9250, those of Copper 8222, those of Silver 7161, those of Lead 6435, those full of Quickfilver 5493, and those of Gold 3895.

2. *In like bodies of several Metals and equal magnitude, having the weight of one, to find the weights of the rest.*

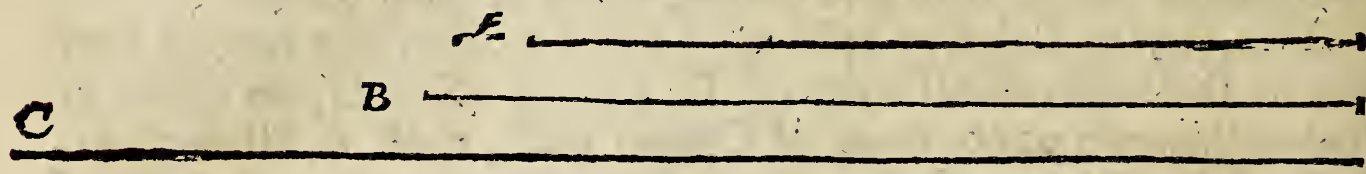
This Proposition is the converse of the former, the proportion not direct, but reciprocal, wherefore having two like bodies, take the given weight of the one out of the Lines of *Solids*, and to it open the *Sector* in the Points belonging to the Metal of the other body: so the Parallel taken from the Points belonging to the body given, and measured in the Lines of *Solids*, shall give the weight of the body required.

As if a Cube of Gold weighed 38 *l.* and it were required to know the weight of a Cube of Lead having equal magnitude. First I take 38 *l*, for the weight of the golden Cube out of the Lines of *Solids*, and put it over in the Points of ♄ belonging to Lead: so the Parallel taken from between the Points of ☉ standing for Gold, and measured in the Lines of Solids, doth give the weight of the leaden Cube required to be 13 *l.*

Thus if a Sphere of Gold shall weigh 10000, we shall find that a Sphere of the same Diameter full of Quickfilver shall weigh 7143, a Sphere of Lead 6053, a Sphere of Silver 5438, a Sphere of Copper 4737, a Sphere of Iron 4210, and a Sphere of Tin 3895.

3. A body being given of one Metal, to make another like unto it of another Metal, and equal weight.

Take out one of the ſides of the body given, and put it over in the Points belonging to his Metal: ſo the Parallel taken from between the Points belonging to the other metal, ſhall give the like ſide, for the body required. If it be an irregular body, let the other like ſides be found out in the ſame manner.



Let the body given be a Sphere of Lead containing in Magnitude 16 *d*, whoſe Diameter is A, to which I am to make a Sphere of Iron, of equal weight: If I take out the Diameter A, and put it over in the Points of ♄ belonging to Lead, the Parallel taken from between the Points of ♂, ſtanding for Iron, ſhall be B, the Diameter of the Iron Sphere required. And this compared with the other Diameter, in the Lines of *Solids*, will be found to be 23 *d*. in magnitude.

4. A body being given of one Metal, to make another like unto it of another Metal, according to a weight given.

Firſt, find the ſides of a like body of equal weight, then may we either augment or diminiſh them according to the proportion given, by that which we ſhewed before in the ſecond and third *Prop.* of *Solids*.

As if the body given were a Sphere of Lead, whoſe Diameter is A, and it were required to find the Diameter of a Sphere of Iron, which ſhall weigh three times as much as the Sphere of Lead: I take A, and put it over in the Points of ♄, his Parallel taken from between the Points of ♂, ſhall give me B for the Diameter of an equal Sphere of Iron: if this be augmented in ſuch proportion as 1 unto 3, it giveth C, for the Diameter required.

CHAP.

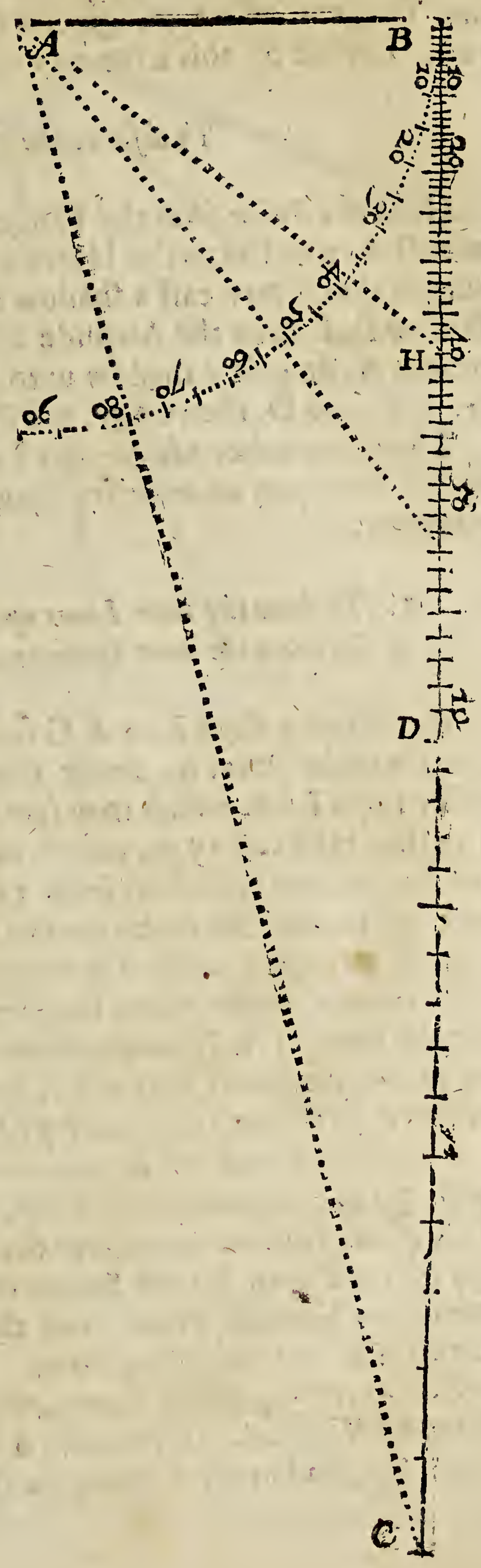
CHAP. VI.

Of the Lines on the edges of the Sector.

HAving ſhewed ſome uſe of the Lines on the flat ſides of the *Sector*, there remain only thoſe on the edges. And here one half of the outward edge is divided into inches, and numbred according to their diſtance from the ends of the *Sector*. As in the Sector of fourteen inches long, where we find 1 and 13, it ſheweth that diviſion to be 1 inch from the nearer end, and 13 inches from the farther end of the *Sector*.

The other half containeth a Line of leſſer Tangents, to which the Gnomon is Radius; They are here continued to 75 *gr.* And if there be need to produce them farther, Take 45 *d*, out of the number of degrees required, and double the remainder: ſo the Tangent and Secant of this double remainder being added, ſhall make up the Tangent of the degrees required.

As if A B being the Radius, and B C the Tangent Line, it were required to find the Tangent of 75 *gr.* If we take 45 *gr.* out of 75 *gr.* the remainder is 30 *gr.* and the double 60 *gr.* whoſe Tangent is B D, and the Secant is A D: if then we add A D to B D, it maketh B C, the Tangent of 75 *gr.* which was required. In like ſort, the Secant of 61 *gr.* added to the Tangent of 61 *gr.* giveth



giveth the Tangent of 75 *gr.* 30 *m.* and the Secant of 62 *gr.* added to the Tangent of 62 *gr.* giveth the Tangent of 76 *gr.* and so in the rest. The use of this Line may be,

To observe the Altitude of the Sun.

Hold the *Sector* so as the Tangent B C, may be Vertical, and the Gnomon B A, parallel to the Horizon; then turn the Gnomon toward the Sun, so that it may cast a shadow upon the Tangent, and the end of the shadow shall shew the Altitude of the Sun. So if the end of the Gnomon at A, do give a shadow unto H, it sheweth that the Altitude is 38 *gr.* $\frac{1}{2}$, if unto D, then 60 *gr.* and so in the rest.

There is another use of this Tangent Line, for the drawing of the hour Lines upon an ordinary Plane, whereof I will set down these Propositions.

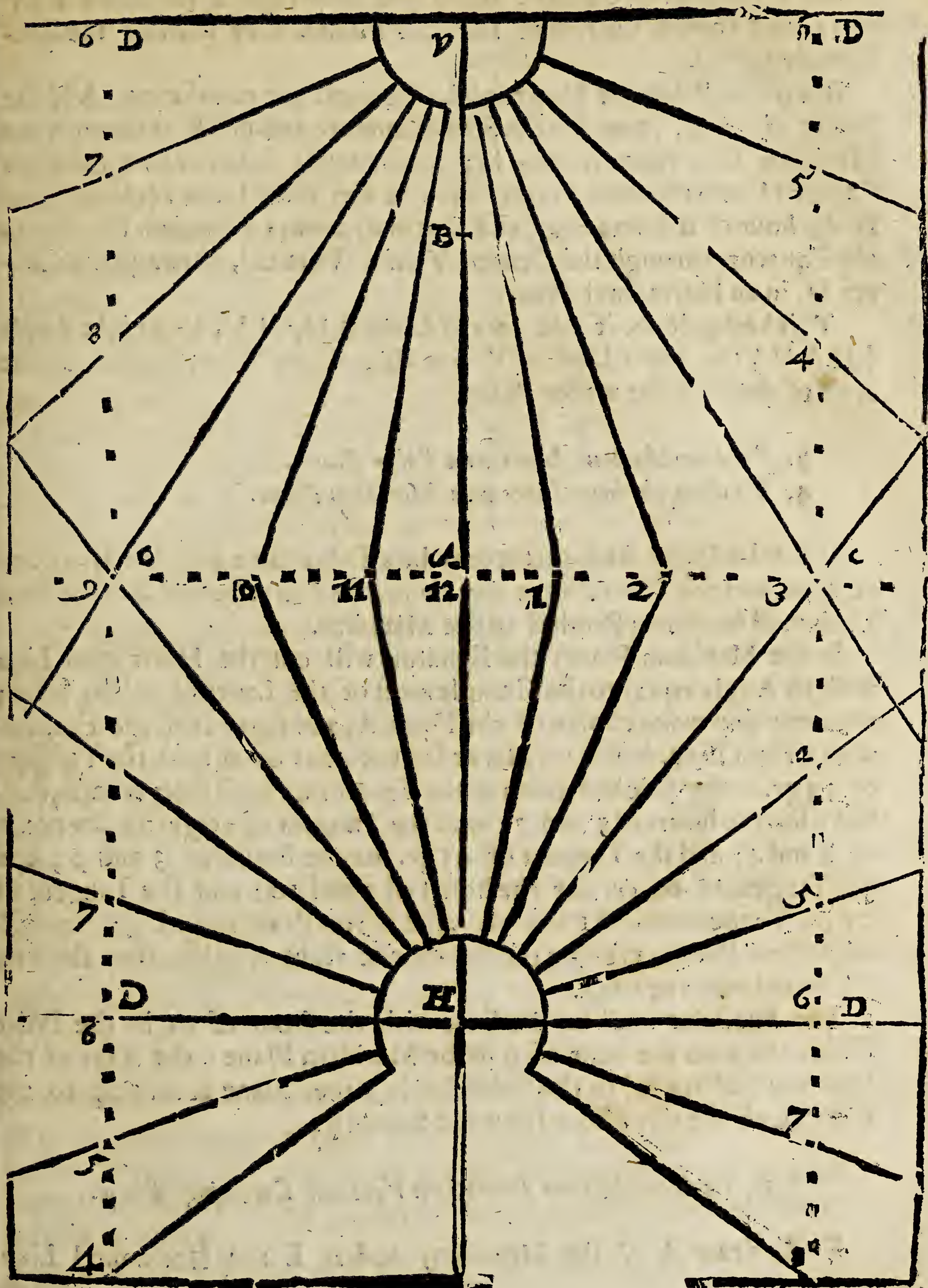
1. *To draw the hour Lines upon an Horizontal Plane.*
2. *To draw the hour Lines upon a direct Vertical Plane.*

First draw a right Line A C for the Horizon, and the Equator, and cross it at the Point A, about the middle of the Line, with A B another right Line, which may serve for the Meridian, and the hour of 12; then take out 15 *gr.* out of the Tangents, and prick them down in the Equator on both sides from 12: so the one Point shall serve for the hour of 11, and the other for the hour of 1. Again, take out the Tangent of 30 *gr*, and prick it down in the Equator on both sides from 12: so the one of these Points shall serve for the hour of 10, and the other for the hour of 2. In like manner may you prick down the Tangent of 45 *gr.* for the hours of 9 and 3, and the Tangent of 60 *gr.* for the hours of 8 and 4, and the Tangent of 75 *gr.* for the hours of 7 and 5.

Or if any please to set down the parts of an hour, he may allow 7 *gr.* 30 *m.* for every half hour, and 3 *gr.* 45 *m.* for every quarter. This done, you are to consider the Latitude of the place, and the quality of the Plane. So the Secant of the Latitude shall be the Semidiameter in a Vertical Plane, and the Secant of the Complement of the Latitude in an Horizontal Plane.

For example, about *London*, the Latitude is 51 *gr*, 30 *m.* and let the Plane be Vertical. If you take A V, the Secant of 51 *gr.* 30 *m.* out of the *Sector*, and prick it down in the Meridian Line from A to V, the

Point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Point V shal be the Center: and if you draw right Lines from V unto 11, and 10, and the rest of the hour Points, they shall be the hour Lines required.

But if the Plane be Horizontal, then you are to take out A H the Secant of 38 *gr.* 30 *m.* for the Semidiameter, and prick it down in the Meridian Line from A unto H; so the right Lines drawn from the Center H unto the hour Points, shall be the hour Lines required; only the hour of 6 is wanting, and that must always be drawn Parallel to the Equator, through the Center V in a Vertical, through the Center H, in an Horizontal Plane.

This being done, if you set the Lines A H, H V, to a right Angle (H A V) the right Line H V the Base of this Triangle shall be the Axis of the style for either Plane.

3. *To draw the hour Lines on a Polar Plane.*
4. *To draw the hour Lines on a Meridian Plane.*

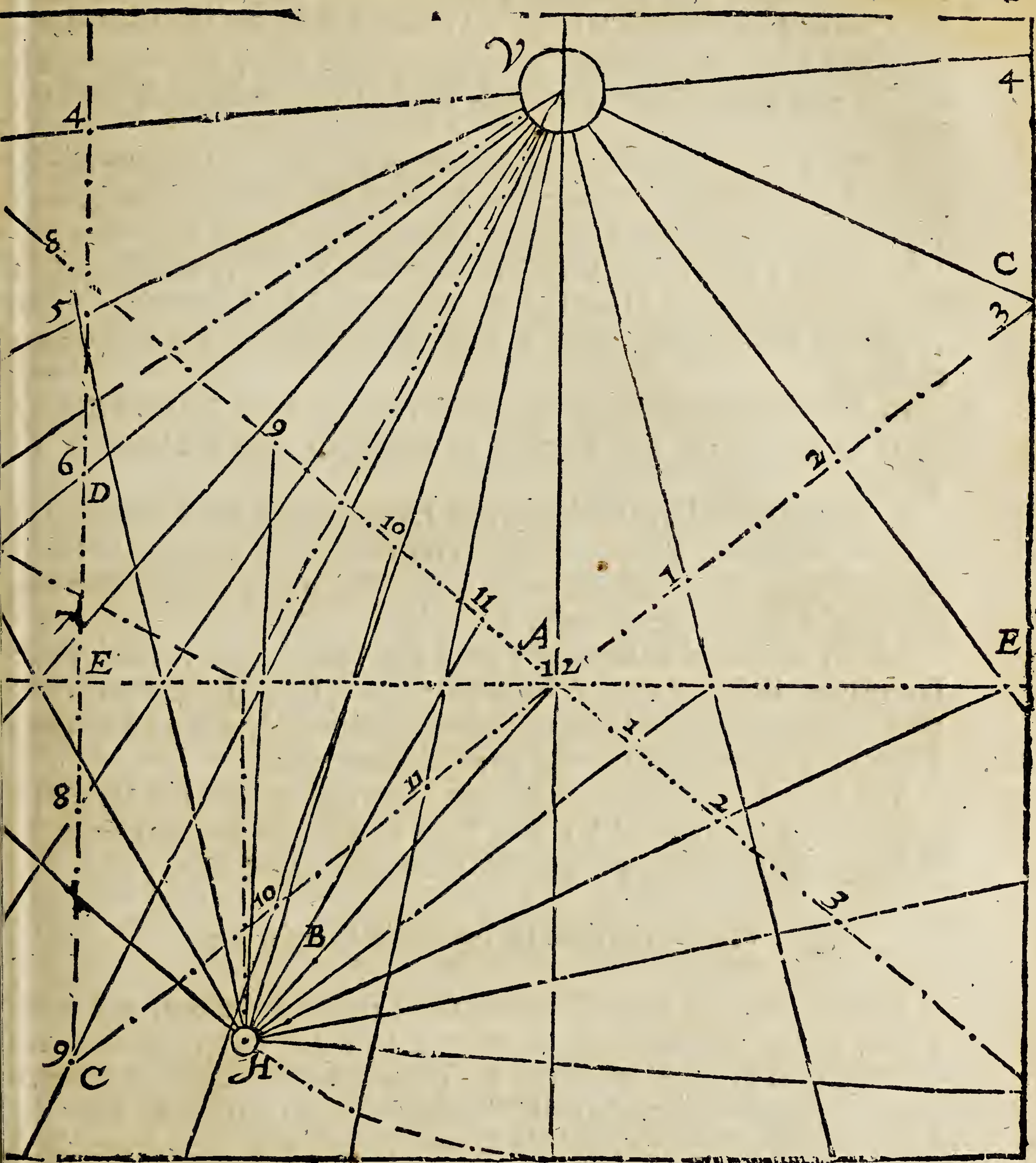
In a Polar Plane the Equator may be also the same with the Horizontal Line, and the Hour Points may be pricked on as before, but the hour Lines must be drawn Parallel to the Meridian.

In the Meridian Plane, the Equator will cut the Horizontal Line with an Angle equal to the Complement of the Latitude of the place; then may you make choice of the Point A, and there cross the Equator with a right Line, which may serve for the hour of 6: so the Tangent of 15 *gr.* being pricked down in the Equator on both sides from 6, shall serve for the hours of 5 and 7; and the Tangent of 30 *gr.* for the hours of 8 and 4, and the Tangent of 45 *gr.* for the hours of 3 and 9; and the Tangent of 60 *gr.* for the hours of 2 and 10; and the Tangent of 75 *gr.* for the hours of 1 and 11. And if you draw right Lines through these hour Points, crossing the Equator at right Angles, they shall be the hour Line required.

The Substilar will be the same with the hour of 12 in the Polar Plane, and with the hour of 6 in the Meridian Plane: the Axes of the stile may be Parallel to the Substilar in either Plane according to the distance of the third hour from the Substilar.

5. *To draw the hour Lines in a Vertical Declining Plane.*

First, draw A V the Meridian, and A E the Horizontal Line, crossing



crossing one the other at right Angles in the point A.

2. Then take out A V, the Secant of the Latitude of the place, which you may suppose to be 51. *gr.* 30 *m.* and prick it down in the Meridian Line from A unto V:

3. Because it is a declining Plane, and you may suppose it to decline

40*gr.* Eaſtward, you are to make an Angle of the Declination upon the Center A, below the Horizontal Line, and to the left hand of the Meridian Line, becauſe the declination is Eaſtward, for otherwiſe it ſhould have been to the right hand, if the Declination had been Weſtward.

4. Take A H, the Secant of the Complement of the Latitude out of the Sector, and prick it down in the Line of Declination from A unto H, as you did before for the Semidiameter in the Horizontal Plane.

5. Draw a Line at full length through the Point A, which muſt be Perpendicular unto A H, and cut the Horizontal Line according to the Angles of Declination, and it will be as the Equator in the Horizontal Plane.

6. Take the hour Points out of the Tangent Line in the *Sector* and prick them down in this Equator on both ſides from the hour of 12 at A.

7. Lay your Ruler, and draw right Lines through the Center H, and each of theſe hour Points: ſo have you all the hour Lines of an Horizontal Plane, only the hour of 6 is wanting, and that may be drawn through H Perpendicular to H A.

Laſtly, you are to obſerve and mark the Interſections, which theſe hours lines do make with A E the Horizontal Line of the Plane: and then if you draw right Lines through the Center V, and each of theſe Interſections, they ſhall be the hour Lines required.

The Line H F draw up to the Horizon, and Parallel to the Meridian, will give the Subſtilar V F: The Line F G drawn Perpendicular to V F, and equal to F H, will give V G, the Axis of the ſtile.

6. To prick down the hour Points another way.

Having drawn a right Line for the Equator as before, and made choice of the Point A, for the hour of 12: you may at pleaſure cut off two equal Lines A 10, and A 2. Then upon the diſtance between 10 and 2, make an Equilateral Triangle, and you ſhall have B for the Center of your Equator, and the Line A B ſhall give the diſtance from A to 9, and from A to 3. That done, take out the diſtance between 9 and 3, and this ſhall give the diſtance from B unto 8, and from B unto 4; again, from 4 to 11, and from 8 unto 1, and alſo from 8 to 7. So have you the hour Points, and if you take out the diſtance B 1, B 3, B 5, *&c.* You may find the Points not only for the half hours, but alſo for the quarters.

But

But if it so fall out, that some of these hour Points fall out of your Plane, you may help your self by the larger Tangent, both in the Vertical, and Horizontal Planes.

For if at the hour Points of 3 and 9, in the Scheme of the Horizontal and Vertical Dials, you draw occult Lines Parallel to the Meridian; the distances D C between the hour Line of 6, and the hour Points of 3 and 9, will be equal to the Semidiameter A V in a Vertical, and A H in a Horizontal Plane, and if they be divided in such sort as the Line A C is divided, you shall have the Points of 4, and 5, and 7, and 8, with their halfs and quarters.

As in the Horizontal Plane, take out the Semidiameter A H, and make it a Parallel Radius by fitting it over in the Sines of 90 and 90: Then take 15 *gr.* out of the larger Tangent and lay them on the Lines of Sines, where they will reach from the Center unto the Sines of 15 *gr.* 32 *m.* therefore take out the Parallel Sine of 15 *gr.* 32 *m.* and it shall give the distance from 6 unto 5, and from 6 unto 7, in your Horizontal Plane. That done, take out 30 *gr.* out of the larger Tangent, and lay them on the Sines, from the Center unto the Sines of 35 *gr.* 16 *m.* and the Parallel Sine of 35 *gr.* 16 *m.* shall give you the distance from 6 unto 4, and from 6 unto 8, in your Horizontal Plane. The like may be done for the half hours and quarters.

So also in the Vertical declining Plane. If you first take out the Secant of the declination of the Plane, and prick it down in the Horizontal Line from A unto E, and through E draw right Lines Parallel to the Meridian, which will cut the former hour Lines of 3 and 9, or one of them in the Point C; then take out the Semidiameter A V, and and prick it down in these Parallels from C unto D, and draw right Lines from A unto C, and from V unto D, the Line V D shall be the hour of 6, and if you divide these Lines A C and D C, in such sort as you divided the like Line D C in the Horizontal Plane, you shall have all the hour Points required.

Or you may find the Point D, in the hour of 6, without knowledge either of H or C. For having prickt down A V in the Meridian Line, and A E in the Horizontal Line, and drawn Parallels to the Meridian through the Points at E, you may take the Tangent of the Latitude out of the Sector, and fit it over in the Sines of 90 and 90: so the Parallel Sine of the Declination measured in the same Tangent Line, shall there shew the Complement of the Angle D V A, which the hour Line of 6 maketh with the Meridian; then having the Point D, take out the Semidiameter V A, and prick it down in those Parallels from D unto C: so shall you have the Lines D C and A C to be divided as before.

The like might be used for the hour Lines upon all other Planes. But I must not write all that may be done by the Sector. It may suffice that I have wrote something of the Use of each Line, and thereby given the ingenuous Reader occasion to think of more.

The Conclusion to the Reader.

IT is well known to many of you, that this Sector was thus contrived, the most part of this Book written in Latine, many Copies transcribed and dispersed more than sixteen years since. I am at the last contented to give way that it come forth in English. Not that I think it worthy either of my labour, or the publick view, but partly to satisfie their importunity, who not understanding the Latine, yet were at the charge to buy the Instrument, and partly for my own ease. For as it is painful for others to transcribe my Copy, so it is troublesome for me to give satisfaction herein to all that desire it. If I find this to give you content, it shall incourage me to do the like for my Cross-staff, *and some other Instruments. In the mean time bear with the Printers faults, and so I rest.*

Gresham Coll. 1 Maij. 1623. *E. G.*

FINIS.

THE SECTOR ALTERED; AND Other SCALES ADDED:

With the Deſcription and Uſe thereof.

Invented and Written by Mr. Samuel Foſter, *ſometime publick Profeſſor of* Aſtronomy, *in* Greſham Colledge *in* London.

And now Publiſhed by *VV. L.*

LONDON,
Printed by *Andrew Clark*, 1673.

THE SECTOR ALTERED.

CHAP. I.

Of the Sector in general.

AMongst the many Writers that have been upon the Sector, Mr. *Gunter* hath done best, the Lines of his Instrument being most in number, and of the most formal contrivance, and most largely Commented upon : yet some Inconveniences have been found in the Use of that Instrument : Partly because his Lines of Tangents, Secants, and Rumbs, or Meridional parts come not from the Center, and so could not in all cases with convenience admit of proportional Works ; and partly because he had no Line of versed Sines, (of which in his Book there is good Use, and might have been much more) but instead thereof he is compelled to use the Line of right Sines, which is but half of the whole Scale of versed Sines, and besides the parts of it stand the contrary way, so that the fitting of the proportional terms whereby to work with halfe the Scale instead of the whole, and then the application of the parts from one end to the other, will be not a little troublesome.

To remedy these and other like defects, I have altered the form of the Sector once more.

1. By diminishing the number of the old Scales, for instead of two of each kind, here is but one.

2. By taking the Meridian Line quite away; and supplying the Use thereof by other means.

3. By bringing the Scales of Tangents and Secants to the Center.

4. By adding a Line of the Versed Sines, and some other Scales of good use.

5. And by changing the form of working upon the Instrument: of all which things I shall give an account in this following Treatise: but first it will be requisite to describe the order and disposition of the Lines, how each of them is to be placed.

CHAP. II.

How the several Lines are disposed upon the Sector.

WHereas in other Sectors there are always two Lines of one kind, upon each Leg one, answering to the like Scale upon the other Leg, in this there is but one Line of one kind, from whence it comes to pass, that one side or flat of this Sector holds all the Scales that are drawn from the Center, and do fill up both sides of the other, and by this means the other side is free for other Scales.

Upon one Leg therefore of the first side are:

1. A Line of equal parts.

2. A Line of Solids, and between these two Scales and the edge, there are inserted two particular Scales more. Namely,

3. Of Inscribed bodies.

4. Of Æquated bodies, with a Scale of Metals. So again, upon the other Leg, there are:

5. The Lines of Sines.

6. The Lines of Superficies, and between them two Scales and the edge are inscribed two other particular Scales: as,

7. The Line of Quadrature. And,

8. Segments. All these Scales are drawn from the Center, and being measured from thence, are all of one length: and do lie at such Angles

Angles one from another, and to the edges of the Sector, will give them convenient distance. So that this one side of the Instrument doth now contain so many Lines of Scales, coming from the Center, as were before on both sides.

Upon the second side of the Sector are four Scales, two upon one Leg, and two upon the other. As namely upon one Leg,

9. Versed Sines, with a Zodiack Line annexed to it.

10. A Line of Tangents going up to 63. *gr.* 26 *m.*

11. A Line of Secants going up to 60 *gr.*

12. A Line of Chords going up to 90 *gr.*

All these are drawn from the Center, and all of one length with those on the other side of the Instrument. The Radius of the Versed Sines, Tangents, and Secant Lines are just half of the whole inscribed Lines, and so will be of very good use in the working of proportions, and in the projecting of the Sphere very commodious.

The descriptions of each Scale may be made by those Tables, and in that manner that Mr. *Gunter* hath directed.

Between these four Scales may be placed other Scales of good use, tending towards (though not running up to) the Center, as a Tangent of three hours of good use in Dialling, and other the like Lines.

Of the other Lines inscribed on the edges and spare places of the Sector.

If the Sector be made of wood, it will require some competent thickness, so that the edges will be large enough to receive some useful Scales also.

The Sector then being opened, and so made a streight Rular; the outer edge hath inscribed upon it the three usual Scales of Logarithmeticall Numbers, Sines and Tangents. The inner edge hath two Scales upon each Leg, one pair of those Scales upon one Leg, is to find the mean Diameter, and one of them is divided into 14 equal parts, the other (of the same length with it) is divided into 20 equal parts, each of them subdivided decimally. The other pair of Scales upon the other Leg is also divided equally, one of them containing four parts, which are to represent feet, and the other being of the same length is divided into 400 parts, representing Inches of the former Feet, and each of these representative, both feet and inches are subdivided decimally. And again, upon the two flat edges of the Sector thus opened (near the outer edge) are inscribed two peculiar Scales (upon one edge) of equal parts for Wine and Ale measure.

Upon the other flat ſide are two Scales more, each equal to the other, both of a juſt foot in length; one is divided into 12 inches, and each inch ſudivided Decimally, the other is divided into 10 equal parts, and each of them again into 10. Theſe two Scales ſerve for true inch and true foot meaſure.

In this manner are the Lines diſpoſed, now follows,

CHAP. III.

The general uſe of the Sector, and the manner of working upon it.

THoſe works that are performed upon the other Sector when it is ſhut, are alſo performed by this, and in the very ſame manner.

But the chief uſe of the Sector is, by having three terms given to find a fourth proportional that the fourth may be to the third as the ſecond is to the firſt. And if the ſecond and third terms fall out to be the ſame, then the proportion is called continual, becauſe the ſecond term is twice repeated, and ſo the next term continued in the ſame proportion to it, that the ſecond was to the firſt. But if the ſecond and third terms be different, then is the proportion called diſcontinual, becauſe the proportion that is between the firſt and the ſecond, though it be made good again between the third and the fourth, yet it diſcontinued between the ſecond and the third terms. Now becauſe this kind is moſt frequent, and the former may be referred to this, (if the ſecond term being twice repeated, be taken as two, namely, as the ſecond and third:) I will ſhew in general

How by three terms given in any kind, to find a fourth.

Firſt, Diſpoſe the three terms given ſo, as that when they are of divers kinds, the firſt and the third may be of one kind, and the ſecond and fourth of another, though this diſpoſition be not always neceſſary, yet for the working upon the Sector it will for the moſt part be convenient.

When the terms are ſo ordered there will three things hence follow. The firſt will be to know upon what Scale the Work will be performed, when the terms are not all of one kind, the other two will be Rules and Directions in what manner to work.

Firſt, Therefore you muſt refer each of the two firſt terms to its proper

proper Scale, then comparing the fecond term with the firft, fee which of them is greateft: For upon that Scale to which the longeft of the two terms belongeth, muft the whole work be performed. Then the two Rules for the manner of working are thefe.

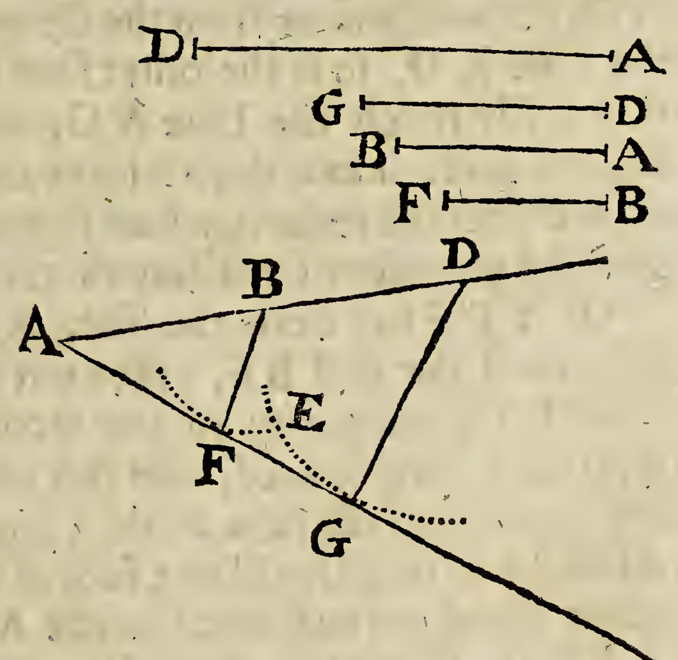
1. If the fecond term be lefs than the firft, you muft then count the firft and the third terms (being both of one kind) laterally upon their proper Scale, and the fecond term being taken out of his proper Scale, and put over parallelly in the term of the firft, fhall open the Leg of the Sector fo, as the fourth term may be taken parallelly over from the term of the third: and being fo taken, it muft be meafured upon the Scale from whence the fecond term was taken, and fo it fhall receive its juft value.

2. But if the fecond term be greater than the firft; you muft then count the fecond laterally, and in the term of it put over the firft parallelly, each being taken in his proper Scale, and this work fhall open the Legs of the Sector fo, as that the third term being taken out of the fame Scale with the firft, and entred parallelly, fhall ftay in that point of the Scale on which the fecond term was counted, and will give the quantity of the fourth term required.

The manner of working then in general, according to thefe two Rules, will be this:

In the firft cafe; where the fecond term is lefs than the firft, let A D be the firft term, D G the fecond, and A B the third.

Count A D the firft term, upon his proper Scale, then with your Compaffes take the fecond term, which we fuppofe to be D G, and fetting one foot of that extent in D, the end of the firft, turn the other foot about, and open or fhut the Sector, till the foot being turned about in the Ark E G, do only touch fome one Line in the other Leg of the Sector, neither going beyond it, nor fhort of it, as here it doth at G, fo is the Sector opened to a true Angle for this Work.



Again, upon the fame Scale A D, whereon the firft term was numbred, count the third term A B; and laftly from B, the extremity thereof, take the leaft diftance to the Scale A G, as here is expreffed by B F. So fhall B F be the length of the fourth term, this Line B F therefore being meafured upon the fame Scale from whence the fecond term D G was taken, fhall give the quantity of the fourth term required.

Or if A B had been the firft term, and B F the fecond, then the Sector muft have been opened by putting over B F from the term B (till the Ark or Motion of the foot of the Compaffes, E F, had only touched the Line A F.) And when the Legs of the Sector are fo opened, count A D the third term upon the fame Scale whereon the firft term A B was counted, and from the extremity of it, at D, take the leaft diftance from the fame Leg A F, which here will be D G, fo fhall D G (being meafured upon the fame Scale of the fecond term) give the quantity of the fourth term required.

In the fecond cafe, when the fecond term is greater than the firft. Suppofe D G be the firft, and A D the fecond, B F the third, take the firft term D G, out of his proper Scale, and count A D the fecond in his proper Scale, then from D the extremity of the fecond term, open the Sector, making D G (when it is turned about in E G) only to touch any one Scale in the other Leg of the Sector, as A G, when the Sector is thus opened, take B F the third term, out of the fame Scale from whence the firft term D G was taken (which is his proper Scale) and keeping one foot of the extent always upon the Line A D, remove it to or from the Center A, till it ftand in fome point of the Line A D, fo as the other foot being turned about in the Ark E F, may juftly touch the Line A G, upon the other Leg, and when you have fo fitted it exactly, obferve the Point in A D, in which the foot of the Compafs refteth, which fuppofe to be the Point B. So fhall A B give the quantity of the fourth term required.

Or if B F had been the firft, A B the fecond, and D G the third, then muft the firft B F, taken out of its proper Scale, have been fet upon B the extremity of the fecond A B, and by it the Line A F G muft have been opened, and this being done, the third term D G being taken from the fame Scale (from whence the firft term B F was taken) and fitted in till one foot of it ftanding upon the Scale A B, the other being turned about in the Ark E G, will only touch the Line A F in G, fo fhall the Point D, wherein then it ftands, give the quantity of A D the fourth term required.

This may serve for a general view of the manner of working upon these single Scales, and how one of a kind may serve to perform any work in this Sector, as well as two have done formerly in other Sectors.

As also here may be seen the manner of Lateral and Parallel entrance, and finding known and unknown quantities: It may likewise be here known what is meant by these Phrases.

1. Opening the Sector to any Line, length, or distance, namely, to open or shut any two Scales upon several Legs of the Sector, till one foot of that length being set in some Point of one Scale, the other foot when it is turned about, may only touch the other Scale, so as not to go beyond it, nor fall short of it.

2. By taking any Line, length, or distance, namely, from some Point in one of the two opened Scales, to take the least from the other Scale. Distance

3. Entring any length or distance, namely, to carry one foot of a length taken in your Compass upon one Scale (from or towards the Center) till the other being turned about, may justly touch the other Scale. These terms are used to avoid needless circumlocution.

It may farther also be observed, that this way of working is more speedy than that upon other Sectors, as by a little practice will quickly be found.

And lastly, the truth of the work will easily appear, if it be considered that in every work thus performed, A B F and A D G are two like Rectangles; as in the other Sector the work went upon two like Equicrural Triangles, in both therefore the ground of the work is alike good, both being grounded upon the similitude of two plain Triangles.

*Now to this general direction for working, I have added examples in several kinds, whereby the Rules before given may the better be understood, and what Mr.*Gunter *and others have published in their Books, may the more easily be applied to this Instrument.*

CHAP.

CHAP. IV.

Examples in ſeveral kinds.

1. *Three numbers 52, 39, 44, being given, to find a fourth proportional.*

THis is wrought upon the Line of equal parts, and becauſe the firſt number is greater than the ſecond, therefore I count the firſt number 52, upon the Line of equal parts, and from the ſame Line I take the ſecond term 39, and ſet it upon 52, and turning the other foot about, I open the other Leg of the Sector, till the ſame foot do juſtly touch ſome one Line on the other leg of the Sector which iſſueth from the Center, neither going beyond it, nor falling ſhort of it, ſo are thoſe two Scales opened fitly to perform the work, then I count the third term of 44 upon the ſaid Line of Lines, and from the end of it to the ſame Scale on the other Leg, I take the leaſt diſtance, this being meaſured in the Scale of Lines, giveth 18 for the fourth term; ſo that as 52 is to 39, ſo 44 to 33.

But if the given numbers had ſtood thus, As 24 to 52, ſo 18 to what? Here becauſe the ſecond number is greater than the firſt, I take 24 out of the Line of Lines, and ſet one foot of it in 52, counted upon the ſame Line, and I open or ſhut the other Leg of the Sector, till the other Foot being turned about, will only touch ſome one Line on the other Leg of the Sector which iſſueth from the Center: When the Sector is thus opened, I take the third number 18, out of the Line of Lines, and keeping one foot always upon the Line of Lines, I remove it ſo long till the other foot being turned about, will only touch the former Line on the other Leg: Then ſhall I find it to ſtay upon the Line of Lines, at the number 39, which is the fourth proportional.

In the ſame manner all proportions in numbers may be wrought by the Lines of Solids and Superficies. But if you had three Lines given, and were to find a fourth proportional Line, you muſt then work upon the Line of Lines only.

2. *Three*

2. *Three Sines being given, to find a fourth proportional Sine.*

THis is to be wrought upon the Line of Sines only. Let the Sines given be 90, 30, and 23 ½; here becauſe the firſt term is greater; therefore I muſt work by the firſt Rule; and ſo the fourth term being taken and meaſured upon the Line of Sines, will be the Sine of 11 ½ *gr.* required.

But if the Sines were of 36, 72, 18 *gr.*. then work by the ſecond Rule, becauſe the ſecond term is greater than the firſt, ſo ſhall your Compaſſes ſtand at the laſt of your work, at the Sine of 30 *gr.* Or becauſe all the four terms are of one kind, you may change the places of the ſecond and third, thus: 36, 18, 72, and ſo working accordingly by the firſt Rule (becauſe the firſt term is greater than the ſecond) you ſhall find the fourth proportional Sine to be 30 *gr.* as before.

In this manner you muſt work when all the four terms are of one kind, and ſo wrought upon one Scale alone. But if the terms be of ſeveral ſorts, then muſt two of the four terms be taken from one Scale, and two from another. As in the examples following will appear.

3. *As the Sine of* 60 *gr. is to the number* 35, *ſo the Sine of* 48 *to what number?*

IN ſolutions of this kind (becauſe the firſt and ſecond, and alſo the third and fourth, are counted upon ſeveral Scales, as here the firſt and third are taken upon the Scale of Sines, and the ſecond and fourth are taken upon the Scale of equal parts:) You muſt firſt trie which is greateſt of the firſt or ſecond terms. As here take the ſecond term 35, out of the Scale of equal parts, and meaſure it upon the firſt term of the Sine of 60. Now becauſe the Sine of 60 is greater, therefore the lateral work muſt be done upon the Scale of Sines, and the ſecond and fourth terms muſt be taken in your Compaſſes from the Scale of equal parts, which is their proper Scale; wherefore in this example take 35 out of the Scale of equal parts, and with one foot of that length ſet in the Sine of 60, open the other Leg till that extent will juſt touch ſome one Line on the other Leg of the Sector which iſſueth from the Center, the Sector being ſo opened, take from the Sine of 48 the leaſt diſtance, to the former Line on the other Leg,

this

this diſtance meaſured upon the Scale of equal parts, ſhall give 30 the number required: Therefore as 60 *gr.* to 35, ſo 48 *gr.* to 30.

But if it had been as the Sine 60 is to the number 90; ſo the Sine of 48 to what? Here if you meaſure the number 90 with the Sine of 60, you ſhall find the number 90 to be the longeſt extent. So that now the lateral work muſt be upon the Line of Lines; I take therefore the Sine of 60 out of the Sines, and ſetting one foot of that extent upon the the number 90 in the equal parts, with the other foot turned about, I open the other Leg, till I ſee the ſame foot only to touch ſome one Line on the other Leg of the Sector, which iſſueth from the Center.

Note that what Line ſoever I begin to work with, I muſt be ſure always to continue and end with the ſame, but that Line on the other Leg, which lieth next the inner edge of the Sector, is always moſt convenient.

Then again I take the Sine of 48, and keeping one foot of that extent continually upon the Line of Lines, I remove the ſame till I find the other foot juſtly to touch the former Line on the other Leg; and then I ſee the other foot to ſtay upon the Line of Lines on the number 77 $\frac{1}{5}$, which is the number ſought.

4. *As the Sine of 60, is to the Tangent of 55 gr. So the Sine of 50 to the Tangent of what Ark?*

First, to know upon what Line to work, I take the Tangent of 55 *gr.* and ſet it to the Sine of 60 *gr.* and becauſe I ſee the Sine of 60 to be the greater, I know that the work muſt be done upon the Line of Sines. And by the firſt Rule accordingly therefore I take the Tangent of 55 *gr.* and from the Sine of 60 I open the Sector to ſome one Line on the other Leg of the Sector, which iſſueth from the Center according to that extent; then I take the leaſt diſtance from the Sine of 50 to the former Line on the other Leg, and meaſuring it upon the Tangent, I find it to reach to the Tangent of 51 $\frac{1}{2}$, which is the Tangent required.

But if the terms were as the Sine of 40 is to the Tangent of 55 *gr.* ſo is the Sine of 50 to what? Then meaſuring the Tangent of 55 *gr.* upon the Line of Sines, and finding the firſt term 40 to be leſſer, I ſee that the work muſt be done upon the Line of Tangents: Wherefore I take the firſt term the Sine of 40, and ſetting one foot of that extent

extent upon the Tangent of 55. by turning the other foot of the Compaſſes, I open the other Leg of the Sector, till the other foot do juſtly touch ſome one Line on the other Leg of the Sector, which iſſueth from the Center; then I take the ſecond term, the Sine of 50 *gr.* and ſetting one foot of that extent upon the Scale of Tangents, untill the other foot being turned about, will juſtly touch the former Line on the other Leg: I find the Compaſſes to ſtay upon the Tangent of 59 ½ *gr.* which is the Tangent required.

The like may be done upon the Sines and Secants, or Tangents or Secants, when any ſuch queſtion ſhall be required. And the like may be done in Tangents and Numbers, or equal parts, by the joynt uſe of theſe two Scales, which is frequent in Menſurations of upright buildings.

5. *Having three numbers, to find a fourth in duplicated proportion.*

THis work is performed by the two Scales of Superficies and Lines joyntly uſed. Let the example be as 32 to 24: ſo 64 to what number in a duplicated proportion? Here the two firſt terms are of one kind, and the two latter will therefore be of one kind. Wherefore to know upon what Scale to work, it will be beſt to change the places of the ſecond and third terms, that ſo the firſt and third may be of one kind, as alſo the ſecond and fourth. Thus as 32 to 64, ſo 24 to what? Now in this diſpoſition of terms you muſt firſt meaſure 32 (taken out of the equal parts) upon the Line of Superficies, and ſo doing you ſhall find it fall far ſhort of the number 64, the ſecond term; therefore it is evident the work muſt be done upon the Line of Superficies, ſo that I take 32 from the Line of equal parts, and putting one foot of that extent upon 64 in the Line of Superficies; I thereby open the Sector to ſome one Line on the other leg of the Sector, which iſſueth from the Center. Then again, I take the third number 24 out of the Line of Lines, and enter it Parallelly between the Superficies and the former Line on the other Leg (in the manner that hath been ſhewed before) and I find it to ſtay at 36 in the Line of Superficies: So that I conclude, as 32 to 64, ſo 64 to 36 in duplicated proportion. That is, ſo is the ſquare of 64 (namely 8) to the ſquare root of 36 (namely 6) in a ſimple proportion.

6. Having two numbers, to find a mean proportional.

THis is performed by the joynt use of Superficies and Lines: Let the Numbers given be 20 and 45. First, I count the first number 20 upon the Line of Superficies, then I take the same number 20 out of the Line of Lines, with this length I open the Sector from the Point 20 in the Line of Superficies, to some one Line on the other Leg, which issueth from the Center; afterwards I count 45 the other given number upon the same Line of Superficies; and from thence take over the least distance to the former Line on the other Leg, this measured upon the Line of Lines, gives 30 for a mean proportional between 20 and 45.

7. Having three Numbers given, whereof the two first are supposed to be in a duplicated proportion, how to find a fourth, unto which the third shall be in the simple proportion of the former; that is, As the square root of the first to the square root of the second.

THis is likewise to be performed by the joynt help of the Lines of Superficies and equal parts. Let the Numbers be, as 25 to 16, so 40 to what? The two first terms are to be counted upon the Line of Superficies, because between them the duplicated proportion is contained; and the other two must be taken upon the Line of Lines, because between them is the simple proportion contained. And to know upon what Line to work, I order the terms so, as the first and third may be of one kind, thus, as 25 to 40, so 16 to what? Now because 25 upon the Line of Superficies (if two upon that Line be taken for 20 as we do here take it) is greater than 40 upon the Line of Lines, therefore the lateral work must be done upon the Line of Superficies. So that I take 40 out of the Line of Lines, and put over that length from 25 in the Line of Superficies unto the Line of Lines upon the other Leg of the Sector. And the Sector being so opened, I count the third Number 16 upon the Line of Superficies, and take over from thence to the same Line of Lines. This length I measure upon the Line of Lines (from whence the second Term was taken) and it reacheth to 32. So that as 25 to 16, supposed to be in a duplicated proportion one to the other, so is 40 to 32 in the simple proportion, whereof that other is duplicated.

8. *Having*

8. *Having three Numbers, to find a fourth in a Triplicated proportion.*

THis work is to be done upon the two Scales of equal parts and Solids joyntly taken together. Let an example be, As 55 to 88, so 125 to what in a triplicated proportion? Here the first two terms are of one kind, and so the two latter are also. Therefore (as before) change the places of the second and third thus; As 55 is to 125, so is 88 to what? The terms then being thus disposed, you must measure 55 (taken out of the Line of equal parts) upon 125 counted in the Line of Solids, and you shall find it of greater length than 125, whereby it is evident, that the work must be done upon the Line of equal parts. Accordingly therefore, take 125 out of the Line of Solids, and setting one foot of that extent upon the Number 55 counted in the Line of Lines, with the other turned about you must open the Line of Sines upon the other Leg of the Sector, as the manner is. Which done, set one foot of your Compass upon 88 in the Line of Lines, and from thence take the least distance from the Line of Sines: This distance being measured upon the Line of Solids, sheweth 512, so that, as 55 to 88 so is 125 to 512 in a triplicated proportion. That is, As 55 is to 88, so is the Cubic root of 125 (namely 5) to the Cubic root of 512 (namely 8) in a simple proportion.

9. *How to find two mean proportionals between two Numbers given.*

THis is done upon the Line of Solids and equal parts joyned in use together; let the two extremes or given Numbers be 512, and 216, between which there are required two mean proportionals. First from the Line of Lines I take 512, and with that extent I open the Point 512 (accounted in the Line of Solids) I take over the least distance of the Line of Sines, and measuring the same upon the Scale of equal parts, I find it to make 384. This last length 384 I take again, and put it over from 512 in the Line of Solids to the Line of Sines. And then I take in length from 216 again (counted in the Line of Solids) unto that same Line of Sines; and measuring this length upon the Line of equal parts, I find it to reach 288, which is the second mean proportional, so that I conclude, as 512 is to 384, so 384 to 288, and so 288 to 216.

10. *Having three Numbers given, whereof the two first are supposed to be in a triplicate proportion, how to find a fourth, unto which the third shall be in a simple proportion, that is, as the Cubic root of the first to the Cubic root of the second.*

THis is to be performed by joynt use of Solids and equal parts. Suppose the three Numbers given to be 704, 297, 98: and let it required to find a fourth Number, unto which 98 shall bear that simple proportion whereof 704 to 297 is the triplicated or Cubical proportion. First, that I may know upon what Line the lateral work is to be performed, I alter the order of the second and third terms, thus, 704, 98, 297: and in this order I compare the first and second terms together; that is, I take 704 out of the Line of Solids which (in this work) is the proper Scale of it, and measure it upon the Line of Lines, which is the proper Scale of the second Number 98; and thereby I find that 98 is the longer, whereby it appears (by the former directions) that the Lateral work is to be done upon the Line of Lines. Wherefore accordingly I take 704 out of the Line of Solids and set one foot of that extent upon 98, counted (as the second term) upon the Line of Lines, and from it I open the Line of Sines upon the other Leg of the Sector. And when the Sector is thus opened, I take the third term (297) out of the Line of Solids again, and put the same over till it fit from the Line of Lines to the Line of Sines; so at last I find one foot to stay upon the foot 73 ½ in the Line of Lines. Whence I conclude, that as 704 is to 297, supposed to be in a triplicated proportion one to the other; So is 98 to 73 ½, which two Numbers do comprehend the fundamental and simple proportion, whereof that other is the triplicated.

CHAP. V.

Of the Scale of Chords.

THough the Scale of Sines will perform all the uses of the Lines of Chords, if every Sine be counted by the double number (as Mr. *Gunter* hath shewed) yet because mistakes are easily made by that numeration;

numeration; therefore it will be more convenient to use the Chords themselves. The uses are chiefly,

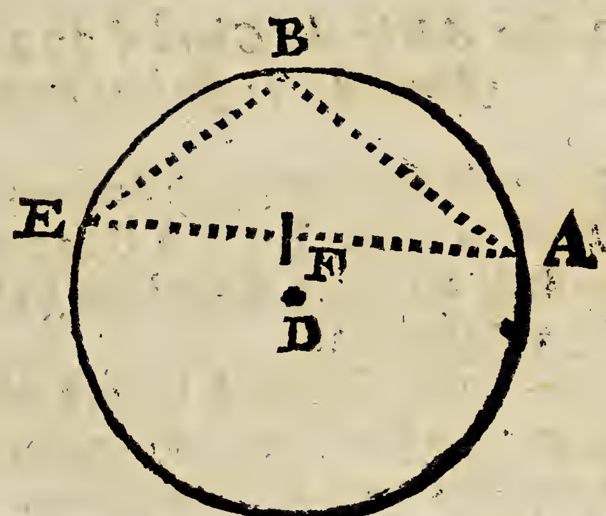
1. To find any Chord, or to set off any Ark or Angle, upon a Circle, whose Radius is given.

2. Having a Radius and any Chord or Ark assigned, to find the Ark which the assigned Chord subtendeth.

3. By having any Chord assigned, to find the Radius according to which the assigned Chord is to be estimated.

1. *The Line of Chords is numbred up to 90, and will therefore set off or measure any Ark within 90 gr. But if the Ark be more, it must do it at twice or more times.*

LEt the Radius A D be given, and let the Circle D A B E be described with it; and let it be required to set off 79 *gr.* from the Point A. First therefore the Sector must be opened to the Radius D A, setting one foot of that extent upon 60 in the Line of Chords, and opening the other Leg till the Compasses being turned about, do only touch some one Line on the other Leg of the Sector, which issueth from the Center: Then from 79 upon the Line of Chords, take the least distance to the former Line on the other Leg, the same being set upon the Circle from A towards B; A B will be an Ark of 79 *gr.* and A D B an Angle of 79 *gr.*



Suppose again that upon the same Circle I should set off 139 *gr.* because this exceeds 90; therefore I divide it into two lesser Arches; namely 79 and 60: First therefore I set off from A to B 79, and then from B to E 60 *gr.* more; which together do make up 139 *gr.* and so those Arks that are greater, may be set off at three or four times.

2. *Having*

2. Having the Radius D A, *and the Chord* A B *assigned, I would know to what number of degrees that Chord answereth.*

I Therefore open the Line of Chords, as before, to the Radius A D, and then enter the Chord B A between the Scale of Chords and some one Line on the other Leg of the Sector, which issueth from the Center; and I find it to stay upon 79 degrees: So that A B is a Chord of 79 degrees, being referred to the Radius D A. But if I had the large Chord A E to the same Radius D A, and would know to what Ark it belongeth; I must first describe the Circle with the Radius D A, and then inscribe the Chord A E into the Circle; afterwards I divide the Ark A E into any two parts, as at B, then take the Chord A B, and enter it upon the Scale of Chords (the Sector being set to the Radius A D) and find it to subtend 79 *gr.* Again, I take B E and do the same thing with that Chord, and find it to answer to 60 *gr.* Then lastly I add 79 to 60, the sum is 139; which gives the whole Ark A E answering to the Chord A E, so if A E had relation to the greater Ark A H E, then you must work as here you did by the Ark A B E, and when that is found to be 139 take it out of a whole Circle, or 360, so shall you find the greater Ark A H E to be 221 *gr.*

3. Let A B be the Chord of 79 gr. *given, and the Radius to which it is estimated to be such a Chord required.*

BEcause the number of degrees is less than 90, therefore the work will be easie. For if A B be put over in 79 in the Line of Chords, then shall 60 in the same Line (rightly taken over) give the Radius A D required.

But if the Chord A E were given, and counted as subtending the 139 *gr.* then it will not be so easie, for if the Line of Chords be used, there will be need of protraction. The better way therefore will be to do it upon the Line of Sines, thus, Take half the Chord A E, namely, A F, and count that as the Sine of half 139 *gr.* that is, 69 ½ *gr.* so putting over this length A F in the Sine of 69 ½ *gr.* you may from 90 take off the Radius required.

Note that if it be required to open any two Lines of the Sector to any number of *gr.* less than 90, or if when the Lines be opened, it be required to know at what Angle they stand. Then it will be the best

way

way to use the Scale of Sines in this manner. Because every Line hath a Point at the very extremity of it, therefore if you take any Sine out of the Scale of Sines, without the forementioned doubling of the Numbers; and from the extremity of one Line, do enter that Sine according to the least distance from the other Line, then shall those two Lines stand at the Angle required.

Or if from the extremity of one Line, you take the least distance of the other, the same measured upon the Line of Sines, shews the Angle at which these two Lines stand. Or for most of the Lines, if it be needful, you may use the way that Mr. *Gunter* shews, *Lib.* 2. *c.* 2. *Prop.*7. *Art.*8.

CHAP. VI.

Of the Tangents and Secants.

BEsides other uses of these two Scales, they serve for Projections, and for Dialling to any bigness greater or smaller, (of which see Mr. *Gunters* Book, concerning Projections of the Sphere, and of Dialling by the Sector, and other ways by a double Tangent of 45.) Hereunto these two Propositions tend.

1. *Having any Line given, as a known Tangent, or Secant: To find the Radius belonging to it.*

SUppose I would know to what Radius any given Line should represent a Tangent of 40 *gr.* I take the Line given, and set one foot of it in 40, in the Scale of Tangents: And from it I open the Sector to some one Line on the other Leg which issueth from the Center, according to the least distance, Then from the Tangent of 45 (which is equal to the Radius) I take the least distance to the former Line on the other Leg, and that length is the Radius required.

The same work is to be done by the Secants, where the Radius is understood to be at the beginning of the Scale.

2. *Having*

2. Having the Radius, to find any Tangent or Secant belonging thereto.

THis is easie and like the former, if first the Sector be opened to the Radius, *&c.*

COROLLARY.

By these two Propositions a way may be found, how by having a Tangent, to find any Secant, or Sine belonging to the same Radius.

FOr first, you must find the Radius, and then the Secant, or Sine afterwards. So also by having the Secant, may be found the Sine or Tangent, or having a Sine, a Tangent or Secant may be found.

And so to a Radius of any length, you may proportion any Sine, Tangent, or Secant. And note, that for pricking down the hour Points in Dials, the Tangents of 15, 30, 45, and 60, will be of frequent use. And if the Tangent or Secant Scales be not far enough extended, Mr. *Gunter* hath given rules how to enlarge them.

CHAP. VII.

How to supply the Meridian Line or Line of Rumbs, by the Scale of Secants.

1. How to make a Sea-Chart, after Mercators *Projection.*

THis Proposition is the chief, that the Meridian Line upon the Sector doth, concerning Navigation, and therefore Mr. *Gunter* makes it his first Proposition. And this is performed by the Line of Secants. For if it were required to project such a Chart as is in his Book. Having drawn his Line A B, and crossed it with the Parallels 50 and 50, at right Angles, you must then take the Secant of 51 *gr.* from the Sector opened to the length you desire, according to the least distance, (the manner whereof hath been shewed enough already,) and set from 50 to 51, on both sides of the Chart, and draw 51 and 51 : Again, take the Secant of 52, from the Sector, and set it upon the Chart from 51

to

to 52, and so draw the Parallel 52, 52. And thus you may draw all the rest of the Parallels. Then for the divisions or Meridians of the Line B C, they are all equal to the Radius, if therefore you take the Radius, and turn it above and below, you shall make the spaces or distances of the Meridians, such as in the bottom of his Chart are figured with 1, 2, 3, 4, 5, 6.

These degrees thus set on, may be subdivided into equal parts, which in the graduations above and below ought so to be, but in the graduation upon the sides of the Chart, they ought as they grow higher, still to grow greater. Yet the difference is so small, that it cannot produce any sensible errour though the divisions be all equal. Divide them therefore equally into 60 minutes, or English miles, or into 20 Leagues, or into 100 parts of degrees, as shall best be liked of.

If a little more curiosity should be stood upon for the graduations of the Meridian, then instead of the Secants, 51, 52, 53. you may take 50 $\frac{1}{2}$, 51 $\frac{1}{2}$, 52 $\frac{1}{2}$, always half a *gr.* less than the Latitude that is to be put in.

2. *The uses of the Sea-Chart, and some other Propositions that concern Navigation, are set down by Mr.* Gunter lib. 2. cap. 6. *of his Sector, which may be here also done.*

THe manner of working upon the Chart (which is the best way) his directions will shew, and how to work them upon this Sector, the former directions in this Treatise will be sufficient. So by this means the use of the Meridian Line is fully supplied, because each Degree may be very large, which in the other Sector could not be so without each part many times repeated, which thing will produce as much errour as this way by the Secants can do.

CHAP. VIII.

The uses of the Line of Versed Sines.

THe Use of it is generally as much as the fourth Axiom of *Spherical Trigonometrie.* Wherefore, I will first shew how the two cases of that Axiom may be performed by this Line. And afterwards how some particular Problems of more frequent use may be performed..

1. *Having two ſides of a Spherical Triangle, and the Angle comprehended, to find the Baſe.*

First, find the ſum and difference of the two ſides or Legs, then count that ſum and difference upon the verſed Scale, and with your Compaſſes, take their diſtance, with this diſtance from the end of the verſed Line, open the Sector to ſome one Line on the other Leg, which iſſueth from the Center afterwards upon the verſed Line, count the Angle given, and ſetting one foot in that number, take the leaſt diſtance from it to the former Line on the other Leg, this length being added to the difference of the Legs upon the verſed Scale, gives the Baſe required.

So if the Legs were 38 *gr.* 30 *m.* and 95 *gr.* their ſum would be 133 *gr.* 30 *m.* their difference would be 56 *gr.* 30 *m.* And now the diſtance of theſe being taken, and the Sector opened as is preſcribed, ſuppoſe the Vertical Angle were 75 *gr.* from 75, therefore take the leaſt diſtance, to the former Line on the other Leg, and ſet that diſtance on the verſed Scale from the former difference 56 *gr.* 30 *m.* numbred thereon, it will fall upon 84 *gr.* 42 *m.* which is the quantity of the Baſe of the Triangle.

An Example.

Two places differing both in Longitude and Latitude, to find their Diſtance.

Let the two places be *London* and *Hieruſalem*, the Latitude of *London* is 51 *gr.* 30 *m.* the Latitude of *Hieruſalem* is 32 *gr.* the difference of Longitude 47 *gr.* their diſtance is required,

The ſum of the Complement of the two Latitudes is	96 *gr.*—30 *m.*
Their difference	19——30.

Take the diſtance between 96 *gr.* 30 *m.* and 19 *gr.* 30 *m.* with this extent of the Compaſſes, open the Sector from the end of the verſed Scale; the Sector thus reſting, take the neareſt diſtance from the difference of Longitude 47 *gr.* this diſtance being applied to 19 *gr.* 30 *m.* on the verſed Scale, will reach to 39 *gr.* 14 *m.* the diſtance required.

2. *Having*

2. *Having the three sides of any Spherical Triangle to find the Vertical Angle.*

THat Angle that is required,is called the Vertical Angle; The side opposed to it.is called the Base; and the other two sides are called the Legs.

1. Find the sum and difference of the two Legs, then count both the sum and difference upon the Scale of versed Sines,and with this distance taken in your Compasses, from the furthest end of the versed Scale, open the Sector to some Line on the other Leg, which issueth from the Center, as the manner hath been. Afterward take the distance from the forenamed difference of the Legs to the Base, counted upon the same Scale, this distance is to be applied to the two Scales before opened, and now appointed for the work, so as that the Compass foot be removed upon the versed Line, till the other being turned about may just touch the former Line on the other Leg, and where the foot of the Compass (with this condition) shall stay upon the versed Line, there shall you see the quantity of the Vertical Angle required. Or if after the Sector be opened, you take the distance, not from the difference, but from the sum of the Legs to the Base, that distance will find the Supplement of the Vertical Angle, which in some cases is most required.

So if the two Legs were 38 $\frac{1}{2}$ *gr.* and 95 *m.*the Base 84 *gr.* 42 *m.* the Vertical Angle will be found to be 75 *gr.* Or the Supplement will be found to be 105 Degrees.

These two Propositions thus generally propounded, do (in brief) set forth two of the principal uses of this Scale of Versed Sines. And to these two I will add one more, which is done without opening the Sector at all.

3. *Having a proportion to be wrought in Sines alone, wherein the Radius leads in the proportion, how to find a fourth proportional Sine upon this Versed Scale.*

TAke the sum and difference of the second and third Arks; count them upon the Scale and take the difference of them; if you set this distance equally remote from 90 upon the Scale on both sides of it, you shall see both the feet of the Compasses to stay upon the fourth proportional Sine required. Suppose the proportion to stand thus. As the Radius, to the Sine of 60 *gr.* so the Sine of 40 *gr.* to what Sine? The sum and difference of the two given Arks, 60 and 40 are 100 and 20. I take the distance of these two counted upon the Scale, and set it equally distant on both sides from 90, and I find it to stay in 33 $\frac{1}{6}$ from 90. Wherefore I conclude that as the Radius is in proportion to the Sine of 60*gr.* so the Sine of 40*gr.* to the Sine of 33 $\frac{1}{6}$*gr.*

As for the former general Problems, that their usefulness may be more manifest, I will here add three Propositions deduced from them, which are of daily use, and by other general Instruments performed with much difficulty.

The first shall be, *To get the Suns Azimuth.*

The second, *To find the hour of the day.*

The third, *To find the Suns Altitude at any hour.*

The first of these is,

1. *Having the Latitude of the place, the degree of the Sun in the Zodiac, and the Suns Altitude above the Horizon, to find the Suns Azimuth either from the North, or from the South.*

BEcause this Proposition is so very useful in many particulars, therefore principally is the Zodiac Line annexed to this versed Scale, and therefore also do I set it in the first place.

This falls under the second general case delivered before; The two Legs of the Triangle are the Complement of the Latitude, and Complement of the Suns Altitude. The Base is the Suns place in the Zodiac, taken from the beginning of the Line. The Vertical Angle in these Northern Latitudes, is the Azimuth from the North.

Take the Sum and difference of the Complement of the Latitude, and the Complement of the Suns Altitude, and count this sum and differende upon the versed Scale, and with your Compasses take their distance, with that distance open the Sector to some one Line on the other Leg, which issueth from the Center, from the end of the versed Line. Afterwards take the distance from the aforenamed difference, to the Suns place, and enter it between the former Line on the other Leg, and the versed Scale, and note the Point of the versed Scale on which the foot of the Compass stays, the same Point shews the Azimuth from the North. Or when the Sector is opened, take the distance from the sum, to the Suns place, the same entred as before, will give the Azimuth from the South. Note that if the sum do exceed 180 *gr.* then you are to account 170 as 190, 160 as 200, 150 as 210, *&c.*

Example.

IN the Latitude 51 *gr.* 30 *m.* The Sun being in the begining of *Taurus*, and the Altitude 35 *gr.* The Complement of the Latititude is 38½ *gr.* The Complement of the Altitude is 55 *gr.* The sum of these two is 93 *gr.* 30 *m.* their difference is 16 *gr.* 30 *m.* I take the distance of these two counted upon the versed Scale, and with it do open the Sector to some one Line on the other Leg which issueth from the Center, from the end of the versed Scale: then I take the distance from the sum, 93½ *gr.* to the Suns place oo *Taurus*, and enter it upon the versed Scale, to the former Line on the other Leg, and find the foot of the Compass to stay at 60 *gr.* 42 *m.* which is the Azimuth required.

COROLLARY.

The same things given, *To find the Amplitude of the rising and setting of the Sun.*

IF you suppose the Sun to have no Altitude above the Horizon, and so the Complement of it to be 90, and then work by the former precept, the first way shews the Amplitude from the North, and the second way shews the Amplitude from the South. And if either of these two numbers be numbred from the middle of the Line noted with 90, you shall have the Amplitude from the East or West. So to the beginning of *Taurus*, I shall find the Amplitude to be 108 *gr.* 41 *m.* from the South: 71 *gr.* 19 *m.* from the North: and 18 *gr.* 41 *m.* from the East or West.

A ſecond Example is,

The Latitudes of two places, and their diſtance being given, to find their Difference of Longitude.

LEt the two places be *London* and *Hieruſalem,* the Latitude of *London* 51 *gr.* 30 *m.* of *Hieruſalem* 32 *gr.* their diſtance 39 *gr.* 14 *m.* and their difference of Longitude required.

The ſum of the Complements of the two Latitudes, 96 *gr.*—30 *m.*
Their difference 19——30

Take the diſtance between this ſum and difference, and open the Sector from the end of the Verſed Scale, then take the diſtance from the difference 19 *gr.* 30 *m.* to the diſtance given, *viz.* 39 *gr.* 14 *m.* where that fits over from the Verſed Scale, which will be at 47 *gr.* is the difference of Longitude required. The next thing is,

Having the Latitude, the Suns place in the Zodiac, and the Altitude above the Horizon, to find the hour from the South.

THis alſo falls under the ſecond general caſe before delivered. The two Legs of the Triangle, are the Complement of the Latitude, and the Suns diſtance from the elevated Pole : the Baſe is the Complement of the Suns Altitude above the Horizon ; the Vertical Angle in theſe Northern Latitudes is the hour from the South, or Mid-day.

Firſt, Count the number of *gr.* from the beginning of the Scale to the Suns place, this number compare with the Complement of the Latitude, and find the ſum and difference of them. Then upon the Verſed Scale count this ſum and difference, and take the diſtance of them, and with that diſtance open the Sector, as is preſcribed in the former Propoſition. Then take the diſtance of the Complement of the Suns Altitude, and the afore-named difference, and entring it upon the Verſed Scale from ſome one Line on the other Leg, which iſſueth from the Center, you ſhall find it to ſtay upon a number of degrees, which turned into time, gives the hour required. One hour anſwers to 15 *gr.* one degree makes four minutes of time. Note here alſo,

also, that if the sum do arise to above 180 *gr.* you are then to account 170 as 190, 160 as 200, 150 as 210, as was before intimated.

Example. From the beginning of the Scale to oo ♉ is 78 ½ *gr.* this I compare with 38 ½ *gr.* and I find the sum of them to be 117 *gr.* and the difference to be 40 *gr.* then I count these two numbers upon the Versed Scale, and take their distance; with this distance, I open the Sector from the end of the Versed Scale, to some one Line on the other Leg. Again, I take the length from 40 *gr.* (the fore-named difference) to 55 *gr.* the Complement of the Altitude (which I suppose to be the same that was in the former example) and this length I enter upon the Versed Scale, from the former Line on the other Leg, and find the foot of the Compasses to stay in 46 *gr.* 48 *m.* This Ark contains 3 hours and 7 ⅕ *m.* of an hour, and so much is the hour at that time from noon. If the Altitude therefore were observed in the morning, it must be 53 *m.* past 8 of the clock; if in the after-noon, it is 7 *m.* past 3 a clock.

Or if I had taken the length from the fore-named sum 117 *gr.* to 55 *gr.* the Complement of the Altitude, and had entered the length, as before, I should then have found the Supplement of the former, namely, 133 *gr.* 12 *m.* which is the hour from mid-night, namely, eight hours and 53 *m.* which is the hour of the day, if the Suns Altitude were taken in the morning, or else the Complement of that hour to 12, namely, 3 hours 7 minutes, is the hour of the day, if the observation were made in the afternoon.

1. Corollary.

To find the Semidiurnal and Seminocturnal Arks.

If you suppose the Altitude to be 00, and so the Complement of it to be 90, and then work by this Precept, you shall find the Semidiurnal Ark from the beginning of the Line, and the Seminocturnal Ark from the end of the Line, numbered in degrees, and each of those turned into hours and minutes, and doubled, will shew the length of the day and night.

And if from the degrees of the Semidiurnal Ark, you take 90, you shall have the Ascensional difference in degrees; or if you take six hours out of the Semidiurnal hours, you shall have the same Ascensional difference in time.

Example.

Example. In the same Latitude, and the 00 *gr.* of ♉, the Semidiurnal Ark will be 104 *gr.* 49 *m.* These doubled, make the length of the day 209 *gr.* 38 *m.* Or the same turned into hours, make 6 hours 59 minutes, and these doubled, make the length of the day 13 hours 58 ½ minutes. The Seminocturnal Ark is 75 *gr.* 11 *m.* or 4 hours 0 ¼ minutes. These doubled, make the length of the night 150 *gr.* 22 *m.* or 10 hours 1 ½ minutes of an hour.

The Ascensional difference is 14 *gr.* 59 *m.* or 00 hours 59 ¼ of an hour.

2. *COROLLARY.*

To find the moment of time, when the Crepusculum *begins and ends.*

IF you suppose the Sun to be 18 *gr.* below the Horizon, and so take from the former difference of the Legs, down to 108, and enter that length as before, you shall find what time from the mid-day the two twy-lights begin and end.

Example. In the beginning of *Taurus*, the morning twy-light begins 139 *gr.* 40 *m.* before noon, that is 9 hours 18 ½ minutes, or at 41 ⅔ minutes past two a clock in the morning, and the evening twy-light ends 139 *gr.* 40 *m.* after-noon, or at 18 ⅔ past nine a clock at night.

3. *COROLLARY.*

The Suns place being assigned in any Point of the Zodiac, to find his Altitude at all hours.

THis Problem falls under the first general Case before delivered. The two Legs of the Triangle are the Complement of the Latitude, and the Suns distance from the Elevated Pole. The Angle intercepted between them is each hour from the South, whose Altitudes are required. The Base is the Complement of the Altitude sought for.

First, Find the sum and difference of the Complement of the Latitude, and the Suns distance from the elevated Pole, count both this sum and difference upon the Versed Scale, and take the distance of them, and open the Sector to some one Line on the other Leg, which

issueth

iſſueth from the Center: to that diſtance, from the end of the verſed Scale. Then count every hour upon the verſed Scale (allowing 15 *gr.* to an hour) and from thoſe hour-points take the leaſt diſtance to the former Line on the other Leg, theſe diſtances being ſet from the afore-named difference of the Legs outword upon the verſed Scale, will give the Complement of the Altitudes to each ſeveral hour from the Meridian. Or if they be numbered from 90 in the Scale, to the foot of the Compaſſes neareſt to 180 upon the Scale, you ſhall have the Altitudes themſelves.

Note, that if you go quite through every fifteenth degree, or every of the twelve hours upon the Scale, you ſhall go beyond 90, and thoſe degrees beyond 90 are the profundities of the night-hours, the Sun being in that degree of the Zodiac. And they are alſo Altitudes of the hours for the Suns being in the oppoſite degree of the Zodiac. So that one opening of the Sector will ſerve to find the Altitudes of all the hours in any two oppoſite Signs or Points in the Zodiac. Note alſo, thot the difference of the Legs is the Complement of the Suns Altitude at 12 a clock at noon, and the ſum of them being diminiſhed by 90 *gr.* is the depth at mid-night or the mid-day Altitude of the Sun, when he is in the oppoſite Sign or degree.

Note laſtly, (as formerly) that if the ſum of the two Legs do amount to above 180, you muſt then count 170 for 190, 160 for 200, 150 for 210, *&c.* as was noted before.

Becauſe this Propoſition is ſo frequent in uſe for the making of Tables of the Suns Altitude in every Sign, or any Parallel of Declination, which ſerve for drawing particular inſtrumental Dials, as Quadrants, Rings, and Cylinders, and for all other purpoſes alſo, I will therefore add one example at large, to make it the more plain.

Example. In our Latitude 51 *gr.* 30 *m.* the Sun being 00 *Taurus*, I would know the Suns Altitudes at every hour of the day, and the profundities of the Sun at every hour in the night. The Complement of the Latitude is 38 *gr.* 30 *m.* and the Suns diſtance from the North Pole is 78 *gr.* 30 *m.* the ſum of theſe is 117 *gr.* the difference of them is 40 *gr.* Firſt then, I count theſe two numbers upon the verſed Scale, and take their diſtance, with this diſtance I open the Sector to ſome one Line on the other Leg, which iſſueth from the Center, from the end of the Verſed Scale. Then I count 15 upon the verſed Scale, and from thence I take the leaſt diſtance, to the former Line on the other Leg. One foot of this diſtance I ſet upon the difference of the Legs (which was 40 degrees.) The other I ſet forward upon the verſed Scale, and where it falls, it ſhews 41 *gr.* 48 *m.* the Complement of the Suns Altitude at 11 and 1 a clock, or counting it from 90, it ſhews 48 *gr.* 12 *m.* the Altitude it ſelf at 11 and 1 a clock.

So again, I count 30 upon the verſed Scale, and take the leaſt diſtance to the former Line on the other Leg, and ſet one foot upon the difference of the Legs (*viz.* 40. *gr.*) the other forwards upon the verſed Scale. I find it to fall upon 46 *gr.* 48 *m.* which is the Complement of the Suns Altitude at 10 and 2 a clock; or counting it from 90, it falls upon 43 *gr.* 12 *m.* the Altitude it ſelf, at 10 and 2 a clock, or from 90 *gr.* it falls upon 43 *gr.* 12 *m.* the Altitude it ſelf.

In the ſame manner taking the leaſt diſtance from 45 *gr.* to the former Line on the other Leg, and ſetting one foot of that diſtance to the difference of the Legs, you ſhall find the other to fall upon 54 degrees, which is the Complement for the Altitude for 9 and 3 a clock.

And ſo working from 60, the Compaſſes will ſhew the Complement of the Altitude 62 *gr.* 29 *m.* and the Altitude it ſelf 27 *gr.* 31 *m.* for the hour of 8 and 4.

And at 75 degrees, having with your Compaſſes taken the leaſt diſtance, and ſet it as before to the difference of the Legs, will give 18 *gr.* 18 *m.* for the Altitude of 7 and 5 a clock.

And at 90, or 6 a clock, the Altitude will be 9 *gr.* 00 *m.*

So working ſtill in the ſame manner, from 105, upon the Verſed Scale, you ſhall find your Compaſſes to reach beyond 90, namely, to 90 *gr.* 06 *m.* for 5 in the morning, and 7 after noon. From which, if you take 90 *gr.* the remainder ſhews how much the Sun is below the Horizon at 5 in the morning, and 7 at night; namely, 6 minutes. Or it ſhews how high the Sun will be, when it is in the beginning of *Scorpio*, the oppoſite ſign to *Taurus*, at 7 in the morning, and at 5 after noon: and doing the like from 120, you ſhall find the Compaſſes to ſhew 98 *gr.* 33 *m.* from whence taking 90, there will remain 8 *gr.* 33 *m.* for the Suns profundity at 4 in the morning, and 8 at night, the Sun being in 00 of *Taurus*, or 8 *gr.* 33 *m.* for the Suns Altitude at 8 in the morning, and 4 after noon in 00 of *Scorpio*. At 135 *gr.* the profundity for 3 and 9, or the Altitude for 9 and 3, will be 15 *gr.* 58 *m.* At 150, the Profundity for 2 and 10, or the Altitude for 10 and 2. will be 21 *gr.* 51 *m.* At 165, the Profundity at 1 and 11 in 00 of *Taurus*,

rus, or the Altitude of 11 and 1 in 00 of *Scorpio*, will be 25 *gr.* 40 *m.* And laftly, whereas the difference of the Legs was found 40 *gr.* by what was formerly intimated, the fame 40 degrees, do fhew the Complement of the Suns Altitude at 12 a clock, when the Sun is in 00 of *Scorpio.*

By this appears the manner of refolving this Propofition, and how thefe Tables may be made to other Signs or Points of the Ecliptick, or Declination from the Equinoctial.

Note alfo that the work may begin with the Winter figns, and end with the Summer, as here it may begin with *Scorpio*, and end with *Taurus*, thus. From the beginning of the Line to the beginning of *Scorpio*, are 101 *gr.* 30 *m.* This diftance compared with 38 *gr.* 30 *m.* makes the fum 140 *gr.* and the difference 63, the Complement of this difference is 27, the Altitude of 12 at noon in the beginning of *Scorpio*, and the Excefs of 140 above 90 is 50, which gives the midnights Profundity at the beginning of *Scorpio*, or the mid-days Altitude in the beginning of *Taurus*. And if you work for the other hours (as in the laft example was largely fhewed) you fhall find the Altitudes pointed out by the other foot of the Compaffes, for each hour in 00 of *Scorpio*, untill you come towards 90, and when you come beyond 90, the Excefs fhews the Profundity for the reft of the hours of the night in *Scorpio*; but the Altitudes for the anfwerable hours in the beginning of *Taurus*. And fo all other Signs and Parallels of Declination.

Thefe are the particulars in which I intended to exemplifie, becaufe their ufes are more frequent than the others are.

By the like work, having the Declination and Reclination of any Plane, may be found. Firft, *The Poles Altitude* above the Plane; then in proportions in Signs alone may be found, *the Planes difference of Longitude*, with *the departure of the Subftyle from the Vertical Line*: and by thefe the Dial may be made, and the Lines placed in a right pofition.

So, by the like work, having the difference of Longitude of any City, or remarkable place from yours, and the Latitude of the fame place, you may find in what Pofition a Plane is to be fet in your Horizon, in refpect of Declination and Reclination, or Inclination, that may reprefent the Horizon of the fame place, and accordingly you may put on the hours that belong to that Plane or place, with all the other furniture whereby the Pofitions of the Sun, in refpect of the place, may be reprefented to your view upon the Plane.

And befides thefe, there are many other particulars which may be

 performed

performed upon this Scale, namely, all that fall under the fore-mentioned fourth Axiom of Spherical Trigonometry.

CHAP. X.

Of the other Scales on the edges and spare places of the Sector.

WHen then the Sector is opened into a straight Ruler, then will the divisions of the outer edges of the two Legs, and upon the two flat sides, (which in the former usage of the Sector appeared to be divided or broken) be made up, and become as entire Scales.

1. Those on the outer edge are the three usual Logarithmetical Lines of Numbers, Sines, and Tangents. The use of those are shewed most largely by Mr. *Gunter* in the use of the Cross-staff, and therefore I shall need to say no more of them.

2. The two Scales of Foot and Inch measure, upon one of the flat sides, will serve to measure small lengths, that reach not above one foot, or else to make longer Scales of foot or inch measure, whereby greater lengths may be measured.

And this is all I shall need to mention of these two Scales, the others that remain, require somewhat more to be said of them.

CHAP. XI.

Of the two Scales of Wine and Ale measure.

It is here sup- { Wine Gallon contains 231 } Cubical
posed that a { Ale Gallon contains 288 } Inches.

THese two Scales serve for the speedy gauging of Wine, Ale, or Beer Vessels; and therefore you must first prepare a Staff of convenient length, whereby to take the Dimensions of any vessel. And upon the Staff.

For Wine, you must set on the length of 4 foot and $\frac{11}{100}$ parts of an inch, which length is to be divided into 30 equal parts, with decimal subdivisions sutable to each part, and continue the same parts quite through the length of the Staff. For

For Ale, set on the length of 4 foot and two inches justly, and divide the same into 25 equal parts, and sub-divide those parts decimally into as many sub-divisions as those parts will contain.

With these Scales you are to measure the Wine or Ale vessels, that is, you must take their Diameters at the bung and head, and measure the length of them each with his proper Scale.

Then to find the Content in Gallons.

Count the length of the Diameter at the bung, upon the proper Scale (that is, upon the Scale of Wine-measure upon the Sector, if it be for Wine, or upon she Scale of Ale-measure, if it be for Ale) and taking the same in your Compasses, apply it to the Line of Superficies, setting one foot in the Center of the Sector, and mark where the other foot falls, and noting the number, write it down twice. In this work the whole Line of Superficies is supposed to contain but ten parts only. Again, count the Diameter at the head, upon the proper Scale of the Sector, and apply that length likewise to the Line of Superficies, and note what number it falls upon, end write it under the two former, only once: then add these three Numbers together, and keep the sum. Afterwads going to the Line of Numbers. Say,

As 2, to the length of the Vessel.
So the former sum, to the content of the Vessel in Gallons.

Example.

Suppose a Vessel whose Diameter at the Bung, contained 22^{44} parts of the Scale of Wine measure: the Diameter at the head 18^{10} of the same parts; the length of the Vessel 30 parts of such a Scale as is formerly prescribed for Wine measure, from which these measures here supposed are taken.

The first number of the Bungs measure I take from the Scale of Wine measure upon the Sector, and applying it to the Line of Superficies, I find it then to fall upon 4^{70}, which number I set down twice.

Then again I take the head number from the same Scale of Wine measure upon the Sector, and when it is applied to the Line of Superficies it reacheth to (about) 3^{25} which I set under the two former numbers in right order, as in the Margin

4. 70
4. 70
3. 25
11. 65

the

the ſum of theſe three Numbers is $12\frac{65}{}$. Then upon the Line of Numbers I work this proportion;

As 2 is to the ſum $12\frac{65}{}$: ſo is the length 30 to 190 near upon. So that the content of ſuch a Veſſel is near 190 Wine Gallons. And if it had been computed by Numbers, the content would be about $\frac{1}{10}$ of a Gallon leſs.

¶ If the ſame Veſſel were meaſured by the fore-mentioned Staff made for Ale meaſure, the Diameter at the Bung would be 18 of thoſe parts, the Diameter at the head 15, the length of the Veſſel 24. And the two former Diameters being taken upon the Scale of Ale meaſure upon the Sector, and applied to the Line of Superficies will produce the ſame three Numbers as before. Then as 2 to the ſum of them $12\frac{65}{}$: ſo is 24 the length to about 152, which is the content in Ale Gallons.

CHAP. XII.

How to perform the ſame work of Gauging by Feet or Inches.

FOr this purpoſe there are two Scales upon the inner edge of one of the Leggs of the Sector, called Feet and Inches, which Scales cannot be true Feet or Inches, as by their length will eaſily appear, but for this work of Gauging (whereunto they are chiefly intended) they are to repreſent Feet and Inches, and accordingly are here called repreſentative Feet, and repreſentative Inches.

Now to take the Dimenſions of a Veſſel, namely, the length with the Diameters at head and Bung, you muſt have a ſtaff divided,

Either into true Feet, and each foot decimally ſubdivided:

Or into true Inches, and each Inch decimally again ſubdivided.

Theſe Scales may be made from the true Foot and Inch Scales inſcribed upon the flat of the Sector, as was intimated before in the tenth Chapter.

Either of theſe two Scales will perform what is here intended.

With one of theſe Staves (which you have moſt mind to) you are to meaſure the Veſſel; and knowing of what Numbers each Dimenſion is, you may caſt up the content of the Veſſel thus.

To

To find the content in Gallons.

Count the Number of the Diameter at the Bung, upon the Scale of representative { Feet, / Inches } upon the Sector, and take that length and set it upon the Line of Superficies from the Center forwards, and see what Number it there falls upon, write that Number twice. [Remember here again that the Superficial Line in this work is to be esteemed of 10 parts only.] So do also with the Diameter at the head. and write that Number under the other two once only. And add these three Numbers and keep the sum.

Then look what the Vessels length was in true { Feet, / Inches, } count the same number upon the Scale of representative { Feet, / Inches, } and when you have taken it off from thence, measure it,

Either upon the Line of { Wine / Ale } measure, and so what Number it reacheth thereon. Then upon the Line of Numbers say, as 2 is to this last Number, so is the sum before found, to the Number of { Wine / Ale } Gallons contained in the Vessel.

Or without measuring it upon those Lines of Wine or Ale Measure, you may do in this manner. Find the sum of the three Numbers, as before. Then upon the Line of Numbers say, as 2 to that sum, so the Number of the Vessels length in Feet to a 4th. And as 1 to { 7.481 for Wine, / 6.000 for Ale, } so the 4th Number to the content in { Wine / Ale } Gallons.

If the Vessels length be taken in Inches, then thus: As 2 to the sum, so the Vessels length in Inches to a 4th. And again, as 10 to { 6.234 for Wine, / 5.000 for Ale, } so is the 4th Number to the content in { Wine / Ale } Gallons.

CHAP.

CHAP. XIII.

Of the two Scales upon the inner edge of the other Leg, which are divided the one into 14, the other into 20 equal parts.

THese two Scales ſerve further for Gauging of Veſſels by the mean Diameter, and the uſe of them is.

To find ſuch a mean Diameter between the Diameters of the head and Bung, as ſhall reduce the Veſſel to an equal Cylinder of the ſame length with the Veſſel.

This way of Gauging is in uſe, and for our common Veſſels may ſerve as coming ſomewhat near the truth.

Having meaſured, either in Feet or Inches, the two Diameters at Head and Bung, take their difference and count it upon the Scale of 20. And taking that length from thence, and applying it to the Scale of 14, ſee what Number it cuts there. Add that Number to the leſſer Diameter, which is the Diameter at the Head, and the ſum will be the mean Diameter.

To Gauge by the mean Diameter.

You muſt firſt meaſure the Dimenſions of the Veſſel with a Scale of Foot or Inch meaſure.

Then take the Number of the mean Diameter out of the repreſentative {Feet, Inches,} and meaſure it upon the Line of Superficies (eſteeming the ſaid whole Line but 10) and keep the Number that it falls upon. Then upon the Line of Numbers ſay, As 1 to the Number kept; ſo the length in {Feet, Inches,} to a fourth.

Then if the length be given in Inches, and the former work were alſo for inches, ſay, as 8 to {7.48 for Wine, 6.00 for Ale,} or as 10 to {9.35 for Wine, 7.50 for Ale,} ſo the former fourth Number to the Number of Gallons in the Veſſel.

Or

Or if the length were given in foot measure, and the former work were also for Feet, say, As 1 to { 11. 22 for Wine, / 9. 00 for Ale, } so the former fourth Number to the content in Gallons.

Or else take the Vessels length out of the representative { Feet, / Inches, } and apply them to the { Wine, / Ale } measure Scales, and observe what Numbers they there fall upon. Then say, as $\frac{2}{3}$ to this new length ; so the Number kept before, to the Number of { Wine, / Ale } Gallons in the Vessel.

This last way is performed by one work upon the Line of Numbers, whereas the other requires two.

CHAP. XIV.

How to measure Cartridges of Gunpowder to know how many pound are contained in them.

1. *If the Cartridge be of a Cylinderical form.*

FIrst measure the Diameter and length of it with a Scale of Decimal Inches. Then count the Diameters length upon the Line of Lines, (counting the whole Line of Lines as 10 representative inches) amd with your Compasses, take the length of that number from the Line of Lines, and apply it to the Line of Superficies, (which now in this work must be supposed to contain 100 parts) and note the Number which it reacheth unto. Then upon the Line of Numbers, say, As $40\frac{1}{2}$ is to that Number noted, so the length of the Cylinder, to the Number of the pounds of Gunpowder.

2. *If the Cartridge be of a Conical form.*

Measure the Diameter of the Base, as before, and the length of the Cone likewise, both with a Scale of Decimal Inches, and count the Diameter upon the Line of Lines, and apply it to the Line of Superficies,

ficies, noting the Number thereon, as was done before. Then say, As 121 ½ to the Number before noted: So the length, to the content in pounds of powder.

3 *If the Cartridge be a resected Cone,*

Measure the Diameters of both Bases, and the length by a Scale of Decimal Inches: Then count the greater Diameter upon the Line of Lines, and measure it upon the Line of Superficies, noting the Number, as was done in the two former ways. Afterwards, upon the Line of Numbers, say; As the greater Diameter to the lesser; So the noted Number to a second, and so that second to a third. Add these three the first, second and third Numbers together, and keep the sum. Then say again, As 121 ½ is to the said sum; So is the length, to the content in pounds of powder.

Or you may count both the Diameters upon the Line of Lines, and transfer them both to the Line of Superficies, and note what two Numbers they cut, count the same two Numbers upon the Line of Numbers, and bisect the distance between them, so shall you find a middle Number, which, with the two former, will make up three Numbers, the same which were found, the other way. Then (as before) add these three Numbers together, and keep the sum, and again say, As 121 ½ is to the said sum; So is the length to the content in pounds of powder.

The End of Mr.* Fosters *alteration of the Sector.

Poſtſcript.

Upon the Scheme of this Sector, as Mr. Foſter *hath contrived it, there are ſome other Lines inſerted in the ſpare places thereof, which do not go up to the Center: As, Firſt, a Line of three hours, with their halves and quarters, which Line is noted at every whole hour with* ♣, *and at every half and quarter, with a little Line thus* |. *The uſe of this Line is chiefly in Dialling, and the manner of uſing it is ſufficiently ſhewed in other of his Works, and it moſt excellently and expediciouſly performeth that manner of Dialling, which Mr.* Gunter *teacheh at the end of his third Book of the Sector.*

There are alſo other Scales, as one of Metals, and another of Segments of a Circle, the uſes whereof are the ſame, as Mr. Gunter *hath ſhewed at large: and there is alſo added another Line by Mr.* Foſter, *which is alſo called a Line of Segments; that of Mr.* Gunter *repreſenting the Segments of a Circle; the other of Mr.* Foſter, *the Segments of a Sphere, and hath like uſe in Spheres, as the other hath in Circles.*

Vale.

THE CROSS-STAFF;

In Three Books:

The First,

Containing its Description, and the Use thereof in taking of Heights and Distances.

The Second,

Contains the Use of the Lines thereon in the Measuring of all manner of Superficies, and Solids, as Board, Glass, Land, Timber, Stone, and Gauging of Vessels, as also in the famous Art of NAVIGATION.

The Third,

Contains the Use of the Lines of Numbers, Sines, and Tangents in Dialling, an Excellent and Compendious TREATISE, fully teaching, and amply explaining the Grounds and Reasons thereof, from a Projection of the Sphere *in Plano*.

To which is added,

An APPENDIX, containing the Description and Use of a small Portable Quadrant, for the more easie finding the Hour and Azimuth, and other Solar Conclusions of more frequent Use.

By *Edmund Gunter*.

LONDON,

Printed by *A. C.* for *Francis Eglesfield*, and are to be sold at the Marigold in St *Pauls* Church-yard. 1673.

THE FIRST BOOK OF THE CROSS-STAFF.

CHAP. I.

Of the Description of the Staff.

THe *Cross-Staff*, is an Instrument well known to our Seamen, and much used by the ancient Astronomers and others, serving Astronomically for observation of the Altitude and Angles of distance in the Heavens; Geometrically for Perpendicular Heights and Distances on Land and Sea.

The Description and several Uses of it are extant in Print, by *Gemma Frisius* in Latine, in English by *D. Hood.* I differ something from them both, in the Projection of this Staff, but so as their Rules may be applyed unto it, and all their Propositions be wrought by it; and therefore referring the Reader to their Books, I shall be brief in the Explanation of that which may be applied from theirs unto mine, and so come to the Use of those Lines which are of my Addition, not extant heretofore.

The necessary parts of this Instrument are Five; (1.) The Staff; (2.) The Cross; And (3.) the three Sights. The Staff which I made for my own use, is a full Yard in length, that so it may serve for measure.

The

The Crofs belonging to it is 26 Inches $\frac{1}{5}$ between the two outward fights, If any would have it in a greater form, the proportion between the Staff and the Crofs, may be fuch as 360 unto 26 $\frac{1}{5}$.

The Lines infcribed on the Staff are of four forts: One of them ferves for Meafure and Protraction: One for obfervation of Angles: One for the Sea-Chart; and the four other for working of Proportions in feveral kinds.

The Line of Meafure is an *Inch Line*, and may be known by his equal quarts, the whole Yard being divided equally into 36 Inches, and each Inch fubdivided firft into ten parts, and then each tenth part into halfs.

The Line for obfervation of Angles may be known by the double Numbers, fet on both fides of the Line, beginning at the fide at 20, and ending at 90: on the other fide at 40, and ending at 180: and this being divided according to the degrees of a quadrant, I call it the *Tangent Line on the Staff.*

The next Line is the Meridian of a Sea-chart, according to *Mercators* Projection from the Equinoctial to 58 *gr.* of Latitude, and may be known by the letter *M*, and the Numbers 1, 2, 3, 4, unto 58.

The Lines for working of proportions may be known by their unequal divifions, and the numbers at the end of each Line.

1. The Line of numbers noted with the Letter N, divided unequally into 1000 parts, and numbred with 1, 2, 3, 4, unto 10.

2. The Line of Artificial Tangents is noted with the letter T, divided unequally into 45 degrees, and numbred both ways, for the Tangent and the Complement.

3. The Line of Artificial Sines noted with the letter S, divided unequally into 90 degrees, and numbred with 1, 2, 3, 4, unto 90.

4. The Line of Verfed Sines for more eafie finding the hour and Azimuth, noted with V, divided unequally into about 164 *gr.* 50 *m.* numbred backward with 10, 20, 30, unto 164.

Thus there are feven Lines infcribed on the Staff: there are Five Lines more infcribed on the Crofs.

1. A Tangent Line of 36 *gr.* 3 *m.* numbred by 5, 10, 15, unto 35; the midft whereof is at 20 *gr.* and therefore I call it the Tangent of 20; and this hath refpect unto 20 *gr.* in the Tangent on the Staff.

2. A Tangent Line of 49 *gr.* 6 *m.* numbred by 5, 10, 15, unto 45; the midft whereof is at 30 *gr.* and hath refpect unto 30 *gr.* in the Tangent on the Staff, whereupon I call it the Tangent of 30.

3. A Line of Inches numbred with 1, 2, 3, unto 26; each Inch equally subdivided into ten parts, answerable to the Inch Line upon the Staff.

4. A Line of several Chords, one answerable to a Circle of twelve Inches Semidiameter, numbred with 10, 20, 30, unto 60, another a Semidiameter of a Circle of six Inches; and the third to a Semidiameter of a Circle of three Inches, both numbred with 10, 20, 30, unto 90.

5. A continuation of the Meridian Line from 57 *gr.* of Latitude unto 76 *gr.* and from 76 to 84 *gr.*

For the Inscription of these Lines. The first for measure is equally divided into Inches, and tenth parts of Inches.

The Tangent on the Staff for observation of Angles, with the Tangent of 20 and the Tangent of 30 on the Cross, may all three be inscribed out of the ordinary Table of Tangents.

The Staff being 36 Inches in length; the Radius for the Tangent on the Staff will be 13 Inches and 103 parts of 1000: so the whole Line will be a Tangent of 70 *gr.* and must be numbred by their Complements, and the double of their Complements, the Tangent of 10 *gr.* being numbred with 80 and 160.

The Radius for the Tangent of 20 on the Cross, will be 36 Inches, and the whole Line between the Sights a Tangent of 36 *gr.* 3 *m.* according as it is numbred, The Radius for the Tangent of 30 *gr.* on the Cross, will be 22 inches and 695 parts of 1000: so the whole Line between the sights will contain a Tangent of 49 *gr.* 6 *m.* in such sort as they are numbred.

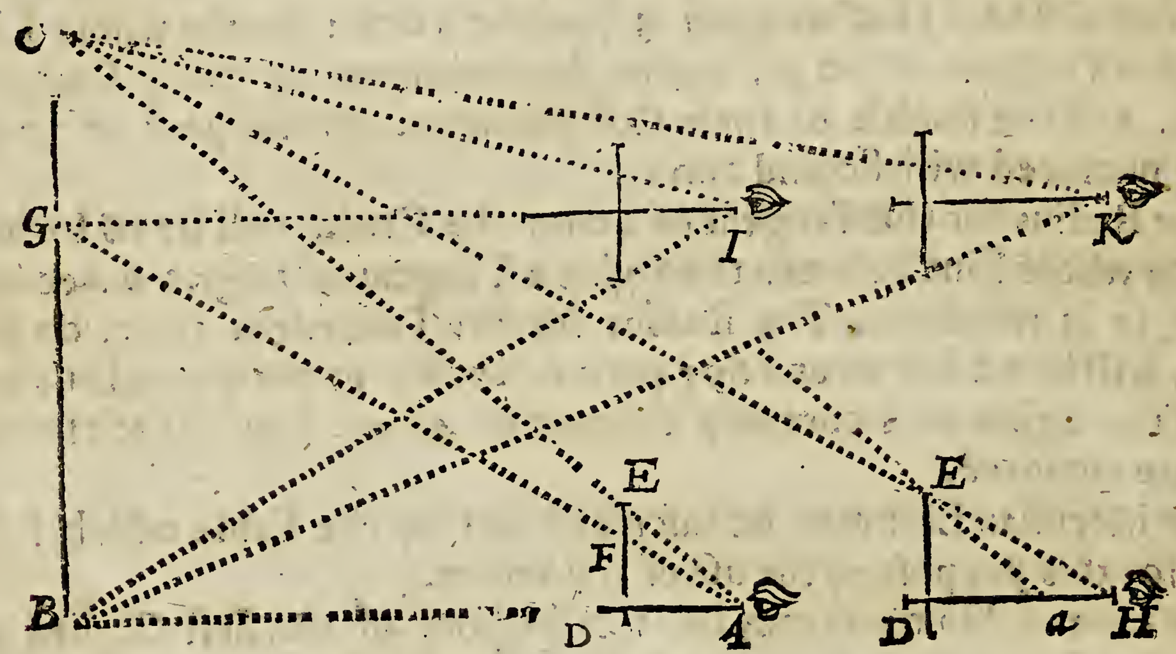
The Meridian Line may be inscribed out of the Table which I set down for this purpose in the use of the Sector.

The Line of Numbers may be inscribed out of the first Chiliad of Mr. *Briggs* Logarithms: and the rest of the Lines of proportion out of my Canon of Artificial Sines and Tangents, and in recompence thereof this Book will serve as a Comment to explain the use of my Canon.

CHAP. II.

The use of the Lines of Inches for perpendicular heights and distances.

IN taking of heights and distances, the Staff may be held in such sort, that it may be even with the distance, and the Cross parallel with the height: and then if the eye at the beginning of the Staff shall see his marks by the inward sides of the two first sights, there will be such proportion between the distance and the height, as is between the parts intercepted on the Staff and the Cross. Which may be farther explained in these Propositions.



1. *To find an height at one station, by knowing the distance.*

Set the middle sight unto the distance upon the Staff, the height will be found upon the Cross. For,

As the Segment of the Staff
  unto the Segment on the Cross :
So is the distance given,
  unto the height.

As

As if the distance A B being known to be 256 feet, it were required to find the height B C: first I place the middle sight at 25 inches and 6 parts of 10; then holding the Staff level with the distance, I raise the Cross Parallel unto the height, in such sort, as that my eye may see from A the beginning of the inches on the Staff by the sight E, at the beginning of the inches on the Cross unto the mark C: which being done; if I find 19 inches and 2 parts of 10 intercepted on the Cross between the sights at E and D, I would say the height B C were 192 feet.

Or if the observation were to be made before the distance were measured, I would set the middle sight either unto 10 inches, or 12 or 16, or 20, or 24, or some such other nnmber as might best be divided into several parts, and then work by proportion. As if in the former example the middle sight were at 24 on the Staff, and 18 on the Cross, it should seem that the height is $\frac{3}{4}$ of the distance; and therefore the distance being 256, the height should be 192.

2. *To find an height by knowing some part of the same height.*

As if the height from G to C were known to be 48, and it were required to find the whole height B C, either put the third sight, or some other running sight upon the Cross between the eye and the mark G. For then,

As the difference between the sights
unto the whole Segment of the Cross:
So is the part of the height given,
unto the whole height.

If then the difference between the sights at E and F, shall be 45, and the Segment of the Cross E D 180, the whole height B C will be found to be 192.

3. *To find an height at two stations, by knowing the difference of the same stations.*

As the difference of Segments on the Staff,
unto the difference of stations:
So is the Segment of the Cross,
unto the height.

 Suppose

Suppose the first station being at H, the Segment of the Cross E D were 180, and the Segment of the Staff H D 300: then coming 64 feet nearer unto B, in a direct Line unto a second station at A, and making another observation, suppose the Segment of the cross E D were 180 as before, and the Segment of the Staff A D 240; take 240 out of 300, the difference of Segments will be 60 parts. And

As 60 parts unto 64 the difference of stations:
So D E 180 unto B C 192 the height required.

In these three Propositions there is a regard to be had of the height of the eye. For the height measured, is no more then from the level of the eye upwards.

4. To find a distance, by knowing the height.

As the Segment of the Cross,
unto the Segment of the Staff:
So is the height given,
unto the distance.

So the Segment E D being 18, and D A 24, the height C B 192, will shew the distance A B to be 256.

5. To find a distance, by knowing part of the height.

As the difference between the sights,
unto the Segment of the Staff:
So is the part of the height given,
unto the distance.

And thus the difference between E and F being 45, and the Segment D A 240; the part of the height G C 48, will give the distance A B to be 256.

6. To find a distance at two stations, by knowing the difference of the same stations.

As the difference of Segments on the Staff,
unto the difference of stations:

So

So is the whole Segment,
unto the distance.

And thus the Segment of the Cross being 180, the Segment of the Staff at the first station 240, at the second 300, the difference of the Segments 60, and the difference of stations 64, the distance A B at the first station will be found to be 256, and the distance H B at the second station 320.

7. *To find a breadth, by knowing the distance perpendicular to the breadth.*

This is all one with the first Proposition. For this breadth is but an height turned sideways; and therefore

As the Segment of the Staff,
unto the Segment of the Cross;
So is the distance
unto the breadth.

And thus the Segment of the Staff being 24, and the Segment of of the Cross 18, the distance A B 256, will give the breadth B C to be 192.

8. *To find a breadth at two stations in a Line Perpendicular to the breadth, by knowing the difference of the same stations.*

This is also the same with the third Prop. and therefore

As the difference of Segments on the Staff,
unto the difference of stations:
So the Segment on the Cross between the two sights,
unto the breadth required.

And thus the difference between the stations at A and H being 64, the difference of Segments on the Staff 60, the Segment of the Cross 180, the breadth B C will be found to be 192.
In like manner may we find the breadth G C, for having found the breadth B C, the proportion will hold;

As D E is unto F E, so B C unto G C. Or otherwise,
As H *a* unto H A, so F E unto G C.

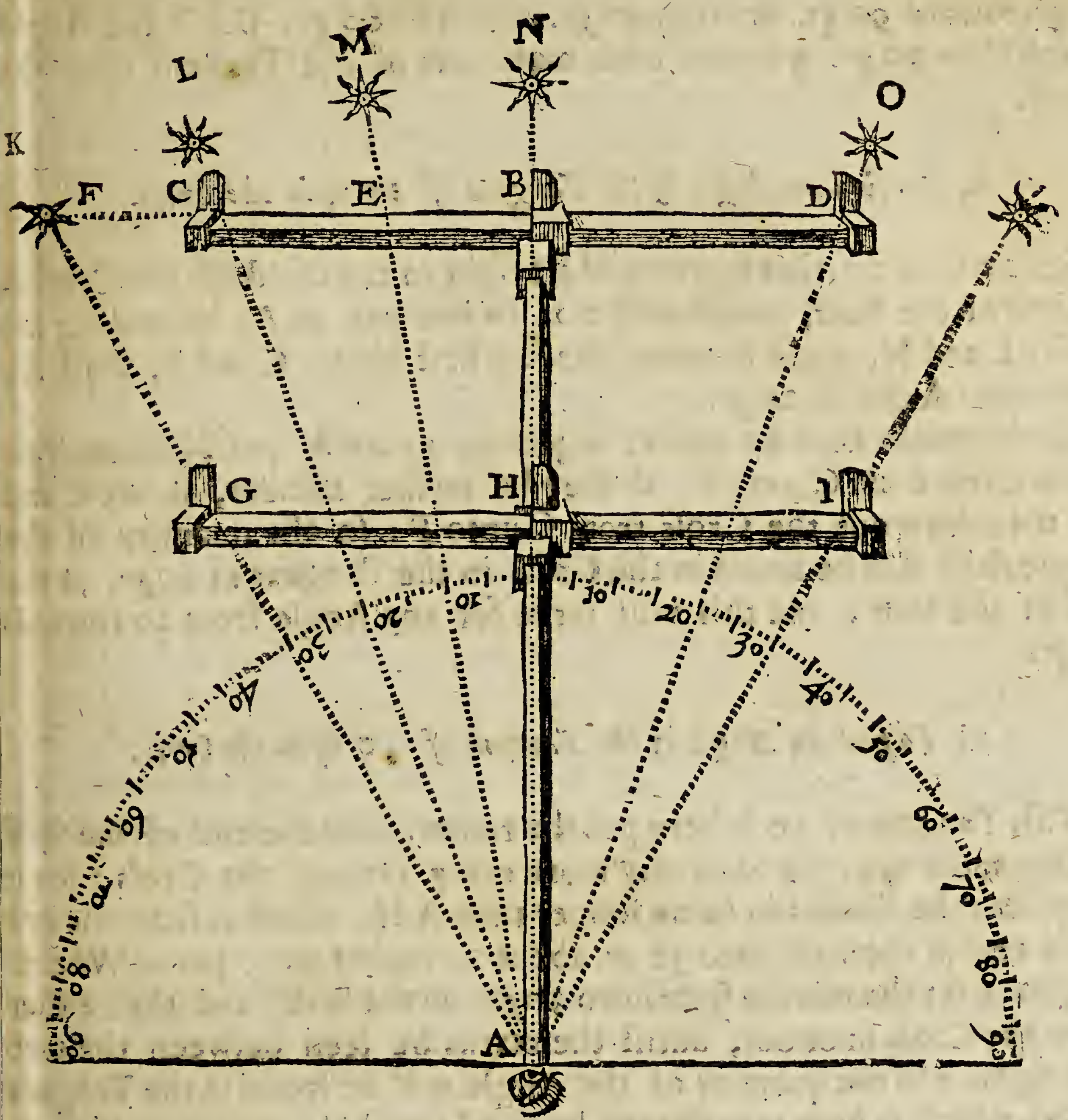
Neither is it material whether the two stations be chosen at one end of the breadth proposed, or without it, or within it, if the Line between the stations be Perpendicular unto the breadth; as may appear, if in stead of the stations at A and H, we make choice of the like stations at I and K.

There might be other ways proposed to work these Propositions, by holding the Cross even with the distance, and the Staff parallel with the height: but these would prove more troublesome, and those which are delivered are sufficient, and the same with those which others have set down under the name of the *Jacobs Staff.*

CHAP

CHAP. III.

The use of the Tangent Lines in taking of Angles.



1. *To find an Angle by the Tangent on the Staff.*

LEt the middle sight be always set to the middle of the Cross, noted with 20 and 30, and then the Cross drawn nearer the eye, untill the marks may be seen close within the sights. For so if the eye at at A (that end of the Staff which is noted with 90 and 180) beholding the mark K and N between the two firſt ſights, C and B, or the marks K and P between the two outward ſights, the Croſs being drawn down unto H, ſhall ſtand at 30 and 60, in the Tangent on the Staff: it ſheweth the Angle K A N is 30 *gr.* the Angle K A P 60 *gr.* the one double to the other; which is the reaſon of the double numbers on this Line of the Staff: and this way will ſerve for any Angle from 20 *gr.* toward 90 *gr.* or from 40 *gr.* toward 180 *gr.* But if the Angle be leſs than 20 *gr.* we muſt then make uſe of the Tangent upon the Croſs.

2. *To find an Angle by the Tangent of* 20 *upon the Croſs.*

Set 20 unto 20, that is, the middle ſight to the midſt of the Croſs at the end of the Staff, noted with 20, ſo the eye at A, beholding the marks L and N, cloſe between the two firſt ſights, C and B, ſhall ſee them in an Angle of 20 *gr.*

If the marks ſhall be nearer together, as are M and N, then draw in the Croſs from C unto E: if they be farther aſunder, as are K and N, then draw out the Croſs from C unto F; ſo the quantity of the Angle ſhall ſtill be found in the Croſs in the Tangent of 20 *gr.* at the end of the Staff: and this will ſerve for any Angle from 20 towards 35 *gr.*

3. *To find an Angle by the Tangent of* 30 *upon the Croſs.*

This Tangent of 30 is here put the rather, that the end of the Staff reſting at the eye, the hand may more eaſily remove the Croſs: for it ſuppoſeth the Radius to be no longer than A H, which is from the eye at the end of the Staff unto 30 *gr.* about 22 inches and 7 parts. Wherefore here ſet the middle ſight unto 30 *gr.* on the Staff, and then either draw the Croſs in or out, untill the marks be ſeen between the two firſt ſights; ſo the quantity of the Angle will be found in the Tangent of 30, which is here repreſented by the Line G H; and this will ſerve for any Angle from 0 *gr.* toward 48 *gr.*

4. *To obſerve the Altitude of the Sun backward.*

Here it is fit to have an horizontal ſight ſet to the beginning of the Staff,

Staff, and then may you turn your back toward the Sun, and your Crofs toward your eye. If the Altitude be under 45 *gr.* fet the middle fight to 30 on the Staff, and look by the middle fight through the Horizontal unto the Horizon, moving the Crofs upward or downward, untill the upper fight do fhadow the upper half of the horizontal fight: fo the Altitude will be found in the Tangent of 30.

If the Altitude fhall be more than 45 *gr.* fet the middle fight unto the midft of the Crofs, and look by the inward edge of the lower fight through the Horizontal to the Horizon, moving the middle fight in or out, untill the upper fight do fhadow the upper half of the Horizontal fight: fo the Altitude will be found in the degrees on the Staff between 40 and 180.

5. *To fet the Staff to any Angle given.*

This is the converfe of the former Propofition: For if the middle fight be fet to his place and degree, the eye looking clofe by the fights as before, cannot but fee his object in the Angle given.

6. *To obferve the Altitude of the Sun another way.*

Set the middle fight to the middle of the Crofs, and hold the Horizontal fight downward, fo as the Crofs may be parallel to the Horizon, then is the Staff Vertical; and if the outward fights of the Crofs do fhadow the Horizontal fight: the Complement of the Altitude will be found in the Tangent on the Staff.

7. *To obferve an Altitude by Thread and Plummet.*

Let the middle fight be fet to the midft of the Crofs, and to that end of the Staff which is noted with 90 and 180; then having a Thread and a Plummet at the beginning of the Crofs, and turning the Crofs upward, and the Staff towards the Sun, the Thread will fall on the Complement of the Altitude above the Horizon. And this may be applied to other purpofes.

8. *To apply the Lines of Inches to the taking of Angles.*

If the Angles be obferved between the two firft fights, there will

 be

be such proportion between the parts of the Staff and the parts of the Crofs, as between the Radius and the Tangent of the Angle.

As if the parts intercepted on the Staff were 20 inches, the parts on the Crofs 9 inches. Then by proportion as 20 unto 9, so 10000 unto 45000 the Tangent of 24 *gr.* 14 *m.*

But if the Angle shall be observed between the two outward sights, the parts being 20 and 9 as before, the Angle will be 48 *gr.* 28 *m.* double unto the former.

In all these there is a regard to be had to the Parallax of the eye, and his height above the Horizon in observations at Sea; to the Semidiameter of the Sun, his parallax and refraction, as in the use of other staves. And so this will be as much, or more than that which hath been heretofore performed by the Crofs-Staff.

CHAP. IV.

The use of the Lines of equal parts joyned with the Lines of Chords.

THe Lines of equal parts do serve also for protraction, as may appear by the former Diagrams, but being joyned with the Lines of Chords, which I place upon one side of the Crofs, they will farther serve for the protraction and resolution of right Lined Triangles; whereof I will give one example in finding of a distance at two stations otherwise than in the Second Chapter.

Let the distance required be A B, at A the first station, I make choice of a station Line towards C, and observe the Angle B A C by the Tangent Lines, which may be 43 *gr.* 20 *m.* then having gone an hundred paces towards C, I make my second station at D, where suppose I find the Angle B D C to be 58 *gr.* or the Angle B D A to be 112 *gr.* this being done, I may find the distance A B in this manner.

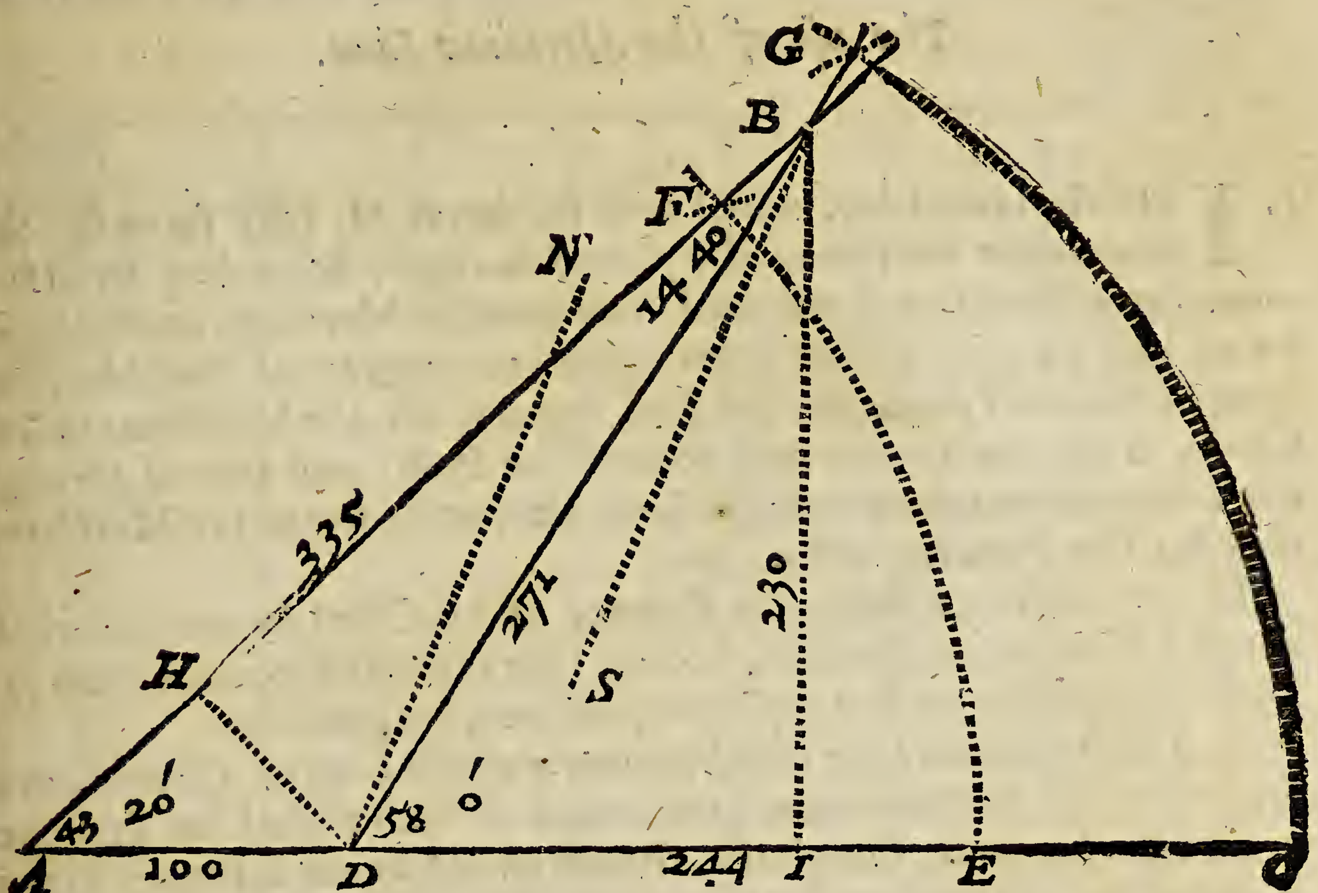
1. I draw a right Line A C, representing the station Line.

2. I take 100 out of the Lines of equal parts, and prick them down from A the first station unto D the second.

3. I open my Compasses to one of the Chords of 60 *gr.* and setting one foot in the point A, with the other I describe an occult Ark of a Circle intersecting the station Line in E.

4. I

4. I take out of the same Line of Chords a Chord of 43 *gr.* 20 *m.* (because such was the Angle at the first station) and this I inscribe into that occult Ark from E unto F, which makes the Angle F A D equal to the Angle observed at the first station.



5. I describe another like Ark upon the Center D, and inscribe into it a Chord of 53 *gr.* from C unto G, and draw the right Line D G, which doth meet with the other Line A F in the Point B, and makes the Angle B D C equal to the Angle observed at the second station. So the Angles in the Diagram being equal to the Angles in the field, their sides will be also proportional; and therefore,

6. I take out the Line A B with my Compasses, and measuring it in the same Line of equal parts, from which I took A D, I find it to be 335, and such is the distance required.

CHAP. V.

The use of the Meridian Line.

1. THe Meridian Line, noted with the letter M, may ſerve for the more eaſie diviſion of the plain Sea-chart, according to *Mercators* Projection,For if you ſhall draw parallel Meridians,each degree being half an inch diſtant from other, the degree of this Meridian Line on the Staff ſhall give the like degrees for the Meridians on the Chart, from the Equinoctial toward the Pole: and then if through theſe degrees you draw ſtraight Lines Perpendicular to the Meridians, they ſhall be Parallels of Latitude.

If any deſire to have the degrees of his Chart larger than thoſe which I have put on the Staff, he may take theſe and increaſe them in a double, or treble, or a decuple proportion at his pleaſure.

2. This Meridian Line being joyned with the Line of Chords, may ſerve for the protraction and reſolution of ſuch right Line Triangles as concern Latitude, Longitude, Rumb and Diſtance in the practice of Navigation. As may appear by this example.

Suppoſe two places given, A in the Latitude of 50 *gr.* D in the Latitude of 52 *gr.* ½, the difference of Longitude between them being 6 *gr.* and let it be required to know, firſt, what Rumb leadeth from the one place to the other; ſecondly, how many degrees diſtant they are aſunder.

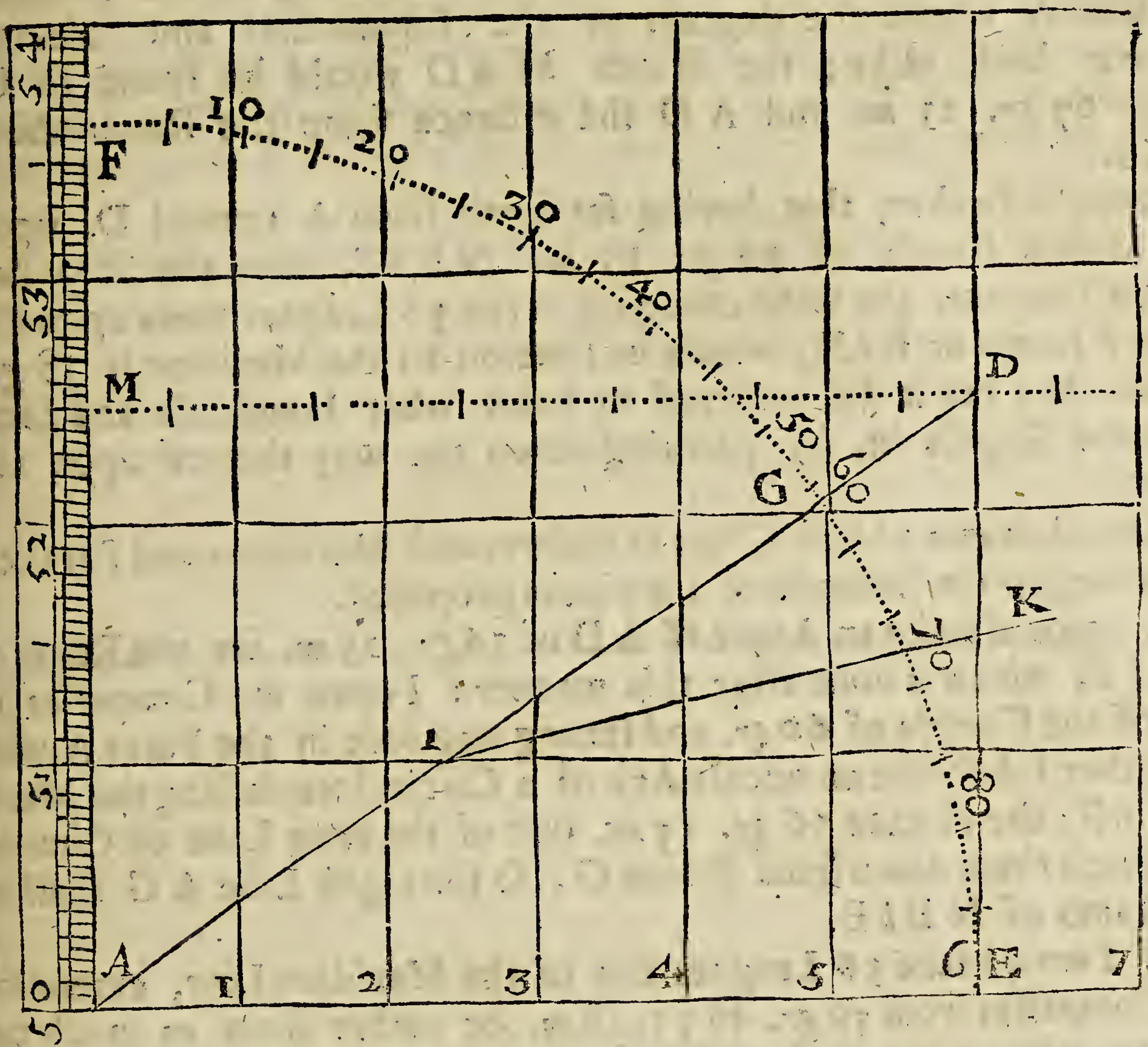
1. I draw a right Line A E, repreſenting the Parallel of the place from whence I depart.

2. I take 6 *gr.* for the difference of Longitude, either out of the Line of Inches, allowing half an inch for every degree, or out of the beginning of the Meridian Line; (for there the Meridian degrees differ very little from the Equinoctial degrees) and theſe 6 *gr.* I prick down in the Parallel from A to E.

3. In A and E, erect two Perpendiculars, A M and E D repreſenting the Meridians of both places.

4. I take the difference of the Latitude from 50 *gr.* to 52 *gr.* 30 *m.* out

out of the Meridian Line, and prick it down in the Meridians from A unto M, and from E to D, and draw the right Line M D for the Parallel of the second place, and the right Line A D for the Line of distance between both places: so the Angle M A D shall give the Rumb that leadeth from the one place to the other.



5. To find the quantities of this Angle M A D, I may either make use of the Protractor, or else of a Line of Chords, and so I open my Compasses unto one of the Chords of *60 gr.* and setting one foot in the Point A, with the other I describe an occult Ark of a Circle, intersecting the Meridian in F, and the Line of distance in G; then I take the Chord of F G with my Compasses, and measuring it in the same Line of Chords as before, I find it *56 gr.* $\frac{1}{4}$: and such is the Inclination of the Rumb to the Meridian, which is the first thing that was required.

6. To

6. To find the quantity of the Line of diſtance A D, I take it out with my Compaſſes, and meaſuring it in the Meridian Line, ſetting one foot beneath the leſſer Latitude, and the other foot as much above the greater Latitude, I find about 4 *gr.* ½ intercepted between both feet: and ſuch is the diſtance upon the Rumb, which is the ſecond thing that was required.

But if this example were protracted according to the common Sea-Chart, where the degrees of the Equinoctial and Meridian are both alike; the Rumb M A D would be found to be above 67 *gr.* 23 *m.* and A D the diſtance upon the Rumb about 6 *gr.* ½.

Suppoſe farther, that having ſet forth from A toward D, upon the former Rumb of 56 *gr.* 15 *m.* N E *b* E, after the Ship had run 36 Leagues, the wind changing, it ran 50 Leagues more upon the ſeventh Rumb of E *b* N, whoſe inclination to the Meridian is 78 *gr.* 45 *m.* And let it be required to know what Longitude and Latitude the Ship is in, by pricking down the way thereof upon the Chart.

Havind drawn a blank Chart as before, with Meridians and Parallel, according to the Latitude of the places propoſed.

1. I would make an Angle M A D of 56 *gr.* 15 *m.* for the Rumb of N E *b* E, which is done after this manner: I open my Compaſſes to one of the Chords of 60 *gr.* and ſetting one foot in the Point A, with the other I deſcribe an occult Ark of a Circle, interſecting the Meridian in F: then I take 56 *gr.* 15 *m.* out of the ſame Line of Chords, and prick them down from F unto G: ſo the right Line A G ſhall be the Rumb of N E *b* E.

2. I would take 36 Leagues out of the Meridian Line, extending my Compaſſes from 50 *gr.* to 51, 48 *m.* or rather from as much below 50 as above 51, and prick them down upon the Rumb from A unto I; ſo the Point I ſhall repreſent the place wherein the Ship was when the wind changed. And this is in the Latitude of 51 *gr.* 0 *m.* and in the Longitude of 2 *gr.* 21 *m.* Eaſtward from the Meridian A M.

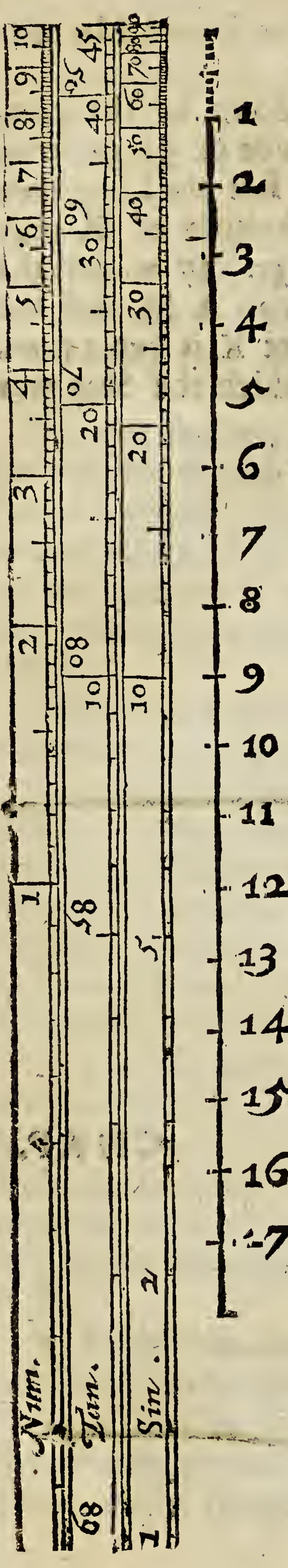
3. By the ſame reaſon, I may draw the right Line I K for the Rumb of E *b* N, and prick down the diſtance of 50 Leagues from I unto K: ſo the Point K ſhall repreſent the place whither the Ship came, after the running of theſe 50 Leagues: and this is in the Latitude of 51 *gr.* 30 *m.* and in Longitude 6 *gr.* 16 *m.* Eaſtward from the firſt Meridian

dian AM; and therefore 16 *m.* Eastward from the second Meridian ED.

But if these two courses were to be pricked down by the common Sea-Chart, the Point I would fall in the Latitude of 51 *gr.* 0 *m.* and the Point K in the Latitude of 51 *gr.* 30 *m.* But the Longitude of I would be only 1 *gr.* 30 *m.* and the Longitude of K only 3 *gr.* 57 *m.* more: both these do make but 5 *gr.* 27 *m.* for the difference of Longitude between the first Meridian A M, and the Point K: whereby it should seem that the Point K is yet 33 *m.* Westward from the Meridian of the place to which the Ship was bound.

Such is the difference between both these Charts.

CHAP.



CHAP. VI.

The uſe of the Line of Numbers.

THe Line of Numbers here noted with 1,2,3,4, unto 10, is compleat in thoſe diviſions which are between 1 and 10: the other like diviſions at the beginning of the Line do ſerve rather to anſwer to the firſt degrees of the two other Lines of Sines and Tangents, than for any neceſſity, which is the cauſe why ſome of them are omitted. And here, as in the uſe of other Scales, the figures 1, 2,3,4, that are ſet down upon the Line, do ſometimes ſignifie themſelves alone, ſometimes 10, 20, 30,40, ſometimes 100, 200, 300, 400, and ſo forward, as the matter ſhall require. The firſt figure of every Number is always that which is here ſet down, the reſt muſt be ſupplied according to the nature of the queſtion.

1. *Having two Numbers given, to find a third in continual proportion, a fourth, a fifth, and ſo forward.*

Extend the Compaſſes from the firſt Number unto the ſecond; then may you turn them from the ſecond to the third, and from the third to the fourth, and ſo forward.

Let the two numbers given be 2 and 4, extend the Compaſſes from 2 to 4, then may you turn them from 4 to 8, and from 8 to 16, and from 16 to 32, and from 32 to 64, and from 64 to 128.

Or if one foot of the Compaſſes being ſet to 64, the other fall out of the Line, you may ſet it to another 64 nearer the beginning of the Line, and there the other foot will reach to 128, and from 128 you may turn them to 256, and ſo forward.

Or

Or if the two firſt Numbers given were 10 and 9: extend the Compaſſes frow 10 at the end of the Line, back unto 9, then may you turn them from 9 unto 8, 1, and from 8, 1, unto 7,29. And ſo if the two firſt Numbers given were 1 and 9, the third would be found to be 81, the fourth 729, with the ſame extent of the Compaſſes.

In the ſame manner, if the two firſt Numbers were 10 and 12, you may find the third proportional to be 14,4, the fourth 17,28. And with the ſame extent of the Compaſſes, if the two firſt Numbers were 1 and 12, the third would be found to be 144, and the fourth to be 1728.

2. *Having two extreme Numbers given, to find a mean proportional between them.*

Divide the ſpace between the extreme Numbers into two equal parts, and the foot of the Compaſſes will ſtay at the mean proportional. So the extreme Numbers given being 8 and 32, the mean between them will be found to be 16, which may be proved by the former Propoſition, where it was ſhewed, that as 8 to 16, ſo are 16 to 32.

3. *To find the ſquare Root of any Number given.*

The ſquare Root is always the mean proportionll between 1 and the number given, and therefore to be found bv dividing the ſpace between them into two equal parts; So the Root of 9 is 3, and the Root of 81 is 9, and the Root of 144 is 12, and the Root of 1440 almoſt 38.

If you ſuppoſe Pricks under the Number given, (as in Arithmetical Extraction) and the laſt Prick to the left hand ſhall fall under the laſt figure, which will be as oft as there be odd figures, the unity will be beſt placed at 1 in the middle of the Line: ſo the Root and the Square will both fall forward toward the end of the Line. But if the laſt Prick ſhall fall under the laſt figure but one, which will be as oft as there be even figures, then the unity may be placed at 1 in the beginning of the Line, and the Square in the ſecond length, or rather the unity may be placed at 10 in the end of the Line of the Root, and the Square will both fall backward toward the middle of the Line, in the ſecond length.

4. *Having two extreme Numbers given, to find two mean Proportionals between them.*

Divide the ſpace between the two extreme Numbers given into three equal parts. As if the extreme Numbers given were 8 and 27, divide the ſpace between them into three equal parts, the feet of the Compaſſes will ſtand in 12 and 18.

5. *To find the Cubic Root of a Number given.*

The Cubic Root is always the firſt of two mean Proportionals between 1 and the Number given, and therefore to be found by dividing the ſpace between them into three equal parts.

So the Root of 1728 will be found to be 12. The Root of 17280 is almoſt 26: and the Root of 172800 is almoſt 56.

If you ſuppoſe a Prick under the Number given after the manner of Arithmetical extraction, and the laſt Prick to the left hand ſhall fall under the laſt figure, as it doth in 1728, the unite will be beſt placed at 1 in the middle of the Line, and the Root, the Square, and the Cube, will all fall forward toward the end of the Line.

If the laſt Prick ſhall fall under the laſt figure but one, as in 17280, the unite may be placed at 1 in the beginning of the Line, and the Cube in the ſecond length, or the unite may be placed at 10 in the end end of the Line: and the Cube in the firſt length; or if the Cube fall out of the Line, you may help your ſelf, as in the firſt Propoſition.

But if the laſt Prick ſhall fall under the laſt figure but two, as in 172800, then place the unite always at 10 in the end of the Line: for the Root, the Square, and the Cube, will all fall backward, and be found in the ſecond Length between the middle and end of the Line.

6. *To multiply one number by another.*

Extend the Compaſſes from 1 to the Multiplicator; the ſame extent applied the ſame way, ſhall reach from the Multiplicand to the Product.

As if the Numbers to be multiplied were 25 and 30: either extend the

the Compasses from 1 to 25, and the same extent will give the distance from 30 to 750; or extend them from 1 to 30, and the same extent shall reach from 25 to 750.

7. *To divide one Number by another.*

Extend the Compasses from the Divisor to 1, the same extent shall reach from the Dividend to the Quotient.

So if 750 were to be divided by 25, the Quotient would be found to be 30.

8. *Three Numbers being given, to find a fourth Proportional.*

This golden Rule, the most useful of all others, is performed with like ease. For extend the Compasses from the first Number to the second, the same extent shall give the distance from the third to the fourth.

As for example, the proportion between the Diameter and the Circumference, is said to be such as 7 to 22: if the Diameter be 14: how much is the Circumference? Extend the Compasses from 7 to 22, the same extent shall give the distance from 14 to 44: or extend them from 7 to 14, and the same extent shall reach from 22 to 44.

Either of these ways may be tried on several places of this Line; but that place is best, where the feet of the Compasses may stand nearest together.

9. *Three Numbers being given, to find a fourth in a duplicated proportion.*

If any have daily use of this Proposition, he may cause another Line of Numbers to be made.

This Proposition concerns questions of proportion between Lines and Superficies; where if the denomination be of Lines, extend the Compasses from the first to the second Number of the same denomination: so the same extent being doubled, shall give the distance from the third Number unto the fourth.

The Diameter being 14, the content of the Circle is 154: the Diameter being 28, what may the content be? Extend the Compasses from 14 to 28, the same extent doubled will reach from 154 to 616.

For firſt, it reacheth from 154 unto 308; and turning the Compaſſes once more: it reacheth from 308, unto 616; and this is the content required.

But if the firſt denomination be of the ſuperficial content, extend the Compaſſes unto the half of the diſtance, between the firſt Number and the ſecond of the ſame denomination: ſo the ſame extent ſhall give the diſtance from the third to the fourth.

The content of a Circle being 154, the Diameter is 14: the content being 616, what may the Diameter be? Divide the diſtance between 154, and 616 into two equal parts, then ſet one foot in 14, the other will reach to 28, the Diameter required.

10. *Three Numbers being given, to find a fourth in a triplicated proportion.*

This Propoſition concerneth queſtions of proportion between Lines and Solids; where if the firſt denomination be of Lines, extend the Compaſſes from the firſt Number to the ſecond of the ſame denomination: ſo the extent being tripled, ſhall give the diſtance from the third Number unto the fourth,

Suppoſe the Diameter of an Iron Bullet being 4 inches, the weight of it was 9 *l.* the Diameter being 8 inches, what may the weight be? Extend the Compaſſes from 4 to 8, the ſame extent being tripled, will reach from 9 unto 72. For firſt, it reacheth from 9 unto 18; then from 18 unto 36; thirdly, from 36 unto 72. And this is the weight required.

But if the firſt Denomination ſhall be of the Solid content, or of the weight, extend the Compaſſes to a third part of the diſtance between the firſt Number and the ſecond of the ſame Denomination; ſo the ſame extent ſhall give the diſtance from the third Number unto the fourth.

The weight of a Cube being 72 *l.* the ſide of it was 8 inches: the weight being 9 *l.* what may the ſide be? Divide the diſtance between 72 and 9, into three equal parts; then ſet one foot to 8, the other will reach to 4, the ſide required.

CHAP.

CHAP. VII.

The use of the Line of Artificial Sines.

THis Line of Sines hath such use in finding a fourth Proportional, as the ordinary Canon of Sines; and the manner of finding it is always such, as in this example.

As the Sine of 90 *gr.*
unto the Sine of 30 *gr.*
So the Sine of 20 *gr.*
unto a fourth Sine.

Extend the Compasses from the Sine of 90 *gr.* unto the Sine of 30 *gr.* the same extent will reach from the Sine of 20 *gr.* unto the Sine of 9 *gr.* 50 *m.*

Or you may extend them from the Sine of 90 *gr.* unto the Sine of 20 *gr.* the same extent will reach from the Sine of 30 *gr.* unto the Sine of 9 *gr.* 50 *m.* and such is the fourth proportional sine required.

In like manner if the question proposed were,

As the Sine of 30 *gr.*
unto the Sine of 52 *gr.*
So the Sine of 38 *gr.*
to a fourth Sine.

Extend the Compasses in the Line of Sines from 30 *gr.* unto 52 *gr.* the same extent shall give the distance from 38 *gr.* unto 76 *gr.* Or, extend them from 30 *gr.* unto 38 *gr.* the same extent will reach from 52 *gr.* unto 76 *gr.* which is the fourth proportional Sine required.

And thus may the rest of all Sinical proportions be wrought two ways. The minutes which are wanting in the first degree of the Sines may be supplied by the Line of Numbers, as I shew in the next Chapter.

CHAP.

CHAP. VIII.

The use of the Line of Artificial Tangents.

THis Line of Tangents hath like use, but commonly joyned with the Line of Sines: the manner of working by it, may appear by this example:

As the Tangent of 38 *gr.* 30 *m.*
is the Tangent of 23 *gr.* 30 *m.*
So is the Sine of 90 *gr.*
to a fourth Sine.

This Proposition, and such others upon two Lines, may be wrought two ways. For extend the Compasses from the Tangent of 38 *gr.* 30 *m.* the Tangent of 23 *gr.* 30 *m.* the same extent shall give the distance from the Sine of 90 *gr.* to the Sine of 33 *gr.* 8 *m.* Or else extend them from 38 *gr.* 30 *m.* in the Tangents unto 90 *gr.* in the Line of Sines; the same extent from the Tangent of 23 *gr.* 30 *m.* shall reach to the Sine of 33 *gr.* 8 *m.* which is the fourth proportional Sine required.

And this Cross work in many cases is the better, in regard the Tangents which should pass on from 40 *gr.* to 50 *gr.* and so forward, do turn back at 45 *gr.* These two Lines of Sines and Tangents, may serve for the resolution of all Spherical Triangles, according to those Canons which I have set down in the use of the Sector. Only two cases the 19 and 20 will be more easily resolved by that which followeth in the last Chapter of this book.

Or if at any time one meet with a Secant, Let him account the Sine of 80 *gr.* for a Secant of 10 *gr.* and the Sine of 70 *gr.* for a Secant of 20 *gr.* and so take the Sine of the Complement instead of the Secant.

As if the Proposition were,

As the Radius
to the Secant of 51 *gr.* 30 *m.*
So the Sine of 23 *gr.* 30 *m.*
to a fourth Sine.

Extend

Extend the Compaſſes from the Radius that is the Sine of *90 gr.* to the Sine of 38 *gr.* 30 *m.* the ſame extent will give the diſtance from the Sine of 23 *gr.* 30 *m.* both to the Sine of 14 *gr.* 22 *m* to the Sine of 39 *gr.* 50 *m.* But in this caſe, the Sine of 39 *gr.* 50 *m.* is the fourth required. For the firſt number being leſs than the ſecond, that is, the Radius leſs than the Secant, the Sine of 23 *gr.* 30 *m.* which is the third, muſt alſo be leſs than the fourth.

If the fourth proportional number ſhall at any time fall out of the Line, by reaſon of the minutes that are wanting in the firſt degree, it may be ſupplied by reſolving the third Number given into minutes, and then working by the Line of Numbers.

As if the Propoſition were,

As the Sine of *90 gr.*
to the Sine of 10 *gr.*
So the Sine of 5 *gr.*
to a fourth Sine.
Or the Tangent of 5 *gr.*
to a fourth Tangent.

Extend the Compaſſes from the Sine of *90 gr.* unto the Sine of 10 *gr.* the ſame extent will reach from the Sine or Tangent of 5 *gr.* beyond the end of the Staff. Wherefore I reſolve theſe 5 *gr.* into 300 *m.* and find the former extent to reach in the Line of Numbers from 300 *m.* unto 52 *m.* and ſuch is the fourth proportional required.

If the extent from the Sine of *90 gr,* unto the Sine of 10 *gr.* be too large for the Compaſſes, we may uſe the Sine of 5 *gr.* 44 *m.* inſtead of the Sine of *90 gr.*

And ſo extending the Compaſſes from the Sine of 5 *gr.* 44 *m.* unto the Sine of 10 *gr.* we ſhall find the ſame extent to reach in the Line of Numbers from 300 unto 52 as before.

And by the ſame reaſon we may uſe the Tangent of 5 *gr.* 43 *m.* inſtead of the Tangent of 45 *gr.* as I further ſhew in the next Chapter.

CHAP. IX.

The use of the Line of Sines and Tangents joyned with the Line of Numbers.

THe Lines of Sines and Tangents have another like use joyned with the Line of Numbers, especially in the resolution of right Lined Triangles, where the Angles are measured by degrees and minutes, and the sides measured by absolute Numbers, whereof, I will set down these Propositions.

1. *Having three Angles and one side, to find the two other sides.*

If it be a Rectangle Triangle, wherein one side about the right Angle being known, it were required to find the other. This may be found by the Line of Tangents, and Line of Numbers. For,

As the Tangent of 45 *gr.*
To the Tangent of the Angle opposite to the side required;
So the Number belonging to the side given,
To the Number belonging to the side required.

As in the Rectangle A B C, knowing the Angle C A B to be 9 *gr.* 15 *m.* and the side A B to be 135 parts, if it were required to find the other side B C about the right Angle,

Extend the Compasses from the Tangent of 45 *gr.* unto the Tangent of 9 *gr.* 15 *m,* the same extent will reach in the Line of Numbers, from 135 unto 22, and such is the length of the side B C. Or in the cross work, extend the Compasses from the Tangent of 45 *gr.* unth 135 in the Line of Numbers, the same extent will reach from the Tangent of 9 *gr.* 15 *m.* unto 22 in the Line of Numbers.

If this extent from the Tangent of 45 *gr.* to 9 *gr.* 15 *m.* or, 135 parts, be too large for the Compasses, you may use the Tangent of 5 *gr.* 43 *m.*

43 *m.* instead of the Tangent of 45 *gr.* because both alike answer to 10, *&c.* parts in the Line of Numbers.

And then either extend the Compass from 5 *gr.* 43 *m.* unto 9 *gr.* 15 *m* in the Line of Tangents, the same extent will reach from 135 unto 22 in the Line of Numbers: or else extend them from the Tangent of 5 *gr.* 43 *m.* unto 135 in the Line of Numbers, the same extent will reach from the Tangent of 9 *gr.* 15 *m.* unto 22 in the Line of Numbers, as before.

In like manner, in the same Rectangle A B C, knowing the Angle A C B to be 80 *gr.* 45 *m.* and the side B C to be 22 parts, it were required to find the other side B A. You may use the Tangent of 84 *gr.* 17 *m.* instead of the Tangent of 45 *gr.* and so the side B A will be found to be 135 parts.

This holdeth for finding of the sides of Rectangle Triangles, but generally in all Triangles, whether they be right or obtuse Angles, having three Angles and one side, we may find the two other sides by the Line of Sines and Line of Numbers.

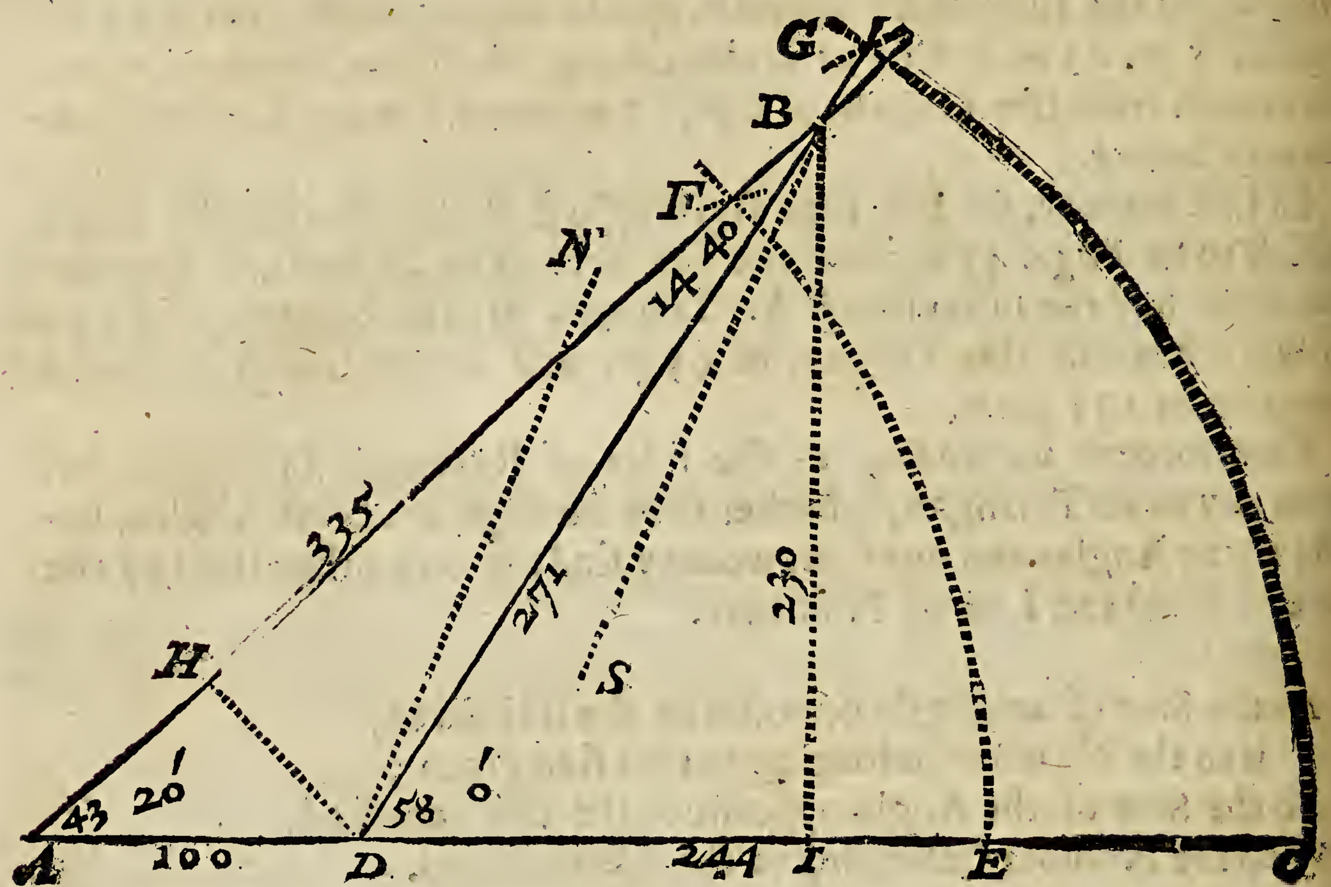
As the Sine of an Angle opposite to the side given,
is to the Number belonging to that side given;
So the Sine of the Angle opposite to the side required,
to the Number belonging to the side required.

As in the example of the fourth Chapter of this Book, where knowing the distance between two stations at A and D to be 100 paces, the Angle B A C to be 43 *gr.* 20 *m.* and the Angle B D C to be 58 *gr.* it was required to find the distance A B.

First, having these two Angles, I may find the third Angle A B D to be 14 *gr.* 40 *m.* either by substraction or by Complement unto 180. Then in the Triangle B A D, I have three Angles, and one side, whereby I may find both A B and D B.

I know the Angle A B D opposite to the measured side A D, to be 14 *gr.* 40 *m.* and the Angle A D B opposite to the side required, to be 122 *gr.* wherefore I extend the Compasses in the Line of Sines, from 14 *gr.* 40 *m.* unto 122 *gr.* or (which is all one) to 58 *gr.* (for after 90 *gr.* the Sine of 80 *gr.* is also the Sine of 100 *gr.* and the Sine of 70 *gr.* the Sine of 110 *gr.* and so in the rest) so shall I find the same extent to reach in the Line of Numbers, from 100 unto 335. And such is the distance required between A and B.

In like manner I extend my Compasses from the Sine of 14 *gr.* 40 *m.* to the Sine 43 *gr.* 20 *m.* the same extent will reach in the Line of Numbers from 100 to 271. And such is the distance between D and B.



Or in cross work, I may extend the Compasses from 14 *gr.* 40 *m.* in the Sines, unto 100 parts in the Line of Numbers, so the same extent will give the distance from 58 *gr.* to 335 parts, and from 43 *gr.* 20 *m* to 271 parts.

2. *Having two sides given, and one Angle opposite to either of these sides, to find the other two Angles and the third side.*

As the side opposite to the Angle given,
is to the Sine of the Angle given:
So the other side given,
to the Sine of that Angle to which it is opposite.

So in the former Triangle, having the two sides, A B 335 paces, and A D 100 paces, and knowing the Angle A D B, which is opposite to the side A B, to 122 *gr.* I may find the Angle A B D, which is opposite to to the other ſide A D. For if I extend the Compaſſes from 335 unto 100 in the Line of Numbers, I ſhall find the ſame extent to reach in the Line of Sines from 122 *gr.* to 14 *gr.* 40 *m.* and therefore ſuch is the Angle A B D.

Then knowing theſe two Angles A B D and A D B, I may find the third Angle B A D either by ſubtraction or by Complement to 180, to be 43 *gr.* 20 *m.* and having three Angles and two ſides, I may well find the third ſide D B, by the former Propoſition.

This may be done more readily by croſs work. For if I extend the Compaſſes from 335 parts, in the Line of Numbers, to the Sine of 122 *gr.* the ſame extent will reach from 100 parts to the Sine of 14 *gr.* 46 *m.* and back from 43 *gr.* 20 *m.* to 271 parts; and ſuch is the third ſide D E.

3. *Having two ſides and the Angle between them, to find the two other Angles and the third ſide.*

If the Angle contained between the two ſides be a right Angle, the other two Angles will be found readily by this Canon.

As the greater ſide given,
 is to the leſſer ſide:
So the Tangent of 45 *gr.*
 to the Tangent of the leſſer Angle.

So in the Rectangle triangle, A I B, knowing the ſide A I to be 244, and the ſide I B 230: if I extend the Compaſſes from 244 to 230 in the Line of Numbers, the ſame extent will reach from 45 *gr.* to about 43 *gr.* 20 *m.* in the Line of Tangents; and ſuch is the leſſer Angle B A I, and the Complement 46 *gr.* 40 *m.* ſhews the greater Angle A B I. The Angles being known, the third ſide A B may be found by the firſt Propoſition.

So likewiſe in the example of the third Chapter of this Book, concerning taking of Angles by the Line of Inches, where the parts intercepted on the Staff being 20 Inches, and the parts on the Croſs 9 Inches, it was required to find the Angle of the Altitude. For,

I may extend the Compaſſes in the Line of Numbers, from 20 unto 9, the ſame extent will reach in the Line of Tangents from 45 *gr.* to 24 *gr.* 14 *m.*

Or in cross work,

I may extend the Compasses from 20 parts in the Line of Numbers, to the Tangent of 45 *gr.* the same extent shall give the distance from 9 parts, unto the Tangent of 24 *gr.* 14 *m.*

And such is the Angle of the Altitude required.

If the parts intercepted on the Staff being 20 Inches, and the parts on the Cross 9 tenth parts of an inch, it were required to find the Angle of the Altitude. Here the Angle would be much less, and the 9 would fall out of the Line of Numbers.

To supply this defect, I use the Tangent of 5 *gr.* 43 *m.* instead of the Tangent of 45 *gr.* And then if I extend the Compasses in the Line of Numbers from 20 unto 9, the same extent will reach in the Line of Tangents from 5 *gr.* 43 *m.* unto 2 *gr.* 35 *m.*

Or in Cross work, if I extend them from 20 parts in the one Line of Numbers, unto the Tangent of 5 *gr.* 43 *m.* the same extent will give the distance from 9 in the Line of Numbers, unto the Tangent of 2 *gr.* 35 *m.*

And such is this Angle of the Altitude required.

But if it be an oblique Angle that is contained between the two sides given, the Triangle may be reduced into two Rectangle Triangles, and then resolved as before.

As in the Triangle A D B, where the side A B is 335, and the side A D 100, and the Angle B A D 43 *gr.* 20 *m.* If I let down the Perpendicular, D H upon the side A B, I shall have two Rectangle Triangles, A H D, D H B; and in the Rectangle A H D, the Angle at A being 43 *gr.* 20 *m.* the other Angle A D H will be 46 *gr.* 40 *m.* and with these Angles and the side A D, I may find both A H and D H, by the first Proposition.

Then taking A H out of A B, there remains H B for the side of the Rectangle D H B, and therefore with this side H B and the other side H D, I may find both the Angle at B, and the third side D B, as in the former part of this Proposition.

Or I may find the Angles required, without setting down any Perpendicular. For,

As the sum of the sides;
  is to the difference of the sides:
So the Tangent of the half sum of the opposite Angles,
  to the Tangent of half the difference between those Angles.

As

As in the former Triangle A D B, the sum of the sides A B, A D, is 435, and the difference between them 235; the Angle contained 43 *gr.* 20 *m.* and therefore the sum of the two opposite Angles 136 *gr.* 40 *m.* and the half sum 68 *gr.* 20 *m.* Hereupon I extend the Compasses in the Line of Numbers from 435 to 235, and I find them to reach in the Line of Tangents from 68 *gr.* 20 *m.* unto 53 *gr.* 40 *m.* and such is the half difference between the opposite Angles at B and D. This half difference being added to the half sum, doth give 122 *gr.* for the greater Angle A D B: and being subtracted, it leaveth 14 *gr.* 40 *m.* for the lesser Angle A B D, then the three Angles being known, the third side B D may be found by the first Proposition.

4. *Having the three sides of a right Line Triangle, to find the three Angles.*

Let one of the three sides given be the Base, but rather the greater side, that the Perpendicular may fall within the Triangle; then gather the sum, and difference of the two other sides, and the proportion will hold.

As the Base of the Triangle,
is to the sum of the sides:
So the difference of the sides
to a fourth, which being taken forth of the Base, the Perpendicular shall fall on the middle of the remainder.

As in the former Triangle A D B, where the Base A B is 335, the sum of the sides A D and D B 371, and the difference of them 171. If I extend the Compasses in the Line of Numbers from 335 unto 371, I shall find the same extent to reach from 171 unto 189. 4. This fourth Number I take out of the Base 335. 0. and the remainder is 145. 6, the half whereof is 72. 8, and doth shew the distance from A unto H, where the Perpendicular shall fall, from the Angle D, upon the Base A B, dividing the former Triangle A D B into two right Angle Triangles, D H A and D H B, in which the Angles may be found by the second Proposition.

And this may suffice for the right Line Triangles, but for the more easie protraction of these Triangles, I will set down one Proposition more concerning Chords.

5. *Having*

5. *Having the Semidiameter of a Circle, to find the Chords of every Ark.*

As the Sine of the Semiradius of 32 *gr.*
to the Sine of half the Ark propoſed:
So is the Semidiameter of the Circle given,
to the Chord of the ſame Ark.

As if in the protracting the former Triangle A D B, it were required to find the length of a Chord of 43 *gr.* 20 *m.* agreeing to the Semidiameter A E, which is known to be three Inches. The half of 43 *gr.* 20 *m.* is 21 *gr.* 40 *m.* wherefore I extend the Compaſſes from the Sine of 30 *gr.* to the Sine of 21 *gr,* 40 *m.* and I find the ſame extent to reach in the Line of Numbers from 3.000 parts to 2.215. which ſhews, that the Semidiameter being three Inches, the Chord of 43 *gr.* 20 *m.* will be 2 Inches and 215 parts of 100.

In like manner the Chord of 58 *gr.* agreeing to the ſame Semidiameter, would be found to be 2 inches and 909 parts. For the half of 58 being 29; if I extend the Compaſſes in the Line of Sines from 30 *gr.* to 29 *gr.* the ſame extent will reach in the Line of Numbers from 3.000. unto 2.909.

Or in Croſs work, if I extend the Compaſſes from the Sine of 30 *gr.* to 3.000 in the Line of Numbers, I ſhall find the ſame extent to reach from 21 *gr:* 40 *m.* to 2.215 parts, and from 29 *gr.* to 2.909 parts, and from 7 *gr.* 20 *m.* to 795 parts; for the Chord of 14 *gr.* 40 *m.* for the third Angle A B D.

CHAP

CHAP. X.

The use of the Line of versed Sines.

THis Line of versed Sines is no necessary Line. For all Triangles, both right lined and spherical may be resolved by the three former Lines of Numbers, Sines and Tangents; yet I thought good to put it on the Staff for the more easie finding of an Angle having three sides, or a side having three Angles of a spherical Triangle given.

Suppose the three sides to be, one of them 100 *gr.* the other 78 *gr.* and the third 38 *gr.* 30 *m.* and let it be required to find the Angle, whose Base is 110 *gr.*

I first add them together, and from half the sum subtract the Base, noting the difference after this manner.

The Base	110 *gr.*	0 *m.*
The one side	78	0
The other side	38	30
The sum of all three	226	30
The half sum	113	15
The difference	3	15

For so the proportion will hold.

1 As the Radius
to the Sine of one of the sides:
So the Sine of the other side,
to a fourth Sine.

2 As this fourth Sine,
to the Sine of the half sum:
So the Sine of the difference
to a seventh Sine.

3 The

3 The mean proportional between this seventh Sine and the Radius, will shew the Sine of the Complement of half the Angle required.

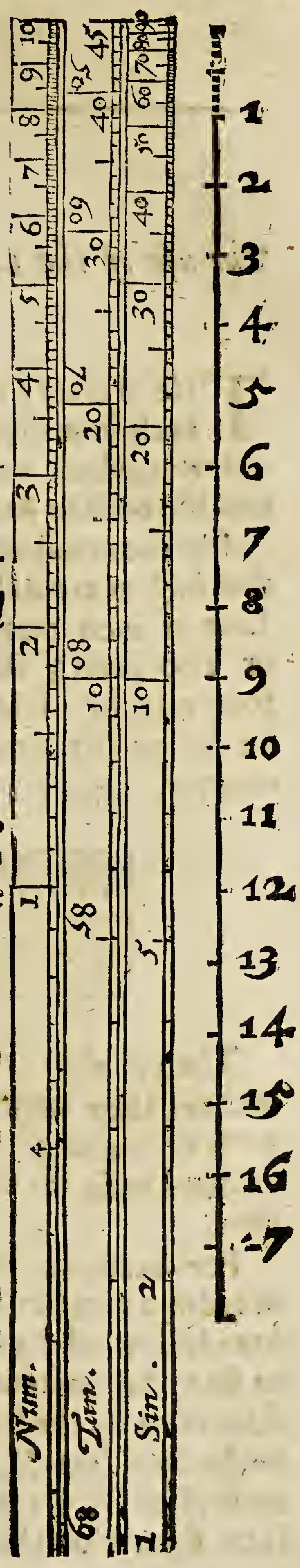
This done, I come to the Staff, and extend the Compasses from the Sine of 90 *gr.* to the Sine of 78 *gr.* which is one of the sides; and applying this extent from the Sine of the other side 38 *gr.* 30 *m.* I find it to reach to a fourth Sine, about 37 *gr.* 30 *m.* From this fourth Sine of 37 *gr.* 30 *m.* I extend the Compasses again, to the Sine of the half sum 113 *gr.* 15 *m.* (which is all one with the Sine of 66 *gr.* 45 *m.*) and this second extent will reach from the Sine of the difference 3 *gr.* 15 *m.* to the Sine of 4 *gr.* 54 *m.*

Then to find the mean proportional Sine between this seventh Sine of 4 *gr.* 54 *m.* and the Sine of 90 *gr.* I might divide the space between them into two equal parts, and so I should find the Compasses to stay at 17 *gr.* whose Complement is 73 *gr.* and the double of 73 *gr.* is 146 *gr.* the Angle opposite to 110 *gr.* which was required.

But because this division is somewhat troublesome I have therefore added this Line of Versed Sines, that having found the seventh Sine you might look over against it, and there find the Angle. And so in this example having found the seventh Sine to be 4 *gr.* 54 *m.* over against this Sine you shall find 146 *gr.* in the Line of Versed Sines for the Angle required as before.

THE

THE SECOND BOOK OF THE CROSS-STAFF.



Of the use of the former Lines of Proposition more particularly exemplified in several kinds.

THe former Book containing the general use of each Line of proportion, may be sufficient for all those which know the Rule of Three, and the doctrine of Triangles.

But for others, I suppose it would be more difficult to find either the Declination of the Sun, or his Amplitude, or the like, by that which hath been said in the use of the Line of Sines, unless they may have the particular proportions, by which such propositions are to be wrought.

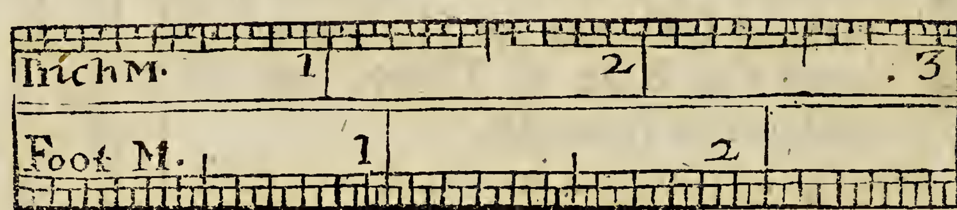
And therefore for their sakes I have adjoyned this second Book, containing several proportions for propositions of ordinary use, and set them down in such order, that the Reader considering which is the first of the three Numbers given, may easily apply them to the Sector, and also resolve them by Arithmetick, beginning with those which require help only of the Line of Numbers.

CHAP. I.

The uſe of the Line of Numbers in broad meaſure, ſuch as Board, Glaſs, and the like.

THe ordinary meaſure for breadth and length are feet and inches, each foot divided into 12 inches, and every inch into halves and quarters, which being parts of ſeveral denominations doth breed much trouble both in Arithmetick and the uſe of inſtruments.

For the avoiding whereof, where I may prevail I give this counſel, that ſuch as are delighted in meaſure would uſe ſeveral Lines, firſt a Line of inch meaſure, wherein every inch may be divided into 10 or 100 parts; ſecondly, a Line of foot meaſure, wherein every foot may be divided into 100 or 1000 parts, both which Lines may be ſet on the ſame ſide of a two foot Ruler, after this or the like manner.



Then if they be to give the content of any Superficies or Solid in inches, they may meaſure the ſides of it by the Line of inches and parts of inches; but if they be to give the content in feet, it would be more eaſie for them to meaſure thoſe ſides by the foot Line and his parts.

For example, let the length of a Plain be 30 inches, and the breadth 21 inches and $\frac{6}{10}$ of an inch; this length multiplied into the breadth, would give the content to be 648 inches: but if I were to find the content of the ſame Plane in feet, I would meaſure the ſides of it by the foot Line and his parts; ſo the length would prove to be two feet $\frac{50}{100}$, and the breadth one foot $\frac{80}{100}$, and the length multiplied by the breadth, cutting off the four laſt figures, for the four figures of the parts, would give the content to be 4.5000, which

which is 4 foot and 5000 parts of a foot, divided into 10000 parts.

21. 6	2. 50
30. 0	1. 80
948,00	20000
	250
	4.5000

☞ The like reason holdeth for Yards and Ells, and all other measures divided into 10. 100 or 1000 parts.

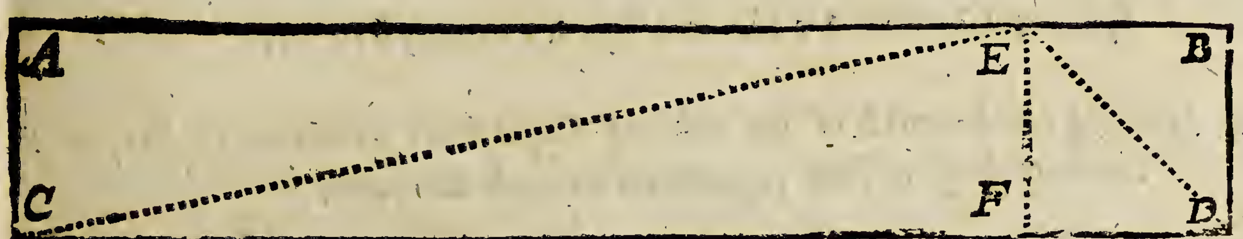
This being presupposed, the work will be more easie both by Arithmetick and the line of Numbers, as may appear by these Propositions.

SECT. I.

Of the Mensuration of Oblong Superficies, and Triangles.

1. *Having the breadth and length of an Oblong Superficies given in inch-measure, to find the content in inches.*

As one inch unto the breadth in inches,
So the length in inches unto the content in inches.



Suppose in the Plane A D, the breadth A C to be 30 inches, and the length A B to be 183 inches; extend the Compasses from 1 unto 30, the same extent will reach from 183 unto 5490; or extend them from 1 unto 183 the same extent will reach from 30 unto 5490. So both ways the content required is found to be 5490 inches.

As 1 unto 30: so are 183 unto 5490.

 2. *Having*

2. *Having the breadth and length of any Oblong Superficies given in inches, to find the content in feet.*

As 144 inches unto the breadth in inches:
So the length in inches unto the content in feet.
And thus in the former Plane A D, working as before, the content will be found to be 38. 125, which is 38 foot and $\frac{1}{2}$ of a foot.
As 144 unto 30, so are 183 unto 38. 125.

3. *Having the length and breadth of any Oblong Superficies given in foot measure, to find the content in feet.*

As 1 foot unto the breadth in foot measure:
So the length in feet unto the content in feet.
And thus in the former Plane A D, the breadth will be two feet 50 parts, and the length 15 feet 25 parts; then working as before, the content will be found to be 38. 125.
As 1 unto 2. 50: so are 15. 25 unto 38. 125.

4. *Having the breadth of any Oblong Superficies given in inches, and the length in foot measure, to find the content in feet.*

As 12 inches to the breadth in inches:
So the length in feet to the content in feet.
So also in the former Plane, the content will be found to be 38. 125.
As the 12 unto 30: so are 15. 25 unto 38. 125.

5. *Having the breadth of an Oblong Superficies given in inches, to find the length of a foot superficial in inch measure.*

As the breadth in inches, unto 144 inches:
So 1 foot unto the length in inch measure.
So the breadth being 30 inches, the length of a foot will be found to be 4 inches 80 parts, the length of two feet 9 inches 60 parts.
As 30 unto 144; so are 1 unto 480.

6. Having

6. *Having the breadth of an Oblong Superficies given in feet, to find the length of a foot superficial in foot measure.*

As the breadth in foot measure to 1 foot:
So the number of feet to the length in foot measure.

So the breadth being 2 feet 50 parts, the length of a foot will be found to be 40 parts, the length of 2 feet 80 parts, and the length of 3 feet 1 foot 20 parts, &c.

As 250 unto 1 : so are 1 unto 0,40.

7. *A four sided Superficies having any of the two sides Parallels, to find the Area.*

Add the two Parallel sides together, and take the half, then say,

As 1
is to the half sum of the two Parallel sides :
So is the breadth (or length)
to the Area,

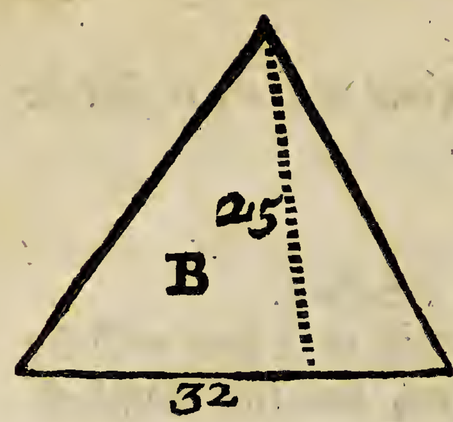
So in the four sided figure A having one of the parallel sides in length 23. 25 foot, and the other, 19. 75 foot, the sum of them is 43. 00 foot, (the half whereof is 21. 50 foot) and the breadth 14. 5. the Area or content of this Superficies will be found to be 311. 75 foot.

Extend the Compasses from 1 to 21. 50 (the mean length) the same extent will reach from 14.5 (the breadth) to 311. 15 the Area or content.

8. *To find the Area or content of a Triangle, the longest side and the Perpendicular being given.*

As 1
is to the half length of the Base :
So is the length of the Perpendicular
to the Content or Area.

So



So the Triangle B, having the Base 32 foot, and the Perpendicular 25 foot, the Area will be found to be 400 foot.

Extend the Compasses from 1 to 16, (half the Base) the same extent will reach from 25 (the Perpendicular) to 400 the Area.

Or, extend the Compasses from 1 to 12.5; (half the length of the Perpendicular) the same extent will reach from 32 (the whole Base) to 400 as before.

Or, extend the Compasses from 1 to 32, the same extent will reach from 25 to 800, the double Area.

9. The side of an Equilateral Triangle being given, to find the Area.

As 1000,
is to 433.01,
So is the Square of the side of the Triangle,
to the Area.

So the side of an Equilateral Triangle being 17.5 foot, the Area will be found to be 132.61 foot.

Extend the Compasses from 1000 to 433.01, the same Extent will reach from 306.25 (the Square of the side of the Triangle) to 132.61, the Area.

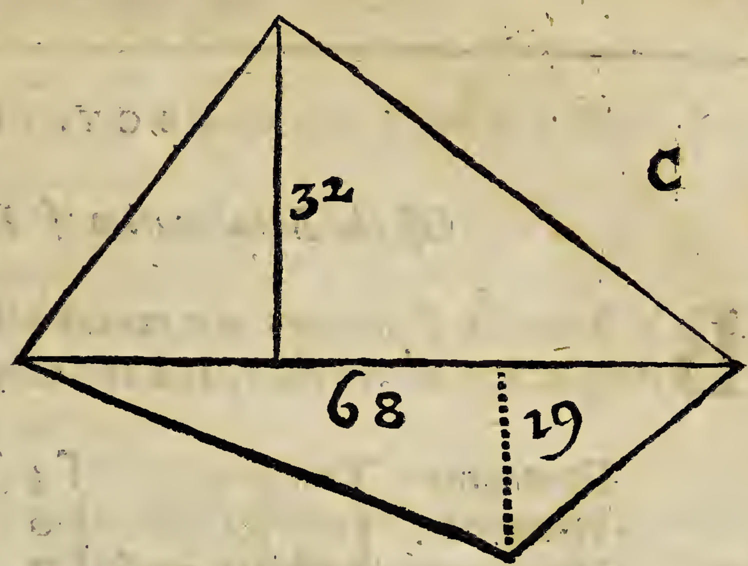
10. *To find the Area of a four sided figure, whose sides are neither equal nor parallel one to the other, which figures are called Trapezias.*

All four sided figures whose sides are neither equal nor parallel, must be reduced into two Triangles, by drawing a Diagonal Line from any one Angle to its opposite, upon which Diagonal two Perpendiculars must be let fall; then,

As 1,
is to half the length of the Diagonal:
So is the length of both the Perpendiculars,
To the Area, or Content.

So

So in the Trapezia C, the Diagonal is 68, (the half of it is 34) and the two Perpendiculars are 32 and 19, their Sum is 51. Then,



Extend the Compasses from 1 to 34. (half the Diagonal) the same extent will reach from 51 (the sum of the Perpendiculars) to 1734 the Area.

Or, extend the Compasses from 1 to 68 (the Diagonal) the same extent will reach from 25. 5 half the sum of the two Perpendiculars) to 1734 as before.

In all other right lined figures, of how many sides, or how irregular soever, before they can be measured they must (by drawing of Lines from Angle to Angle) be reduced into Triangles or Trapezias, and so be measured by these two last Precepts.

And here note, That when any irregular figure is thus reduced into Triangles, the number of Triangles will be less by two than the number of the sides of the irregular figure.☞

11. *Having the length and breadth of an Oblong Superficies, to find the side of a Square equal to the Oblong.*

Divide the space between the length and the breadth into two equal parts, and the foot of the Compasses will stay at the side of the Square.

So the length being 183 inches, and the breadth 30 inches, the side of the square will be found to be 74 inches, and almost 10 parts of 100.

Or the breadth being 2 foot and 50 parts, the length 15 foot and 25 parts, the side of the square will be found to be about 6 feet and 17 parts.

As 30 unto 74, 10 : so are 74, 10 unto 183, 027.
And as 2, 50 unto 6, 174 : so are 6, 174 unto 15, 247.

Sect. II.

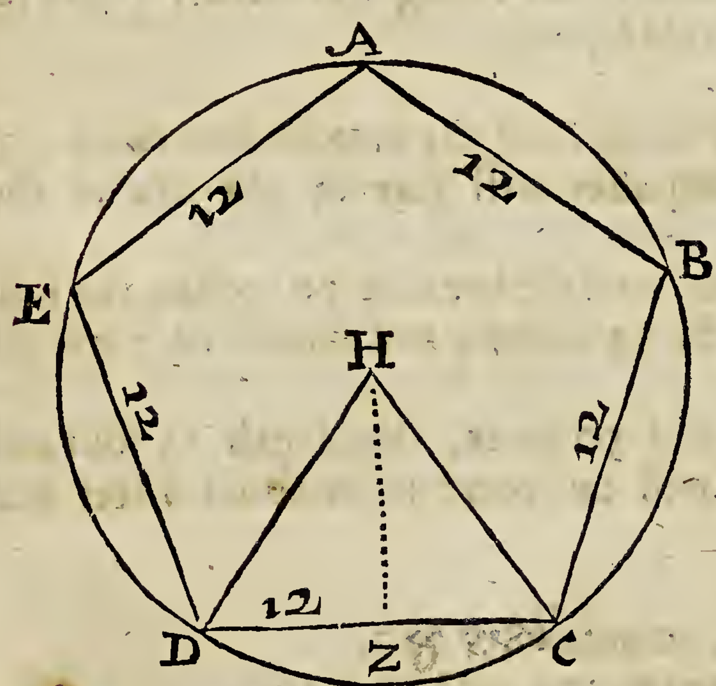
Of the Mensuration of Regular Polygons.

BY Regular Polygons are meant all such figures whose sides and Angles are above four, and are all equal. As the

Pentagon	which consi-steth of	5	Equal sides and Angles.
Hexagon		6	
Heptagon		7	
Octagon		8	
Nonagon		9	
Decagon		10	

And the Area of any of these Regular Polygons is equal to a Paral-lelogram, whose length shall be equal to half the Perimeter, and whos breadth equal to a Perpendicular drawn from the Center of the figur to the middle of any of the sides of the Polygon.

1. *The side and Perpendicular of a Pentagon being given, to find th Area.*



As 1
is to the Perpendicula (8, 25
So is half the Perimete (30 i
To the Area 247. 7 (inche

So in this Pentagon, whe the side C D (and so all th rest) contains 12 inches, an the Perpendicular H Z, 8, 25 inches, the Area will be foun to be 247. 74.

Extend the Compasses from 1. to 8. 258, (the Perpendicular) th same extent will reach from 30 (which is half the length of all the sides) to 247. 74 the Area. 2. 7

2. *The Side and Perpendicular of an Octagon (or figure of 8 ſides) being given, to find the Area.*

As 1
is to the Perpendicular 14. 48.
So is the Semi-Perimeter 48.
to 695. 04 the Area.

So a regular Polygon of 8 ſides, each ſide containing 12 inches and the Perpendicular 14 48 inches, the Area thereof will be found to be 695. 04.

Extend the Compaſſes from 1 to 14. 48 the Perpendicular, the ſame extent will reach from 48 (half the Perimeter) to 695 04. the Area.

And in this nature, may any Regular Polygon, of what number of ſides ſoever, be meaſured.

Sect. III.

Of the Menſuration of Circles.

THe Proportion of the Diameter of a Circle to its Circumference as 7 is to 22. but *Ludolph Van Culen* comes ſomewhat nearer, allowing the Diameter to the Circumference to be (near) as 113 to 355. which proportions I ſhall uſe in the following Problems.

1. *The Diameter of a Circle being given, to find the Circumference.*

As 113 is to 355:
So is the Diameter to the Circumference.

So the Diameter of a Circle being 15 inches, the Circumference will be found to be 47.12.

Extend the Compaſſes from 113 to 355. the ſame extent will reach from 15 the Diameter, to 47. 12 inches the Circumference.

2. The Circumference of a Circle being given, to find the Diameter.

As 355 is to 113 :
So is the Circumference to the Diameter.

So the Circumference being 47. 12. the Diameter will be found to be 15 inches.

Extend the Compasses from 355 to 113, the same extent will reach backwards from 47. 12 to 15.

3. The Diameter of a Circle being given, to find the Area.

As 28
is to 22 :
So is the square of the Diameter 225
to the Area 176. 61.

So the Diameter being 15 inches, the Area will be found to be 176. 61.

Extend the Compasses from 28 backwards to 22, the same extent applied (the same way) will reach from 225 (the Square of the Diameter) to 176. 61, the Area.

4. The Area of a Circle being given, to find the Diameter.

As 22
is to 28 :
So is the Area 176. 61,
to the Square of the Diameter 225.

So the Area of a Circle being 176. 61 inches, the Diameter will be found to be 15 inches.

Extend the Compasses from 22 to 28, the same extent will reach from 176. 61 to 2247. 08 the Square of the Diameter, the middle way upon the Line between 2247. 08 and 1, is 15 the Diameter.

5. The Circumference of a Circle being given, to find the Area.

As 88
is to 7:
So is the Square of the Circumference 2220. 29
to the Area 176. 61.

So the Circumference of a Circle being 47. 12, the Area will be found to be 176. 61 inches.

Extend the Compasses from 88 to 7, the same extent will reach from 2220. 29 (the square of the Circumference) to 176. 61 the Area.

6. The Area of a Circle being given, to find the Circumference.

As 7
is to 88:
So is the Area 176. 61.
to the Square of the Circumference 2220. 29.

So the Area of a Circle being 176. 61, the Circumference will be found to be 47. 12.

Extend the Compasses from 7 to 88, the same extent will reach from 176. 61, the Area to 2220. 29, the Square of the Circumference, the half distance between 1 and 2220. 29, is 47. 12 the Circumference.

7. Having the Diameter of a Circle, to find the side of a Square equal to that Circle.

As 10000 to the Diameter:
So 8862. unto the side of the Square.

So the Diameter of a Circle being 15 inches, the side of the square will be found about 13 inches and 29 parts.

As 10000 unto 8862: so are 15 unto, 29.

8. *Having the Circumference of a Circle, to find the side of a Square equal to the same Circle.*

As 10000 to the Circumference:
So 2821 to the side of the Square.

So the Circumference of a Circle being 47 inches 13 parts, the side of the Square will be about 13 inches 29 parts.
As 10000 unto 2821 : so are 47, 13 unto 13,29.

SECT. IV.

Of the Mensuration of Land by Perch and Acres.

1. *Having the breadth and length of an Oblong Superficies, given in Perches, to find the content in Perches.*

As 1 Perch, to the breadth in Perches:
So the length in Perches, to the content in Perches.

So in the former Plane A D, if the breadth A C be 30 Perches, and the length A B 183 Perches, the content will be found to be 5490 Perches.

2. *Having the length and breadth of an Oblong Superficies given in Perches, to find the content in Acres.*

As 160, to the breadth in Perches:
So the length in Perches, to the content in Acres.

So in the former Plane A D, the content will be found to be 34 Acres, and 31 Centesmes, or parts of 100.
As 160, unto 30 : So are 183, unto 34, 31.

To augment a Superficies in a proportion,
To diminish a Superficies in a proportion given.

A Table

A Table for the use of the Chain.

Long.

	Inch.	Cent.*	Foot.	Pace.	Perch.	Chain.	Acre.	Mile.
Inch.	1	7, 92	12	60	198	792	7920	63360
Cent.	62,7264	1	1,515	7,575	25	100	1000	8000
Foot.	144	2, 295	1	5	26,5	66	660	5280
Pace.	3600	57, 485	25	1	3,3	13,2	132	1056
Perch.	39204	625	272,25	10, 89	1	4	40	320
Chain.	627264	10000	4356	17424	16	1	10	80
Acre.	6272640	100000	43560	1742,4	160	0	1	8
Mile.	4014489600	64000000	27878400	1115136	10400	6400	640	1

Square.

* Centesms of a Chain.

3. *Having the length and breadth of an Oblong Superficies given i Chains, to find the content in Acres.*

It being troublesome to divide the content in Perches by 160, w may measure the length and breadth by chains, each chain being Perches in length, and divided into 100 links, then will the work b more easie in Arithmetick. For,

As 10 to the breath in Chains:
So the length in Chains, to the content in Acres.

And thus in the former Plane A D, the breadth A C will be Chains 50 Links, and the length A B 45 Chains 75 Links; then work ing as before, the content will be found as before, 34 Acres 31 parts

4. *Having the Perpendicular and Base of a Triangle given in Perches, find the content in Acres.*

If the Perpendicular go for the breadth, and the Base for th length, the Triangle will be the half of the Oblong, as the Triang C E D is the half of the Oblong A D, whose content was found in th former Proposition. Or without halfing.

As 320 to the Perpendicular:
So the Base, to the content in Acres.

So in the Triangle C E D, the Perpendicular being 30, and the Ba 183, the content will be found to be about 17 Acres and 15 parts.

5. *Having the Perpendicular and Base of a Triangle given in Chains, find the content in Acres.*

As 20 to the Perpendicular:
So the Base, to the content in Acres.

And so in the Triangle C E D, the Perpendicular E F being 7, 5 and the Base C D 45, 75, the content will be found, as before, to b about 17 Acres 15 parts.

6. *Havi*

6. Having the content of a Superficies after one kind of Perch, to find the content of the same Superficies, according to another kind of Perch.

As the length of the second Perch,
to the length of the first Perch:
So the content in Acres to a fourth number;
and that fourth to the content in Acres required.

Suppose the Plane A D measured with a chain of *66* feet, or with a Perch of 16 feet and an half, contained 34 Acres 31 parts; and it were demanded how many Acres it would contain, if it were measured with a chain of 18 foot to the Perch: these kind of Propositions are wrought by the backward Rule of three, after a duplicate proportion. Wherefore I extend the Compasses from 16, 5 unto 18, 0, and the same extent doth reach backward, first from 34, 31 to 31, 45, and then from 31, 45 to 28, 84, which shews the content to be 28 Acres 84 parts.

7. Having the plot of a Plane with the content in Acres, to find the Scale by which it was plotted.

Suppose the Plane A D, contained 34 Acres 31 Centesms; if I should measure it with a Scale of 10 in the inch, the length A B would be 38 Chains, and about 12 Centesms, and the breadth A C, *6* Chains and 25 Centesms; and the content would be found by the third Proposition of this Chapter, to be about 22 Acres 82 parts, whereas it should be 34 Acres 31 parts.

Wherefore I divide the distance between 23, 82 and 34, 31, upon the Line of Numbers, into two equal parts; then setting one foot of the Compasses upon 10, my supposed Scale, I find the other to extend to 12, which is the Scale required.

8. *Having the length of the Furlong, to find the breadth of the Acre.*

As the length in Perches, to 160:
So 1 Acre to the breadth in Perches.

So the length of the Furlong being 40 Perches, the breadth for an

Acre

Acre will hold found to be 4 Perches. If the length be 50, the breadth for one Acre must be 3, 20, the breadth for two Acres 6,40.

Or if the length be measured by chains.

As the length in chains unto 10:
So 1 Acre to his breadth in chain measure.

So the length of the Furlong being 12 Chains 50 Links the breadth for one Acre will be found to be 80 Links, the breadth for two Acres 1 Chain 60 Links.

As 12, 50, unto 10: so 1 unto 0, 80.

Or if the length be measured by feet measure:

As the length in feet, unto 43560:
So 1 Acre, to his breadth in foot measure.

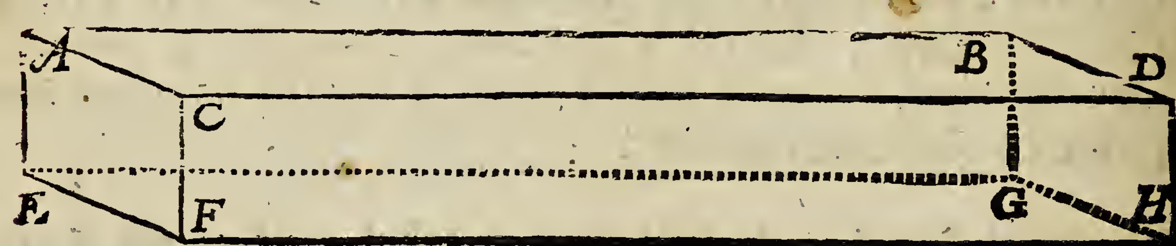
So the length of the Furlong being 792 feet, the breadth for one Acre will be found to be 55 feet, the breadth for two Acres 110 feet.

CHAP. III.

The use of the Line of Numbers in solid measure, such as Stone, Timber, and the like.

Sect. I.

Of the Mensuration of Regular Solids.



1 *Having the side of a Square equal to the Base of any Solid given in inch measure, to find the length of a foot Solid in inch measure.*

The side of a Square equal to the Base of a Solid, may be found by dividing the space between the length and breadth int into two equal parts, as in the seventh Proposition of broad measure.

As the side of the Square in inches, to 41, 57 :
So is 1 foot, to a fourth number ;
and that fourth to the length in inches.

So in the Solid A H, the side of the square equal to the Base E C, being about 25 inches 45 parts, the length of a foot Solid will be found about two inches 67 parts, and the length of two foot Solid 5 inches 34 parts.

As 25, 45, unto 41, 57 : so 1, 00, unto 1, 6 :
and so are 1, 63, unto 2, 67.

2. *Having the side of a Square equal to the Base of any Solid given in foot measure, to find the length of a foot Solid in foot measure.*

As the side of the Square in feet, unto 1 :
So is 1, unto a fourth number :
And that fourth, to the length in foot measure.

So in the Solid A H, the side of the Square equal to the Base E C, being about 2 foot 120 parts, the length of a foot Solid will be found about 222 parts of a foot.

As 2, 120, unto 1,000 : so 1,000, unto 0,471.
and so are 471, unto 222.

3. *Having the breadth and depth of a squared Solid given in foot measure, to find the length of a foot Solid in foot measure.*

As 1, unto the breadth in foot measure :
So the depth in feet to a fourth number :
which is the content of the Base in foot measure, Then

As this fourth number, unto 1 :
So 1, unto the length in foot measure.

So in the Solid A H, the breadth being 2 foot 50 parts, the depth 1 foot 80 parts, the content of the Base E C will be found 4 foot 50 parts, and the length of one foot Solid about 222 parts, the length of two foot Solid about 444 parts of 1000.

As 1,00 unto 2, 50 : so are 1,80 unto 4,50.
As 4, 50 unto 1,00: so 1,000 unto 0, 222.

1. *Having the breadth and depth of a squared Solid given in inches, to find the length of a foot Solid in inch measure.*

As 1 hath to the breadth in inches:
So the depth in inches to a fourth number;
Which is the content of the Base in inches. Then,

As this fourth number unto 1728:
So 1 unto the length of a foot in inch measure.

So in the Solid AH, the breadth A C being 30 inches, and the depth A E 21 inches 60 parts, the content of the Base E C will be found to be 648 inches, and the length of a foot Solid about 2 inches 67 parts, the length of two foot Solid 5 inches 34 parts.

As 1 unto 21, 6 : so 30 unto 648.
As 648 unto 1728 : so 1 unto 267,

Or as 12 to the breadth in inches:
So the depth in inches to a fourth number.

As this fourth number to 144:
So 1 unto the length of a foot Solid in inch measure.

So in the Solid A H, the breadth being 30 inches, the depth 21 inches 6 parts, the fourth number will be found to be 54, and the depth of a foot Solid 2 inches 67 parts.

As 12 unto 21,6: so 30 unto 54.
As 54 unto 144: so 1 unto 2, 67.

5. *Having*

5. *Having the side of a Square equal to the Base of any Solid, and the length thereof given in inch measure, to find the content thereof in feet.*

As 41.57 to the side of the Square in inches:
So the length in inches to a fourth number;
and that fourth to the content in foot measure.

So in the Solid A H, the length A B being 183 inches, and the side of the Square equal to the Base E C about 25 inches 45 parts, the fourth number will be found about 112, and the whole Solid content about 68 feet 62 parts.

As 41.57 unto 25.45 : so 183 unto 112:
and so are 112 unto 68, 62.

6. *Having the side of a Square equal to the Base of any Solid, and the length thereof given in foot measure, to find the content thereof in feet.*

As 1 to the side of the Square in foot measure:
So the length in feet to a fourth number;
and that fourth to the content in foot measure.

So in the former Solid A H, the side of the square equal to the Base A E, being about 2 foot 12 parts, and the length A B 15 foot 25 parts, the content will be found to be about 68 foot 62 parts.

As 1 unto 2.12 : so 15. 25 unto 32. 35:
and so are 32. 35 unto 68.62.

7. *Having the side of a Square equal to the Base of any Solid given in inch measure, and the length of the Solid given in foot measure, to find the content thereof in feet.*

As 12 to the side of the Square given in inches:
So the length in feet to a fourth number;
and that fourth to the content in foot measure.

So in the former Solid A H, the side of the Square being 25 inches 45 parts, the content will be found to be about 68 feet 62 parts.

As 12 unto 25.45 : so 15.25 unto 32.35.
and so are 32.35 unto 68.62.

8. *Having the length, breadth and depth of a squared Solid given in inches, to find the content in inches.*

As 1 unto the breadth in inches :
So the depth in inches unto the Base in inches. Then,

As 1 unto the Base :
So the length in inches unto the Solid content in inches.

So in the Solid A H, whose breadth A C is 30 inches, depth A E 21 inches, and 6 parts of 10, and length A B 183, the content of the Base E C will be found 648 inches, and the whole Solid content about 118584 inches.

As 1 unto 21.6 : so are 30 unto 648 :
As 1 unto 648 : so are 183 to 118,584.

9. *Having the length, breadth, and depth of a squared Solid given in inches, to find the content in feet.*

As 1 to the breadth in inches :
So the depth in inches to the Base in inches.

As 1728 to that Base :
So the length in inches to the content in feet.

So in the Solid A H, the content will be found to be about 68 feet 62 parts.

As 1 unto 21.6 : so 30 unto 648 :
As 1728 unto 648 : so 183 to 68,62.

Or as 12 to the breadth in inches :
So the depth in inches to a fourth number.

As

As 144 to that fourth number:
So the length in inches to the content in feet.

And so also in the same Solid A H, the content will be found to be about 68 feet 62 parts.

As 12 unto 216: so 30 unto 68.62.
As 144 unto 54: so 183 unto 68. 62.

10.*Having the length, breadth, and depth of a squared Solid given in foot measure, to find the content in feet.*

As 1 unto the breadth in foot measure:
So the depth in feet to the Base in feet.

As 1 unto that Base:
So the length in feet to the content in feet.

And thus in the former Solid A H, the breadth A C will be two foot 50 parts, the depth A E, 1 foot 80 parts, and the length A B 15 foot 25 parts; then working as before, the content of the Base A F will be found 4 feet 50 parts, and the whole Solid content about 68 foot 62 parts, which of all others may very easily be tried by Arithmetick.

As 1 unto 2. 50: so 1, 80 unto 4.50.
As 1 unto 4. 50: so 15.25. unto 68. 62.

11. *Having the breadth and depth of a squared Solid given in inches, and the length in foot measure, to find the content thereof in feet.*

As 1 unto the breadth in inches:
So the depth in inches unto a fourth number, which is the content of the Base in inches.

As 144 hath unto that fourth number:
So the length in feet to the content in feet.

And so in the same Solid A H, the content will be found to be about 68 feet 62 parts.

As

As 1 unto 21. 6 : so 30 unto 648.
As 144 unto 15. 25. so 648 unto 68, 62.

Or as 144 unto the breadth in inches :
So the depth in inches unto a fourth number ;
which is the content of the Base in feet.

As 1 hath unto that fourth number :
So the length in feet to the content in feet.

And so in the same Solid A H, the content will be found to be about 68 feet 62 parts.

As 144 unto 21.6 : so 30 unto 4. 50.
As 1 unto 4.50 : so 15.25 unto 68. 62.

Or as 12 unto the breadth in inches:
So the depth in inches unto a fourth number.

As 12 unto this fourth number :
So the length in feet to the content in feet.

And so also in the same Solid A H, the content will be found to be about 68 feet 62 parts.

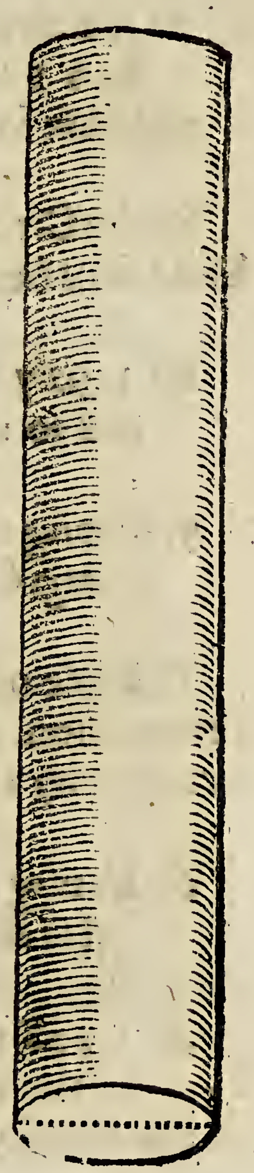
As 12 unto 21. 6 : so 30 unto 54.
As 12 unto 54 : so 15.25 unto 68.62.

All these varieties (and such like not here mentioned) do follow upon the making of the Base of the Solid to be E C; there would be as many more if any shall begin with the Base E H, and so likewise if they make the Base to be F D.

Sect. II.

Of the Menſuration of Cylinders.

1. *Having the Diameter of a Cylinder given in inch meaſure, to find the length of a foot Solid in inches.*



AS the Diameter in inches unto 46 90:
So is 1 unto a fourth number:
And that fourth to the length in inches.

So the Diameter of a Cylinder being 15 inches, the ourth number, will be about 3, 12, and the length of a oot Solid 9 inches 78 parts.

As 15 unto 46. 90: ſo 1 unto 3.127.
and ſo are 3. 127 unto 9. 78.

2. *Having the Diameter of a Cylinder given in foot meaſure, to find the length of a foot Solid in foot meaſure.*

As the Diameter in feet unto 1, 128:
So is 1 unto a fourth number;
and that fourth to the length in foot meaſure.

So the Diameter being 1 foot 25 parts; the length of foot Solid will be found about 8.14 parts of 100c.

As 1 25 unto 1. 128: ſo 1.00 to 0.9027:
and ſo are 9027 unto 8148.

3. *Having the Circumference of a Cylinder given in inches, to find the length of a foot Solid in inch meaſure.*

As the Circumference in inches to 147. 36:
So is 1 to a fourth number;
and that fourth to the length in inches.

So

So the Circumference being 47 inches 13 parts, the length of a foot Solid will be found about 9 inches 74 parts.

As 47. 13 unto 147. 36 : so 1. 00 to 3. 13.
and so are 3, 13 unto 9. 78.

4. *Having the Circumference of a Cylinder given in foot measure, to find the length of a foot Solid in foot measure.*

As the circumference in feet to 3.545 :
So is 1 to a fourth number ;
and that fourth to the length in foot measure.

So the Circumference being 3 foot 927 parts, the length of a foot Solid will be found to be about 815 parts.

As 3. 927 unto 3,545 : so 1.000 unto 0.90. 3.
and so are 903 unto 815.

5. *Having the side of a Square equal to the Base of a Cylinder, to find the length of a foot Solid.*

The side of a square equal to the Circle, may be found by the eighth Proposition of broad measure, and then this Proposition may be wrought by the first and second Proposition of Solid measure.

6. *Having the Diameter of a Cylinder, and the length given in inches, to find the content in inches.*

As 1 128 unto the Diameter in inches :
So the length in inches to a fourth number ;
and that fourth number to the content in inches.

So the Diameter being 15 inches, and the length 105, the content of the Cylinder will be found to be about 18555 inches.

As 1. 1284 unto 15 : so are 105 unto 1395. 87 :
and so are 1395. 87 unto 18555. 34.

7. *Having*

7. *Having the Diameter and length of a Cylinder in foot measure, to find the content in feet.*

As 1. 128 to the Diameter in feet:
So the length in feet to a fourth number;
and that fourth to the content in feet.

So the Diameter being 1 foot 25 parts, and the length 8 foot and 75 parts, the content of the Cylinder will be found about 10 foot 75 parts.

A 1.128 unto 1.25 : so 8.75 unto 9. 69;
and so are 9.69 unto 10. 74.

8. *Having the Diameter of a Cylinder, and the length given in inches, to find the content in feet.*

As 46. 90 to the Diameter in inches:
So the length in inches to a fourth number;
and that fourth to the content in feet.

So the Diameter being 15 inches, and the length 105, the content will be found about 10 foot 74 parts.

As 46. 906 unto 15: so 105 unto 33. 58:
and so are 33. 58 unto 10. 74.

9. *Having the Diameter of a Cylinder, given in inches, and the length in feet, to find the content in feet.*

As 3. 54 to the Diameter in inches:
So the length in feet, to a fourth number;
and that fourth to the content in feet.

So the Diameter being 15 inches, and the length 8 foot 75 parts, the content will be found about 10 foot 74 parts.

As 13.54 unto 15 : so 8.75 unto 9.69 :
and so are 9.69 unto 10.74.

10. *Having the Circumference and length of a Cylinder given in inches to find the content in inches.*

As 3.545 to the Circumference in inches :
So the length in inches to a fourth number ;
and that fourth to the content in inches.

So the Circumference being 47 inches 13 parts, and the length 105 inches, the content will be found about 185. 55 inches.

As 3.545 unto 47. 13 : so 105 unto 1396.
and so are 1396 unto 18555.

11. *Having the Circumference and length of a Cylinder given in inches to find the content in feet.*

As 147. 36 to the Circumference in inches :
So the length in inches to a fourth number ;
and that fourth to the content in feet.

So the Circumference being 47 inches 13 parts, and the length 105 inches, the content will be found about 10 foot 74 parts.

As 147.36 unto 47.13 : so 105 unto 33.58.
and so are 33. 58 unto 1074.

12. *Having the Circumference and length of a Cylinder given in foot measure, to find the content in feet.*

As the 3.545 to the Circumference in feet :
So the length in feet to a fourth number ;
and that fourth to the content in feet.

So the Circumference being 3 foot 927 parts, and the length 8 foot 75 parts, the content will be found to be 10 foot 74 parts.

A

As 3. 545 unto 3, 927 : so 8. 75 unto 9,69.
and so are 9, 69 unto 10, 74.

13. *Having the Circumference of a Cylinder given in inches and the length in foot measure, to find the content in feet.*

As 42, 54. to the Circumference in inches:
So the length in feet to a fourth number;
and that fourth to the content in feet.

So the Circumference being 47 inches 13 parts, and the length 8 oot 75 parts, the content will be found as before 50 foot 74 parts.

As 42, 54 unto 47, 13 : so 8, 75 unto 9, 69 :
and so are 9, 69. unto 10. 74.

Sect. III.

Of the Mensuration of Cones.

1. *The Diameter of the Base and the length of the side of a Cone being given, to find the superficial content thereof.*

As 7 is to 22; Or 113 to 355.
So is ½ the Diameter 6 multiplied in 18 the side,
To the Superficial Content 339.29.

So the Diameter of the Base of a right Cone being 12 inches, and e side thereof 18 inches, the Area will be found to be 339. 29. For, If you extend the Compasses from 7 to 22, or from 13 to 355, the me extent will reach from 108, (which is the half Diameter multipli- in the side) to 339. 29 the Area, or Superficial content.

2. *The Diameter and Axis of a right Cone being given, to find the Solid Content.*

As 28,
Is to 22:

So is the Square of the Diameter 144, multiplied by $\frac{1}{3}$ of the Axis
To the Solid Content of the Cone 678. 85. (*viz.* 86.8

So the Axis of a Cone being 18 inches, and the Diameter 12 inches, the Solid content will be found to be 678.85.

Extend the Compaſſes from 28 to 22. The ſame extent will reach from 864 ($\frac{1}{3}$ of the Axis multiplied in the Square of the Diameter) to 678.85 the ſolid content.

Sect. IV.

Of the Menſuration of Spheres.

1. *The Diameter of a Sphere being given, to find the Superficial content.*

As 7 is to 22, or 113 to 355.
So is the Square of the Diameter 144
To the ſuperficial content.

Thus a Sphere whoſe Diameter is 12 inches, the ſuperficial content thereof will be found to be 452.57.

Extend the Compaſſes from 7 to 22, the ſame extent will reach from 144 (the ſquare of the Diameter) to 452. 57 the ſuperficial content.

2. *The Superficies of a Sphere being given, to find the Axis.*

As 22,
Is to 7:
So is the Superficies
To the ſquare of the Diameter.

So a Sphere whoſe Superficies is 452.57 inches, the Diameter thereof will be found to be 12 inches.

Extend the Compaſſes from 22 to 7, the ſame extent will reach from 452, 57 (the Superficies) to 144, the Square of the Diameter, the $\frac{1}{2}$ diſtance between 144 and 1 is 12 the Diameter.

3. *Th*

3. *The Axis of a Sphere being given, to find the Solid content.*

As 42,
Is to 22:
So is the Cube of the Diameter
To the Solidity.

So if the Axis of a Sphere be 12 inches, the Solid content thereof will be found to be 590. 62.

Extend the Compaſſes from 42 to 22, the ſame extent will reach from 1728 (the Cube of the Diameter) to 905. 14, the Solid content.

4. *The Solidity of a Sphere being given, to find the Axis.*

As 22,
Is to 42:
So is the Solidity
To the Cube of the Diameter, or Axis.

So a Sphere whoſe Solid content is 905. 14. the length of the Axis will be found to be 12 inches.

Extend the Compaſſes from 22 to 42. the ſame extent will reach from 905. 14, the Solidity, to 1728 the Cube of the Axis.

Sect. V.

Of the Menſuration of Priſmes.

A Priſme *is a Solid figure contained under Planes; whereof the two oppoſite are equal, like, and Parallel; but the other are Parallelograms.* Euclid. Defin. 13. Lib. 11.

1. *To find the Solid content of a Triangular Priſme.*

Suppoſe a piece of Timber or Stone to be an Equilateral Triangle at the ends, each ſide thereof being 2. 25 foot, and the length of the piece

piece 17. 75 foot, this is called a Triangular Prisme.

1. Find the content of the Triangle at the end of the piece (by the tenth aforegoing) which will be found to be 2.19. Then say,

As 1,
Is to the Area of the Base:
So is the Length of the Piece
To the content of the Piece in foot measure.

Extend the Compasses from 1 to 2. 19. (the Content of the Area of the Base in feet) the same extent will reach from 17. 75 (the length of the piece in feet) to 38. 87, the content of the Piece in Feet.

2. *To find the Solid Content of a Regular Solid, whose sides at the end thereof are equal, and more than* 3. *As* 4, 5, 6, 7, 8, *or* 10, &c.

Suppose a Regular Solid, as of Timber or Stone, the Plane at the Base or end thereof being a Pentagon, or Figure of 5 equal sides and Angles, each side being 12 inches, or one foot, and the length of the Solid 14 foot.

Note, *The Perpendiculars in these Regular Polygons may be found exact enough for these kinds of Mensurations by taking the least distance from the Center to one of the sides of the Polygon.*

1. Find the Content of the Base (or Pentagon) at the end, by the 1. of the second Section beforegoing, which will be found to be 1.725 foot, the Perpendicular of the Pentagon being 0. 69 parts of a foot: Then say,

As 1,
Is to the Content of the Base in feet 1.725:
So is the length of the piece 14 foot,
To the Content of the Piece in feet 24. 15.

Extend the Compasses from 1 to 1.725, (the Content of the Base) the same extent will reach from 14 foot, the length of the Piece to 24. 15. the Content of the Piece in feet.

And in the same manner, if the side of an Octagon were 12 inches or 1 foot, the Perpendicular would be found to be 1.64, and the length 21.5 feet, the Solidity would be found to be 103.20.

SECT.

SECT. VI.

Of the Menſuration of Pyramides.

A Piramide *is a Solid figure comprehended under divers Planes, ſet upon one Plane, (which is the Baſe of the Pyramide) and gathered together to one Point.* Euclid. Lib. 11. Defin. 12.

The Baſes of Pyramids may be either *Triangles, Squares, Pentagons, Hexagons,* &c. as the Priſmes were; Wherefore to meaſure any Pyramis, you muſt firſt find the Area, or Content of the Baſe, and then ſay,

As 1,
Is to the Area or content of the Baſe 2. 25 :
So is one third part of the height 15 feet,
To the Solid content 33.75 feet.

Suppoſe a Pyramis, whoſe Baſe is a Square, each ſide being 18 inches, or 1. 5 feet, and the height of the ſame Pyramis were 45 feet, and it were required to find the Solidity. The Area of the Baſe by the ſecond of the fifth Section beforegoing, will be found to be 2. 25 feet.

Extend the Compaſſes from 1, to 2.25. (the Content of the Baſe) the ſame extent will reach from 15. (one third part of the height) to 33.75. the Solid content of the Pyramid in feet.

And the like of any other.

SECT. VII.

Of the Menſuration of Fruſtums or Segments of Pyramids or Cones.

THe Solidity of every Cone or Pyramid is found by multiplying the Area of the Baſe (of what form ſoever) into one third part of the Altitude ; Therefore in a Cone whoſe Baſe is Circular, and the Diameter of that Circle is in Foot meaſure 2.50, its Area will be found by what is delivered in the foregoing Sections to be 4.91, and its Altitude 56.25 foot ; I ſay,

As

As 1,
To the Area of the Baſe:
So is one third of the Altitude
To the Solid Content.

So the Area of the Baſe of a Cone or Pyramis being 4. 91, and the Altitude 56. 25, the Solid Content thereof will be found to be 92. 06 foot.

Exend the Compaſſes from 1 to 4. 91 the Area of the Baſe, the ſame extent ſhall reach from 18. 75, the third part of the Altitude, to 92. 06. the Solid Content of the Cone or Pyramis.

But if this Cone or Pyramis were cut off at 18 foot from the Greater end, and then the Leſſer Baſes Area ſhould be found to be in foot meaſure 2.27. what ſhall the Solidity of the Fruſtum be? And in this nature do moſt Timber Trees grow, and ſo being cut off ought to be meaſured, being either Squared or Growing; And no greater Error is here committed in the Meaſuring of Timber, it being in this form, than by the vulgar way of meaſuring ſuch Timber, which is, by finding out the Square in the Middle of the Piece, and taking of that for the true Square, but this always makes the Content of the Piece leſs than it is; The Genuine and true way is this.

Multiply the Area of the two Baſes together, and from the Product extract the Square Root, then add this Root, and the two Area's together, which ſum multiplied by one third part of the Length of the Fruſtum or part ſhall give the Solid Content of that piece.

So a Piece of Stone or Timber whoſe Area at one end is 4. 91. (as in the former Piece) and at the Smaller end 2. 27, and its length 18 foot; the Solidity by the former Rule will be found to be 63. 48 foot. For,

As 1,
Is to the Greater Baſe:
So is the Leſſer Baſe
To a fourth Number,

Whoſe Square Root being Extracted, and added to the two former Area's, will produce another number. Then ſay,

As 1,
Is to this number last found :
So is one third of the length of the Piece,
To the Solid Content of the Piece.

Therefore extend the Compasses from 1 to 4. 91, the greater Base, he same extent shall reach from 2. 27, the lesser Base, to 11. 15. A nean Proportional between 1 and 11. 5 will be found to be 3. 34, which dded to the other two Area's 4. 91, nd 2.27, (as is done in the Margin,) vill produce 10.52. which is your other umber sought for : Then,

Greater Base	4—91
Lesser Base	2—27
Square Root of 11. 15.	3—34
	10—52

Extend the Compasses from 1 to 10.52, the same will reach from 6. the third part of 18 the Length,) to 63. 12. the Solid Content of the Piece which is 63 foot, and half a quarter of a foot.

And now for Proof of this Work to be true, let us find the Solidity f the upper or lesser part of the whole Cone which was 56. 25 foot ong.

The Lesser Base, 18 foot being cut off of the whole Length is found o be 2. 27, and 18 being taken from 56. 25 the whole length, there vill remain 38.25, and third part whereof is 12. 75. which multiplied y 2.27, the Base produceth 28. 94 for the Solidity of the Lesser Cone r Pyramis, and this being added to 63. 12. the Content of the Fru- um produceth 92. 06. the which is equal to the whole Cone or Py- amis, both the parts equal to the whole, which proveth the Work to e true.

CHAP. IV.

The use of the Line of Numbers in Gauging of Vessels.

THe Vessels which are here measured are supposed to be Cylinders, or reduced unto Cylinders, by taking the mean between the Dia- meter at the Head and the Diameter at the Bongue, after the usual anner.

1. Having the Diameter and the length of a Vessel with the Content there-of, to find the Gauge point.

Extend the Compasses in the Line of Numbers to half the distance between the Content and the length of the Vessel, the same extent will reach from the Diameter to the Gauge point.

I put this Proposition first, because these kind of measures are not alike in all places.

Here at *London* it is said that a Wine Vessel being 66 inches in length, and 38 inches the Diameter, would contain 324 Gallons, which if it be true, we may divide the space between 324 and 66 into two equal parts, and the middle will fall about 146, and the same extent which reacheth from 324 to 146, will reach from the Diameter 38 unto 17, 15, the Gauge point for a Gallon of Wine or Oyl after *London* measure.

The like reason holdeth for the like measure in all other places.

2. Having the mean Diameter, and the length of a Vessel, to find the content

Extend the Compasses from the Gauge-point to the mean Diameter, the same extent being doubled, shall give the distance from the length to the content.

So the mean Diameter of a Wine Vessel being 20 inches, and the length 25 inches, the Content will be found to be 34 Gallons after *London* measure.

For extend the Compasses from 17.15 unto 20, the same extent will reach from 23 unto 29, 15, and from 29, 15 unto 34.

In like manner, if the mean Diameter were 16 inches, and the length 23, the Content will be found to be about 20 Gallons.

For the same extent which reacheth back from 17, 15 unto 16, will reach from 23 to 21, 45, and from 21, 45 unto 20.

So that if the mean Diameter shall be 17 inches and 15 Centesmes or parts of 100, the number of inches in the length of the Vessel will give the number of Gallons contained in the same Vessel: if the Diameter shall be more or less than 17, 15, the Content in Gallons will be accordingly more or less than the length in inches.

3. *Havin*

3. *Having the Diameter and Content, to find the length.*

Extend the Compaſſes from the Diameter to the Gauge-point, the ſame extent being doubled, ſhall give the diſtance from the Content to the length of the Veſſel.

So the gauge-point ſtanding as before, if the Diameter be 38 inches, and the Content 324 gallons wine-meaſure, the length of the Veſſel will be found about *66* inches.

4 *Having the length of a Veſſel, and the Content, to find the Diameter.*

Extend the Compaſſes to half the diſtance between the length and the Content, the ſame extent ſhall reach from the Gauge-point to the Diameter.

So the length being *66* inches, and the Content 324 Gallons wine meaſure, the Gauge-point ſtanding as before, the Diameter of the Veſſel will be found to be about 38 inches.

CHAP. V.

Containing ſuch Aſtronomical Propoſitions as are of ordinary uſe in the practice of Navigation.

1. *To find the Altitude of the Sun by the ſhadows of a Gnomon ſet Perpendicular to the Horizon.*

As the parts of the ſhadow,
 are to the parts of the Gnomon:
So the Tangent of 45 *gr.*
 To the Tangent of the Altitude.

Extend the Compaſſes in the Line of Numbers, from the parts of the ſhadow to the parts of the Gnomon; the ſame extent will give the diſtance from the Tangent of 45 *gr.* to the Tangent of the Suns Altitude.

So the Gnomon being 36, and the ſhadow 27, the Altitude will be found

found to be 36 *gr.* 52 *m.* Or the Gnomon being 27, and the shadow 3
the Altitude will be found to be 53 *gr.* 8 *m.* Or the shadow being 20
and the Gnomon 9, the Altitude will be found to be 24 *gr.* 14 *m.* as in
the eighth Proposition of the use of the Tangent-line.

If the Gnomon be 22. and the shadow 135, the Altitude is 9 *gr.*
15 *m.* as I shewed before.

2. *Having the distance of the Sun, from the next Equinoctial point, to find his declination.*

As the Radius is in proportion,
to the Sine of the Suns greatest declination:
So the Sine of the Suns distance from the next Equinoctial Point,
to the Sine of the Declination required.

Extend the Compasses in the Line of Sines, from 90 *gr.* to 23 *gr.* 30 *m.*
the same extent will give the distance from the Suns place unto his Declination.

So the Sun being either in 29 *gr.* of *Taurus*, or 1 *gr.* of *Aquarius*, or
1 *gr.* of *Leo*, or 29 *gr.* of *Scorpio*, that is 59 *gr.* distant from the next
Equinoctial Point, the Declination will be found about 20 *gr.*

If the Sun be so near the Equinoctial Point, that his Declination
fall to be under 1 *gr.* it may be found by the Line of Numbers. As if
the Sun were in 2 *gr.* 5 *m.* of *Aries*, that is 125 *m.* from the Equinoctial Point, the former extent of the Compasses from the Sine of 90 *gr.*
to the Sine of 23 *gr.* 30 *m.* will reach in the Line of Numbers from
125 unto 50, which shews the Declination to be about 50 *m.*

3. *Having the Latitude of the place, and the Declination of the Sun, to find the time of the Suns rising and setting.*

As the Co-tangent of the Latitude.
to the Tangent of the Suns Declination:
So is the Radius,
to the Sine of the Ascentional difference between the hour of 6
and the time of the Suns rising or setting.

Extend the Compasses from the Tangent of the Complement of
the Latitude, to the Tangent of the Declination: the same extent
wi

will reach from the Sine of 90 *deg.* to the Sine of the Ascentional difference.

Or extend the Compasses from the Co-tangent of the Latitude to the Sine of 90 *gr.* the same extent will reach from the Tangent of the Declination, to the Sine of the Ascentional difference.

So the Latitude being 51 *gr.* 30 *m.* Northward, and the Declination 20 *gr.* the difference of Ascension will be found to be 27 *gr.* 14 *m.* which resolved into hours and minutes, doth give 1 hour and almost 49 *m.* for the difference between the Suns rising or setting, and the hour of 6, according to the time of the year.

4. *Having the Latitude of the place, and the distance of the Sun, from the next Equinoctial point, to find his Amplitude.*

As the Co-sine of the Latitude,
to the Sine of the Suns greatest Declination:
So the Sine of the place of the Sun,
to the Sine of the Amplitude.

So the Latitude being 51 *deg.* 30 *m.* and the place of the Sun in 1 *deg.* of *Aquarius*, that is 59 *deg.* distant from the next Equinoctial point, the Amplitude will be found about 33 *deg.* 20 *m.* For extend the Compasses in the Line of *Sines*, from 38 *deg.* 30 *m.* the Sine of the Complement of the Latitude unto 23 *deg.* 30 *m.* the Sine of the Suns greatest Declination; the same extent will reach from 59 *deg.* unto 33 *deg.* 20 *m.* Or extend them from 38 *deg.* 30 *m.* unto 59 *deg.* the same extent will reach from 23 *gr.* 30 *m.* unto 33 *gr.* 20 *m.* as before.

5. *Having the Latitude of the place, and the Declination of the Sun, to find his Amplitude.*

As the Co-sine of the Latitude,
is to the Radius:
So the Sine of the Declination,
to the Sine of the Amplitude.

Extend the Compasses from the Co-sine of the Latitude to the sine of 90 *gr.* the same extent will reach from the Sine of the Suns Declination to the Sine of the Amplitude.

Or

Or extend them from the Tangent of the Latitude to the Sine of the Declination, the same extent will reach from the Sine of 90 *gr.* to the Sine of the Amplitude.

So the Latitude being 51 *gr.* 30 *m.* and the Declination 20 *gr.* the Amplitude will be found to be 33 *gr.* 20 *m.*

6. *Having the Latitude of the place, and the Declination of the Sun, to find the time when the Sun cometh to be due East or West.*

As the Tangent of the Latitude,
is to the Tangent of the Declination:
So the Radius
to the Co-sine of the hour from the Meridian.

Extend the Compasses from the Tangent of the Latitude the Tangent of the Declination, the same extent will reach from the Line of 90 *gr.* to the Sine of the Complement of the hour.

Or extend them from the Tangent of the Latitude to the Sine of 90 *gr.* the same extent will reach from the Tangent of the Declination to the Sine of the Complement of the hour.

So the Latitude being 51 *gr.* 30 *m.* and the Declination 20 *gr.* the Sun will be 73 *gr.* 10 *m.* that is 4 hours, and 53 *m.* from the Meridian, when he cometh to be in the East or West.

7. *Having the Latitude of the place, and the Declination of the Sun, to find what Altitude the Sun shall have, when he cometh to be due East or West.*

As the Sine of the Latitude,
is to the Sine of the Declination:
So the Radius,
to the Sine of the Altitude.

Extend the Compasses in the Line of Sines from the Latitude to the Sine of the Declination, the same extent will reach from the Sine of 90 *gr.* to the Sine of the Altitude.

Or extend them from the Sine of the Latitude to the Sine of 90 *gr.* the same extent will reach from the Sine of the Declination to the Sine of the Altitude.

So the Latitude being 51 *gr.* 30 *m.* and the Declination 20 *gr.* the Altitude will be found about 25 *gr.* 55 *m.*

8. *Having*

8. *Having the Latitude of the place, and the Declination of the Sun, to find what Altitude the Sun shall have at the hour of six.*

As the Radius is in proportion,
to the Sine of the Suns Declination :
So the Sine of the Latitude,
to the Sine of the Altitude.

Extend the Compasses in the Line of Sines, from 90 *gr.* to the Declination ; the same extent will reach from the Latitude to the Altitude.

Or extend them from 90 *gr.* to the Latitude, the same extent will hold from the Declination to the Altitude.

So the Latitude being 51 *gr.* 30 *m.* and the Declination of the Sun 20 *gr.* the Altitude of the Sun will be found to be about 15 *gr.* 30 *m.*

9. *Having the Latitude of the place, and the Declination of the Sun, to find what Azimuth the Sun shall have at the hour of six.*

As the Co-sine of the Latitude,
is to the Radius :
So the Co-tangent of the Suns Declination,
to the Tangent of the Azimuth from the North part of the Meridian.

So the Latitude being 51 *gr.* 30 *m.* and the Declination 20 *gr.* the Azimuth will be found to be 77 *gr.* 14 *m.* For extend the Compasses in the Line of Sines, from 38 *gr.* 30 *m.* to 90 *gr.* the same extent will reach from the Tangent of 70 *gr.* to the Tangent of 77 *gr.* 14 *m.*

10. *Having the Latitude of the place, and the Declination of the Sun, and the Altitude of the Sun, to find the Azimuth.*

First, Consider the Declination of the Sun, whether it be toward the North or the South, so have you his distance from your Pole : then add this distance, the Complement of his Altitude, and the Complement of your Latitude, all three together, and from half the sum subtract the distance from the Pole, and note the difference.

1. As

1. As the Radius is in proportion,
 to the Co-sine of the Altitude:
So the Co-sine of the Latitude,
 to a fourth Sine.

2. As this fourth Sine,
 is to the Sine of the half sum:
So the Sine of the difference,
 to a seventh Sine.

Then find a mean proportional between this seventh Sine and the Radius, this mean shall be the Sine of the Complement of half the Azimuth from the North part of the Meridian.

Suppose the Declination of the Sun being known by the time of the year to be 20 *gr.* Southward, the Altitude above the Horizon found by observation 12 *gr.* and the Latitude Northwards 51 *gr.* 30 *m.* it were required to find the Azimuth.

The Declination is Southward, and therefore the distance from the Pole 110 *gr.* then turning the Altitude and Latitude unto their Complements, I add them all three together, and from half the sum subtract the distance from the Pole, noting the difference after this manner:

Declin.	South	20 *gr.*	0 *m.*	The Distance	110 *gr.*	0 *m.*
Altitude		12	0	The Complement	78	0
Latitude	N	51	30	The Complement	38	30
				The sum of all three	226	30
				The half sum	113	15
				The difference	3	15

This done, I come to the Staff, and extend the Compasses from the Sine of 90 *gr.* to the Sine of 78 *gr.* and find the same extent to reach from the Sine of 38 *gr.* 30 *m.* unto 37 *gr.* 30 *m.* Or if I extend them from 90 *gr.* to 38 *gr.* 30 *m.* the same extent doth reach from 78 *gr.* unto 37 *gr.* 30 *m.* which is the fourth Sine required.

Then I extend the Compasses again, from this fourth Sine of 37 *gr.* 30 *m.* unto the Sine of the half sum 113 *gr.* 15 *m.* that is to the Sine of 66 *gr.* 45 *m.* (for after 90 *gr.* the Sine of 80 *gr.* doth stand for a Sine of of 100 *gr.* and the Sine of 70 *gr.* for a Sine of 100 *gr.* and so the rest for those which are their Complements to 180 *gr.*) and this second extent doth reach from the Sine of the difference 3 *gr.* 15 *m.* to the Sine of 4 *gr.* 54 *m.* Or if I extend them from the fourth Sine of 37 *gr.* 30 *m.* to the Sine of the difference 3 *gr.* 15 *m.* the same extent will reach from the Sine of the half sum 113 *gr.* 15 *m.* unto 4 *gr.* 54 *m.* which is the seventh Sine required.

Lastly, I divide the space between this seventh Sine of 4 *gr.* 54 *m.* and the Sine of 90 *gr.* into two equal parts, and I find the mean proportional side to fall on 17 *gr.* whose Complement is 73 *gr.* the double of 73 *gr.* is 146 *gr.* and such is the Azimuth required.

Or having found the seventh Sine to be 4 *gr.* 54 *m.* I might look over against it, in the Line of *Versed Sines*, and there I should find 146 *gr.* for the Azimuth from the North part of the Meridian; and the Complement of 146 *gr.* to a Semicircle being 34 *gr.* will give the Azimuth from the South part of the Meridian.

But if it were required to find the Azimuth in the same Latitude of 51 *gr.* 30 Northward, with the same Altitude of 12 *gr.* and like Declination of 20 *gr.* to the Northward, it would be found to be only 72 *gr.* 52 *m.* though the manner of work be the same as before.

Declin. North.	20 *gr.*	0 *m.*	The distance is	70 *gr.*	0 *m.*
Altitude	12	0	The Complement	78	0
Latitude North.	51	30	The Complement	28	30
			The sum of all three	186	30
			The half sum	93	15
			The difference	23	15

Here as the Radius is to the Sine of 78 *gr.* so the Sine of 38 *gr.* 30 *m.* to the Sine of 37 *gr.* 30 *m.* which is the fourth Sine, and the same as before.

Then as this fourth Sine of 37 *gr.* 30 *m.* is to the Sine of 93 *gr.* 15 *m.* to the Sine of 23 *gr.* 15 *m.* to the Sine of 40 *gr.* 20 *m.* which is the seventh Sine.

The half way between the seventh Sine and the Sine of 90 *gr.* doth fall at 53 *gr.* 34 *m.* whose Complement is 36 *gr.* 26 *m.* and the double of that is 72 *gr.* 52 *m.* the Azimuth required.

Or I may find this same Azimuth in the Line of *Versed Sines*, ove against the seventh Sine of 40 *gr.* 20 *m.*

11. *Having the Latitude of the place, the Declination of the Sun, an the Altitude of the Sun, to find the hour of the day.*

Add the Complement of the Suns Altitude, and the distance of th Sun from the Pole, and the Complement of your Latitude, all thre together, and from half the sum substract the Complement of the Al titude, and note the difference.

1. As the Radius is in proportion
   to the Sine of the Suns distance from the Pole:
   So the Sine of the Complement of the Latitude,
   to a fourth Sine.

2. As this fourth Sine,
   is to the Sine of the half sum:
   So the Sine of the difference
   to a seventh Sine.

The mean proportional between this seventh Sine and the Sine 90 *gr.* will be the Sine of the Complement of half the hour from th Meridian.

Thus in our Latitude of 51 *gr.* 30 *m.* the Declination of the Sun b ing 20 *gr.* Northward, and the Altitude 12 *gr.* I might find the Sun be 95 *gr.* 52 *m.* from the Meridian.

Altitude	12 *gr.*	0 *m.*	The Complement is	78 *gr.*	0
Declin. North	20	0	The diff. from the Pole	70	0
Latitude	51	30	The Complement is	38	30
			The sum of all three	186	30
			The half sum	93	15
			The difference	15	15

Here as the Radius, is to the Sine of 70 *gr.*
So the Sine of 38 *gr.* 30 *m.* to the Sine of 35 *gr.* 48 *m.*

As this Sine of 15 *gr.* 48 *m.* is to the Sine of 93 *gr.* 15 *m.*
So the Sine of 15 *gr.* 15 *m.* to the Sine of 26 *gr.* 40 *m.*
The half way between this seventh Sine of 26 *gr.* 40 *m.* and the Sine of 90 *gr.* doth fall at 42 *gr.* 4 *m.* whose Complement is 47 *gr.* 56 *m.* and the double of that, 95 *gr.* 52 *m.* which converted into hours, doth give 6 hours and almost 24 *m.* from the Meridian.

Or I might find these 95 *gr.* 52 *m.* in the Line of *Versed Sines*, over against the seventh Sine of 26 *gr.* 40 *m.*

12. *Having the Azimuth, the Suns Altitude, and the Declination, to find the hour of the day.*

As the Co-sine of the Declination,
is to the Sine of the Azimuth:
So the Co-sine of the Altitude,
to the Sine of the hour.

Thus the Declination being 20 *gr.* Southward, the Altitude 12 *gr.* and the Azimuth found by the tenth Proposition 146 *gr.* I might find the time to be 35 *gr.* 36 *m.* that is 2 hours 22 *m.* from the Meridian.

13. *Having the hour of the day, the Suns Altitude, and the Declination, to find the Azimuth.*

As the Co-sine of the Altitude,
is to the Sine of the hour:
So the Co-sine of the Declination,
to the Sine of the Azimuth.

So the Altitude of the Sun being 12 *gr.* and the Declination 20 *gr.* Southward, and the Angle of the hour 35 *gr.* 36 *m.* I should find the Azimuth to be 34 *gr.* And so it is if it be reckoned from the South; but 146 *gr.* if it be taken from the North part of the Meridian.

14. *Having the distance of the Sun from the next Equinoctial point, to find his right Ascension.*

As the Radius,
to the Co-sine of the greatest Declination:
So the Tangent of the distance,
To the Tangent of the right Ascension.

So the Sun being in the first degree of *Aquarius*, that is 59 *gr.* distant from the next Equinoctial point, and the greatest Declination 23 *gr.* 30 *m*: the right Ascension will be found to be 56 *gr.* 50 *m.* short of the beginning of *Aries*, and therefore 303 *gr.* 14 *m.*

15. *Having the Declination of the Sun, to find his right Ascension.*

As the Tangent of the greatest Declination,
  is to the Tangent of the Declination given:
So the Radius
  to the Sine of the right Ascension.

So the greatest Declination being 23 *gr.* 30 *m.* and the Declination of the Sun given 20 *gr.* the right Ascention will be found about 56 *gr.* 50 *m.*

16. *Having the Longitude and Latitude of a Star, to find the right Ascension of that Star.*

17. *To find the Declination of that Star.*

The stars have little or no alteration in their Latitude, in their Longitude they move forward, about 1 *gr.* 25 *m.* in an hundred years. These being known,

As the Radius,
  to the Sine of the Stars Longitude from the next Equinoctial (point:
So the Co-tangent of the stars Latitude,
  to the Tangent of a fourth Ark.

Compare this fourth Ark, with the Ark of distance between the Poles of the world and of the Ecliptick. If the Longitude and Latitude of the Star be both alike, as when the Longitude falleth to be among the Northern Signs, *Aries, Taurus, Gemini, Cancer, Leo, Virgo,* and the Latitude is North from the Ecliptick: or the Longitude among the Southern signs, *Libra, Scorpio, Sagitarius, Capricorn, Aquarius, Pisces,* and the Latitude Southward, then shall the difference between this fourth Ark and the distance of Poles, be your fifth Ark.

But if the Longitude and Latitude shall be unlike, as the Longitude in a Northern sign, and the Latitude South, or the Longitude in a

Southern

Southern sign, and the Latitude North, then add this fourth Ark to the distance of both Poles, the sum of both shall be your fifth Ark. And,

As the Sine of the fourth Ark,
to the Sine of the fifth Ark:
So the Tangent of the stars Longitude,
to the Tangent of the stars right Ascension, from the next Equinoctial point.

As the Co-sine of the fourth Ark,
to the Co-sine of the fifth Ark:
So the Sine of the stars Latitude,
to the Sine of the stars Declination.

Then for proof of the work, if there be no former error, the proportion will hold.

As the Co-sine of the Latitude,
to the Co-sine of the right Ascension:
So the Co-sine of the Declination,
to the Co-sine of the Longitude.

For example, Take the upper of the two former stars in the square of the little bear, which sea-men call the *Former Guard.* This in the year 1655, was in 7 *deg.* 53 *m.* of *Leo*, and so his Longitude from the beginning of *Libra* 52 *deg.* 7 *m.* But his Latitude is still the same 72 *gr.* 51 *m.* Northwards. Wherefore,

As the Sine of 90 *gr.*
is to the Sine of 52 *gr.* 22 *m.*
So is the Co-tangent of 72 *gr.* 51 *m.*
to the Tangent of 13 *gr.* 44 *m.*

Which is the fourth Ark. Then because the Longitude and Latitude are both Northward, the difference between this fourth Ark and 23 *gr.* 31 *m.* the distance of both Poles will give you 9 *gr.* 47 *m.* for the fifth Ark. And,

As

As the Sine of 13 *gr.* 44 *m.*
to the Sine of 9 *gr.* 47 *m.*
So the Tangent of 52 *gr.* 22 *m.*
to the Tangent of 42 *gr.* 56 *m.*

Which is the right Aſcenſion of this ſtar, from the beginning of *Libra*, but 222 *gr.* 56 *m.* from the beginning of *Aries.*

As the Co-ſine of 13 *gr.* 44 *m.*
to the Co-ſine of 9 *gr.* 47 *m.*
So the Sine of 72 *gr.* 51 *m.*
to the Sine of 75 *gr.* 46 *m.*

Which is the Declination of this ſtar from the Equator.

As the Co-ſine of 72 *gr.* 51 *m.*
to the Co-ſine of 42 *gr.* 56 *m.*
So the Co-ſine of 75 *gr.* 46 *m.*
to the Co-ſine of 52 *gr.* 7 *m.*

Which agreeing ſo well with the Longitude of the ſtar propoſed is a good proof, that the right Aſcenſion and Declination were truly found.

Theſe are ſuch Aſtronomical Propoſitions, as I take to be uſeful for Sea-men. For the firſt and ſecond will help them to find their Latitude, the third to find the Suns riſing and ſetting, the 4, 5, 6, 7, 8, 9, 10, 13 *Prop.* to find the variation of their Compaſs, the 11 and 12 *Prop* to find the hour of the day; and the reſt toward the finding of the hour of the night. For having the Latitude of the place, with the Declination and Altitude of any ſtar, they may find the hour of the ſtar from the Meridian, as in the 11 *Prop.* Then comparing the right Aſcenſion of the ſtar, with the right Aſcenſion of the Sun, they may have to the hour of the night.

All theſe Propoſitions, and ſuch others, may be wrought alſo by the Table of Sines and Tangents. For where four Numbers do hold in proportion; as the firſt to the ſecond, ſo the third to the fourth; there if we multiply the ſecond into the third, and divide the Product by the firſt, the Quotient will give the fourth required. As in the exampl example of the 15 *Prop.* where the Declination being given, it was required to find the right Aſcenſion. The Tangent of 20 *gr.* the Declination given is 3639702, which being multiplied by the Radius, the Product is 36397020000000, and this divided by 4348124, the Tangent of 23 *gr.* 30 *m.* the Quotient is 8370741, the Sine of 56 *gr.* 50 *m.* for the right Aſcenſion required.

Or if any will uſe my Tables of Artificial Sines and Tangents, they may add the ſecond and third together, and from the ſum ſubtract the firſt, the remainder will give the fourth required. And ſo my Tangent of 20 *gr.* is 9561, 0658, which being added to the Radius, makes 19561, 0658, from this if they ſubtract 9638, 3019, the Tangent of 23 *gr.* 30 *m.* they ſhall find the remainder to be 9922, 7639, which in my *Canon* is the Sine of 56 *gr.* 49 *m.* 56 *ſeconds*; and ſuch is the right Aſcenſion required, if it be reckoned from the next Equinoctial point.

The like reaſon holdeth for all other Aſtronomical Propoſitions, as I will farther ſhew by thoſe two examples which I gave before, for the finding of the Azimuth in the 10 *Prop.* becauſe they are thought to be harder than the reſt, and require three operations.

In the firſt Example.

Declin. South	20 *gr.*	0 *m.*	The diſtance	110 *gr.*	0 *m.*
Altitude	12	0	The Complement	78	0
Latit. North	51	30	The Complement	38	30
			The ſum of all three	226	30
			The half ſum	113	15
			The difference	3	15

The firſt operation will be to find the fourth Sine; and that is done by adding the Sine of the Complement of the Altitude to the Sine of the Complement of the Latitude, and ſubtracting the Radius: ſo adding 9990, 4044 the Sine of 78 *gr.* unto 9794, 1495 the Sine of 38 *gr.* 30 *m.* the ſum will be 19784, 5539. And the Radius being ſubtracted, the remainder 9784, 5539 is the fourth Sine, and belongeth to 37 *gr.* 30 *m.*

The ſecond operation will be to find the ſeventh Sine, and that is done

done by adding the fine of the half fum to the fine of the difference, and fubtracting the fourth fine. So the half fum being 113 *gr.* 15. I take his Complement to a Semi-circle, and fo find his fine to be 9663, 2168, to which I add 8753, 5278, the fine of the difference 3 *gr.* 15 *m.* and the fum is 18716, 7446. From this I take the fourth fine 9784, 5539, and the remainder will be 8932, 1907, which is the feventh fine, and belongeth to 4 *gr.* 54 *m.*

The third operation will be to find the mean proportional fine between the feventh fine and the Radius. This in common Arithmetick is done by multiplying the two extreams, and taking the fquare root of the Product. As in finding a mean proportional between 4 and 9, we multiply 4 into 9, and the Product is 36, whofe fquare root is 6, the mean proportional between 4 and 9. But here it is done by adding the fine and the Radius, and taking the half of them. So the fum of the laft feventh fine and the Radius is 18932, 1907, and the half of that 9466, 0953, which is the mean proportional fine required, and belongeth to 17 *gr.* whofe Complement is 73 *gr.* and the double of that 146 *gr.* the fame Azimuth as before.

In the fecond Example.

Declin. North	20 *gr.*	0 *m.*	The diftance	70 *gr.*	0 *m.*
Altitude	12	0	The Complement	78	0
Latitude N.	51	30	The Complement	38	30
			The fum of all three	186	30
			The half fum	93	15
			The difference	23	15

The firft operation will be to find the fourth fine, and that is here 9784, 5539, as in the former Example.

The fecond operation will be to find the feventh fine; and fo here the fine of the half fum 93 *gr.* 15 *m.* being the fame with the fine of 86 *gr.* 45 *m.* his Complement to 180 *gr.* I find it to be 9999, 3009, to which I add 9596, 3153, the fine of the difference 23 *gr.* 15 *m.* and the fum is 19595, 6162. From this I take the fourth fine 9784, 5539, and the remainder will be 9811, 0623 for the feventh fine, and belongeth to 40 *gr.* 20 *m.*

The

The third operation will be to find the mean proportional Sine be-
ween the seventh Sine and the Radius. And so here the Radius being
dded to the seventh Sine, the sum will be 19811,0623, and the half of
hat 9905, 5311, doth give the mean proportional Sine belonging to
bout 53 *gr.* 34 *m.* whose Complement is 36 *gr.* 26 *m.* and the double
f that 72 *gr.* 52 *m.* the same Azimuth as before.

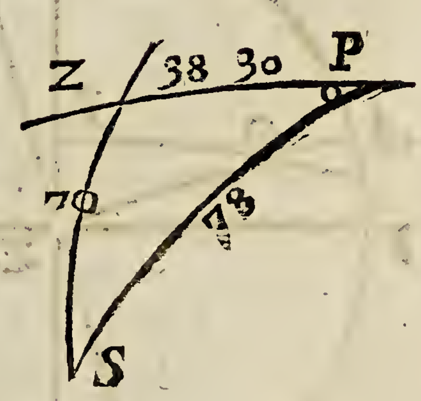
I have set down these three Examples thus particularly, that I might
hew the agreement between the *Staffe* and the *Canon.* But otherwise I
night deliver both the Precept and the Work, for the two last, more
ompendiously. For generally in all Spherical Triangles, where three
ides are known, and an Angle required, make that side which is oppo-
te to the Angle required, to be the Base; and gather the sum, the half
um, and the difference as before.

As the Rectangle contained under the Sines of the sides,
is to the Square of the whole Sine:
So the Rectangle contained under the Sines of the half sum and the (difference,
to the square of the Co-sine of the half of the Angel.

Then for the work, we may for the most part leave out the two last
gures; and if they be about 50, put an unite to the sixth place, after
his manner.

The second Example.

70 *gr.*	0 *m.*		
78	0	9990	40
38	30	9794	15
186	30	19784	55
93	15	9999	30
23	15	9596	32
		20000	00
		39595	62
		19811	07
36	26	9905	53
72	52		



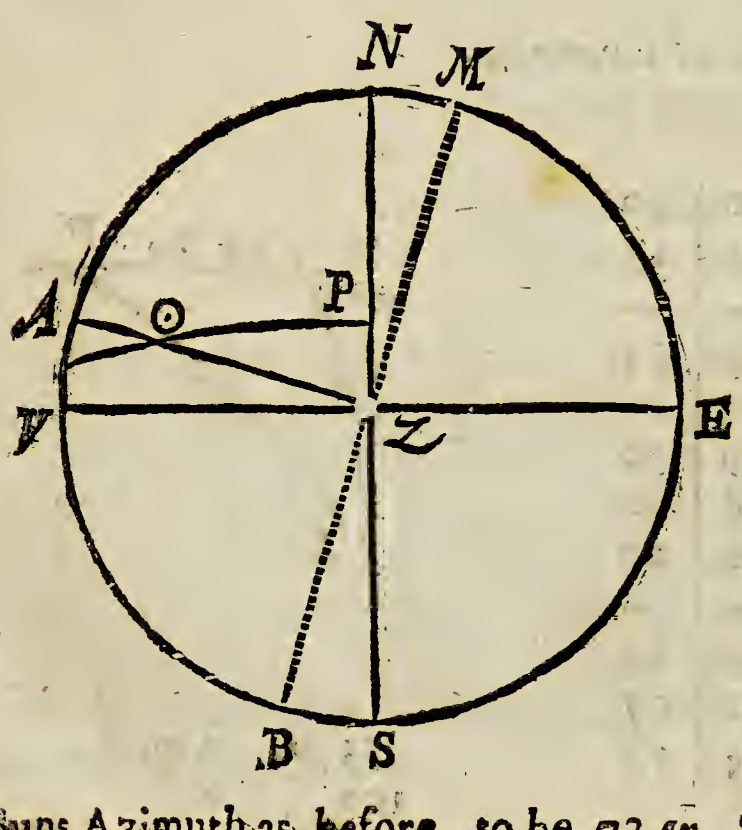
53 *gr.* 34 *m.*
107 8

O o

Or

Or for such Numbers as are to be subtracted, I may take them out of the Radius, and write down the residue, and then add them together with the rest. As in the same second Example, the Sines of 78 *gr.* and of 38 *gr.* 30 *m.* being the Numbers to be subtracted; if I take 9990, 4044 the Sine of 78 *gr.* out of the Radius 10000, 0000, the residue is 9. 5956: and so the residue of 9794, 1495 is 205. 8505. Wherefore instead of subtracting those Sines, I may add these residues after this manner:

70 *gr.*	0 *m.*			
78	0	9	59	
38	30	205	85	
186	30			
93	15	9999	30	
23	15	9596	32	
		19811	c6	
36	26	9905	53	53 *gr.* 34 *m.*
72	52			107 8

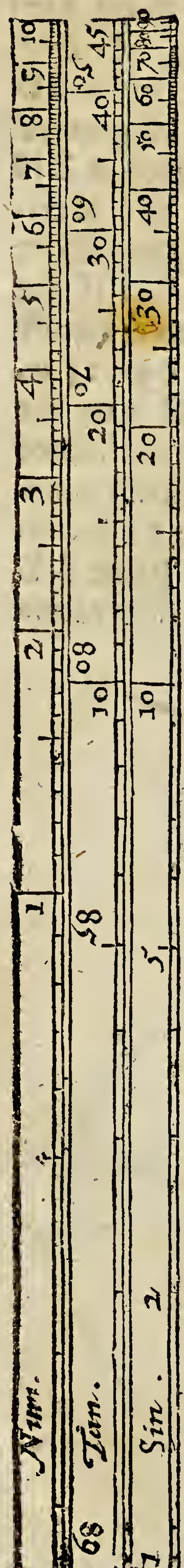


Having these means to find the Suns Azimuth, we may compare it with the Magnetical Azimuth, and so find the variation of the Needle.

For let the Circle A M B, drawn by the Center Z, be a Plane, parallel to the Horizon; A the Point whereon the Sun bears from us, M the North point of the Magnetical Needle, and the Angle A Z M, the Magnetical Azimuth. If we find the Suns Azimuth as before, to be 72 *gr.* 52 *m.* from the North to the Westward;

Westward, we may allow so many *gr.* from A unto N, and so we have he true North point of the Meridian, and consequently, the East, outh, and West Points of the Horizon, and the distance between N nd M shall be the variation of the Needle. So that if the Magnetical Azimuth A Z M shall be 84 *gr.* 7 *m.* and the Suns Azimuth A Z N 72 *gr.* 52 *m.* then must N Z M the difference between the two Meridians, give the variation to be 11 *gr.* 15 *m.* as Mr. *Borough* heretofore ound it by his observations at *Limehouse* in the year 1580. But if the Magnetical Azimuth Z M shall be 79 *gr.* 7 *m.* and the Suns Azimuth A Z N 72 *gr.* 52 *m.* then shall the variation N Z M be only 6 *gr.* 15 *m.* s I have sometimes found it of late. Hereupon I enquired after the lace where Mr. *Borough* observed, and went to *Limehouse* with some f my Friends, and took with us a Quadrant of three foot Semidiameter, and two Needles, the one above 6 inches, and the other 10 inches ong, where I made the Semidiameter of my Horizontal Plane A Z 2 inches: and towards night the 13 of *June* 1622, I made observation n several parts of the ground, and found as followeth:

Alt. ☉		*A Z M.*		*A Z N.*		*Variat.*	
Gr.	*M.*	*Gr.*	*M.*	*Gr.*	*M.*	*Gr.*	*M.*
19	0	82	2	75	52	6	10
18	5	80	50	74	44	6	6
17	34	80	0	74	6	5	54
17	0	79	15	73	20	5	55
16	18	78	12	72	32	5	40
16	0	77	50	72	10	5	40
10	20	71	2	64	49	6	13
9	25	70	12	64	25	5	47



CHAP. VI.

Containing such nautical questions, as are of ordinary use, concerning Longitude, Latitude, Rumb, and Distance.

1. To keep an account of the Ships way.

THe way that the Ship maketh, may be known to an old Sea-man by experience, by others it may be found for some small proportion of time, either by the Log Line, or by the distance of two known marks on the Ships side.

The time in which it maketh this way, may be measured by a Watch, or by a Glass, or by the Pulse, or by repeating a certain number of words. Then as long as the wind continueth at the same stay, it followeth by proportion,

As the time given, is to an hour:

So the way made, to an hours way.

Suppose the time to be 15 seconds, which make a quarter of a minute, and the way of the Ship 88 feet: then because there are 3600 seconds in an hour, I may extend the Compasses in the Line of Numbers, from 15 unto 3600, and the same extent will reach from 88 unto 21120. Or I may extend them from 15 unto 88, and this extent will reach from 3600 unto 21120, according to the ordinary work in Arithmetick,

As 15, unto 3600:

So 88, unto 21120.

Which shews that an hours way came to 21120 feet.

But this were an unnecessary business, to hearken after feet or fathoms. It sufficeth our Sea-men to find the way of their Ship in Leagues or Miles.

And they say that there are 5 feet in a pace, 1000 paces in a Mile, and 60 miles in a degree, and therefore 300000 feet

feet in a degree. Yet comparing ſeveral obſervations, and their meaſures with our feet uſual about *London*, I find that we may allow 352000 feet to a degree; and then if I extend the Compaſſes in the Line of Numbers from 352000 unto 21120, I ſhall find the ſame extent to reach from 20 Leagues, the meaſure of one degree, to 1,2, and from 60 miles o 3, 6, according to Arithmetick, which ſhews the hours way to be 1 league, and 2 tenths of a league, or 3 miles and 6 tenths of a mile.

As 352000, unto 21120,:
So 20, 00, unto 1, 20.
and 60, 00, unto 3, 60.

But to avoid theſe fractions, and other tedious reductions, I ſuppoſe it would be much better to keep this account of the Ships way (as alſo of the difference of Latitude, and the difference of Longitude) by *deg.* and parts of *deg.* allowing in 100 parts to each *deg.* which we may therefore call by the name of *Centeſms.* For ſo doing there would be ſome agreement between the account and the days ſayling. Ordinarily the ſhip goes a degree in a day, as it may appear by comparing ſeveral Journals to the Eaſt and Weſt *Indies.* The time of paſſage between the Lizard and the Southermoſt Cape of *Africa*, is commonly ſaid to be about 3 months, and the diſtance is not much different from 90 degrees.

Again, this account by degrees and Centeſms would be more exact; and the addition, ſubtraction, multiplication, diviſion of them more eaſie. Neither would this be hard to conceive. For,

	Centeſms,		*Minutes*,	*Leagues.*
If	100	do equal	60 and	20,
then	50	ſhall equal	30 and	10,
and	5	be equal	3 and	1.

And ſo in the former example of 82 feet in 15 ſeconds, having firſt found that the hours way is about 21120 feet.

If I extend the Compaſſes from 352000, unto 21120, as before, I ſhall find the ſame extent to reach from 100 unto 6, as before, which ſhews that the hours way required is 6 *Cent.* ſuch as 100 do make a degree, and 5 do make an ordinary league.

This might alſo be done at one operation. For upon theſe ſuppoſitions, divide 44 feet into 45 lengths, and ſet as many of them as you may conveniently between two marks on the ſhips ſide, and note the ſeconds of the time in which the ſhip goeth theſe lengths, ſo the proportion will hold,

As

As the seconds, to the lengths:
So 1 hour, unto the Centesms.

The lengths divided by the time, shall give the *Cent.* which the ship goeth in an hour.

Suppose the distance between the two marks to be *60* lengths (which are 58 feet and 8 inches) and let the time be 12 seconds: extend the Compasses from 12 to 1, in the Line of Numbers; so the same extent will reach from *60* unto 5. Or extend them from 12 unto *60*, and the same extent will reach from 1 unto 5. This shews that the ships way is according to 5 *Cent.* in an hour.

This may be found yet more easily, if the Log-line shall be fitted to the time. As if the time be 45 seconds, the Log-line may have a knot at the end of every 44 feet; then doth the ship run so many *Cent.* in an hour as there are knots vered out in the space of 45 seconds. If 30 seconds do seem to be a more convenient time, the Log-line may have a knot at the end of every *29* feet and 4 inches; and then also the *Cent.* will be as many as the knots: Or if the knots be made to any set number of feet, the time may be fitted unto the distance. As if the knots be made at the end of every 24 feet, the Glass may be made 24 seconds, and somewhat more than an half of a second, and so these knots will shew the *Cent.* If there be 5 knots vered out in a Glass, then 5 *Cent.* if *6* knots, then the ship goeth 6 *Cent.* in the space of an hour, and so in the rest. For upon this supposition, the proportion between the time and the feet will be as 45 unto 44. But according to the common supposition, it should seem to be as 45 unto $37\frac{1}{2}$, or in lesser terms, as *6* unto 5.

Those which are upon the place may make proof of both, and follow that which agrees best with their experience.

2. *By the Latitude and difference of Longitude, to find the distance upon a course of East and West.*

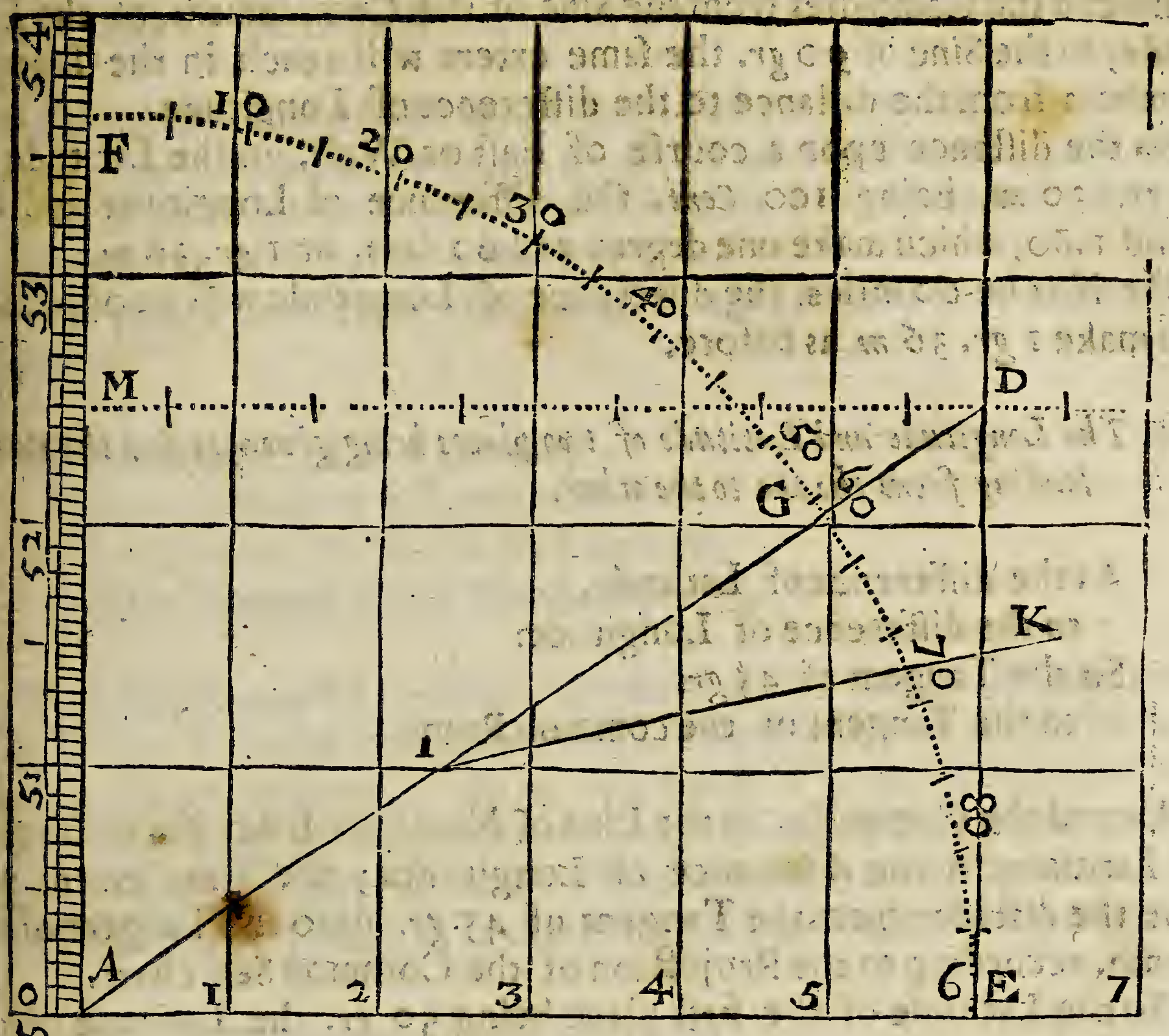
As the Sine of *90 gr.*
to the Co-sine of the Latitude:
So the difference of Longitude at the Equator,
to the distance required on the parallel.

Extend the Compasses from the Sine of *90 gr.* unto the Sine of the Complement of the Latitude; the same extent shall reach in the Line of Numbers, from the difference of Longitude to the distance.

So

So the meaſure of one degree in the Equator being 100 *Cent.* the diſtance belonging to one *deg.* of Longitude in the Latitude of 51 *gr.* 30 *m.* will be found about 62 *Cent.* and $\frac{1}{4}$.

Or if the meaſure of a degree be 60 miles, the diſtance will be found about 37 miles and $\frac{1}{3}$. If the meaſure be 20 Leagues, then



almoſt 12 Leagues and $\frac{1}{2}$. if the meaſure be 17 $\frac{1}{2}$, as in the Spaniſh Charts, then ſomewhat leſs than 11 Leagues ſailing upon this parallel, will give an alteration of one degree of Longitude.

3. By the Latitude and diſtance upon a courſe of Eaſt or Weſt, to find the difference of Longitude.

If the diſtance be given in Leagues or Miles, reduce them into Centeſms, then will the proportion hold.

As

As the Co-sine of the Latitude,
to the Sine of 90 *gr*
So the distance on the Parallel
to the difference of Longitude.

Extend the Compasses from the Sine of the Complement of the Latitude, to the Sine of 90 *gr.* the same extent will reach in the Line of Numbers from the distance to the difference of Longitude.

So the distance upon a course of East or West, in the Latitude of 51 *gr.* 30 *m.* being 100 *Cent.* the difference of Longitude will be found 1,60, which make one degree and 60 *Cent.* or 1 *gr.* 36 *m.*

Or if it be 60 miles, the difference of Longitude will be 96, which also make 1 *gr.* 36 *m.* as before.

4. *The Longitude and Latitude of two places being given, to find the Rumb leading from the one to the other.*

As the difference of Latitude,
to the difference of Longitude:
So the Tangent of 45 *gr.*
to the Tangent of the common Rumb.

Extend the Compasses in the Line of Numbers from the difference of Latitudes to the difference of Longitudes; the same extent will give the distance from the Tangent of 45 *gr.* unto the Tangent of the Rumb, according to the Projection of the Common Sea-chart.

So the Latitude of the first place being 50 *gr.* the Latitude of the second 52 *gr.* 30 *m.* and the difference of Longitude 6 *gr.* the Rumb will be found to be about 67 *gr.* 23 *m.* which is near the inclination of the sixth Rumb to the Meridian. But this Rumb so found is always greater than it should be, and therefore to be limited; which may be done sufficiently for the Sea-mans use, after this manner:

As the Sine of 90 *gr.*
to the Co-sine of the middle Latitude:
So the Tangent of the common Rumb
to the Tangent of the Rumb required.

Extend

Extend the Compasses either from the Sine of 90 *gr.* unto the Sine of the Complement of the middle Latitude, the same extent will reach from the Tangent of the Rumb before found, unto the Tangent of the Rumb limited.

Or else extend them from the Sine of 90 *gr.* unto the Tangent of the Rumb before found; the same extent will reach from the Sine of the Complement of the middle Latitude, unto the Tangent of the Rumb limited.

So the middle Latitude between 50 *gr.* and 52 *gr.* 30 *m.* being 51 *gr.* 5 *m.* and the Rumb before found 67 *gr.* 23 *m.* the Rumb limited will be found to be about 56 *gr.* 20 *m.* which is but 5 *m.* more than the inclination of the fifth Rumb to the Meridian.

If any please to work by the *Canon*, he may joyn both these in one operation.

As the difference of Latitude,
  to the difference of Longitude:
So the Co-sine of the middle Latitude,
  to the Tangent of the Rumb required.

2. This Rumb may be found by the help of the *Meridian Line* upon the Staff. For if I take the difference of Latitude out of the *Meridian Line* from 50 *gr.* unto 52 *gr.* 30 *m.* and measure it in his Equinoctial, or at the beginning of the *Meridian Line*, I shall find it there to be equal to 4 *gr.* which may be called the difference of the Latitude inlarged. Wherefore I work as if the difference of Latitude were 4 *gr.*

As the difference of Latitude inlarged,
  to the difference of Longitude:
So the Tangent of 45 *gr.*
  to the Tangent of the Rumb required.

And extend the Compasses in the Line of Numbers from 4 unto 6: so shall I find the same extent to reach from the Tangent of 45 *gr.* unto the Tangent of 56 *gr.* 20 *m.* and this is the inclination of the Rumb required.

6. By the Rumb and both Latitudes, to find the distance upon the Rumb.

As the Co-sine of the Rumb from the Meridian,
to the Sine of 90 *gr.*
So the difference between both Latitudes,
to the distance upon the Rumb.

Extend the Compasses from the Sine of the Complement of the Rumb, unto the Sine of 90 *gr.* the same extent in the Line of Numbers shall reach from the difference of Latitude unto the distance upon the Rumb.

So the Latitude of the first place being 50 *gr.* the Latitude of the second 52 *gr.* 30 *m.* and the Rumb the fifth from the Meridian. If I extend the Compasses from 33 *gr.* 45 *m.* unto the Sine of 90 *gr.* I shall find the same extent in the Line of Numbers to reach from 2 *gr.* 50 *Cent.* to 4 *gr.* 50 *Cent.* and such is the distance required.

7. By the distance and both Latitudes to find the Rumb.

As the distance on the Rumb,
to the difference between both Latitudes:
So the Sine of 90 *gr.*
to the Co-sine of the Rumb from the Meridian.

Extend the Compasses in the Line of Numbers from the distance unto the difference of Latitudes; the same extent will reach in the Line of Sines from 90 *gr.* unto the Complement of the Rumb.

So the one place being in the Latitude of 50 *gr.* the other in the Latitude of 52 *gr.* 30 *m.* and the distance between them 4 *gr.* 50 *Cent.* If I extend the Compasses from 4. 50 unto 2. 50 in the Line of Numbers, I shall find the same extent to reach from the Sine of 90 *gr.* unto the Complement of 56 *gr.* 15 *m.* and such is the inclination of the Rumb required.

8. B

8. *By one Latitude, Rumb, and distance, to find the difference of Latitudes.*

As the Sine of 90 *gr.*
to the Co-sine of the Rumb from the Meridian:
So the distance upon the Rumb,
to the difference between both Latitudes.

Extend the Compasses in the Line of Sines, from 90 *gr.* unto the Complement of the Rumb, the same extent in the Line of Numbers, will reach from the distance, unto the difference of Latitudes.

So the lesser Latitude being 50 *gr.* and the distance 4 *gr.* 50 *Cent.* upon the fifth Rumb from the Meridian: If I extend the Compasses from the Sine of 90 *gr.* to 33 *gr.* 45 *m.* I shall find the same extent to reach from 4. 50 in the Line of Numbers unto 2. 50; and therefore the second Latitude to be 52 *gr.* 30 *m.*

9. *By the Rumb and both Latitudes, to find the difference of Longitude.*

As the Tangent of 45 *gr.*
to the Tangent of the Rumb from the Meridian:
So the difference of Latitude,
to the difference of Longitude in the common Sea-chart.

Extend the Compasses from the Tangent of 45 *gr.* unto the Tangent of the Rumb; the same extent will reach in the Line of Numbers from the difference of Latitudes unto the difference of Longitude, according to the Projection of the Common Sea-chart.

So the first Latitude being 50 *gr.* and the second 52 *gr.* 30 *m.* and the Rumb the fifth from the Meridian: if I extend the Compasses from the Tangent of 45 *gr.* unto 56 *gr.* 15 *m.* I shall find the same extent to reach from 2. 50 in the Line of Numbers to be about 3. 75, which make 3 *gr.* 45 *m.* But this difference of Longitude so found, is always lesser than it should be, and therefore to be enlarged, which may be done sufficiently for the Sea-mens use after this manner:

As the Co-sine of the middle Latitude,
to the Sine of 90 *gr.*
So the difference of Longitude in the common Sea-chart,
to the difference of Longitude inlarged.

 Extend

Extend the Compasses from the Sine of the Complement of the middle Latitude, unto the Sine of 90 *gr.* the same will reach in the Line of Numbers from the difference of Longitude before found, unto the difference of Longitude inlarged.

So the middle Latitude in this example being 51 *gr.* 15 *m.* and the difference of Longitude before found, 3 *gr.* 75 *Cent.* the difference of Longitude inlarged will be found about 5 *gr.* 99 *Cent.* which are near 6 *gr.*

If any please to work by the *Canon*, he may joyn both these in one operation.

As the Co-sine of the middle Latitude,
to the Tangent of the Rumb from the Meridian:
So the difference of Latitude,
to the difference of Longitude required.

2. This difference of Longitude may be found by help of the Meridian Line upon the Staff. For if I take the proper difference of Latitude out of the Meridian Line, and measure it in his Equinoctial, or at the beginning of the Meridian Line, I shall find the Latitude inlarged to be equal to four of those degrees.

As the Tangent of 45 *gr.*
to the Tangent of the Rumb from the Meridian:
So the difference of Latitude inlarged,
to the difference of Longitude required.

Wherefore having extended the Compasses, as before, from the Tangent of 45 *gr.* unto the Tangent of 56 *gr.* 15 *m.* the same extent will reach from 400 in the Line of Numbers, unto 5.99, which shews the difference of Longitude to be about 5 *gr.* 99 *Cent.* or about half a minute short of 6 degrees.

10. *By the Rumb and both Latitudes, to find the distance belonging to the Chart of* Mercators *Projection.*

Take the proper difference of Latitudes out of the Meridian Line of the Chart, and measure it in his Equinoctial, or one of the Parallels, and it will there give the difference of Latitude inlarged.

As

As the Co-ſine of the Rumb from the Meridian,
to the Co-ſine of *90 gr.*
So the difference between both Latitudes,
to the diſtance upon the Rumb.

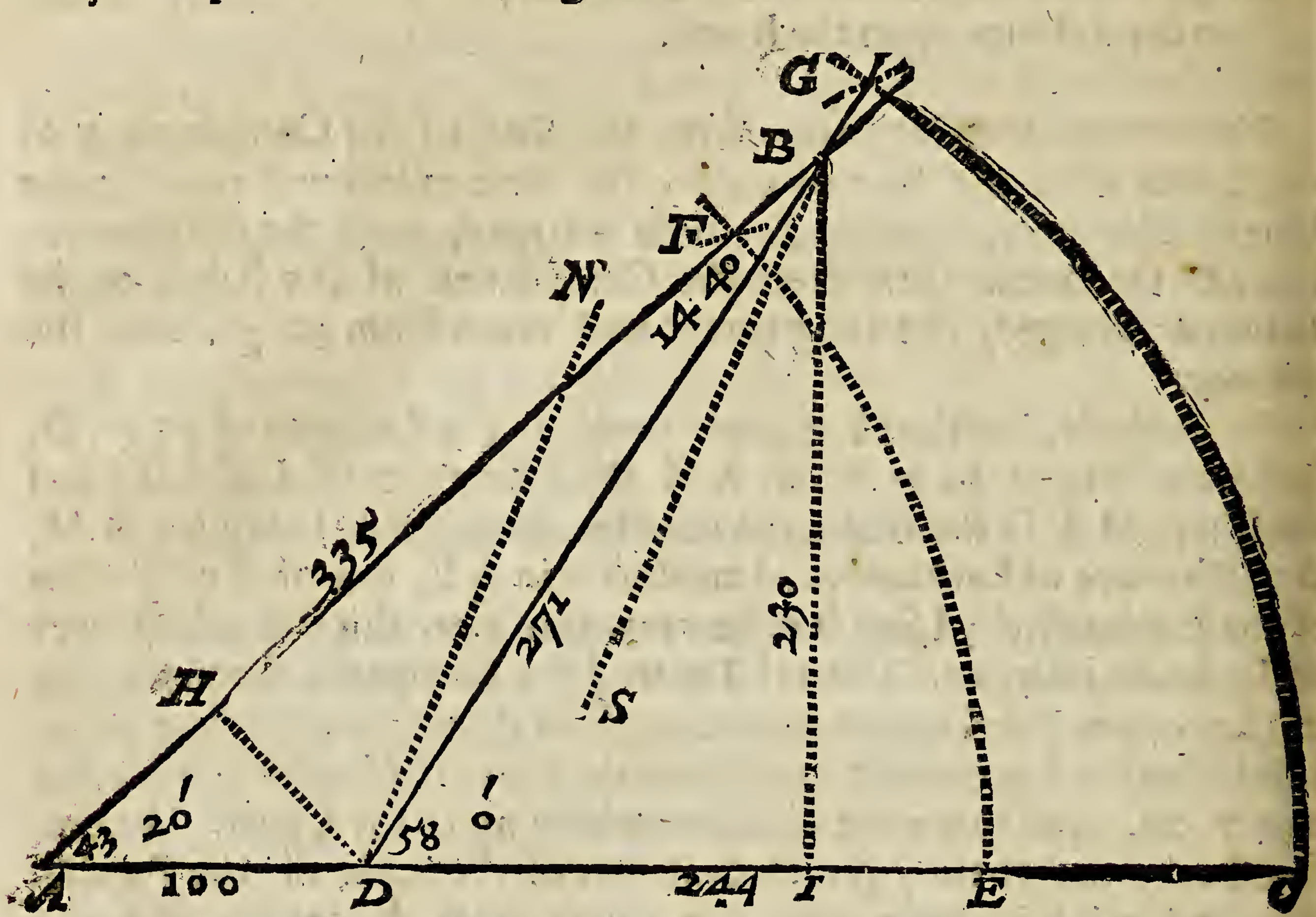
Then extend the Compaſſes from the Sine of the Complement of the Rumb unto the Sine of 90 *gr.* the ſame extent will reach in the Line of Numbers, from the Latitude inlarged, unto the diſtance required. Or extend them from the Complement of the Rumb to the Latitude inlarged, the ſame extent will reach from 90 *gr.* unto the diſtance.

For example, Let the place given be A, in the Latitude of 50 *gr.* D, in the Latitude of 52 *gr.* 30 *m.* A M the difference of Latitudes, and the Rumb M A D the fifth from the Meridian. Firſt, I take out A M, the difference of Latitudes, and meaſure it in A E, one of the Parallels of the Equinoctial; I find it to be very near 4 *gr.* this is the difference of Latitudes inlarged. Then if I extend the Compaſſes from the Sine of 33 *gr.* 45 *m.* the Complement of the fifth Rumb, unto the Sine 90 *gr.* I ſhall find the ſame extent to reach in the Line of Numbers, from 400 unto 7. 20, And this is the diſtance belonging to the Chart. Wherefore I take out theſe 7 *gr.* 20 *Cent.* out of the Scale of the Parallel A E, and prick it down upon the Rumb from A unto D, where it meeteth with the Parallel of the ſecond Latitude. Laſtly, I meaſure it in the Meridian Line, ſetting one foot of the Compaſſes as much below the leſſer Latitude, as the other above the greater Latitude, and find it to be 4 *gr.* 50 *Cent.* which is the ſame diſtance that I found before in the 5 *Prop.*

11. *By the way of the ſhip, and two Angles of poſition, to find the diſtance between the Ship and the Land.*

The way of the Ship may be known as in the firſt *Prop.* The Angles may be obſerved either by the Staff, or by a Needle ſet on the Staff. For example, ſuppoſe that being at A, I had ſight of the Land at B, the Ship going Eaſt Northeaſt from A toward C, and the Angle of the Ships Poſition B A C being 43 *gr.* 20 *m.* and after that the Ship had made 10 *Cent.* or two Leagues of way from A unto D, I obſerved

served again, and found the second Angle of the Ships Position B D C to be 58 *deg.* or the inward Angle B D A, to be 112 *deg.* then may I find the third Angle A B D, to be 14 *deg.* 40 *m.* either by Subtraction, or by Complement unto 180 *gr.*



In this and the like cases, I have a right Line Triangle, in which there is one side and three Angles known, and it is required to find the other two sides, and the *Canon* for it is this:

As the Sine of an Angle opposite to the known side,
is to that known side:
So the Sine of the Angle opposite to the side required,
is to the side required.

Wherefore I extend the Compasses from 14 *gr.* 40 *m.* in the Sines to 10 in the Line of Numbers, and this extent doth reach from 58 *gr* to 33 ½, and such is the distance between A and B, and it reacheth from 43 *gr.* 20 *m.* unto 27 in the Line of Numbers; and such is the distance from D to B.

These two distances being known, I may set out the Land upon the Chart

Chart. For having set down the way of the Ship, from A to D, by that which I shewed before in the use of the Meridian Line, I may by the same reason set off the distance A B and D B, which meeting in the Point B, shall there resemble the Land required.

12. *By knowing the distance between two places on the Land, and how they bear one from the other, and having the Angles of Position at the Ship, to find the distance between the Ship and the Land.*

If it may be conveniently, let the Angle of Position be observed at such time as the Ship cometh to be right over against one of the places. As if the places be East and West, seek to bring one of them South or North from you, and then observe the Angle of Position, so shall you have a right Line Triangle, with one side and three Angles, whereby to find the two other sides. First, you have the Angle or Position at the Ship, then a right Angle at the place that is over against you, and the third Angle at the other place is the Complement to the Angle of Position. Wherefore,

As the Sine of the Angle of Position,
is to the distance between the two places:
So the Co-sine of the Angle of Position,
to the distance between the Ship and the nearer place.
And so is the Sine of 90 *gr.*
to the distance from the Ship to the farther place:

So the places being 15 *Cent.* or three Leagues one from the other, and the Angle of Position 29 *gr.* the nearer distance will be found about 72 *Cent.* and the further distance about 31 *Cent.*

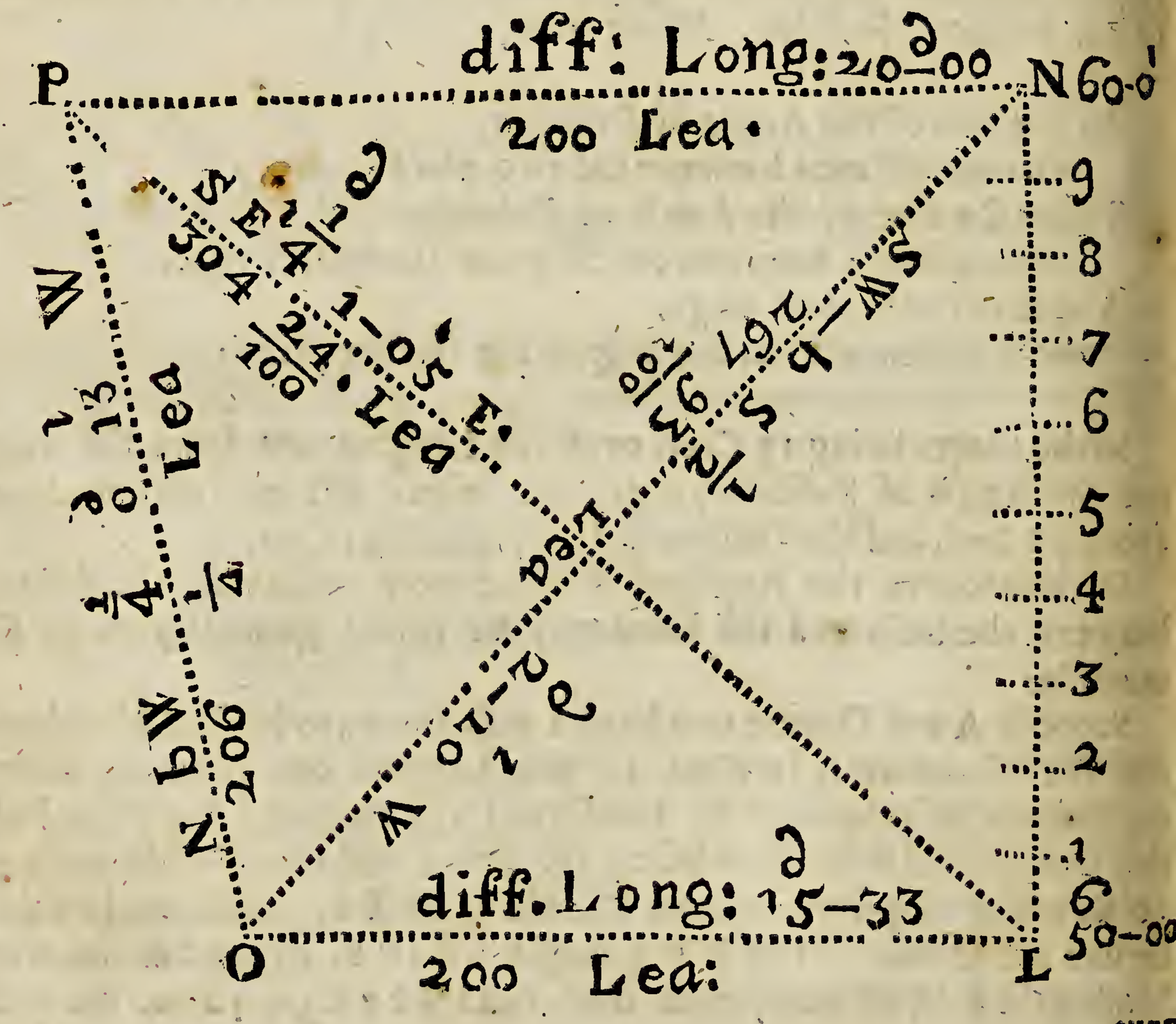
Or howsoever the Angle of Position were observed, the distance between the Ship and the Land may be found generally as in this example:

Suppose A and D were two head Lands known to be East Northeast, and West Southwest, 10 *Cent.* or two Leagues one from the other; and that the Ship being at B, I observed the Angle of the Ships Position D B A, and found it to be 14 *gr.* 40 *m.* and that D did bear 9 *gr.* 30 *m.* and A 24 *gr.* 10 *m.* from the Meridian B S, this example would be like the former. For if the Angle S B D be 9 *gr.* 30 *m.* from the South to the Westward, then shall N D B be 9 *gr.* 30 *m.* from the North

North to the Eastward. Take these 9 *gr.* 30 *m.* out of the Angle N D E, which is 67 *gr.* 30 *m.* because the two head Lands lie East Northeast, and there will remain 58 *gr.* for the Angle B D E, and the inward Angle B D A out of 180 *gr.* Take these two Angles A B D and B D A out of 180 *gr.* and there will remain 43 *gr.* 20 *m.* for the third Angle B A D. Wherefore here also are three Angles and one side, by which I may find the two other sides, as in the last *Prop.*

These Propositions thus wrought by the Staff, are such as I thought to be useful for Sea-men, and those that are skilful may apply the example to many others. Those that begin, and are willing to practice, may busie themselves with this which followeth.

Suppose four Ports, L, N, O, P, of which L is in the Latitude of 50 *gr.* N is North from L 200 Leagues or 1000 *Cent.* O West from L 1000 *Cent.* and P West from N 1000 *Cent.* so that L and O will be in the same Latitude of 50 *gr.* N and P both in the Latitude of 60 *gr.* Then let two Ships depart from L, the one to touch at O, the other at N, and then both to meet at P, there to Lade, and from thence to re-



turn

turn the nearest way unto L. Here many questions may be proposed:

1. What is the Longitude of the Port at O from L?
2. What is the Longitude of P from N? And why O and P should not be the same Longitude?
3. What is the Rumb from O unto P?
4. What is the distance from O unto P? And why the way should be more from L unto P, going by O, then by N?
5. What is the Rumb from P unto L?
6. What is the distance from P unto L?
7. What is the Rumb from N unto O?
8. What is the distance from N unto O? And why it should not be the like Rumb and distance from N unto O, as from P unto L?

These questions well considered, and either resolved by the Staff, or pricked down on the Chart, and compared with the Globe and the common Sea-chart, shall give some light to the direction of a course, and reduction of places to their due Longitude, which are now fully destorted in the common Sea-charts.

Here follows all the usual Problems of sailing, according to *Mercator*, which are resolved Arithmetically by the Table of Logarithm Tangents, without the Table of Meridional parts, and may also be performed Geometrically, by the Tangent Line upon the Cross-staff if it be large.

First, we are to know that the Logarithm Tangents from 45 *gr.* 00 *m.* upwards, do increase in the same manner, that the Secants added together do, if we account every half degree above 45 *gr.* 00 *m.* to be one whole degree of *Mercators* Meridional Line; and so the Table of Logarithm Tangents, is a Table of Meridional parts, to every two minutes of the Meridian Line, leaving out the Radius in every Line.

The manner of making use of it thus, (as it is shall more plainly appear in the Examples of the following Problems) because the Tables begin at 45 *gr.* 00 *m.* and that every 30 *m.* is for a whole degree, when one, or both Latitudes are given in any questions, take the ½ of each Latitude, and add 45 *gr.* 00 *m.* to each of them, and take the Tangent of the sum of each, for the equal parts of the Latitudes given (neglecting the Radius as beforesaid) then subtract the lesser sum of equal parts from the greater, and the remainer divide by the Tangent of 45, 30, the Radius neglected, the Quotient shall be the equal or

Equinoctial degrees contained between the two Latitudes, or else multiply the foresaid remainer by 10, and divide it by the half of the foresaid Tangent of 45 *gr.* 30 *m.* and the Quotient shall be the equal or Equinoctial Leagues contained between the two Latitudes.

Example.

LEt one Latitude be 45 *gr.* 30 *m.* the ½ is 22 *gr.* 45 *m.* unto which add 45 *gr.* 00 *m.* the sum is 67 *gr.* 45 *m.* the Tangent above the Radius is 3881591. let the other Latitude be 40 *gr.* 00 *m.* the ½ is 20 *gr.* 00 *m.* unto which add 45 *gr.* 00 *m.* the sum is 65 *gr.* 00 *m.* the Tangent above the Radius is 3313275, which subtracted from the former, the remainder is 56831: which being divided by 75803 the Tangent of 45 *gr.* 30 *m.* above the Radius, the Quotient is 7 *gr.* 497 parts, the Equinoctial degrees contained between the two Latitudes, or else multiply the remainder or difference 568316 by 10 and divide it by 37901, the ½ of the Tangent of 45 *gr.* 30 *m.* above the Radius, and the Quotient is 149 *Lea.* 94 parts, the equal or Equinoctial Leagues contained between the two Latitudes, and the like of any other.

PROBL. I.

The course and distance that the Ship hath run or sailed, being given, to find the true place or point where the Ship is in Mercators *Chart.*

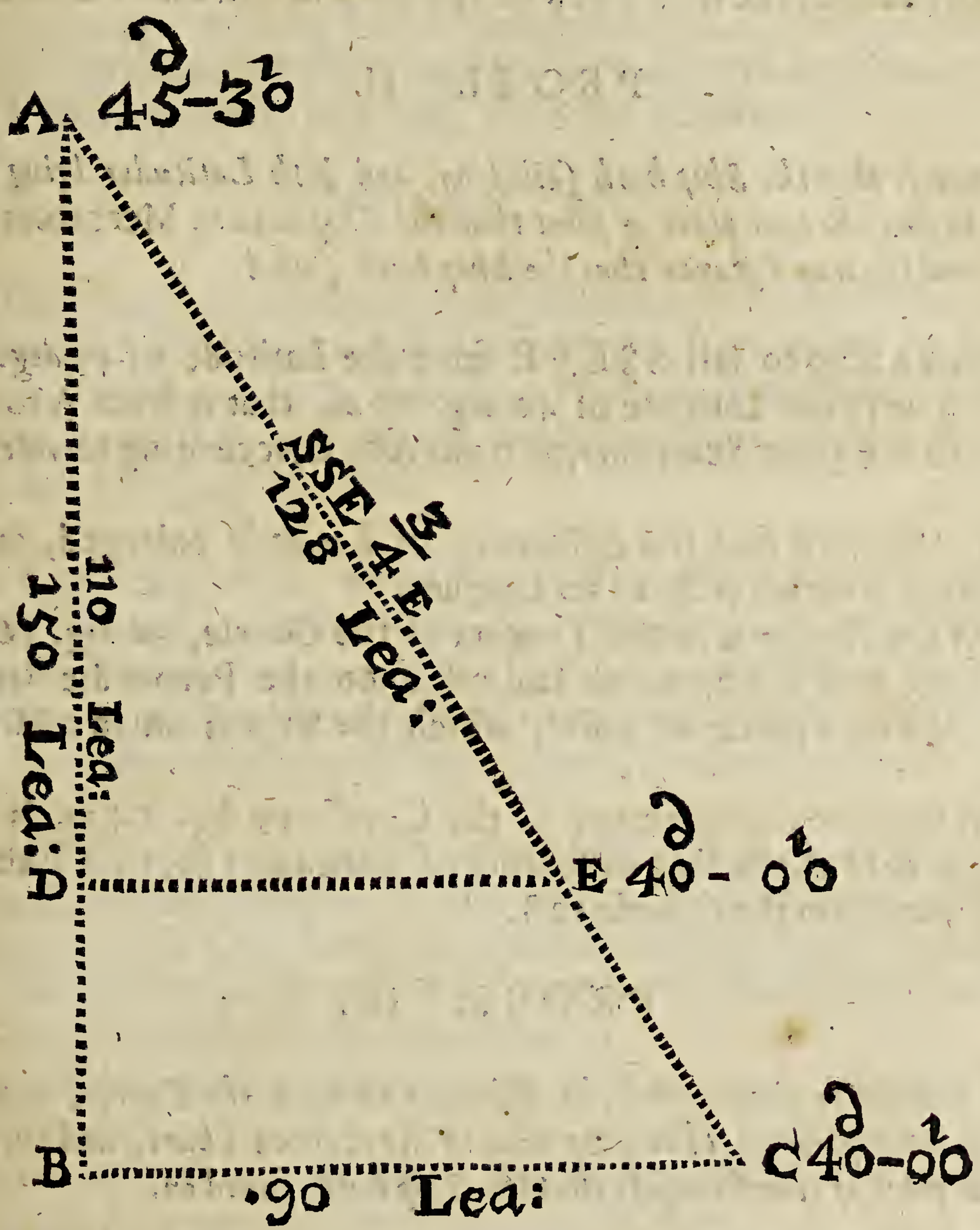
ADmit a Ship sail S S E ¼ E 128 Leagues from Latitude 45 *gr.* 30 *m.* North Latitude, that is from A to E, according to the plain Sea-chart, I demand the true place or point that the Ship is at, according to *Mercators* Chart.

Before this question can be resolved, we must find what Latitude the Ship is in, which is thus found:

As the Radius is to the Sine of the Complement of the course 59 *deg.* 04 *m.* So is E A the distance upon the course 128, to A D the true difference of Latitude in Leagues, which is 110. This being converted into *deg.* and *min.* is 5 *deg.* 30 *m.* and because the Latitude decreaseth, or the Pole is depressed, we subtract it from 45 *d.* 30 *m.* and the remainder is 40 *deg.* 00 *m.* the Latitude the Ship is in, that i[s]

a

at E, according to the plain Sea-chart, or at C according to *Mercator:* but before we can find the point C, we must find the distance of the point B in the Meridian Line from A: the manner how to do it is shewed in the Example before this Problem, and it is there found to be 150 Leagues near. Now the point C, the true place of the Ship in *Mercators* Chart may be found two several ways.



First, As D A the true distance of Latitude 110, is to A E the true distance run upon the course, so is B A the difference of Latitude enlarged 150, to A C $174\frac{5}{11}$, the enlarged distance, which being laid off upon the Line of the course, gives the point C, the true place of the Ship in *Mercators* Chart.

Here we may take notice, that the true point of any place, according to the plain Sea-chart, or according to *Mercators* Chart, is always upon one and the same right Line of the course.

Secondly, As the Radius is to the Tangent of the Course 30 *d.* 56 *m.* so is A B the difference of Latitude enlarged 150, to B C 90 the difference of Longitude, which being laid off upon the Perpendicular B C, gives the point C, the true place of the Ship in *Mercators* Chart.

PROBL. II.

The course that the Ship hath sailed on, and both Latitudes being known, to find the true place or point that the Ship is on in Mercators *Chart, and the true distance that the Ship hath sailed.*

Suppose a Ship to sail S S E ¾ E from the Latitude of 45 *deg.* 30 *m.* until it be in the Latitude of 40 *deg.* 00 *m.* that is from A to E, according to the plain Sea-chart, or from A to C, according to *Mercators* Chart.

First, We must find the difference of Latitude enlarged, as is before directed, which is A B 150 Leagues.

1. As the Radius is to the Tangent of the Course, 30 *deg.* 56 *m.* so is A B 150, to B C 90, which laid off upon the Perpendicular B C, gives C the true place or point, which the Ship is on in *Mercators* Chart.

2. As the Sine Complement of the Course 59 *deg.* 04 *m.* is to the Radius, so is D A the true difference of Latitude 110, to A E the true distance run upon the Course 128.

PROBL. III.

Both Latitudes given, and the distance run upon the Course, to find the point or place that the Ship is on in Mercators *Chart, and the course or point of the Compass that the Ship hath sailed on.*

Suppose a Ship to sail 128 Leagues, between South and East, from A in the Latitude of 45 *deg.* 30 *m.* and at the end of her distance, it be in the Latitude of 40 *deg.* 00 *m.*

First, Find the difference of Latitude enlarged, as is before directed, which is A B 150.

1. As

1. As D A 110, the true difference of Latitude, is to A E 128 the true distance run, so is B A 150 the difference of Latitude enlarged, to A C 174 $\frac{1}{10}$ the distance enlarged, which laid off upon the Line A E, from A to C it shall be the true point or place that the Ship is on in *Mercators* Projection.

2. As A E 128, the true distance run, is to A D 110, the true difference of Latitude, so is the Radius to the Sine of the Complement of the course 59 *deg.* 04 *m.* (which Complement 59 *deg.* 04 *m.* subtract out of 90 *deg.* 00 *m.* and the remainder is 30 *deg.* 056 *m.* the course, and being it is between South and East, it is S S E $\frac{3}{4}$ Easterly.

PROBL. IV.

Both Latitudes, and the departure or distance of the Meridian you are upon, and the Meridian you began your course on, to find the point or place where you are in Mercators *Chart, also the course that you have made good, and the distance that you have run from the place, where you began your course.*

THis Problem is chiefly useful for the Navigator, when he hath cast up his traverse. Admit a Ship to sail upon the Southeast quarter of the Compass, from Latitude 45 *deg,* 30 *m.* unto Latitude 40 *deg.* 00 *m.* and the departure from the Meridian it went from, be 65 $\frac{8}{10}$ Leagues.

First, Find the difference of Latitude enlarged, as is before directed 150 Leagues.

1. As A D 110 the true difference of Latitude, is to D E 65 $\frac{8}{10}$ the departure from the Meridian, so is A B 150, the difference of Latitude enlarged, to B C 90 Leagues, the difference of Longitude, which laid off upon the Perpendicular B C, from B to C shall be the point or place in *Mercators* Chart, where the Ship is.

2. As A D 110 the true difference of Latitude, is to D E 65 $\frac{8}{10}$ the departure from the Meridian, so is the Radius to the Tangent of the course 30 *d.* 56 *m.* that is two points $\frac{3}{4}$ from the South to the Eastward, that is S S E $\frac{3}{4}$ E the course that the Ship hath kept.

3. As the Sine of the course 30 *deg.* 56 *m.* is to the Radius, so is D E 65 $\frac{8}{10}$ the departure from the Meridian, to E A 128 the distance run.

PROBL.

PROBL. V.

Both Latitudes being given, and the difference of Longitude, to find the distance the Ship hath kept, and the distance it hath run.

ADmit a Ship to be at A in North Latitude 45 *deg.* 30 *m.* and to sail Southeastwards, untill it be at E in Latitude 40 *deg.* 00 *m.* according to the plain Chart, and the point C be the place in *Mercators* Chart where the Ship is, and the difference of Longitude be B C 90 Leagues.

First, Find the difference of Latitude enlarged, as is before directed 150 Leagues.

1. As A B 150 the difference of Latitude enlarged, is to B C 90, so is the Radius to the Tangent of the course, 30 *deg.* 56 *m.* which is two points $\frac{3}{4}$ that is S S E $\frac{3}{4}$ E.

2. As the Sine Complement of the course 59 *deg.* 04 *m.* is to the Radius, so is D A 110 the true difference of Latitude, to A E the true distance run 128.

PROBL. VI.

One Latitude, with the course, and the difference of Longitude given, to find the other Latitude, and the distance run.

SUppose a Ship to be in the Latitude of 45 *deg.* 30 *m.* North Latitude, and to sail S S E $\frac{3}{4}$ E (until the difference of Longitude be 90 Leagues) that is from A to C, which is the point or place of the Ship in *Mercators* Chart.

1. As the Radius is to the Tangent Complement of the course 59 *deg.* 04 *m.* so is C B the difference of Longitude 90, to A B 150 the difference of the Latitude enlarged, by which multiply 37901 the $\frac{1}{2}$ of the Tangent of 45 *deg.* 30 *m.* above the Radius, and divide the Product by 10, and the Quotient is 568515. Then take $\frac{1}{2}$ the Latitude given, the $\frac{1}{2}$ of 45 *deg.* 30 *m.* which is 22 *deg.* 45 *m.* unto which add 45 *deg.* 00 *m.* the sum is 67 *deg.* 45 *m.* then seek the Tangent of 67 *deg.* 45 *m.* above the Radius, which is 3881591, and subtract the former Quotient 568515 from it, and the remainer is 3313076, which seek in the Tangent, and you shall find it at 65 *deg.* 00 *m.* from which subtract

45 *deg.*

45 *deg.* 00 *m.* the remainer is 20 *deg.* 00 *m.* which being doubled, is 40 *deg.* 00 *m.* the Latitude required. Here we are to note, that if the Latitude had increased, we must have added the Quotient 568515 to the Tangent of 67 *deg.* 45 *m.* and so sought the sum in the Tangents, to have found the Latitude required.

2. As the Sine of the Complement of the course 59 *deg.* 04 *m.* is to the Radius, so is D A the true difference of Latitude 110, to A E, the true distance run 128.

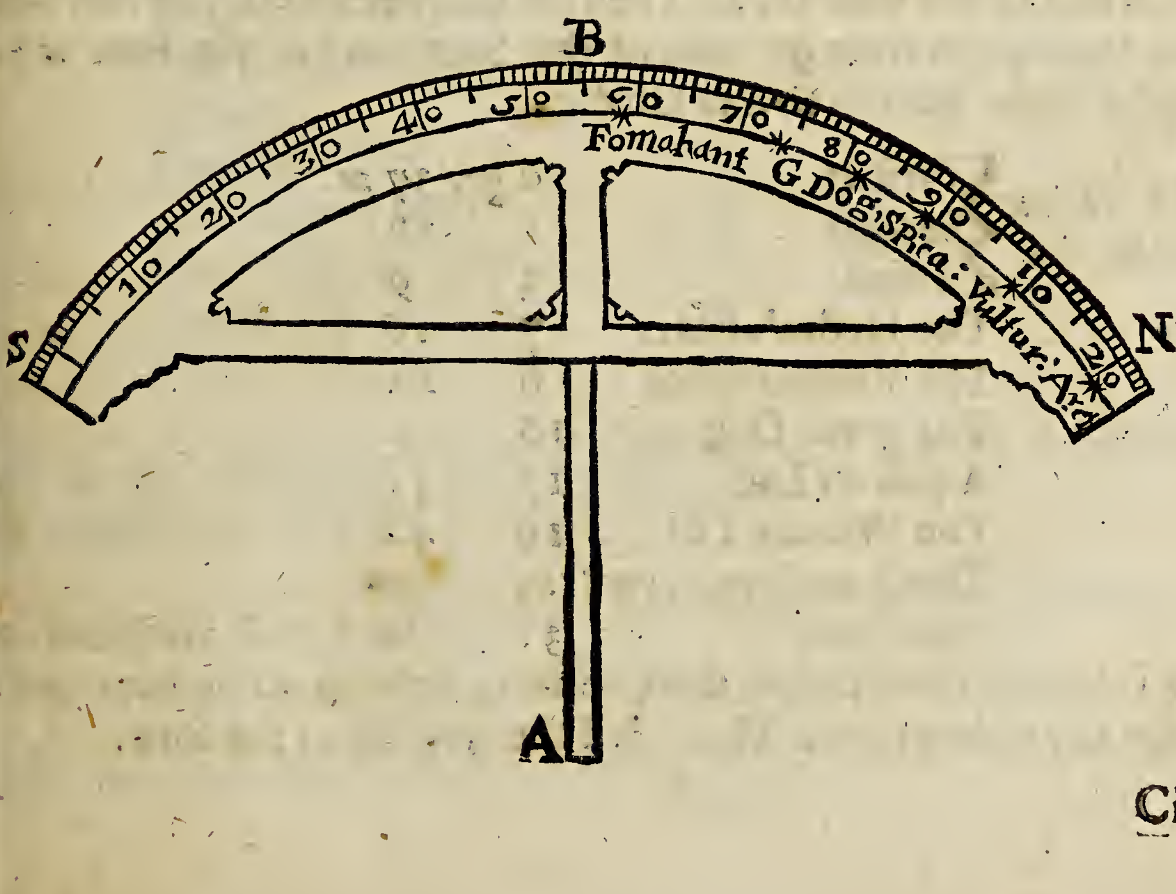
Although I have set down but the proportions and the answers to each question, they may all be calculated by the Canon, and the Chiliad of Logarithms in this Book.

AN APPENDIX,

Concerning the description and use of an Instrument, made in form of a Cross-bow, *for the more easie finding of the Latitude at Sea.*

THe former *Prop.* suppose the Latitude to be known, I will here shew it how it may be easily observed.

Upon the Center A, and Semidiameter A B, describe an Ark of a



Circle

Circle S B N. The same Semidiameter will set off 60 *gr.* from B unto S for the South end, and other 60 *gr.* from B unto N for the North end of the Bow: so the whole Bow will contain 120 *gr.* the third part of a Circle. Let it therefore be divided into so many degrees, and each degree subdivided into six parts, that each part may be ten minutes: but let the numbers set to it be 5, 10, 15, unto 90 *gr.* and then again 5, 10, 15, unto 25, that 55 may fall in the middle, as in this Figure.

The Bow being thus divided and numbred, you may see the months and days of each month upon the back, and such stars as are fit for observation upon the side of the Bow.

If you desire to make use of it in North Latitude, you may number 23 *gr.* 30 *m.* from 90 towards the end of the Bow at N, and there place the tenth day of *June*, And 23 *gr.* 30 *m.* from 90 towards S; and there at 66 *gr.* 30 *m.* place the tenth day of *December*. And so the rest of the days of the year, according to the declination of the Sun at the same days.

The stars may be placed in like manner according to their Declinations, to the year 1670.

Arcturus	20 *gr.*	57 *m.*
The Bulls Eye	15	47
The Lions Heart	13	32
The Vultures Heart	8	8
The little Dog	6	0

from 90 toward the North end of the Bow at N. Then for Southern stars, you may number their declination from 90 toward the South end of the Bow at S. At first the three Stars in *Orions* Girdle,

In *Orions* Girdle the First at	0 *gr.*	37 *m.*
Second	1	26
Third	2	9
The Hydra's heart	7	16
The Virgins Spike	9	24
The great Dog	16	9
Aquaries Leg	17	33
The Whales Tail	19	48
The Scorpions Heart	25	37
Fomahant	31	16

And so the South crown, the Triangle, the Clouds, the Crosiers, or what other Stars you think fit for the observation. This I call the foreside of the Bow.

If you desire to make use of it in South Latitude, you may turn the ow, and divide the back side of it, and number it in like manner, d then put on the months and days of the year, placing the tenth of *ecember* at the South end, and the tenth of *June* toward the middle of e Bow, and the rest of the days according to the Suns declination as fore.

The chiefest of the Northern Stars may here be placed in like anner, according to their declination, *Anno* 1670.

The Pole Star at	87 *gr.*	32 *m.*
The first Guard	75	34
The second Guard	73	15
The great Bears back	63	30
In the great Bears Tail — First	57	47
In the great Bears Tail — Second	56	40
In the great Bears Tail — Third	51	2
The side of Perseus	48	36
The Goat	45	37
The Tail of the Swan	44	9
The Head of Medusa	39	41
The Harp	38	32
Castor	32	33
Pollux	28	47
The North-Crown	27	50
The Rams Head	21	30
Arcturus	20	47
The Bulls Eye	15	37
The Lions Heart	12	32
The Vultures Heart	8	08
Orions right Shoulder	7	19
Orions left Shoulder	6	2

And so any other Star whose declination is known unto you; hich being done. The use of this Bow may be.

1. *The day of the month being known, to find the declination of the Sun.*

2. *The declination being given, to find the day of the month.*

These two *Prop.* depend on the making of the Bow. If the day be

 known,

known, look it out in the back of the Bow: so the declination will ap-
pear in the side. Or if the declination be known, the day of the month
is set over against it. As if the day of the month were the 14 of *July*
look for this day in the back of the Bow, and you shall find it over
against 20 *gr.* of North declination. If the declination given be 20 *gr.*
to the Southward, you shall find the day to be either the eleventh of
November, or the eleventh of *January*.

3. *To find the Altitude of the Sun, or Stars.*

Here it is fit to have two running sights which may be easily moved
on the back of the Bow. The upper sight may be set either to 60 *gr.* or
to 70 *gr.* or to 86 *gr.* as you shall find to be most convenient: the other
sight may be set on to any place between the middle and the other end
of the Bow. Then with the one hand hold the Center of the Bow to
your eye, so as you may see the Sun or Star by the upright sight, and
with the other hand move the lower sight up or down until you have
brought one of the edges of it, to be even with the Horizon (as when
you observe with the Cross-staff) so the degrees contained between
that edge and the upper sight, shall shew the Altitude required.

Thus if the upper sight shall be at 80 *gr.* and the lower sight at 50 *gr.*
the Altitude required is 30 *gr.*

4. *To find any North Latitude, by the Meridian Altitude of the Sun at a forward observation, knowing either the day of the month, or the declination of the Sun.*

As oft as you are to observe in North Latitude, place both the sights
on the foreside of the Bow, the upper sight at the Declination of the
Sun, or the day of the month at the North end, and lower sight toward
the South end. Then when the Sun cometh to the Meridian, turn your
face to the South, and with the one hand hold the Center of the Bow to
your Eye, so as you may see the Sun by the upper sight; with the other
hand move the lower sight, until you have brought one of the edges of
it to be even with the Horizon: so that edge of the lower sight shall
shew the Latitude of the place in the fore-side of the Bow.

Thus being in North Latitude upon the ninth of *October*: if I set
the upper sight to this day, at the fore-side and North end of the Bow
I shall find it to fall to the Southward of 90 upon 80 *gr.* and therefore
a

: 10 *gr.* of South declination. Then the Sun coming to the Meridian, I ay set the Center of the Bow to mine eye, as if I went to find the Al-tude of the Sun, holding the North end of the Bow upward, with the pper sight between mine eye and the Sun, and moving the lower sight, ntil it come to be even with the horizon. If here the lower sight shall ay at 50 *gr.*. I may well say, that the Latitude is 90 *gr.* For the Me-idian Altitude of the Sun is 30 *gr.* by the third *Prop.* and the Sun ha-ing 10 *gr.* of South declination, the Meridian Altitude of the Equa-or would be 40 *gr.* and therefore the observation was made in 50 *gr.* of North Latitude.

By the same reason, if the lower side had stayed at 51 *gr.* 90 *m.* the atitude must have been 51 *gr.* 30 *m.* and so in the rest.

5. To find any North Latitude, by the Meridian Altitude of the Stars to the Southward.

Let the upper sight be set to the Star, which you intend to observe, ere placed in the fore-side of the Bow. Then hold the North end of he Bow upward, and turning your face to the South, observe the Me-idian Altitude as before: so the lower sight shall shew the Latitude f the place in the fore-side of the Bow.

Thus if in observing the Meridian Altitude of the great Dog-star, he lower sight shall stay at 50 *gr.* it would shew the Latitude to be o *gr.* For this Star being here placed at 73 *gr.* 48 *m.* if we take thence o *gr.* his Meridian Altitude would be 23 *gr.* 48 *m.* to this if we add o *gr.* 12 *m.* for the South declination of this Star, it would shew the Meridian Altitude of the Equator to be 40 *gr.* and therefore the Lati-ude to be 50 *gr.*

6. To find any North Latitude, by the Meridian Altitude of the Stars to the Northward.

If the Bow be intended only for North Latitude it may suffice to ave the degrees divided only on the fore-side, and then the Stars to he Northward may be placed either on the backside or the inside of he Bow by these degrees: the Pole-star at 87 *gr.* 20 *m.* near the 20 ay of *September*, the formost guard at 75 *gr.* 45 *m.* the hindmost guard t 73 *gr.* 25 *m.* and the rest according to their declinations before men-ioned, so the 90 *deg.* shall represent the North Pole of the World.

When any of these stars come to be in the Meridian, and under th
Pole, set the upper sight to that Star, hold the North end of the Bo
upward, and turning your face to the North, observe his Altitude a
before; so the degrees contained between the 90 degrees and the lowe
sight, shall shew the Altitude of the Pole.

Thus the former guard coming to be in the Meridian, under the Pol
if you observe and find the lower sight to stay at 40 *gr.* the Elevatio
of the Pole is 50 *gr.* according to the distance between 40 and 90.

If you would observe any of these Stars, at such time as they com
to be in the Meridian, and above the Pole, you may place these Star
in the Bow above 90 *gr.* the North Star at 2 *gr.* 40 *m.* near the fourt
day of *September*, the formost Guard at 14 *gr.* 15 *m.* the hindmost Guar
at 16 *gr.* 35 *m.* and such others as you think fittest, according to thei
distance from the Pole: then setting the upper sight to the place o
Star above the Pole, the rest of the observation will be the same a
before.

But if the Bow be made to serve at large, both in South and Nort
Latitude, then these Northern Stars would be let placed on the back
side of the Bow, by the degrees on that side, according to the Com
plement of their declinations, that the North Stars may answer to th
North Sun in South Latitude, in such sort as the Southern Stars did t
the South Sun in North Latitude in the former *Prop.* This being done
let the upper sight be set to the Star which you intend to observe, her
placed on the backside of the Bow. Then hold the North end of th
Bow upward, and turning your face to the North, observe the Altitud
of the Star when he cometh to be in the Meridian, and under the Pole
so the lower sight shall shew the Altitude of the Pole in the backsid
of the Bow.

Thus the former guard coming to be in the Meridian under th
Pole, if you observe and find the lower sight to stay at 90 *gr.* such i
the Elevation of the Pole, and the Latitude of the place to the North-
ward. For the distance between the two sights will shew the Altitud
to be 35 *gr.* 45 *m.* and the Star is 14 *gr.* 15 *m.* distant from the Nort
Pole. These two do make up 50 *gr.* for the Elevation of the Nort
Pole, and therefore such is the North Latitude.

7. *To find any South Latitude, by the Meridian Altitude of the Sun at a forward observation, knowing either the day of the month, or the declination of the Sun.*

When you are come into South Latitude, turn both your fights to the back fide of the Bow: the upper fight to the declination of the Sun, or the day of the month at the South end, and the lower fight toward the North end of the Bow. Then the Sun coming to the Meridian, turn your face to the North, and holding the South end of the Bow upward, obferve the Meridian Altitude as before: fo the lower fight fhall fhew the Latitude of the place in the backfide of the Bow.

Thus being in South Latitude, upon the tenth of *May*, if you obferve and find the lower fight to ftay at 30 *gr.* on the backfide of the Bow, fuch is the Latitude. For the declination is 20 *gr.* Northward, the Altitude of the Sun between the two fights 40 *gr.* the Altitude of the Equator 60 *gr.* and therefore the Latitude 30 *gr.*

8. *To find any South Latitude by the Meridian Altitude of the Stars to the Northward.*

Let the upper fight be fet to the Star which you intend to obferve, here placed on the backfide of the Bow. Then hold the South end of the Bow upward, and turning your face to the North, obferve the Meridian Altitude as before: fo the lower fight fhall fhew the Latitude of the place in the back fide of the Bow.

Thus being in South Latitude, and the former guard coming to be in the Meridian over the Pole. If you obferve, and find the lower fight to ftay at 5 *gr.* fuch is the Latitude. For the Star is 14 *gr.* 15 *m.* from the North Pole, the Altitude of the Star between the two fights 9 *gr.* 15 *m.* the North Pole depreffed 5 *gr.* and therefore the Latitude 5 *gr.* to the Southward.

9. *To obferve the Altitude of the Sun by the Bow, or with an Aftrolabe.*

Here it is fit to have a third fight (like to the Horizontal fight belonging to the ftaff) which may be fet to the Center of the Bow.

If the Sun be near to the Zenith, hold the Bow as when you obferve with the *Aftrolabe*, fo as the Center being downward the Line A B may

may be vertical, and the Line S N Parallel to the Horizon, then turning one end of the Bow toward the Sun, you may move one of the sights on the back of the Bow, until the shadow thereof fall on the middle of the Horizontal sight, so the degrees contained between the Vertical A B, and that upper sight shall shew the distance of the Sun from the Zenith.

If the Sun be nearer to the Horizon, you may hold the Bow so as the Line S N may be Vertical, and the Line A B Parallel to the Horizon, then observing, as before, the degrees contained between the Line A B, and the upper sight, shall shew the Altitude of the Sun above the Horizon.

10. *To find a South Latitude by the Meridian Altitude of the Stars to the Southward.*

Let the upper sight be set to the Star which you intend to observe, which might be here placed on the foot side of the Bow by the Complement of their declinations, if we knew the true place of such as are near to the South Pole.

Then hold the South end of the Bow upward, and turning your face to the South, observe the Altitude when he cometh to be in the Meridian, and under the Pole, so the lower sight shall shew the Altitude of the Pole in the foreside of the Bow.

11. *To observe the Altitude of the Sun backward.*

Set the upper sight either to 60, or 70, or 80 *gr.* as you shall find it to be most convenient, the lower sight on any place between the middle and the other end of the Bow, and have an Horizontal sight to be set to the Center. Then may you turn your back to the Sun, and the back of the Bow toward your self, looking by the lower sight through the Horizontal sight, and moving the lower sight up and down until the upper sight do cast a shadow upon the middle of the Horizontal sight: so the degree, contained between the two sights on the Bow, shall give the Altitude required.

Thus if the upper sight shall be at 80 *gr.* and the lower sight at 50 *gr.* the Altitude required is 30 *gr.* as in the third *Prop.*

Or if you turn the other end of the Bow upward, and set the upper sight to the beginning of the Quadrant, and then observe as before, the lower sight will shew the Altitude.

12. *To*

12. To find any North Latitude by the Meridian Altitude of the Sun at a back observation, knowing either the day of the month, or the declination of the Sun.

Place your three sights as before on the fore-side of the Bow: the upper sight to the declination of the Sun, or to the day of the month, at the North end; the lower sight toward the South end of the Bow; and the Horizontal sight to the Center. Then the Sun coming to the Meridian, turn your face to the North, and holding the North end of the Bow upward, the South end downwards, with the back of it toward your self, observe the shadow of the upper sight as in the former part of the fifth *Proposition*, so the lower sight shall shew the Latitude of the place in the foreside of the Bow.

Thus being in North Latitude upon the ninth of *October*, if you observe and find the lower sight to stay at 50 *gr.* on the foreside of the Bow, such is the Latitude. For the declination is 10 *gr.* Southward, and the Altitude of the Sun between the two sights 30 *gr.* the Altitude of the Equator 40 *gr.* and therefore the Latitude 50 *gr.* as in the sixth *Prop.*

13. To find any South Latitude by the Meridian Altitude of the Sun at a back observation, knowing either the day of the month, or the declination of the Sun.

When you observe in South Latitude, place your three sights on the back side of the Bow: the upper sight to the declination of the Sun, or the day of the month at the South end; the lower sight toward the North end of the Bow, and the Horizontal sight to the Center. Then the Sun coming to the Meridian, turn your face to the South, and holding the South end of the Bow upward, with the back of it toward your self, observe the shadow of the upper sight as before: so the lower sight shall shew the Latitude of the place in the back side of the Bow.

Thus being in the South Latitude upon the tenth of *May*, if you observe and find the lower sight to stay at 30 *gr.* on the back of the Bow, such is the Altitude of the Sun between the two sights 40 *gr.* the Altitude of the Equator 60 *gr.* and therefore the Latitude 30 *gr.* as in the seventh *Prop.*

14. *To find the day of the month, by knowing the Latitude of the place, and observing the Meridian Altitude of the Sun.*

Place your three ſights according to your Latitude; the Horizontal ſight to the Center, the lower ſight to the Latitude, and upper ſight among the months. Then when the Sun cometh to the Meridian, obſerve the Altitude, looking by the lower ſight through the Horizontal, and keeping the lower ſight ſtill at the Latitude, but moving the upper ſight until it give ſhadow upon the middle of the Horizontal ſight: ſo the upper ſight ſhall ſhew the day of the month required.

Thus in our Latitude if you ſet the lower ſight to 51 *gr.* 30 *m.* and obſerving find the Altitude of the Sun between that and the upper ſight to be 28 *gr.* 30 *m.* this upper ſight will fall upon the ninth of *October*, and the twelfth of *February*. And if yet you doubt which of them two is the day, you may expect another Meridian Altitude; and then if you find the upper ſight upon the tenth of *October*, and the eleventh of *February*, the queſtion will be ſoon reſolved.

15. *To find the declination of any unknown Star, and ſo to place it on the Bow, knowing the Latitude of the place, and obſerving the Meridian Altitude of the Star.*

When you find a Star in the Meridian that is fit for obſervation. Set the Center of the Bow to your eye, the lower ſight to the Latitude, and move the upper ſight up or down until you ſee the Horizon by the lower ſight, and the Star by the upper ſight, then will the upper ſight ſtay at the declination and place of the Star.

Thus being in 20 *gr.* of North Latitude, if you obſerve and find the Meridian Altitude of the head of the Coſier to be 14 *gr.* 50 *m.* The upper ſight will ſtay at 34 *gr.* 50 *m.* and there may you place this Star. For by this obſervation the diſtance of this Star from the South Pole ſhould be 34 *gr.* 50 *m.* and the declination from the Equator 55 *gr.* 10 *m.* And ſo for the reſt.

The Stars which I mentioned before, do come to the Meridian in this order after the firſt point of *Aries*.

16. *To find any North Latitude on land by obſervation with Thread and Plummet.*

Set the ſight to the day of the month at the foreſide and South end of of the Bow: then when the Sun cometh to the Meridian turning the North end in your left hand toward the South, so as the sight at the Center may shadow the sight at the day, observe where the thread falleth, and abate 20 *gr.* if it fall on 70 *gr.* the Latitude is 50 *gr.* If on 71 *gr.* 30 *m.* in the Latitude is 51 *gr.* 30 *m.* And so in the rest.

If the Bow had been made only for finding the Latitude on Land I might then have set such numbers to it as needed no allowance.

17. *To find any South Latitude on Land, by observation with Thread and Plummet.*

Set the sight to the day of the month, at the backside and North end of the Bow, and when the Sun cometh to the Meridian, turning the South end to your left hand toward the North, observe as before, and abate 20 degrees.

Or you may set the sight to the day of the month, at the foreside, and North end of the Bow, and so observing as before, the Thread will fall on the Complement of the Latitude.

The right Ascension of these Stars is to the year 1670.

	H.	M.		H.	M.
The Pole Star at	0	31	The Lions Heart	9	50
The Rams Head	1	48	The great Bears Back	10	43
The Head of Medusa	2	47	First in gr. Bears Tail	12	39
The Side of Perkus	3	00	The Virgins Spike	13	7
The Bulls Eye	4	17	Second in gr. Bears Tail	13	11
The Goat	4	52	Third in gr. Bears Tail	13	36
Orions left shoulder	5	07	Arcturus	14	02
Orions Girdle, the first	5	15	The formost Guard	14	53
Orions Girdle, the second	5	19	The North Crown	15	21
Orions Girdle, the third	5	24	The hindmost Guard	15	27
Orions right shoulder	5	37	Scorpions heart	16	10
The great Dog	6	31	The Harp	18	25
Castor	7	13	Vulturs heart	19	35
The little Dog	7	22	Swans tail	20	31
Pollux	7	25	Fomahant	22	38
The Hydra's Heart	9	11			

Anno 1670	R. Ascen.		Declin.		M
Pole Star	07	33	87	35	2
	262	26	82	25	4
	294	58	86	28	4
Little Bear	240	24	78	52	4
	247	24	76	30	5
First Guard	223	37	75	34	2
Second Guard	231	48	73	17	3
Great Bear					
Snout	118	48	61	26	4
Eye	121	00	64	17	4
Forehead {	124	45	68	16	4
	127	10	67	47	4
Ear	131	45	71	02	5
Neck {	32	10	63	37	4
	142	50	62	02	4
Breast	144	36	61	12	4
Knee	141	10	59	49	4
Right Foot {	136	10	13	27	3
	129	40	18	36	3
In the square {	27	40	48	26	3
	160	48	63	31	2
	160	08	58	09	2
	173	54	55	35	2
	179	50	58	53	2
	190	00	57	49	2
	197	50	56	41	2
In the Tail	203	42	51	13	2
Cassiopea.	R. Ascen.		Declin.		M
Head	4	44	52	00	4
Breast	5	31	54	39	3
Waste	7	19	55	50	4
Belly	9	18	58	48	3
Knee	16	24	58	25	3
Thigh	22	49	62	13	3
Foot	30	14	65	45	4
Chair {	3	24	61	00	4
	357	43	57	15	3

ADVERTISEMENT
CONCERNING THE
LOGARITHMS,
Rendring them useful to 100000.

A Number that consisteth of five places being given, to find the Logarithm thereof.

I Find the Logarithm of the first four Figures, rejecting the Characteristick; then observe the difference between that and the next following, which multiply by the last Figure of the Number given, and cut off one Figure from the Product towards the right hand; the rest add to the Logarithm of the first four Figures. Lastly, if you prefixt the proper Characteristick for the Number given, that Logarithm so ordered, is the number required.

Example. 19438 being propounded, I demand the Logarithm thereof: By the direction fore-going I find the Logarithm of the first four Figures, *viz.* 1943, to be (rejecting the Characteristick) 2884728, also I see the difference between that Number and the Number following to be 235, which I multiply by the last figure of the Number propounded, being 8; and that sum is 17880. Wherefore I add 1788 to 2884728, and prefix before it the proper Characteristick for the Number given, which must be 4 — because that is the Characteristick for all Numbers from 10000 to 100000, so is produced at last 4, 2886516, which is the Logarithm for 19438, as was required.

Yyyyy Again,

Again,

Let it be required to find the Logarithm for 56724.

Having found the Logarithm of the first four figures t
be 7537362, and the difference between that and the nex
766, and multiplied the difference by 4, the last figure o
the sum propounded, of which adding 306 to 7537362
they make 7537668, before which prefixing the Characte
ristick 4, the Logarithm for 56724 will be 4, 7537668, th
thing required.

And for 94395, it will be found 4,9749490, *&c.*

FINIS.

These Books following are Printed for, and to be sold by Francis Eglesfield *at the Sign of the Marigold in St.* Paul's *Church-yard.*

Esops Fables in Prose and Verse, Illustrated with Emblems or Pictures, and also Grammatically Translated into Prose, with apt Morals, and Printed according to the order of the Latine Copy; Together with the History of his Life from the best Greek Copies, very useful for all, but especially for young Scholars.

Note, There is lately crept forth a Counterfeit thing under the Title of *Esops* Fables, and the better to colour the Imposture, it is pranked up in the same Volume and Dress with the true: whereas the same is none of *Esop*'s, but a confused Fardle of nonsense scrap'd from *Poggius* and the *Seven wise Masters*, unworthy of *Esop*'s name; and no more his than *Scoggins* Tales, or the Legend of Grannum *Shipton.*

The true one hath *Esop*'s Picture on a Copper Cut, and a Greek Sentence in the Title Page, and is both in Verse and Prose, the Authors Life being at the latter end, and newly reprinted the eighth time by *I. R.* 1673. for *Francis Eglesfield*, and are to be sold by him at the Sign of the *Marigold* in St. *Pauls* Church-yard.

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The danger of being almost a Christian.

A word to *Israel* in the Wilderness; Or the

Araignment of Unbelief: All three by *John Cheshull*, late Minister of *Tiverton* in *Devon*, in Twelves.

The

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R

THE END.

Cassiopea.	R. Ascen.		Declin.		M
Auriga					
Head	82	34	54	15	4
Left shoulder	373	05	45	36	1
Hircus Right shoulder	83	48	44	55	4
Cepheus Girdle	321	28	69	07	3
Right shoulder	316	10	60	22	3
Left shoulder	340	16	64	33	4
Head	329	41	56	23	4
Right foot	304	51	76	52	4
Left foot	351	41	75	32	3
Draco					
Tongue	154	31	54	55	4
Mouth	261	15	55	30	4
Eye	260	34	52	34	4
Head	267	04	51	21	3
In the middle of the first winding	276	41	56	44	5
In the □ of the second winding	288	14	67	05	3
In the first △.	395	14	72	34	4
In the second △.	266	58	73	23	4
In the third winding	3247	20	69	30	3
After the fourth winding	209	31	66	32	2
Last in the Tail	67	9	71	10	3

The End of the Second Book of the Cross-Staff.

The right Aſcenſion, Declination, and Magnitude of ſome principal Fixed Stars.

The Stars Names.	Right Aſcenſi. D.	Right Aſcenſi. M.	Decli-nation. D.	Decli-nation. M.		Magni-tude.
The Pole-Star	7	47	87	27	N	2
The Girdle of *Andromeda*	12	32	33	48	N	2
The former Horn of the Ram	23	38	17	37	N	4
Bright Star in the Ram's Head	26	56	21	48	N	3
The Whale's Jaw	41	3	2	42	N	2
The Head of *Meduſa*	41	27	39	35	N	3
The Bull's Eye	64	0	15	46	N	1
The Goat	72	44	45	35	N	1
The former Shoulder of *Orion*	76	38	4	59	N	2
The latter Shoulder of *Orion*	84	7	7	18	N	2
The great Dog	97	27	16	13	S	1
The uppermoſt Head of the Twins	108	1	32	35	N	2
The little Dog	110	17	6	6	N	2
The lower head of the Twins	111	0	28	49	N	2
The Crib	125	4	20	52	N	Neb.
Hydra's Heart	137	39	7	10	S	2
Lion's Heart	147	27	13	39	N	1
Lion's Loins	163	54	22	26	N	2
Lion's Tail	172	49	16	32	N	1
The Virgin's Girdle	189	32	5	20	N	3
Alioth	189	36	57	36	N	2
Vindemiatrix	191	15	12	51	N	3
The Virgin's Spike	196	44	9	17	S	1
Arcturus	209	56	21	4	N	1
The Southern Balance	217	56	14	32	S	2
The Northern Balance	224	31	8	2	S	2
In the Serpent's Neck	231	49	7	35	N	3
The Scorpion's Heart	242	4	25	34	S	1
Hercules Head	254	40	14	51	N	3
Ophinchus Head	259	41	12	52	N	3
The Harp	276	17	38	30	N	1
The Vulture	293	27	8	1	N	2
The upper Horn of the Goat	299	30	13	32	S	3
Left Hand of *Aquarius*	307	10	10	43	S	4
Left Shoulder of *Aquarius*	318	18	7	2	S	3
Pegaſus Mouth	321	49	8	18	N	3
Right Shoulder of *Aquarius*	326	59	1	58	S	3
Fomahant	339	29	31	23	S	1
Upper Wing of *Pegaſus*	341	53	13	21	N	2
In the tip of *Pegaſus* Wing	358	52	13	15	N	2

THE THIRD BOOK OF

The Use of the Lines of Numbers, Sines and Tangents, for the drawing of Hour-lines on all sorts of Planes.

THere are ten several sorts of Planes, which take their denominati-on from those Great Circles to which they are Parallels, and may sufficiently for our use be represented in this one Funda-nental Diagram, and be known by their Horizontal and Perpendicular ines, of such as know the Latitude of the Place, and the Circles of the phere.

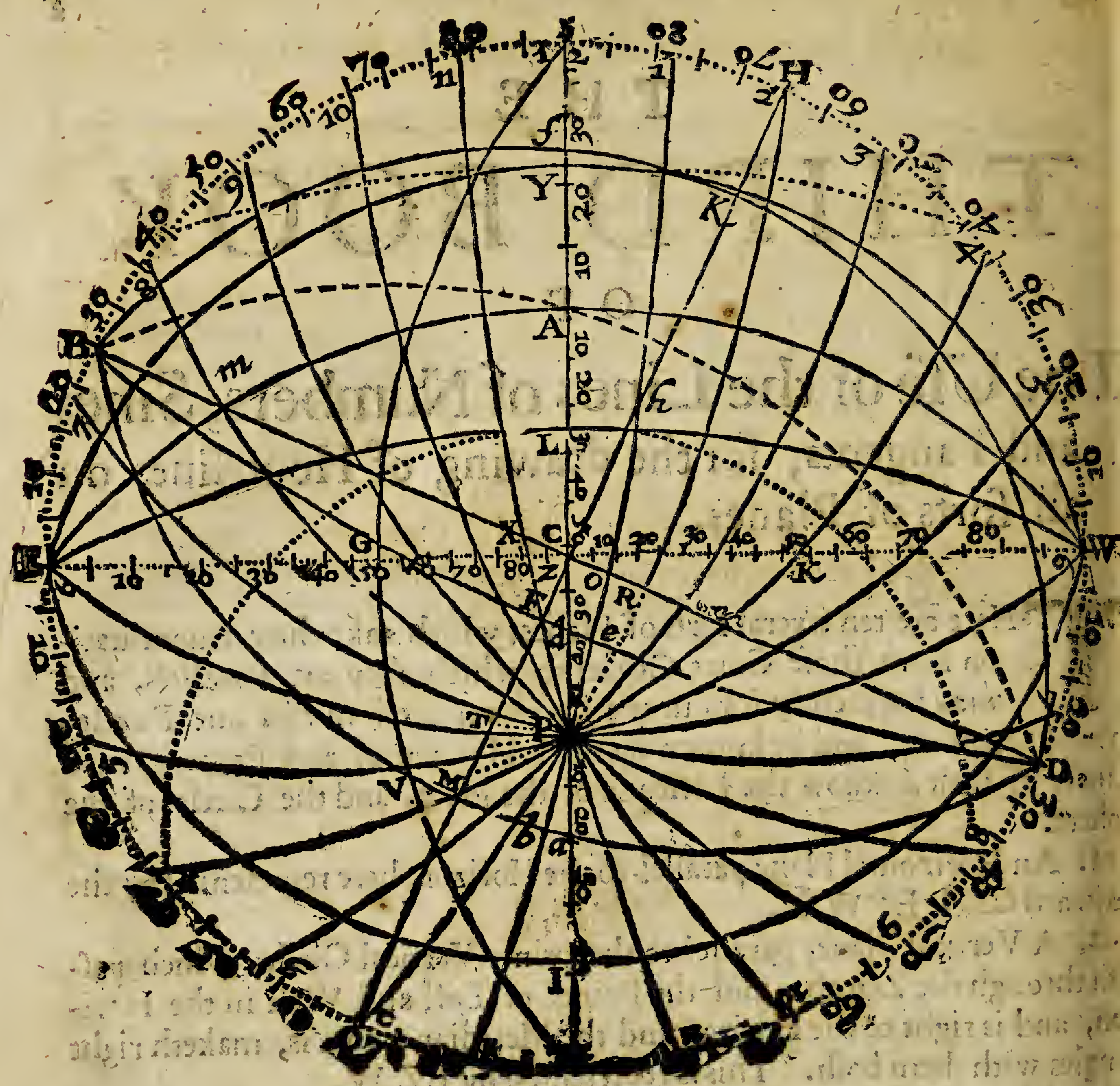
1. An Horizontal Plane parallel to the Horizon, here represented by the utward Circle E S W N.

2. A Vertical Plane, parallel to the prime Vertical Circle, which pas-eth through the Zenith, and the Points of East and West in the Hori-on, and is right to the Horizon and the Meridian; that is, maketh right ngles with them both. This is represented by E Z W.

3. A Polar Plane parallel to the Circle of the Hour of 6, which passeth rough the Pole, and the Points of East and West, being right to the quinoctial and the Meridian, but inclining to the Horizon, with an Angle qual to the Latitude. This is here represented by E P W.

4. An Equinoctial Plane parallel to the Equinoctial, which passeth rough the Points of East and West, being right to the Meridian, but in-ining to the Horizon, with an Angle equal to the Complement of the atitude. This is here represented by E A W.

5. A Vertical Plane inclining to the Horizon, parallel to any Great ircle, which passeth through the Points of East and West, being right the Meridian, but inclining to the Horizon, and yet not passing through e Pole, nor parallel to the Equinoctial. This is here represented either by I W, or E Y W, or E L W.



6. A Meridian Plane parrllel to the Meridian, the Circle of the Ho of 12, which passeth through the Zenith, the Pole, and Points of th South and North, being right to the Horizon, and the prime Vertica This is here represented by S Z N.

7. A Meridian Plane inclining to the Horizon, parallel to any Gre Circle, which passeth through the Points of South and North, being rig to the prime Vertical, but inclining to the Horizon. This is here represen ed by S G N.

8. A Vertical Declining Plane, parallel to any Great Circle, which pa seth through the Zenith, being right to the Horizon, but inclining to t Meridian. This is represented by B Z D.

9. A Polar Declining Plane, parallel to any Great Circle, which passeth through the Pole, being right to the Equinoctial, but inclining to the Meridian. This is here represented by H P Q.

10. A Declining Inclining Plane, parallel to any Great Circle, which is right to none of the former Circles, but declining from the prime Vertical, and inclining both to the Horizon and the Meridian, and all the Hour-circles. This may here be represented either by B M D, or B F D, or B K D, or any such Great Circle, which passeth neither through the South and North, nor East and West points, nor through the Zenith, nor the Pole.

Each of these Planes (except the Horizontal) hath two Faces whereon Hour-lines may be drawn, and so there are nineteen Planes in all. The Meridian Plane hath one Face to the East, and another to the West: The other Vertical Planes have one to the South, and another to the North, and the rest one to the Zenith, and another to the Nadir: but what is said of the one, may be understood of the other.

To describe the Fundamental Diagram.

The Description of this *Diagram* is set down at large in the Use of my *Sector*, *Chap.* 3. But for this purpose it may suffice, if it have the Vertical Circle, the Hour-circles, the Equator, and the Tropicks first drawn in it, other Circles may be supplied afterwards, as we shall have use of them: And those may be readily drawn in this manner.

Let the outward Circle representing the Horizon be drawn, and divided into four equal parts with S N the Meridian and E W the Vertical, and each fourth part into 90 *gr.* That done, lay a Ruler to the point S, and each Degree in the Quadrant E N, and note the Intersections where the Ruler crosseth the Vertical, so shall the Semidiameter E C be divided into other 90 *gr.* and from thence the other Semidiameters may be divided in the same sort. These may be numbred with 10, 20, 30, *&c.* from E toward C, and for variety with 10, 20, 30, *&c.* from C toward W. But for the Meridian, the South part would be best numbred according to the Declination from the Equator, and the North part according to the distance from the Pole.

Then with respect unto the Latitude, which here we suppose to be 51 *gr.* 30 *m.* open the Compasses unto 38 *gr.* 30 *m.* from C toward W, and prick them down in the Meridian from C unto P, so this point P shall represent the Pole of the World, and through it must be drawn all the Hour-circles,

Having three points E, P, W, find their Center, which will fall in th
Meridian a little without the point S, and draw them into a Circle E P W
which will be the Circle of the Hour of *6*.

Through this Center of the Hour of *6*, draw an occult Line at lengt
parallel to E W, so this Line shall contain the Centers of all the othe
Hour-circles. Where the Circle of the Hour of *6* crosseth this occul
Line, there will be the Centers of the Hour-circles of *9* and *3*. The di
stance between these Centers of *9* and *3*, will be equal to the Semidiame
ters of the Hour-circles of 10 and 2 : and where these two Circles of 1
and 2 shall cross this occult Line, there will be the Centers for the Hour
circles of 11 and *7*, and *5* and 1. Again, divide the distance between th
Centers of 10 and 2 into three equal parts, so the feet of the Compass
will rest in two points; the one is the Center of the Hour-circle of 8, an
the other the Center of the Hour-circle of 4 ; and the extent of the Com
passes to one of these third parts shall be the true Semidiameter of the
Circles, if there be no error committed in the finding of the other Cen
ters,

The Hour-circles being thus drawn, take 51 *gr.* 30 *m.* from C towar
W, and prick them down in the South part of the Meridian from C unt
A, and bring the third point E A W into a Circle, this Circle so draw
shall represent the Equator.

The Tropick of ♋ is 23 *gr.* 30 *m.* above the Equator, and 66 *gr.* 30 *m*
distant from the Pole, and so in this Latitude it will cross the South pa
of the Meridian at 28 *gr.* from the Zenith, and the North part of the Me
ridian at 15 *gr.* below the Horizon. Take therefore 28 *gr.* from C towar
W, and prick them down in the Meridian from C unto L, so have you th
South Intersection. Then lay the Ruler to the point E, and 15 *gr.* in th
Quadrant N E, numbred from N toward E, and note where it crosseth th
Meridian, so shall you have the North Intersection. The half way be
tween these two Intersections will fall in the Meridian at the point a a a
and the Circle drawn on the Center a, and Semidiameter a L, shall repre
sent the Tropick of ♋, and here cross the Horizon before 4 in the morn
ing, and after 8 in the evening, about 40 *gr.* Northward from E and W
according to the Rising and Setting of the Sun at his entrance into ♋.

The Tropick of ♑ is 23 *gr.* 30 *m.* below the Equator, and 113 *g*
30 *m.* distant from the North Pole, so that in this Latitude it crosseth th
South part of the Meridian at 75 *gr.* from the Zenith, and the North pa
of the Meridian at 62 *gr.* below the Horizon. Take therefore 75 *g*
from C toward W, and prick them down in the Meridian from C unto T

so have you the South Intersection; then lay the Ruler to the point E, and 62 *gr.* in the Quadrant N E numbred from N toward E, and note where it crosseth the Meridian, so shall you have the North Intersection. The half way between these two Intersections shall be the Center whereon you may describe the Tropick of ♑, and this Tropick will cross the Horizon after 8 in the Morning, and before 4 in the Evening, about 40 *gr.* Southward from E and W, according to the rising and setting of the Sun at his entrance into ♑.

To find the Inclination of any Plane.

For the distinguishing of these Planes, we may find whether they be Horizontal, or Vertical, or inclining to the Horizon, and how much they incline, either by the usual Inclinatory Quadrant, or by fitting a Thred and Plummet unto the Sector.

For let the Sector be opened to a Right Angle, the Lines of Sines to an Angle of 90 *gr.* inward edges of the Sector to 90 *gr.* and let a Thred and Plummet be hanged upon a Line parallel to the edges of one of the Legs, so that Leg shall be vertical and the other Leg parallel to the Horizon.

If the Plane seem to be vertical (like the Wall of an upright Building) you may try it by holding the Sector, so that the Thred may fall upon his Plummet-line: For then if the vertical edge of the Sector shall lie close to the Plane, the Plane is erect, and therefore said to be vertical; and if you draw a Line by that edge of the Sector, it shall be a Vertical Line.

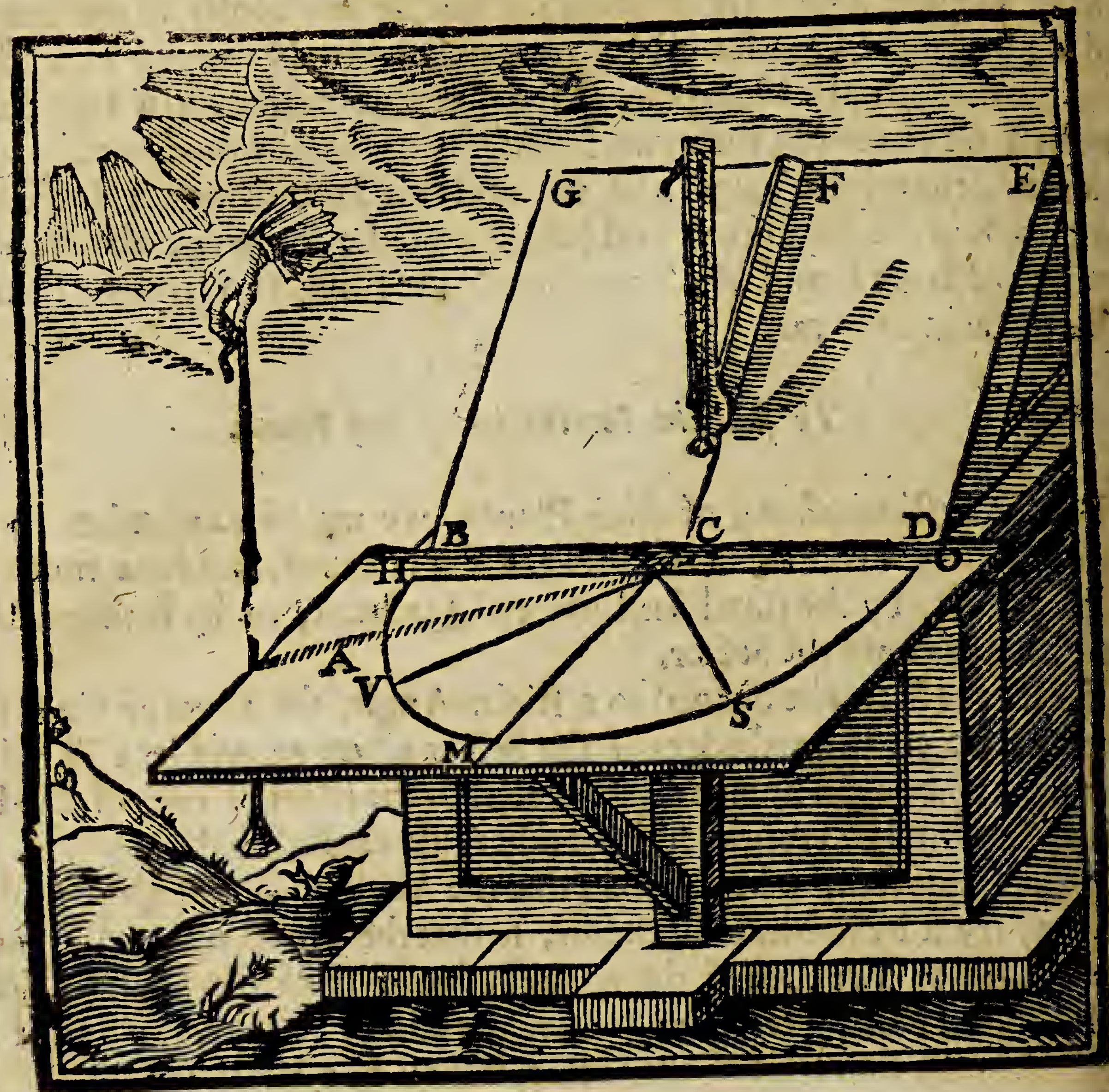
If the Plane seem to be level with the Horizon, you may try it by setting the Horizontal Leg of the Sector to the Plane, and holding the other Leg upright: For then if the Thred shall fall on his Plummet-line, which way soever you turn the Sector, it is an Horizontal Plane.

If the one end of the Plane be higher than the other, and yet not vertical, it is an inclining Plane, and you may find the Inclination in this manner.

First hold the Vertical Leg of the Sector upright, and turn the Horizontal Leg about, until it lie close with the Plane, and the Thred fall on his Plummet-line; so the Line drawn by the edge of that Horizontal Leg shall be an Horizontal Line.

Suppose the Plane to be B G E D, and that B D were thus found to be the Horizontal Line upon the Plane, then may you cross the Horizontal Line at Right Angles with a Perpendicular C F: that done, if you set one of the Legs of the Sector upon the Perpendicular Line C F, and

make



make the other Leg with a Thred and Plummet to become Vertical, y
shall have the Angle between the Vertical Line and the Perpendicular
the Plane, as before in the Use of the Sector, *pag.* 62. and the Comp
ment of this Angle is the Inclination of the Plane to the Horizon.

To find the Declination of a Plane.

The Declination of a Plane is always reckoned in the Horizon betw
the Line of East and West, and the Horizontal Line upon the Plane.
in the Fundamental Diagram, the prime Vertical Line (which is the L
of East and West) is E C W; if the Horizontal Line of the Plane p
posed shall be B C D, the Angle of Declination is E C B.

But because a Plane may decline divers ways, that we may the better
stinguish them, we consider three Lines belonging to every Plane:

rst is the Horizontal Line; the second, the Perpendicular Line, crossing the Horizontal at Right Angles; the third, the Axis of the Plane, crossing both the Horizontal Line, and his Perpendicular, and the Plane it self at Right Angles.

The Perpendicular Line doth help to find the Inclination of the Plane, as before; the Horizontal, to find the Declination; the Axis, to give denomination unto the Plane.

For example: In a Vertical Plane in the Fundamental Diagram, represented by F Z W, the Horizontal Line is E C W, the same with the Line of East and West, and therefore no Declination. The Perpendicular crossing it is C Z, the same with the Vertical Line, drawn from the Center to the Zenith, right unto the Horizon, and therefore no Inclination. The Axis of the Plane is S C N, the same with the Meridian Line, drawn from the South to the North, and accordingly gives the denomination to the Plane. For the Plane having two Faces, and the Axis two Poles, S and N, the Pole S falling directly into the South, doth cause that Face to which it is next, to be called the South Face; and the other Pole at N, pointing into the North, doth give the denomination to the other Face, and make it to be called the North Face of this Plane.

In like manner, in the Declining Inclining Plane in the Fundamental Diagram, represented by B F D, the Horizontal Line is B C D, which crosseth the prime Vertical Line E C W, and therefore it is called a Declining Plane, according to the Angle of Declination E C B or W C D. The Perpendicular to this Horizontal Line is C F, where the point F falleth in the Plane Q Z H perpendicular to the Plane proposed, between the Zenith and the North part of the Horizon; and therefore it is called a Plane inclining to the Northward, according to the Ark F Q, or the Angle F C Q. The Axis of the Plane is here represented by the Line C K, where the Pole K is *90 gr.* distant from the Plane, and so is as much above the Horizon at H, and the other Pole as much below the Horizon at Q, as the Plane at F is distant from the Zenith: And this Pole K here falling between the Meridian and the prime Vertical Circle into the Southwest part of the World, this upper Face of the Plane is therefore called the South-west Face, and the lower the North-east Face of the Plane.

The Declination from the prime Vertical may be found by the Needle in the usual Inclinatory Quadrant, or rather by comparing the Horizontal Line drawn upon the Plane, with the Azimuth of the Sun, and the Meridian Line, in such sort as before we found the Variation of the Magnetical

netical Needle. For take any Board that hath one ſide ſtraight, and draw as in the laſt Diagram the Line H O parallel to that ſide, and the Line Z M perpendicular unto it, and on the Center Z make a Semicircle H M O: this done, hold the Board to the Plane, ſo as H O may be parallel to B D the Horizontal Line on the Plane, and the Board parallel to the Horizon: then the Sun ſhining upon it, hold out a Thred and Plummet, ſo as the Thred being Vertical, the ſhadow of the Sun may fall on the Center Z; and draw the Line of Shadow A Z, repreſenting the common Section which the Azimuth of the Sun makes with the Plane of the Horizon, and let another take the Altitude of the Sun at the ſame inſtant: ſo by reſolving a Triangle, as I ſhewed before, you may find what Azimuth the Sun was in when he gave ſhadow upon A Z.

Suppoſe the Azimuth to be 72 *gr.* 52 *m.* from the North to the Weſtward, and therefore 17 *gr.* 8 *m.* from the Weſt, we may allow theſe 17 *gr.* 8 *m.* from A unto V, and draw the Line Z V, and ſo we have the true Weſt point of the prime Vertical Line: then allowing 90 *gr.* from V unto S, we have the South point of the Meridian Line Z S, and the Angle H Z V ſhall give the Declination of the Plane from the Vertical, and the Angle O Z S the Declination of the Plane from the Meridian.

Or we may take out only the Angle A Z H, which the Line of Shadow makes with the Horizontal Line of the Plane, and compare it with the Angle A Z V, which the Line of Shadow makes with the prime Vertical. And ſo here, if A Z V the Suns Azimuth ſhall be 17 *gr.* 8 *m.* paſt the Weſt, and yet the Line of Shadow A Z 7 *gr.* 12 *m.* ſhort of the Plane, the Declination of the Plane ſhall be 24 *gr.* 20 *m.* as may appear by the ſite of the Plane and the Circles.

If the Altitude of the Sun be taken at ſuch time as the Shadow of the Thread falleth on B D or H O, and then a Triangle reſolved, the Declination of the Plane will be ſuch as the Azimuth of the Sun from the Prime Vertical.

If at ſuch a time as the Shadow falleth on M Z, the Declination will be ſuch as the Azimuth of the Sun from the Meridian.

If it be a fair Summers day, you may firſt find what Altitude the Sun will have when he cometh to the due Eaſt or Weſt, and then expect until he come to that Altitude, ſo the Declination of the Plane ſhall be ſuch as the Angle contained between the Line H O and the Line of the Shadow.

Having diſtinguiſhed the Planes, the next care will be for the placing of the Style, and the drawing of the Hour-lines.

The Style will be as the Axis of the World, ſometimes parallel to the

Plane

Plane, ſometimes perpendicular, ſometimes cut the Plane with Oblique Angles.

The Hour-lines will be either parallel one to the other, or meet in a Center with equal Angles, or meet with unequal Angles. If the Style be perpendicular to the Plane, the Angles at the Center will be equal; and this falls out only in the South and North Face of the Equinoctial Plane. If the Style be parallel to the Plane, the Hour-lines will be alſo parallel one to another; and this falls out in all Polar Planes, as in the Eaſt and Weſt Meridian Planes parallel to the Circle of the Hour of 12, in the upper and lower direct Polars, parallel to the Circles of the Hour of 6, and in the upper and lower declining Polars, which are parallel to any of the other Hour-circles.

But in the Horizontal and all other Planes, the Style will cut the Plane with an acute Angle, and the Hour-lines will meet at the root of the Style, and there make unequal Angles.

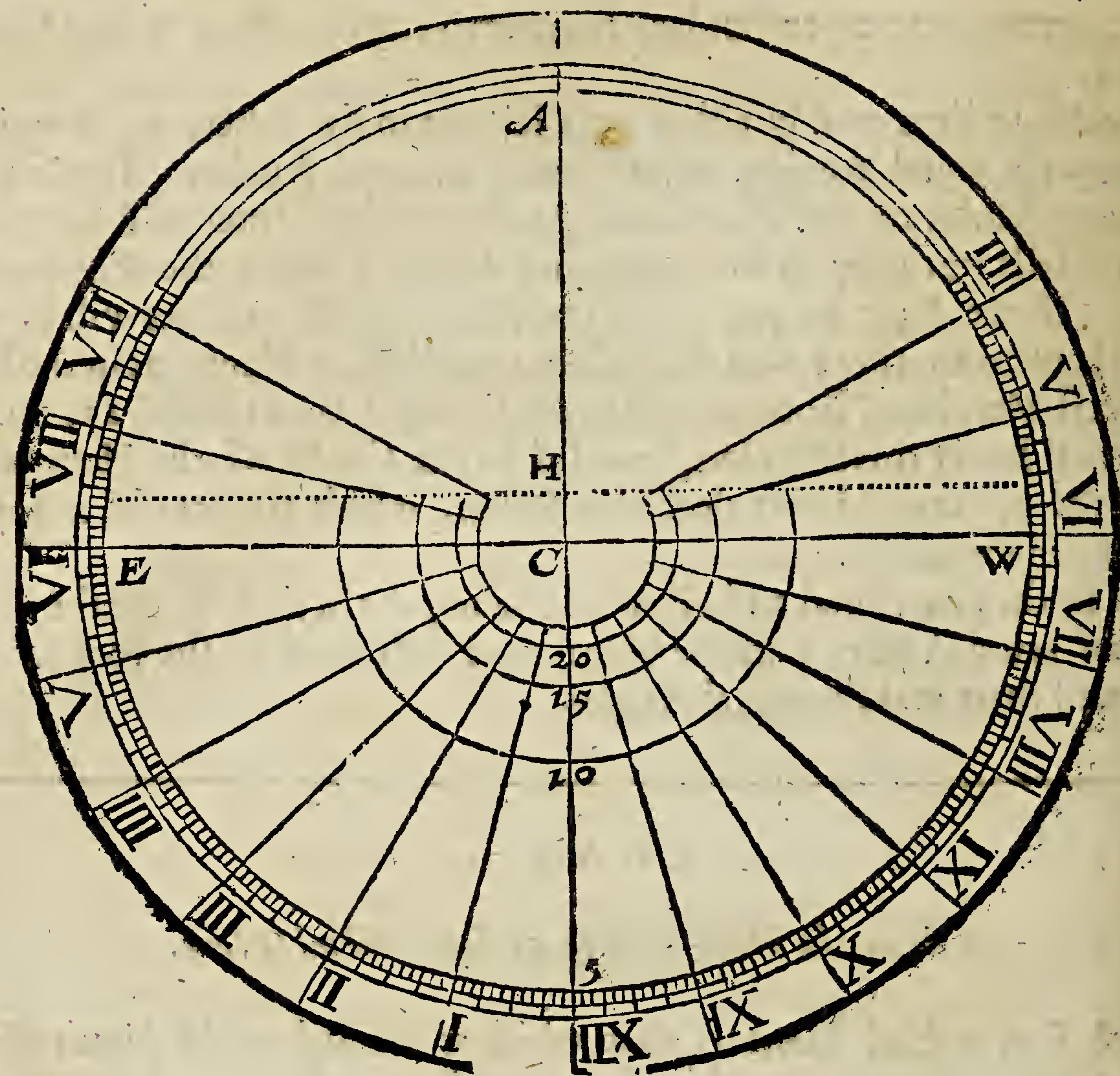
CHAP. I.

To draw the Hour-lines in an Equinoctial Plane.

AN Equinoctial Plane is that which is parallel to the Equinoctial Circle here repreſented by E A Y, wherein the ſpaces between the Hour-circles being equal, there is no need of further Precept, but only to draw a Circle, and to divide it into 24 equal parts from the 24 Hours, and ſubdivide each Hour into Halves and Quarters, and then to ſet up the Style perpendicular to the Plane in the Center of the Circle. The help which theſe Lines of Proportion do here afford us, is only in the diviſion of the Circle, which may be done readily by that which I ſhewed before in the Firſt Book of the *Sector*.

For Example: Suppoſe the Semidiameter of the Equinoctial Circle to be ſix Inches, and that it were required to know the diſtance of the Hour-points each from other; here each Hour being 15 *gr.* diſtant from other, I extend the Compaſſes from the Sine of 50 *gr.* unto the Sine of 7 *gr.* 30 *m.* the half of 15 *gr.* and I find the ſame extent to reach in the Line of Numbers from 6. 00 unto 1. 56.

Or in croſs work I extend them from the Sine of 30 *gr.* unto 6. 00 in the Line of Numbers, the ſame extent will reach from the Sine of 7 *gr.* 30 *m.* unto 1. 56 in the Line of Numbers; which ſhews that in a Circle



of six Inches semidiameter, the distance of the Hour-points each from other will be about 1 Inch and 56 *Centesms* or parts of 100. The like reason holds for the inscribing of all other Chords in the *Prop.* following.

CHAP. II.

To draw the Hour-lines in a Direct Polar Plane.

A Direct Polar Plane is that which is parallel to the Hour of 6, and here represented by E P W; wherein the Style will be parallel to the Plane, and the Hour-lines parallel one to the other; and therefore may be best drawn by that which I have shewed in the Use of the *Sector.* They may be also drawn by the help of these Lines of Proportion, in this manner.

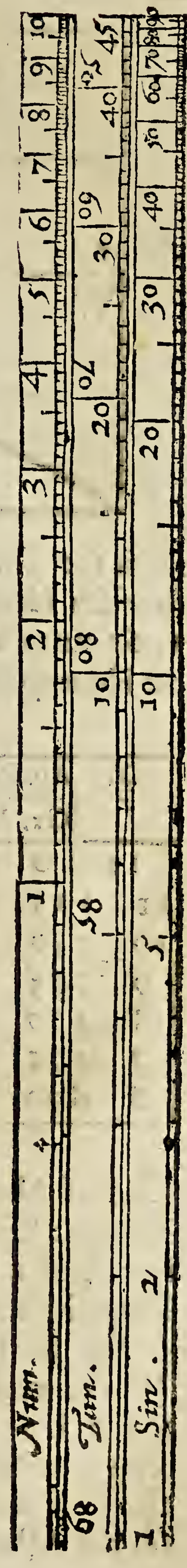
First draw a Right Line W E for the Horizon and the Equator, and cros

:rofs it at the Point C, about the middle of the Line, with C B another Right Line, which may ferve for :he Meridian and the Hour of 12, and muft alfo be the Subftylar Line wherein the Style fhall ftand. Then, to proportion the Style unto the Plane, confider the length of he Horizontal Line, and what Hour-lines you would ave to fall on your Plane.

For the diftance of any one Hour-line from the Meridi- n being known, we may find both the length of the Style, and the diftance of the reft: becaufe,

As the Tangent of the Hour given,
Is to the Diftance of the Meridian:
So the Tangent of 45 gr.
To the Height of the Style.

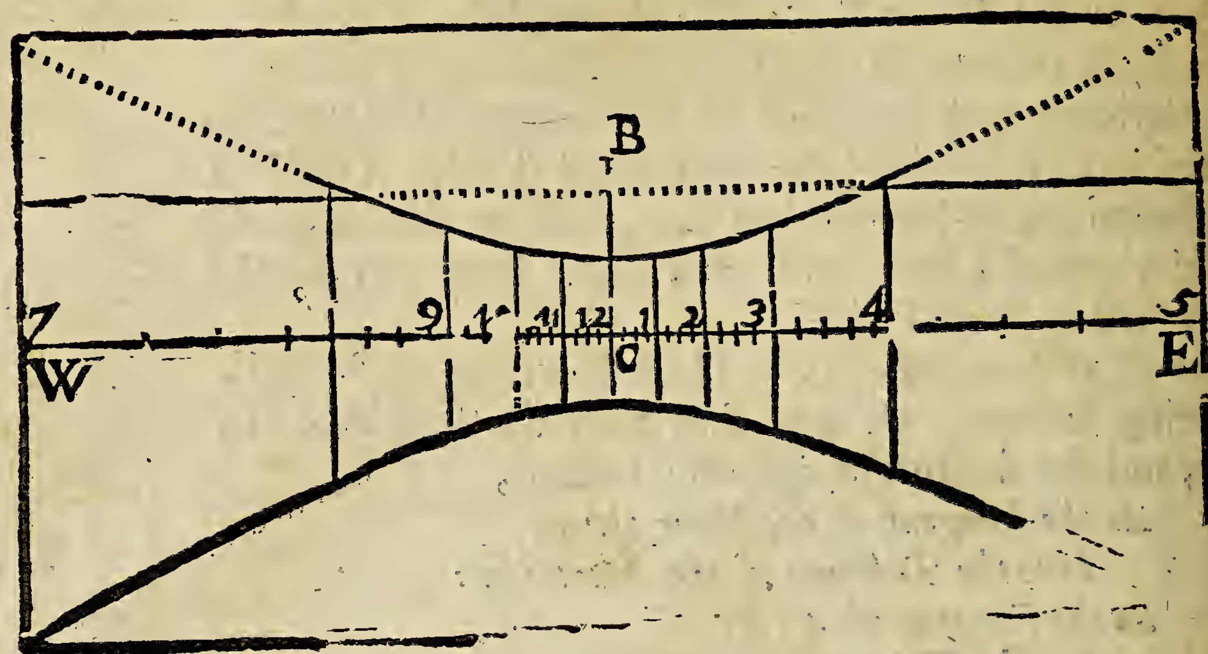
Suppofe the length of the Horizontal Line to be 12 In- :hes, and that it were required to put on all the Hour-lines rom 7 in the Morning unto 5 in the Evening. Here we ave 5 Hours and 6 Inches in either fide of the Meridian: Wherefore I allow 15 *gr.* for an Hour, and extending the Compaffes from the Tangent of 75 degrees, I find the ame extent to reach in the Line of Numbers from 600 to bout 161. This fhews both the height of the Style, and he diftance of the Hour-points of 9 and 3 from the Meri- lian, to be 1 Inch 61 parts.



To find the length of the Tangent between the Subftylar and the Hour-points.

As the Tangent of 45 gr.
To the Tangent of the Hour:
So the Height of the Style
To the length of the Tangent-line between the Sub- ftylar and the Hour points.

Thus having found the length of the Style in our Ex- mple to be 1.61, if I extend the Compaffes from the Tangent of 45 *gr.* unto the Tangent of 15 *gr.* the meafure f the firft Hour from the Subftylar, I fhall find the fame xtent to reach in the Line of Numbers from 1.61 unto . 43, for the length of the Tangent between the Subftylar nd the Hour-points of 11 and 1. If I extend them



from the Tangent of 45 *gr.* unto the Tangent of 75 *gr.* the meafure o the fifth Hour, I fhall find them to reach in the Line of Numbers from **1. 61** unto **6. 00**, for the length of the Tangent from the Subftylar t the Hour-points of 7 and 5. For howfoever it be the fame diftance in th Line of Tangents from 45 to 75, as from 4 unto 15; yet becaufe 75 are more, and 15 le than 45, the Tangent Lines that anfwer t them will be accordingly more or lefs than th length of the Style.

Ho.		An. Po.		Tang.	
		Gr.	M.	In.	Pa.
12		0	0	0	0
11	1	15	0	0	43
10	2	30	0	0	93
9	3	45	0	1	61
8	4	60	0	2	79
7	5	75	0	6	00
6	6	90	0	Infin.	

Again, If I extend them from 45 *gr.* in th Tangents unto 30 *gr.* the meafure of the fecon Hour, I fhall find them reach in the Line o Numbers from **1. 61** unto **0. 93** for the Hour o 10 and 2: If I extend them from the Tange of 45 *gr.* unto the Tangent of 60 *gr.* for t fourth Hour, I fhall find them to reach in t Line of Numbers from **1. 61** unto **2. 79**, and fuch is the length of t Tangent Line from the Subftylar unto the Hour of 8 and 4. And t like Reafon holdeth for the infcribing of all other Tangent Lines in t Propofitions following.

But for fuch Tangents as fall under 45 *gr.* I may better ufe cr Work, and extend the Compaffes from the Tangent of 45 *gr.* unto 1. in the Line of Numbers, fo fhall I find the fame extent to reach fro 30 *gr.* in the Tangents, to 93 parts in the Line of Numbers, for the d ftance of the fecond Hour; and from 15 *gr.* in the Tangents, to 43 pa for the diftance of the firft Hour from the Meridian.

Or if this extent from 45 *gr.* backward to 1. 61 be too large for the Compaſſes, I may extend them forward from the Tangent of 5 *gr.* 43 *m.* to 1. 61 parts in the Lines of Numbers, and the ſame extent ſhall reach from 15 *gr.* in the Tangents, to 43 parts in the Lines of Numbers, for the diſtance of the firſt Hour; and from 30 *gr.* to 93 parts, for the diſtance of the ſecond Hour, as before.

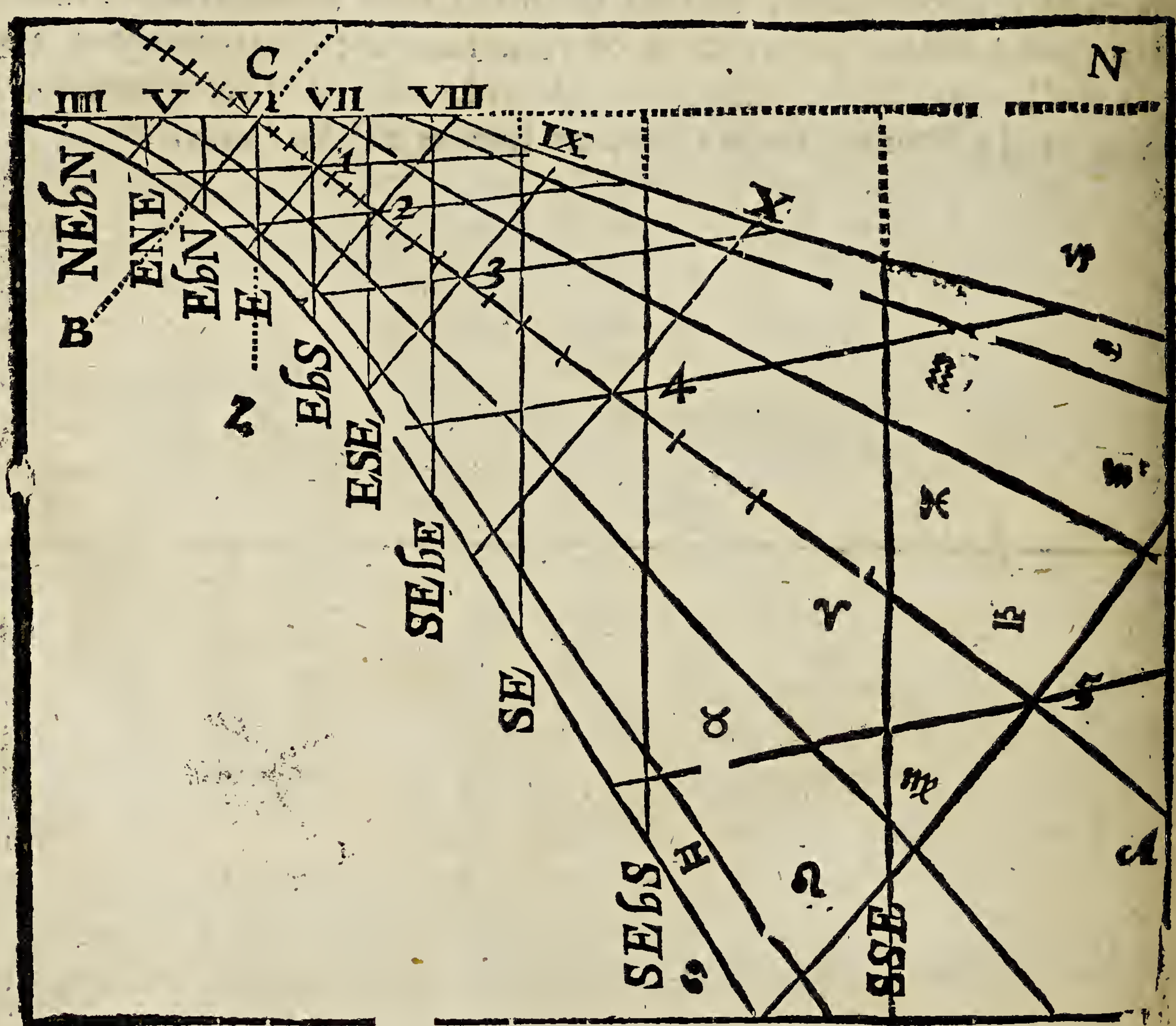
Having found the length of the Tangent Lines in Inches and parts of Inches, and pricked them in the Equator on both ſides of the Meridian, from the Center C; if we draw Right Lines through each of thoſe Points, croſſing the Equator at Right Angles, they ſhall be the Hour-lines required; and if we ſet a Style over the Meridian, ſo as the edge of it be parallel to the Plane, and the height of it be as much above the Meridian, as the diſtance between the Meridian and the Hour-points of 3 and 9, it ſhall repreſent the Axis of the World, and be truly placed for the caſting of the Shadow upon the Hour lines in a Polar Plane.

CHAP.

CHAP. III.

To draw the Hour-lines in a Meridian Plane.

A Meridian Plane is that which is parallel to the Meridian Circle in the Fundamental Diagram, repreſented by S Z N; it hath two Faces, one to the Eaſt, and the other to the Weſt; in each of them the Style will be parallel to the Plane, and the Hour-lines parallel one to the



other, as in a Polar Plane; the difference being only in the placing of the Equator, and in numbring of the Hours.

For in theſe Meridian Planes having drawn an occult Vertical Line C Z, and an occult Horizontal Line C N, croſſing one the other at Right Angles in the Point C, the Equator A C will cut the Vertical with an Angle

ngle Z C A, equal to the Latitude of the lace: then may we cross the Equator at Right ngles with the Line C B for the Hour of 6, nd from this set off the Hour-points in the quator, as in the former Prop.

For, supposing the length of the Style B to be 10 Inches, the length of the Tangent Line belonging to the first Hour will be *In.* 68 *p.* the length of the second 5 *In.* 77 *p.* in the Table. Then the Tangent of 15 *gr.* eing prickt down in the Equator on both des from 6, shall serve for the Hours of 5 nd 7, and the Tangent of 30 *gr.* for the ours of 4 and 8; and so in the rest.

This done, if we draw Right Lines through ach of these Points, crossing the Equator at ight Angles, they shall be the Hour-lines quired: And if we set a Style over the our of 6, so as the edge of it may be parallel to the Plane, and the height of it may e equal to the distance between the Hours 6 and 9 in the Equator, it shall represent the Axis of the World, and be truly aced for the casting of the Shadow upon e Hour-lines in a Meridian Plane.

Ho.	Ang. Po.		Tang.	
	Gr.	M.	In.	Pa.
12		0		0
	3	45	0	55
	7	30	1	32
	11	15	1	99
1	15	0	2	68
	18	45	3	39
	22	30	4	14
	26	15	4	93
2	30	0	5	77
	33	45	6	68
	37	30	7	67
	41	15	8	77
3	45	0	10	00
	48	45	12	40
	52	30	13	03
	56	15	14	97
4	60	0	17	32
	63	45	20	28
	67	30	24	14
	71	15	29	46
5	75	0	37	32
	78	45	50	27
	82	30	75	96
	86	15	152	57
6	90	0	Infinit.	

CHAP. IV.

To draw the Hour-lines in an Horizontal Plane.

AN Horizontal Plane is that which is parallel to the Horizon, represented in the Fundamental Diagram by the outward Circle S W N, in which the Diameter S N drawn from the South to the North, may go both for the Meridian Line and the Meridian Circle, Z r the Zenith, P for the Pole of the World, and the Circles drawn through

Latit.	51			30
Ho.	Ang. Po.		Arc. Pla.	
	Gr.	M.	Gr.	M.
12	0	0	0	0
	3	45	2	56
	7	30	5	52
	11	15	8	51
1	15	0	11	50
	18	45	14	52
	22	30	17	57
	26	15	21	6
2	30	0	24	20
	33	45	27	36
	37	30	31	0
	41	15	34	28
3	45	0	38	3
	48	45	41	45
	52	30	45	34
	56	15	49	30
4	60	0	53	35
	63	45	57	47
	67	30	62	6
	71	15	66	33
5	75	0	71	6
	78	45	75	45
	82	30	80	25
	86	15	85	13
6	90	0	90	0

through P for the Hour-circles of 1, 2, 3, 4, *&c.* as they are numbred from the Meridian.

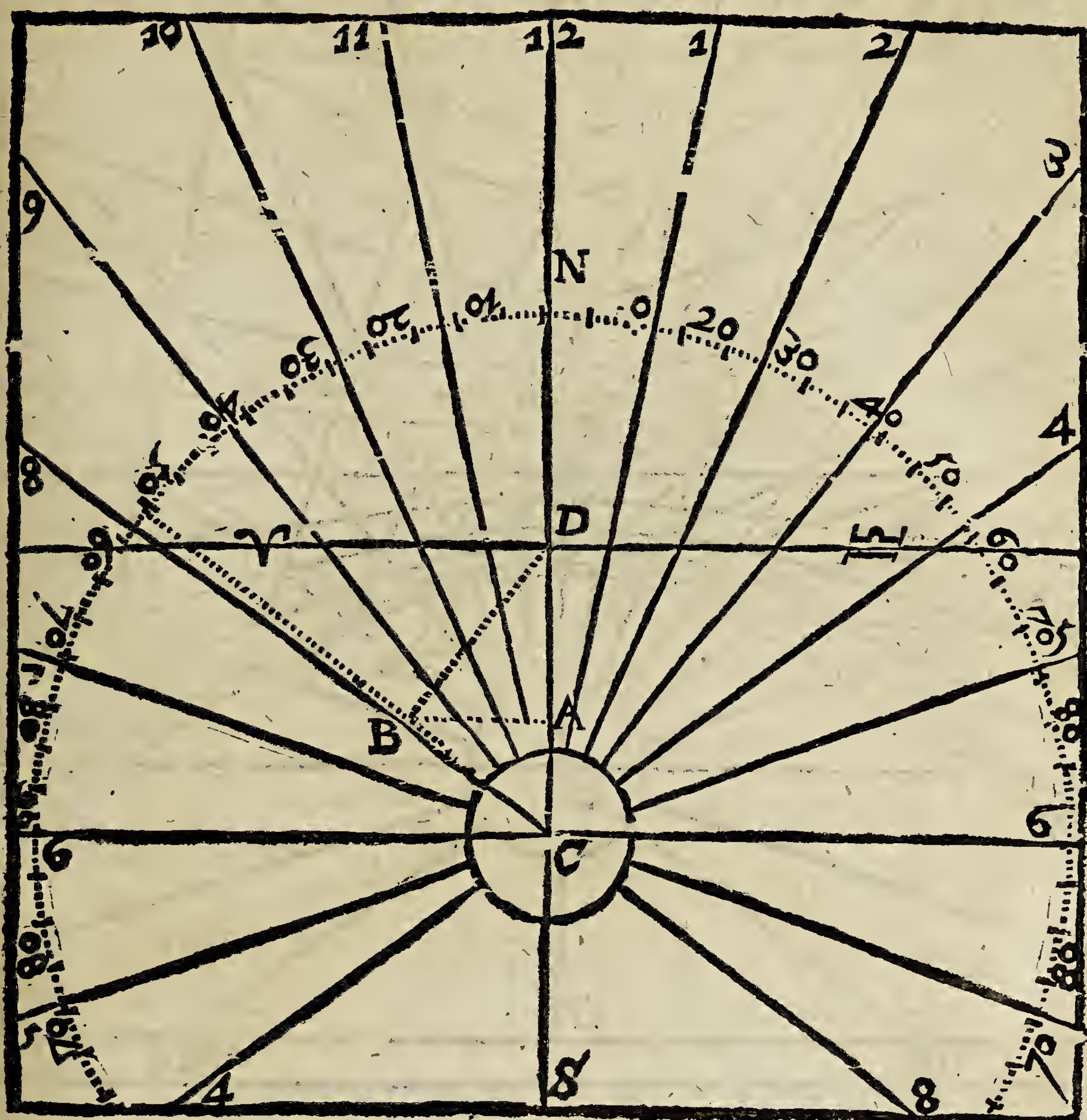
These are equal at the Pole, and at the Equator, but unequally distant at the Horizon; the distance between the Meridian and the first Hour being not full 12 *gr.* the distance between the fifth and sixth Hour about 18 *gr.* which inequality being observed, if you suppose Right Lines drawn from the Center C to the Intersections of these Hour-circles with the Horizon, the Line so drawn shall be the Hour-lines here inquired. And then, if you can, imagine a Line drawn from the Center C, toward P the Pole of the World, and raised above the Meridian Line C N, so as the Angle P C N may be equal to the Latitude of the Place, this Right Line C P shall be the Axis of the Style. And so you have both Style and Hour-lines ready drawn to your hand. But more particularly to our purpose.

These Hour-circles considered, with the Meridian and the Horizon, do make divers Triangles, P N 1, P N 2, P N 3, in which we have known, first, the Right Angle at N, the North Intersection of the Meridian and the Horizon; secondly, the Side P N, the Ark of the Meridian between the Pole and the Horizon, which is always equal to the Latitude of the Place; thirdly, the Angles at the Pole, made by the Meridian and the Hour-circles, the Angle N P 1 being 15 *gr.* N P 2 30 *gr.* each Hour 15 *gr.* more than other each half Hour 7 *gr.* 30 *m.* each quarter 3 *gr.* 45 *m.* as in the second Column of this Table. And these three being known, we may find the Arcs of the Horizon between the Meridian and the Hour-circles N 1, N 2, N 3 *&c.* For,

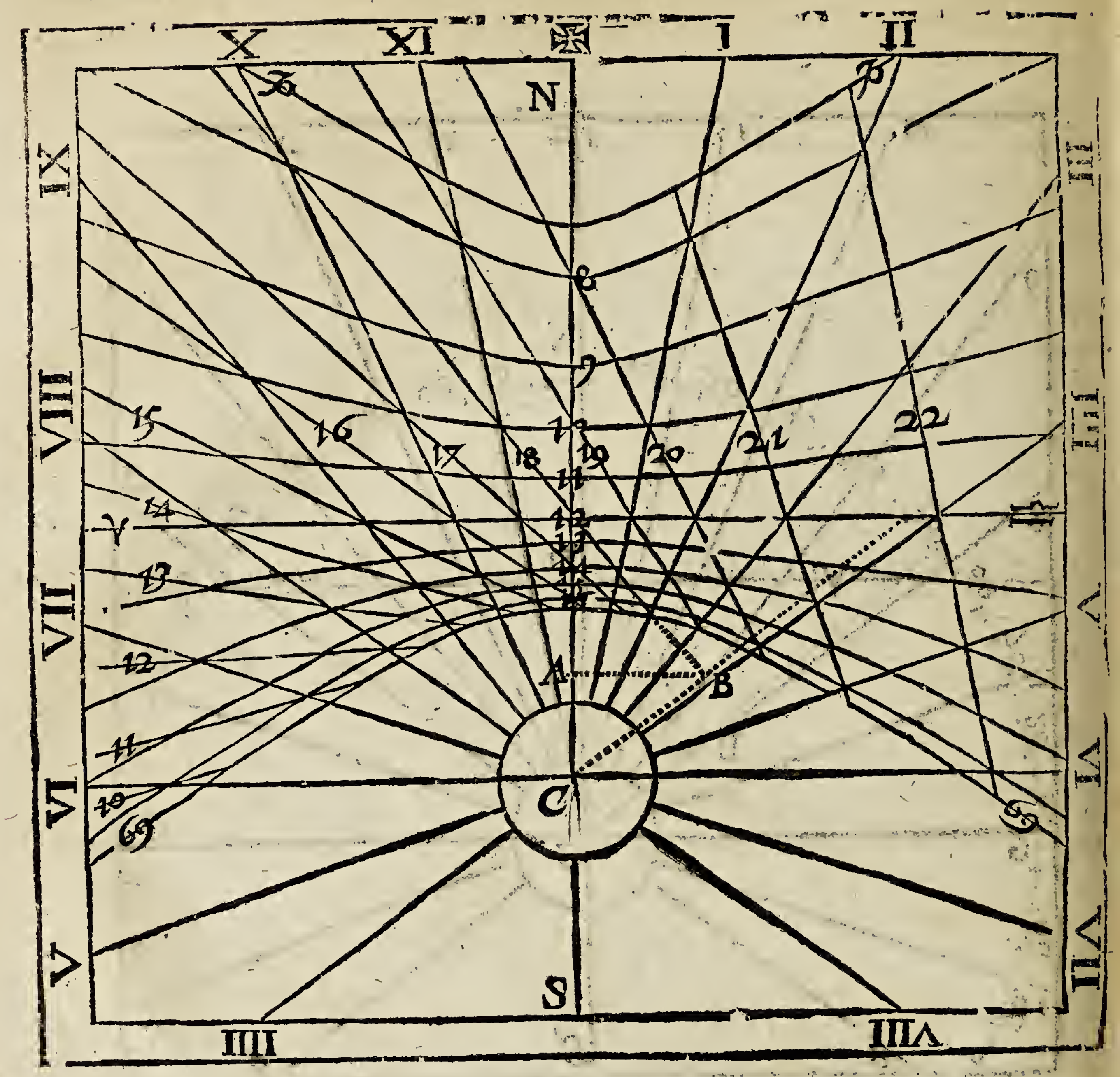
As the Sine of 90 gr.
is to the Sine of the Latitude:
So the Tangent of the Hour,
to the Tangent of the Hour-line from the Meridian.

Extend

Extend the Compasses from the Sine of *90 gr* to the Sine of the Lati-
ude; so the same Extent shall reach from the Tangent of the Hour, to



e Tangent of the Hour-line from the Meridian. Thus the Latitude be-
g 51 *gr.* 30 *m.* I extend the Compasses from the Sine of *90 gr.* to the
ne of 51 *gr.* 30 *m.* and find the same extent to reach from the Tangent
f 3 *gr.* 45 *m.* unto the Tangent of 2 *gr.* 56 *m.* for the distance of the
rst quarter from the Meridian; and from the Tangent of 7 *gr.* 30 *m.*
nto the Tangent of 5 *gr.* 52 *m.* for the half Hour; and from the Tan-
ent of 11 *gr.* 15 *m.* to the Tangent of 8 *gr.* 51 *m.* for the third quarter;
d from the Tangent of 15 *gr.* 0 *m.* unto 11 *gr.* 50 *m.* for the first
our: And so of the rest, as in the third Column of this Table, under
e Title of the *Arks of the Plane.*



Only when I come to set one Foot of the Compasses to 48 *gr.* 45 *m.* for the finding of a quarter past 3, the other Foot will fall out of the Line, and then I may either take out so much as is out of the Line beyond 45 *gr.* and turn it back into the Line, and it will reach from 45 *gr.* to 41 *gr.* 45 *m.* or I may use cross work, extending the Compasses from the Sine of 90 *gr.* to the Tangent of 48 *gr.* 45 *m.* so the same extent will reach from the Sine of 51 *gr.* 30 *m.* to the Tangent of 41 *gr.* 45 *m.* And such is the distance of the Line of 3 Hours $\frac{1}{4}$ from the Meridian.

This done, I come to the Plane, and there according as the Lines do fall in the Fundamental Diagram,

1. I draw the Right Line S N, serving for the Meridian, the Hour of 12, and the Substylar.

2 I

2. In this Meridian I make choice of a Center at C, and there defcribe an occultCircle reprefenting the Horizon.

3. I find a Chord of 11 *gr.* 50 *m.* and infcribe it into this Circle on either fide of the Meridian, for the Hours of 11 and 1; in like manner, a Chord of 24 *gr.* 20 *m.* for the Hours of 10 and 2; and a Chord of 38 *gr.* 3 *m.* for the Hours of 9 and 3: And fo for the reft of the Hours, their Halves, and Quarters.

4. I draw Right Lines through the Center, and the Terms of thefe Chords, and thefe Lines fo drawn are the Hour-lines required.

The Line belonging to the Hour of 6 will be perpendicular to the Meridian, and the Hour-lines before 6 in the Morning, or after 6 in the Evening, may be fupplied by continuing their oppofite Hour-lines beyond the Center; as the Hour-line of 7 in the Morning continued, will be the Hour-line of 7 in the Evening: And fo the reft.

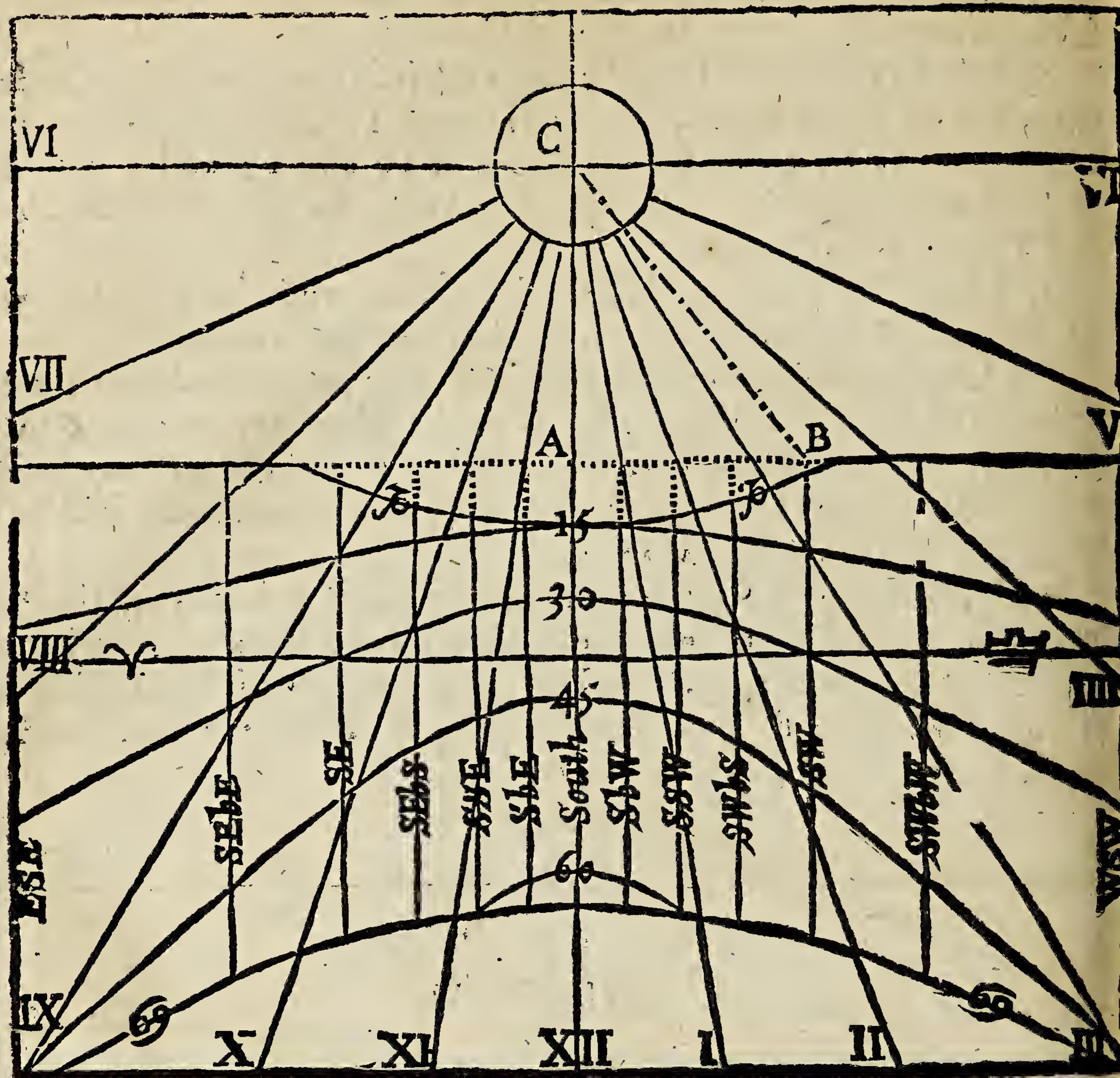
Laftly, I fet up the Style over the Meridian, fo as it may cut the Plane in the Center, and there make an Angle with the Meridian equal to the Latitude of the Place; fo it fhall reprefent the Axis of the World, and be truly placed for cafting of the Shadow upon the Hour-lines in an Horizontal Plane.

CHAP. V.

To draw the Hour-lines in a Vertical Plane.

A Vertical Plane is that which is parallel to the Prime Vertical Circle in the Fundamental Diagram, reprefented by E Z W. It hath two Faces, the one to the North, the other to the South; in each of them the Subftylar will be the fame with the MeridianLine, and the Angle of the Style above the Plane will be equal to Z P, the Complement of the Latitude; and the Hour-lines here inquired may be fupplied by imagining Right Lines drawn from the Center C to the Interfections of the Hour-circles E Z W.

The Triangles here confidered are made by the Vertical, the Meridian, and the Hour-circles, in which we know the Side Z P, the



Angles at the Pole, and the right Angle at the Zenith, and therefo[r]
may find the Arks of the Vertical, between the Meridian and th[e]
Hour-circles, after this manner.

As the Sine of 90 gr.
is to the Co-sine of the Latitude:
So the Tangent of the Hour,
to the Tangent of the Hour-line from the Meridian.

Extend the Compasses from the Sine of *90 gr.* to the Sine of th[e]
Complement of the Latitude, so the same extent shall reach from th[e]
Tangent of the Hour, to the Tangent of the Hour-line from the Meri-
dian.

Th[us]

Thus in the Latitude of 51 *gr.* 30 *m.* I extend the Compasses from the Sine of 90 *gr.* to the Sine of 38 *gr.* 30 *m.* and find the same extent to reach from the Tangent of 15 *gr.* to the Tangent of 9 *gr.* 28 *m.* for the distance of the first Hour from the Meridian; and from the Tangent of 75 *gr.* unto the Tangent of 66 *gr.* 42 *m.* for the fifth Hour: and so in the rest, as in this Table.

These Arks being known, I may come to the Plane, and then by help of a Thread and Plummet draw a Vertical Line, serving both for the Meridian and the Hour of 12, and the Substylar; then may I draw an occult Vertical Circle, and therein inscribe the Chords of those former Arks, and draw the Hour-lines, and set up the Style, as before in the Horizontal Plane.

If it be the South Face of the Plane, the Center will be upward, and the Style will point downward: If the North Face, the Center must be in the lower part of the Meridian Line, and the Style point upward in all such Places as are to the Northward of the Equinoctial Line, as it may appear by considering how the Lines do fall in the Fundamental Diagram.

Latit.	51			30
Ho.	Ang. Po.		Arc. Pla.	
	Gr.	M.	Gr.	M.
12	0	0	0	0
	3	45	2	20
	7	30	4	41
	11	15	7	3
1	15	0	9	28
	18	45	11	56
	22	30	14	27
	26	15	17	4
2	30	0	19	45
	33	45	22	35
	37	30	25	32
	41	15	28	38
3	45	0	31	54
	48	45	35	22
	52	30	39	3
	56	15	42	58
4	60	0	47	9
	63	45	51	36
	67	30	56	20
	71	15	61	23
5	75	0	66	42
	78	45	72	17
	82	30	78	3
	86	15	84	0
6	90	0	90	0

CHAP. VI.

To draw the Hour-lines in a Vertical Inclining Plane.

All those Planes that have their Horizontal Line lying East and West, are in that respect said to be Vertical; if they be also upright and pass through the Zenith, they are direct Verticals; if they incline to the Pole, they are direct Polars; if to the Equinoctial, they are

are properly called Equinoctial Planes, and are described before: if t
none of these three Points, they are then called by the general name c
Inclining Verticals.

These may incline either to the North parts of the Horizon, or to th
South; and each of them hath two Faces, one to the Zenith, the othe
to the Nadir, in which we are first to consider the height of the Po
above the Plane, by comparing the Inclination of the Plane to the Hori
zon with the Latitude of the Place.

As in our Latitude of 51 *gr.* 30 *m.* if the declination of the Plan
E I W in the Fundamental Diagram shall be 13 *gr.* Northward, that i
if I N, the Ark of the Meridian between the Plane and the North pa
of the Horizon, shall be 13 *gr.* we may take these 13 *gr.* out of P N 51 *g*
30 *m.* the Elevation of the Pole above the Horizon, and there will remai
P I 38 *gr.* 30 *m.* for the Elevation of the North Pole above the upper Fa
of the Plane, and therefore 38 *gr.* 30 *m.* for the height of the South Po
above the lower Face of the Plane.

Or if the Inclination of the Plane shall be found to be 62 *gr.* to th
Southward, we may number them in the Meridian from S the South pa
of the Horizon unto L, and there draw the Ark E L W representing th
Plane; so the Ark of the Meridian P L shall give the height of the Nor
Pole above the upper Face of this Plane to be 66 *gr.* 30 *m.* and therefo
the height of the South Pole above the lower Face of the Plane is al
66 *gr.* 30 *m.*

In like manner, if the Inclination of the Plane E Y W shall be 15 *g*
Southward, that is, if S Y the Ark of the Meridian between the Sou
part of the Horizon and the Plane shall be 15 *gr.* the height of the Nor
Pole above the upper Face of the Plane, and the height of the South Po
above the lower Face of the Plane, will be also found to be 66 *gr.* 30 *m*

But if the Plane shall fall between the Zenith and the North Po
then will the North Pole be elevated above the lower Face, and the Sou
Pole above the upward Face of the Plane, as may appear by the Project
on of the Sphere in the Fundamental Diagram.

Then in the Triangles made by the Plane, the Meridian, and t
Hour-circles, we have the side which is the height of the Pole abo
the Plane, together with the Angles at the Pole, and the Right A
gle at the Intersection of the Meridian with the Plane, by which
may find the Arks of the Plane between the Meridian and the Hor
circles, after this manner.

e

As the Sine of 90 gr.
Is to the Sine of the Pole above the Plane:
So the Tangent of the Hour,
To the Tangent of the Hour-line from the Meridian.

Thus in the former Example, where P I the height of the Pole ove the Plane was found to be 38 *gr.* 30 *m.* if you ſhall extend e Compaſſes from the Sine of 90 *gr.* to the Sine of 38 *gr.* 30 *m.* e ſame extent will reach from the Tangent of 15 *gr.* unto the angent of 9 *gr.* 28 *m.* for the diſtance of the firſt Hour from the eridian, and from 30 *gr.* unto 19 *gr.* 46 *m.* for the ſecond Hour, and forward, as in the direct Vertical.

And for the two laſt Examples, you may extend the Compaſſes from e Sine of 90 *gr.* unto the Sine of 66 *gr.* 30 *m.* ſo the ſame extent ſhall ach in the Line of Tangents from 15 *gr.* unto 13 *gr.* 48 *m.* for the firſt our, from 75 *gr.* unto 73 *gr.* 43 *m.* for the fifth Hour, from 30 *gr.* nto 27 *gr.* 54 *m.* for the ſecond Hour, from 60 *gr.* unto 57 *gr.* 48 *m.* r the fourth Hour, and from 45 *gr.* unto 42 *gr.* 31 *m.* for the third our from the Meridian.

Theſe Arks being known, you may firſt draw the Horizontal Line, nd croſs it in the middle with a Perpendicular, that may ſerve both r the Meridian and the Hour of 12, and the Subſtylar; then know- g which Pole is elevated above the Plane, you may accordingly ake choice of a fit Point in the Meridian for the Center of your Hour- nes, and thence deſcribe an occult Ark of a Circle, inſcribe the Chords f thoſe former Arks, and draw the Hour-lines, and ſet up the Style, as ſhewed before in the Horizontal Plane.

CHAP.

CHAP. VII.

To draw the Hour-lines in a Vertical Declining Plane.

ALl upright Planes whereon a man may draw a Vertical Line, are in this respect said to be Vertical; if they shall also stand directly East and West, they are directly Verticals; if directly North and South, they are properly called Meridian Planes, and are described before: if they behold none of these four principal Parts of the World, but shall stand between the prime Vertical and the Meridian, they are then called by the general name of Declining Verticals.

These have two Faces, one to the South, the other to the Northward, which may be distinguished in these Northern parts of the World after this manner. If the Sun coming to the Meridian shall shine upon the Plane, it is the South Face; if not, it is the North Face of that Plane. Again, If the Sun shall shine upon the Plane at High-noon, and yet longer in the Forenoon than in the Afternoon, it is the South-east Face; if longer in the Afternoon than in the Forenoon, it is the South-west Face of the Plane. But how much the Declination cometh to, is best found as before,

When the Declination is found, there be four things more to be considered, before we can come to the drawing of the Hour-lines.

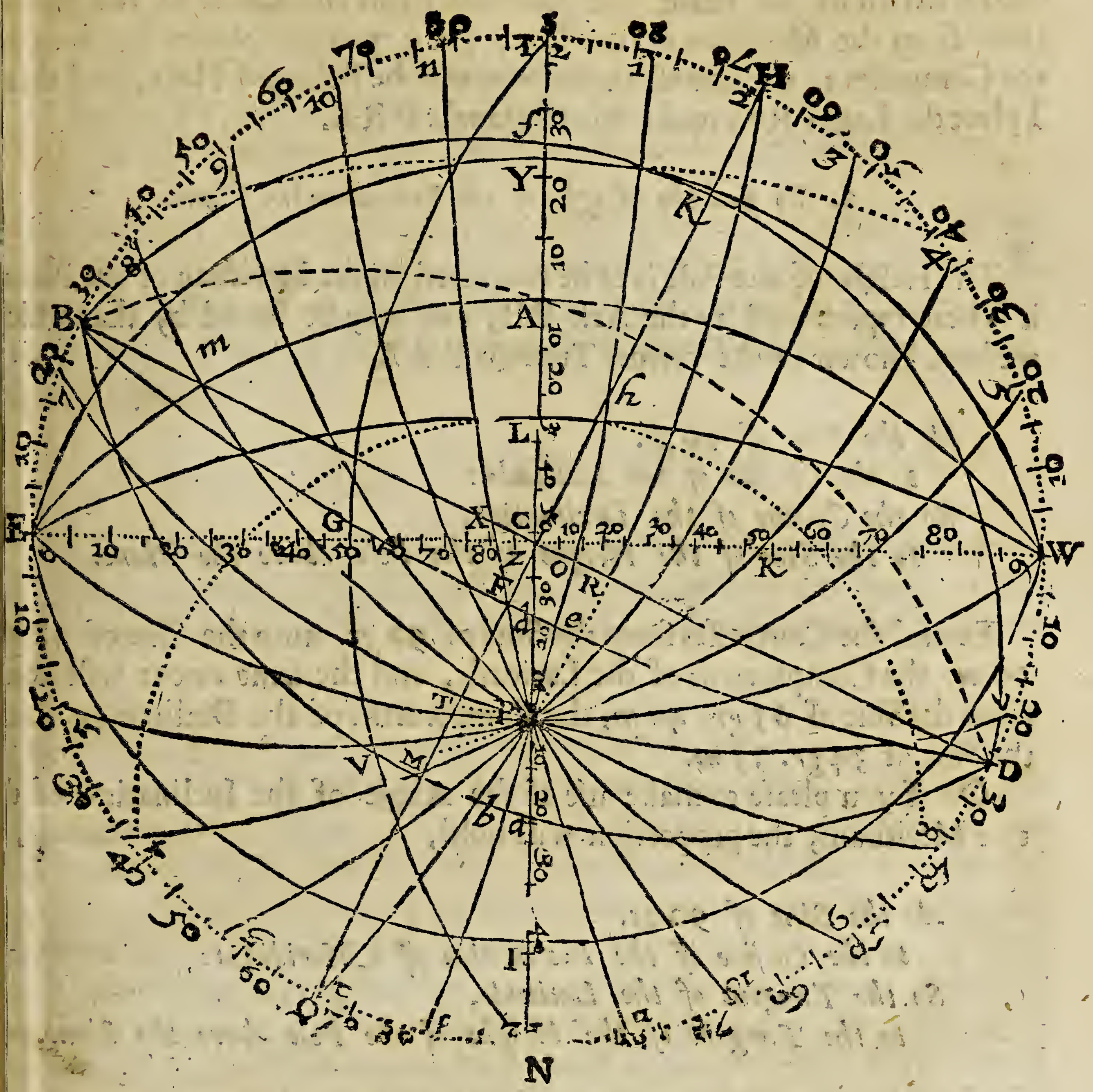
1. The Meridian of the Plane, and his Inclination to the Meridian of the Place.
2. The Height of the Pole above the Plane.
3. The Distance of the Substylar from the Meridian Line.
4. The Distance of each Hour-line from the Substylar.

And these four may all be represented in the Fundamental Diagram, as in this Example.

Suppose that in our Latitude of 51 *gr.* 30 *m.* Northward, the Declination of an upright Plane should be found to be 24 *gr.* 20 *m.*

In the Triangle P R Z we know the Angle at R to be a Right Angle, and the Angle at Z, for it is the Complement of the Declination; and the Base P Z, for it is the Complement of the Latitude. And these three being known, we may find the other Angle R P Z, which is the Angle of Inclination between both Meridians.

A



As the Sine of the Latitude.
Is to the Sine of 90 gr.
So the Tangent of the Declination
to the Tangent of Inclination of Meridians.

Thus in our former Example I extend the Compasses from the Sine of e Latitude 51 *gr.* 30 *m.* unto the Sine of 90 *gr.* the same extent will ach in the Line of Tangents from 24 *gr.* 20 *m.* the Declination given, to out 30 *gr.* and such is Z P R, the Angle of Inclination between the eridian of the Place and the Meridian of the Plane: and thererefore

 the

the Meridian of the Plane will here fall upon the Circle of the second Hour from the Meridian of the Place (as it may also appear by opening the Compasses to the nearest extent between the Pole and Plane) and there I place the Letter R to make this Rectangle P R Z.

2. *To find the Height of the Pole above the Plane.*

The Height of the Pole is to be measured in the Meridian of the Plane; it is here represented by the Ark P R, and may be found by that which we have known in the former Triangle P R Z.

As the Sine of 90 gr.
to the Co-sine of the Latitude:
So the Co-sine of the Declination,
to the Sine of the Height of the Pole above the Plane.

Extend the Compasses from the Sine of *90 gr.* unto the Sine of 38 *gr.* 30 *m.* the Complement of the Latitude, and the same extent will reach from the Sine of *65 gr.* 40 *m.* the Complement of the Declination, unto the Sine of *34 gr.* 33 *m.*

Or if you please to make use of the Angle of the Inclination of the two Meridians, the proportion will hold,

As the Sine of 90 gr.
to the Co-sine of the Inclination of Meridians:
So the Tangent of the Latitude,
to the Tangent of the Height of the Pole above the Plane.

And then you may extend the Compasses from the Sine of *90 gr.* unto the Sine of *60 gr.* the Complement of the Inclination of the Meridians, and the same extent will reach from the Tangent of 38 *gr.* 30 *m.* the Complement of the Latitude, unto the Tangent of *34 gr.* 33 *m.* and such is the Ark P R the Height of the Pole above the Plane.

3. *To find the Distance of the Substylar from the Meridian.*

This is here represented by the Ark Z R, and may be found by that which we have known in the former Triangle P R Z.

As the Sine of 90 gr.
to the Sine of the Declination :
So the Co-tangent of the Latitude,
to the Tangent of the Subſtylar from the Meridian.

Extend the Compaſſes from the Sine of 90 *gr.* unto the Sine of 24 *gr.* 20 *m.* the Declination given, and the ſame extent will reach from the Tangent of 38 *gr.* 30 *m.* the Complement of the Latitude, unto the Tangent of 18 *gr.* 8 *m.* and ſuch is the Ark Z R, the Diſtance of the Subſtylar from the Meridian.

4. *To find the Diſtance of each Hour-line from the Subſtylar.*

The Diſtances of the Hour-lines from the Subſtylar are here repreſented by thoſe Arks of the Declining Vertical belonging to the Plane, which are intercepted between the proper Meridian of the Plane and the Hour-circles.

To this purpoſe we have divers Triangles made by the Declining Plane, together with his proper Meridian and the Hour circles. In theſe we have known, firſt the Right Angle at the Interſection of the proper Meridian with the Plane; then the Side which is the Height of the Pole above the Plane; and thirdly, the Angles at the Pole. For knowing the Angle of Inclination between the Meridian of the Plane and the Meridian of the Place, which is always the Hour of 12, we may find the Angle between the Meridian of the Plane and the Hour of 1, by allowing in 15 *gr.* and the Angle between the Meridian of the Plane and the Hour of 2, by allowing in 30 *gr.* and ſo for the reſt: which being known, and ſet down in a Table, we may find the Arks of the Plane from the Subſtylar to the Hour-circles, in this manner.

As the Sine of 90 gr.
to the Sine of the Height of the Pole above the Plane :
So the Tangent of the Hour from the proper Meridian,
to the Tangent of the Hour-line from the Subſtylar.

Thus in our Latitude of 51 *gr.* 30 *m.* if the Declination of an upright Plane ſhall be found to be 24 *gr.* 20 *m.* from the prime Vertical, the one Face open to the South-weſt, the other to the North-eaſt, I may number these 24 *gr.* 20 *m.* in the Horizon of the Fundamental Diagram from E unto B, according to the situation of the Plane, and there draw the Vertical B Z D, which shall represent the Plane proposed.

The two Poles of this Plane will fall in the Horizon at H and Q, and therefore the proper Meridian drawn through the Poles of the Plane and the Pole of the World must be the Circle H P Q, which here crosseth the Plane at Right Angles in the Point of R, and inclineth to P Z S the Meridian of the Place, according to the Angle R P Z.

The quantity of this Inclination may be readily found by the Hour-circle where the proper Meridian falleth. As here it falleth on the second Hour-circle, and so the Inclination is 30 *gr.*

The height of the Pole above the Plane, which giveth the height of the Style above the Substylar, is here represented by the Ark P R. For as in the Horizontal, so in this and all other Planes, the Line C P the Axis of the World is always the Axis of the Style, and the nearest Line that can be drawn upon the Plane to the Axis of the World is the fittest for the Substylar, and that is the Line C R: so the Angle P C R is the Angle between the Axis and the Plane, commonly called the Height of the Style, and the measure of this Angle is the Ark P R. This Ark is always less than the Complement of the Latitude, and may be estimated by taking the distance P R with the Compasses, and measuring it in the Meridian from P toward Z. So in this Example it will appear to be about 34 *gr.* $\frac{1}{2}$.

The distance of the Substylar from the Meridian is here represented by the Ark Z R: For the Meridian Line upon the Plane is C Z, the Substylar Line is C R; so the Angle contained between them is Z C R, and the measure of this Angle is the Ark Z R, which taken with the Compasses, and measured in the Semidiameter C W, from C toward W, will be found about 18 *gr.*

The distances of each Hour-line from the Substylar are here represented by the Arks of the Plane between the Point R and the Intersections of the Hour-circles: For the Substylar Line is C R, and the Hour-circle of 1 crossing the Plane in the Point O, the Hour-line of 1 upon the Plane must be C O; so the Angle between the Substylar and the Hour-line of 1 is R C O, and the measure of this Angle is the Ark R O. In like manner, the Hour-line of 12 will be C Z, and the distance from the Substylar R Z: the Hour-line of 11 will be C X, and the distance from the Substylar R X: and so the rest. These distances R O, R Z, R X, *&c.* may also be taken with the Compasses, and measured as before.

Besides

Besides these four Representations, the Diagram will shew what Pole is elevated above the Plane, and what time the Sun shineth upon the Plane. If it be the North-east Face of this Plane, you may think P to be the North Pole, and the Hour-circles to be drawn on a Convex Hemisphere; so C R the Substylar, and C P the Axis of the Style, will will both point upward: and having drawn the Tropick of ♋, you shall find by the meeting of the Plane with the Tropick, and the Hour-circles, that the Sun at the highest may shine upon the Plane from the time of the rising until it be past 9 in the morning, and from 7 in the evening unto the time of his setting. But if it be the South-west Face of the Plane, then you may either suppose the Substylar and the Axis to be continued down below the Center, like unto the Hours before and after 6 in an Horizontal Plane; or else you may turn the Diagram, and think P to be the South Pole, and the Hour-circles to be drawn in an Horizontal Concave, so C R the Substylar, C P the Axis of the Style, will both point downward, and so also the Hour-lines from 8 in the morning until after 7 in the evening, as it doth appear by the meeting of the Plane with the Horizon, and the Hour-circles.

Thus with the drawing of one Line in the Diagram, to represent the Plane according to his declination, you may have the Hour-lines fitted to any Declining Vertical, with the Style and Substylar in their due place, which may suffice to free you from gross errour; but for more exactness, we consider three Triangles.

1. *To find the Inclination of Meridians.*

The Meridian of the Place is a Circle passing through the Poles of the World, the Zenith and the Nadir. The proper Meridian of the Plane is a Circle passing through the Poles of the World and the Poles of the Plane. The Circle of the Plane and these two Meridians do make a Triangle, such as P R Z, wherein we know the Angle at R.

I consider the Angle of Inclination of the Meridian R P Z, and there see how that P Z, the Meridian of the Place, which is the Hour of 12, being 30 *gr.* distant from P R the Meridian of the Plane, and that one Face of the Plane being open to the South-west, and the other to the North-east, this Meridian of the Plane falleth to be the same with the Hour of 2, (otherwise with the Hour of 10:) therefore allowing 15 *gr.* for an Hour, the Hour of 1 R P O will be 15 *gr.* and R P X the Hour of 11 will be 45 *gr.* distant from P R the proper Meridian of the Plane:

And

Latitude N.		51	30		
Declinat.		24	20		
Diff. Merid.		30	0		
Alt. Styl.		34	33		
Dist. Subst.		18	8		
Hours.		Ang. Po.		Ar. Pla.	
M.	E	Gr.	M.	Gr.	M.
4	8	90	00	90	0
5	7	75	00	64	42
6	6	60	00	44	30
7	5	45	00	29	33
8	4	30	00	18	8
9	3	15	00	8	38
10	2	Merid.		Subftyl.	
11	1	15	00	8	38
12		30	00	18	8
1	11	45	00	29	33
2	10	60	00	44	30
3	9	75	00	64	42
4	8	90	00	90	0

And so I gather the Inclination of the reft of the Hour-circles towards this Meridian, according to their Angles at the Pole, as in the fecond Column of this Table.

Then taking my Compaffes in my hand, I extend them from the Sine of 90 *gr.* unto the Sine of 34 *gr.* 33 *m.* the height of the Pole above the Plane, and find them to reach in the Line of Tangents from 15 *gr.* the Inclination of the Hour of 1, to 8 *gr.* 38 *m.* for the Ark of 1 from the Subftylar, and from 30 *gr.* unto 18 *gr.* 8 *m.* for the Hour of 12, agreeable to the third Prop. and from 45 *gr.* unto 29 *gr.* 33 *m.* for the Hour of 11, and fo the reft, which I alfo fet down in the third Column of the Table.

Thefe Arks being thus found, will ferve for the drawing of the Hour-lines both on the South-weft Face and the North-eaft Face of this Plane, and alfo on either Face of the like Plane that hath the fame Declination, and the Poles in the South-eaft and North-weft.

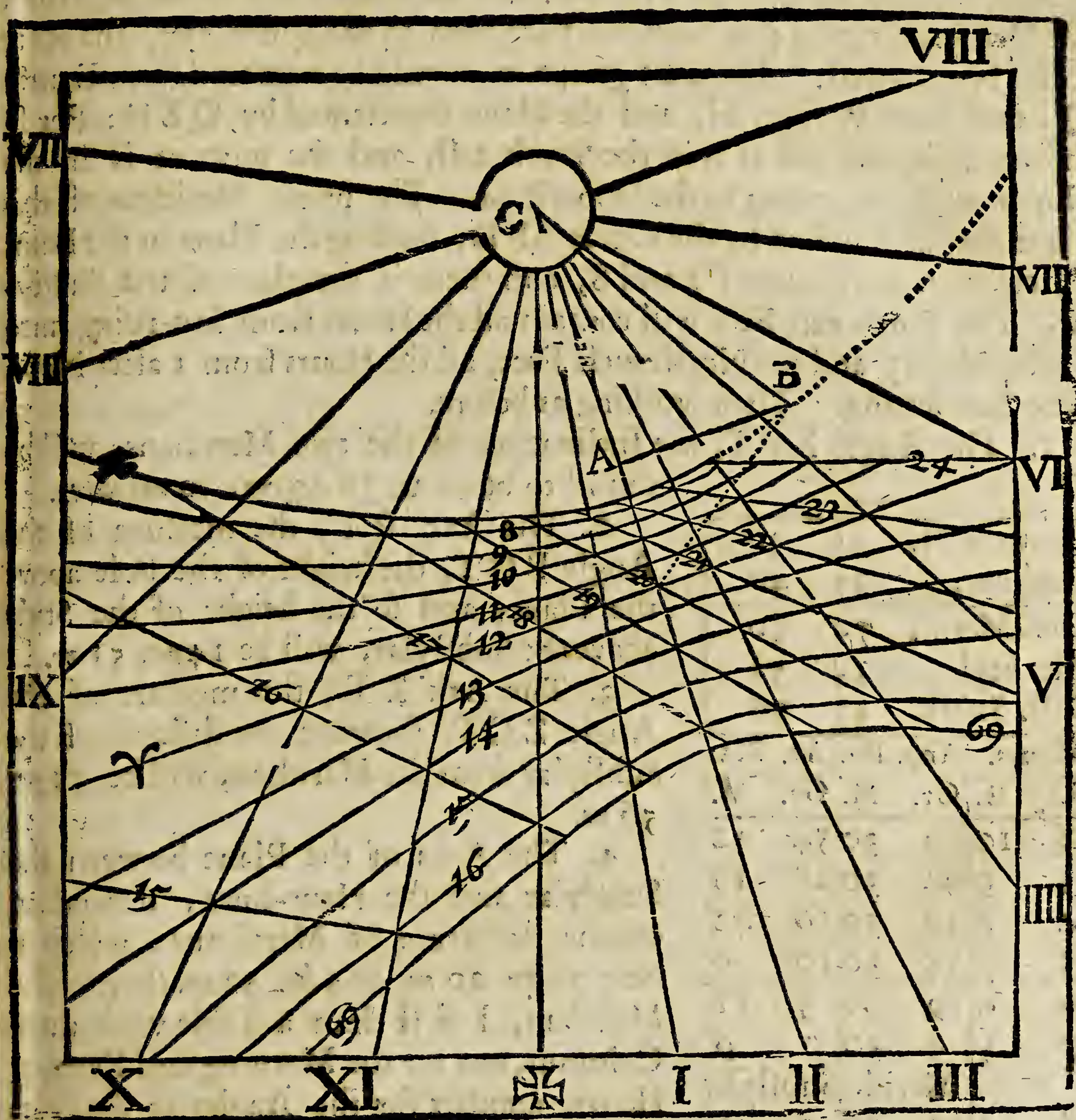
1. By the help of a Thred and Plummet I draw a Vertical Line, ferving both for the Meridian of the Place, and the Hour of 12.

2. In this Meridian Line I make choice of a Center at C, in the upper part of the Line if it be the South Face, as here we fuppofe it, that the Style may have room to point downward: but in the lower part of the Line if it be the North Face of the Plane, for there the Style muft point upward: and upon this Center I defcribe an occult Circle, reprefenting the Declining Vertical belonging to the Plane.

3. I find a Chord of 18 *gr.* 8 *m.* the diftance of the Subftylar from the Meridian of the Place, and infcribe it into this Circle, from the Meridian unto A toward the right hand, becaufe in this Example the Meridian of the Plane falls among the Hours after Noon, (for otherwife it muft have been infcribed toward the left hand) and there I draw the Line C A ferving for the Subftylar.

4. According to the Table of the Arks of the Plane from the Subfty-

lar, I find a Chord of 8 *gr.* 38 *m.* and inſcribe it into this Circle, from the Subſtylar toward the Meridian for the Hour of 1. In like manner a Chord of 29 *gr.* 23 *m.* for the Hour of 11, and a Chord of 44 *gr.* 30 *m.* for the Hour of 10; and ſo for the reſt of the Hours, their Halfs, and Quarters.



5. I draw Right Lines through the Center and the Terms of theſe Chords, and theſe Lines ſo drawn are the Hour-lines required.

Laſtly, I ſet up the Style over the Subſtylar, ſo as it may cut the Plane in the Center, and there make an Angle with the Subſtylar of 34 *gr.* 33 *m.* according to the height of the Pole above the Plane; ſo it ſhall repreſent

represent the Axis of the World, and be truly placed for casting of the Shadow upon the Hour-lines in this Declining Plane.

A second Example.

Suppose another upright Plane in the same Latitude to decline from the Vertical 65 *gr.* 44 *m.* with one Face open to the South-east, the other to the North-west. These 65 *gr.* 40 *m.* would be numbred from E unto Q, and from W unto H, and the Plane represented by QZH: For so the one Pole will fall at B in the South-east, and the other at D in the North-west, according to the supposition. The proper Meridian of this Plane may be supplied by the Circle B P D, crossing the Plane in the Point T, between the Hours of 7 and 8, and there is the place of the Substylar. The South-east Face will contain all the Hours from Sun-rising unto 2 after Noon; and the North-west Face, all the Hours from 1 after Noon unto Sun-setting. Then working as before,

1. The Angle Z P T, the Inclination of the two Meridians, will be found to be about 70 *degrees* 30 *minutes*.

Latitude N.	51	30
Declinat.	35	40
Diff. Merid.	70	30
Alt. Styl.	14	51
Dist. Subst.	35	56

Hours.		Ang. Po.		Ar. Pla.	
M.	E.	Gr.	M.	Gr.	M.
2	10	79	30	54	12
3	9	64	30	28	16
4	8	49	30	16	42
5	7	34	30	10	0
6	6	19	30	5	11
7	5	4	30	1	9
		Merid.		Substyl.	
8	4	10	30	2	43
9	3	25	30	6	58
10	2	40	30	12	21
11	1	55	30	20	28
12		70	30	35	56
1	11	85	30	72	56

2. The Ark P T, the measure of the Angle P C T, the height of the Pole above the Plane, and so the height of the Style above the Substylar, will be 14 *gr.* 51 *m.*

3. The Ark Z T, the measure of the Angle Z C T, shewing the distance of the Substylar from the Meridian, will be 35 *gr.* 56 *m.*

4. The Arks of the Plane between the Substylar and the Hour-lines, depending on the difference of Meridians, which is here 70 *gr.* 30 *m.* or 4 *ho.* 42 *m.* short of the Meridian, I first draw a Table with three Columns, one for the Morning and Evening Hours, another for the Angles at the Pole, and the third for the Arks of the Plane, and there write 70 *gr.* 30 *m.* by the Hour of 12, and place the Meridian and Substylar between the Hours of 7 and 8, according as the Poles of the Plane do fall in the Diagram.

Then

Then will the Angle at the Pole between the proper Meridian and the Hour of 11 be 55 *gr.* 30 *m.* the Hour of 10 will be 40 *gr.* 30 *m.* distant from that Meridian; and the rest in their order: which being noted in the second Column, the Ark of the Plane will be found to be such Numbers as I have noted in the third Column.

With this Table thus made you may draw the Hour-lines, and set up the Style on either Face of this or the like Plane, the difference being only in the placing of the Substylar, and that is resolved by the sight of the Diagram.

A third Example, of a Plane falling near the Meridian.

After the like manner, if in our Latitude an upright Plane shall decline 85 *gr.* for the prime Vertical, the one Face of it being open to the North-west, and the other to the South-east, we may in some sort represent it by the Vertical Q Z H, and then working as before,

1. The Angle Z P T, the Inclination of the two Meridians, will be found to be 86 *gr.* 5 *m.* so that P T the Meridian of this Plane will here fall between the Hour-circles of 6 and 7 from the Meridian.

2. The Ark P T, the measure of the Angle P C T, the height of the Pole above the Plane, will be onely 3 *gr.* 6 *m.*

3. The Ark Z T, the measure of the Angle Z C T, the distance of the Substylar from the Meridian, 38 *gr.* 23 *m.*

Latitude	51	30
Declination	85	0
Diff. Merid.	86	5
Altitude Styl.	3	6
Dist. Substyl.	38	23

4. The Table of the Angles at the Pole will be also gathered, by comparing the Meridian of the Plane with the rest of the Hour-circles: For the Angle T P Z, between T P the Meridian of the Plane, P Z the Meridian of the Place, and the Hour of 12, being 86 *gr.* 5 *m.* allowing 15 *gr.* for an Hour, the Hour of 11 ½ will be 78 *gr.* 35 *m.* and the Hour of 11, 71 *gr.* 5 *m.* distant from the Meridian of the Plane; and so the rest of the Hours. Or because the difference of Meridians 86 *gr.* 5 *m.* resolved into Time, makes 5 *ho.* 44 *m.* and so the Meridian of the Plane falls between the Hours of 6 and 7 from the Meridian. I first place this Meridian between these Hours, and then taking 75 *gr.* the common measure for 5 Hours, out of 86 *gr.* 5 *m.* there remains 11 *gr.* 5 *m.* for the Angle at the Pole between the Meridian of the Plane and the Hour of 7. Again, I take 86 *gr.* 5 *m.* out of 90 *gr.* the common measure of 6 Hours, and

Hours	An. Po.		Ar. Pla.		C I		C G	
	Gr.	M.	Gr.	M.	In.	Par.	In.	Par.
12	86	5	38	23	91	08	79	21
	78	35	15	3	30	92	26	89
11	71	5	9	6	18	42	16	02
	63	35	6	13	12	52	10	89
10	56	5	4	36	9	25	8	05
9	41	5	2	42	5	43	4	72
8	26	5	1	31	3	05	2	65
7	11	5	0	30	1	20	1	05
	Merid.		Subſtyl.		0	0	0	0
6	3	55	0	13	0	44	0	38
5	18	55	1	4	2	15	1	86
4	33	55	2	5	4	18	3	64
3	48	55	3	33	7	13	6	20
2	63	55	6	20	12	77	11	10
	71	25	9	10	18	56	16	14
1	78	55	15	28	31	82	27	67
	86	25	40	55	99	67	86	68

there remains 3 *gr.* 55 *m.* for the Angle at the Pole between the Meridian of the Plane and the Hour of 6. To theſe Angles ſo found, I allow 15 *gr.* for every Hour, as in the ſecond Column of this Table.

Then having the height of the Pole above the Plane, and theſe Angles at the Pole, the Arks of the Plane between the Subſtylar and the Hour-circle will be found as in the third Column.

Theſe Arks being found, will ſerve for the drawing of the Hour-lines on either Face o this or the like Plane.

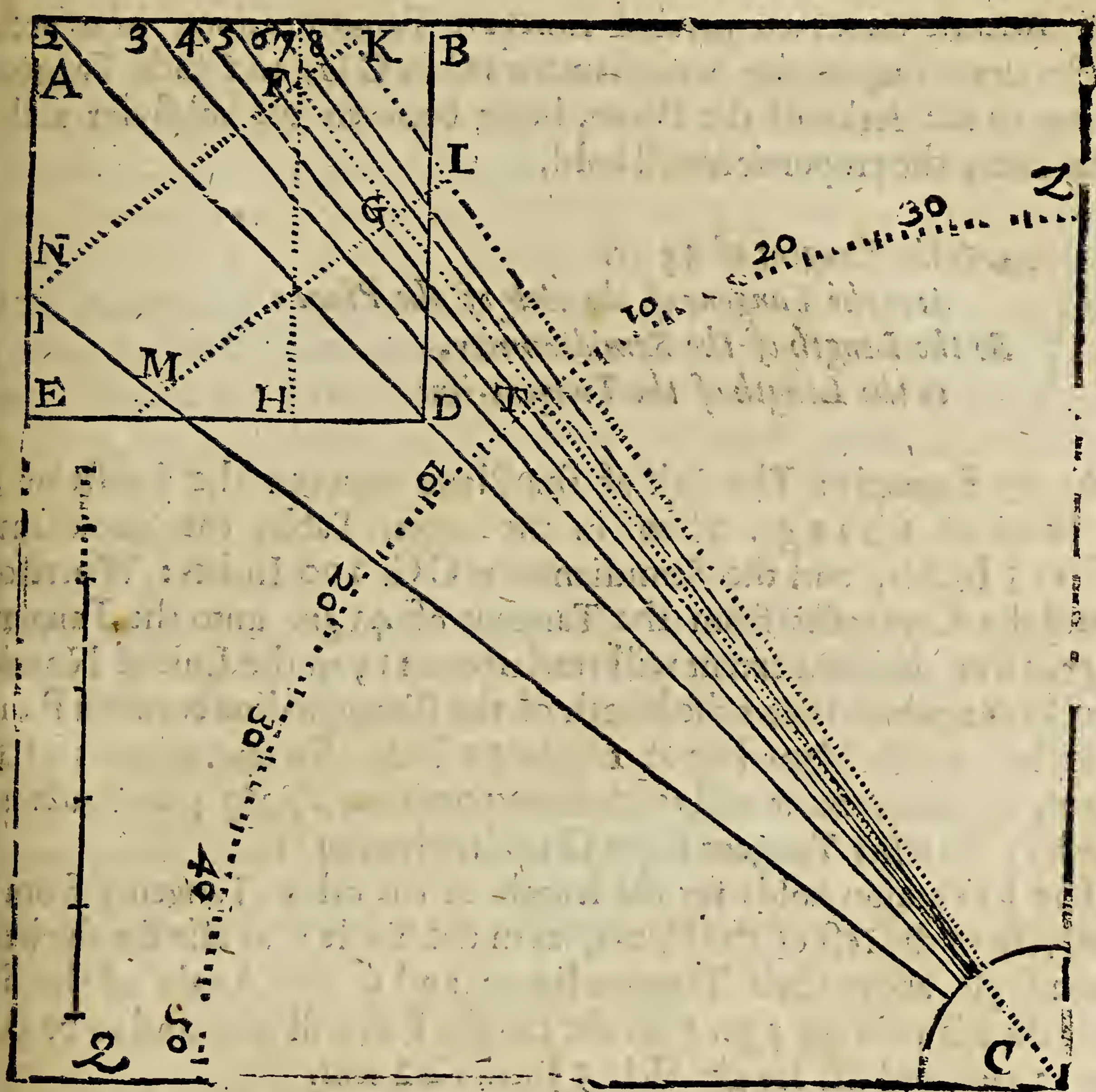
1. By the help of a Thre and Plummet I draw Z C Vertical Line, ſerving bot for the Meridian of the place, and the Hour of 12.

2. In this Meridian Line I make choice of a Center in the upper pa of the Line, if it had been the Southern Face of the Plane; but here i C the lower part of the Line, becauſe we ſuppoſed it to be the North weſt Face of the Plane, and the Style muſt point upward: and upon th Center I deſcribe an occult Circle, repreſenting the Declining Vertic belonging to this Plane.

3. I find a Chord of 38 *gr.* 23 *m.* the diſtance of the Subſtylar fro the Meridian of the Place, and inſcribe it into this Circle, from Z in t Meridian, unto T toward the left hand, according as the proper Meridi P T falls in the Fundamental Diagram; and here I draw the Line C ſerving for the Subſtylar.

4. The Subſtylar being drawn, I may inſcribe the Chords of the Ar of the Plane from the Subſtylar, and draw the Hour-lines, and ſet up t Style, as in the former Plane.

Or the Arks of the Plane from the Subſtylar being found as before, may draw the Hour-line, upon the Plane otherwiſe than by Chords: F having drawn the Hour-lines as in the laſt Figure, upon Paper or Paſt boa board, we ſhall find the moſt part of them, in this and ſuch like Planes that have greater Declination, to fall ſo cloſe together, that they can hardly be diſcerned; wherefore to draw them at large to the beſt advantage of the Plane, I leave out the Center, and draw them by Tangents, as in the Polar Plane.



1. I conſider the length and breadth of the Plane whereon I am to draw the Hour-lines, which I ſuppoſe to be a Square whoſe Side is 36 Inches, and find that the little Square A B D E will contain both the Subſtylar and all thoſe Hour-lines which are required in the great Square A Z C Q.

2. I draw two parallel Lines, F N, G M, croſſing the Subſtylar at Right Angles in the points F and G, as they may beſt croſs all the Hour-lines, and yet the one be diſtant from the other as far as the Plane will give me leave;

 and

and I find by the sight of the Figure, that if A B the Side of the lesser Square shall be 36 Inches, the Line C F will be about 115 Inches, and the Line C G about 100 Inches, and therefore F G 15 Inches. Again, that the Point F will fall about 6 Inches below the upper Horizontal Side A B, and about 12 Inches from the next Vertical Side B D; for I need not here stand upon Parts.

3. Because these two parallel Lines are Tangent Lines, in respect of Circles drawn upon the Semidiameters C F, C G, and such Tangent as belongs to the Arks of the Plane, being between the Substylar and the Hour-lines, the proportion will hold,

As the Tangent of 45 gr.
is to the Tangent of the Ark of the Plane:
So the Length of the Semidiameter,
to the Length of the Tangent-line.

As for Example: The Ark of the Plane between the Substylar and the Hour of 1 is 15 *gr.* 28 *m.* in the former Table, the Semidiameter C F 115 Inches, and the Semidiameter C G 100 Inches: Wherefore I extend the Compasses from the Tangent of 45 *gr.* unto the Tangent of 15 *gr.* 28 *m.* the same extent will reach from 115 in the Line of Numbers unto 31,82, which shews the length of the Tangent-line between F in the Substylar and the Hour-line of 1 to be 31 Inches 82 *cent.* or parts of 100. Again, the same extent will reach from 100 unto 27, 67; and such is the length of the lesser Tangent from G to the Hour of 1.

The like reason holds for the length of the other Tangents from the Substylar to the rest of the Hours, as in the Table; as also for the height of the Style above these Tangent-lines: and so the Angle of the Style above the Plane being 3 *gr.* 6 *m.* the Height F K will be found to be 6 Inches 23 *cent.* and the Height G L 5 Inches 42 *cent.*

Where the Reader may observe, that if the extent from the Tangent of 45 *gr.* to the Tangent of 3 *gr.* 6 *m.* or to 115 in the Line of Numbers be too large for his Compasses, he may use the Tangent of 5 *gr.* 43 *m.* instead of the Tangent of 45 *gr.* as I noted before.

4. Having found these Lengths and Heights, and set them down in a Table, I come to the Plane here resembled by the lesser Square A B D E, where I begin with an occult Vertical F H, about 12 Inches from the Side B D, and upon the Center F, about 6 Inches below the Side A B, describe an occult Ark of a Circle,

5. Into

5. Into this Ark I firſt inſcribe a Chord of 38 *gr.* 23 *m.* the diſtance of the Subſtylar from the Meridian, to make the Angle H F G equal to the Angle Z C T; ſo the Line F G ſhall be the Subſtylar: and then another Chord of 51 *gr.* 37 *m.* the Complement of this Diſtance, to make up the Right Angle G F N; ſo the Line F N ſhall be the greater of the two Tangent-lines before-mentioned.

6. I ſet off 15 Inches from F unto G toward the Center, and through G draw the leſſer Tangent-line G M, parallel to the former.

7. Theſe two occult Tangent-lines being thus drawn, I look into the former Table for the Hour of 1, and there find the Ark of the Plane between the Subſtylar and the Hour of 1 to be 15 *gr.* 28 *m.* and the length belonging to it in the greater Tangent-line to be 31 Inches 82 *cent.* in the leſſer Tangent-line 27 Inches 67 *cent.* wherefore I take out 31 Inches 82 parts, and prick them down in the greater Tangent from F to N, and then 27 Inches 67 Parts, and prick them down in the leſſer Tangent from G to M, and draw the Line MN for the Hour of 1, which if it were produced, would croſs the Subſtylar F G in the Center C, and there make the Angle F C N 15 *gr.* 28 *m.* The like Reaſon holdeth for the drawing of all the reſt of the Hour-lines.

Laſtly, I ſet up the Style right over the Subſtylar, ſo as the Height F K may be 6 Inches 23 *cent.* and the Height G L 5 Inches 42 *cent.* then ſhall K L repreſent the Axis of the World, and if it were produced, would croſs the Subſtylar F G in the Center C, and there make the Angle F C K to be 3 *gr.* 6 *m.* and ſo be truly placed for caſting of the Shadow upon the Hour-lines in this Declining Plane.

CHAP. VIII.

To draw the Hour-lines in a Meridian Inclining Plane.

All thoſe Planes wherein the Horizontal Line is the ſame with the Meridian Line are therefore called Meridian Planes: if they be right to the Horizon, they are called by the general name of Meridian Planes, without farther addition, and are deſcribed before: if they lean to the Horizon, they are then called Meridian Incliners.

Theſe may incline either to the Eaſt part of the Horizon, or to the Weſt, and each of them hath two Faces, the upper towards the Zenith, the lower towards the Nadir, wherein knowing the Latitude of the

Place,

Place, and the Inclination of the Plane to the Horizon, we are to consider,

1. The Inclination of the Meridian of the Plane to the Meridian of the Place.
2. The Height of the Pole above the Plane.
3. The Distance of the Substylar from the Meridian.
4. The Distance of each Hour-line from the Substylar.

And all these four are represented in the Fundamental Diagram, as in this Example.

In our Latitude of 51 *gr.* 30 *m.* a Meridian Plane inclineth Eastward 50 *gr.* these 50 *gr.* I number in the Vertical Circle from E unto G, according to the Inclination of the Plane, and there draw the Ark S G N representing the Plane proposed. Again, I number 50 from Z unto K, so the Point K (being 90 *gr.* from the Plane at G) shall be the Pole of this Plane, and the proper Meridian of this Plane may be supplied by a Circle drawn through K and P. This Meridian doth here fall between the Hours of 4 and 5, and crossing the Plane at Right Angles in the Point V, in the Right Line C V shall be the Substylar, and the Angle P C V the height of the Style above the Plane, and Right Lines drawn from the Center C to the Intersections of the Hour-circles with S G N shall be the Hour-lines here inquired. The lower Face of the Plane will contain all the Hour-lines from Sun-rising unto 11 in the Morning, and the upper Face the Hours from 9 in the Morning unto Sun-setting. Then have I a Rectangle Triangle P V N, wherein the Base P N is the Height of the Pole above the North part of the Horizon, and the Angle P N V the Complement of the Inclination to the Horizon: And these being known,

1. I may find the Angle N P V of Inclination of the two Meridians: For,

As the Cosine of the Latitude,
is to the Sine of 90 gr.
So the Tangent of Inclination to the Horizon,
to the Tangent of Inclination of Meridians.

Extend the Compasses from the Sine of 38 *gr.* 30 *m.* the Complement of the Latitude, unto the Sine of 90 *gr.* the same extent will reach from the Tangent of 50 *gr.* 0 *m.* the Inclination of the Plane to the Horizon unto the Tangent of 62 *gr.* 25 *m.* and such is the Inclination of the Meridian of the Plane to the Meridian of the Place; which being resolved int into Time, doth give about 4 *ho.* and 10 *m.* from the Meridian, for the place of the Subſtylar among the Hour-lines.

2. The Height of the Pole above the Plane is here repreſented by the quantity of the Ark of the proper Meridian P V, between the Pole and the Plane, and may be known by that which we have given in the former Triangle P V N. For,

As the Sine of 90 gr.
to the Sine of the Latitude:
So the Co-ſine of the Inclination to the Horizon,
to the Sine of the Height of the Pole above the Plane.

Extend the Compaſſes from the Sine of 90 *gr.* unto 51 *gr.* 30 *m.* the Sine of the Latitude, the ſame extent will reach from the Sine of 40 *gr.* the Complement of the Inclination of the Plane to the Horizon, unto the Sine of 30 *gr.* 12 *m.* Or,

As the Sine of 90 gr.
to the Co-ſine of Inclination of Meridians:
So the Tangent of the Latitude,
to the Tangent of the Height of the Pole above the Plane.

Extend the Compaſſes from the Sine of 90 *gr.* unto the Tangent of 51 *gr.* 30 *m.* the Latitude of the Place, the ſame extent will reach from the Sine of 27 *gr.* 35 *m.* the Complement of the Inclination of the two Meridians, unto the Tangent of 30 *gr.* 12 *m.* And ſuch is P V the Height of the Pole above the Plane, and ſuch muſt be the Height of the Style above the Subſtylar.

3. The Diſtance of the Subſtylar from the Meridian is here repreſented by N V the Ark of the Plane between the two Meridians, and may be found by that which we have given at the firſt in the former Triangle P V N. For,

As the Sine of 90 gr.
to the Sine of the Inclination to the Horizon:
So the Tangent of the Latitude,
to the Tangent of the Subſtylar from the Meridian.

Extend the Compaſſes from the Sine of 90 *gr.* unto the Tangent of

51 *gr.*

51 *gr.* 30 *m.* the Latitude of the Place, the same extent will reach from the Sine of 50 *gr.* the Inclination of the Plane to the Horizon, unto the Tangent of 43 *gr.* 55 *m.* And such is the Ark N V, the distance of the Substylar from the Meridian.

4. The Distances of the Hour-lines from the Substylar are here also represented by those Arks of the Plane which are here intercepted between the proper Meridian and the Hour-circles, and may be found by that which we have given in the Triangles made by the Plane, with his proper Meridian and the Hour-circles: For the Angle at V, between the Plane and the proper Meridian, is well known to be a Right Angle, and the Side P V is the Height of the Pole above the Plane, and the Angles at the Pole between the proper Meridian and the Hour-circles are easily gathered into a Table. The Angle V P N between V P the proper Meridian of the Plane, and P N the general Meridian of the Place, being 62 *gr.* 25 *m.* the Angle between the proper Meridian and the Circle of the Hour of 11 will be 77 *gr.* 25 *m.* and the Angle belonging to the Hour of 1, 47 *gr.* 25 *m.* and so the rest of the Angles at the Pole. Then,

Latitude	51	30
Inclinat.	50	0
Diff. Merid.	62	25
Alt. Styl.	30	12
Dist. Subst.	43	55

Hours.	Ang. Po. Gr.	Ang. Po. M.	Ar. Pla. Gr.	Ar. Pla. M.
11	77	25	66	4
12	62	25	43	55
1	47	25	28	41
2	32	25	17	43
3	17	25	8	58
4	2	25	1	13
	Merid.		Substyl.	
5	12	35	6	26
6	27	35	14	44
7	42	35	24	48
8	57	35	38	23
9	72	35	58	3
10	87	35	85	12

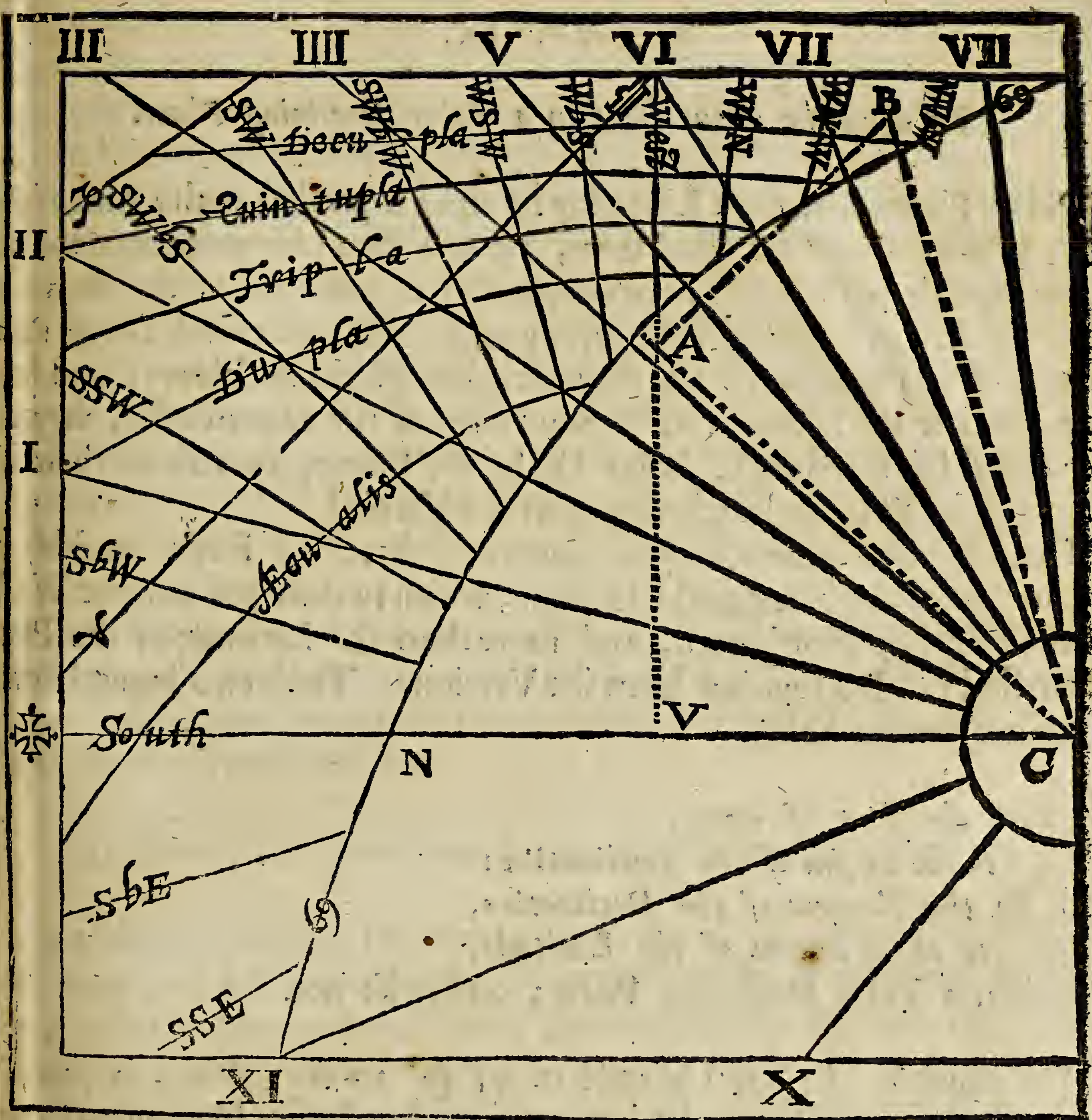
As the Sine of 90 gr.
to the Sine of the Height of the Pole above the Plane:
So the Tangent of the Angle at the Pole,
to the Tangent of the Hour-line from the Substylar.

Wherefore I extend the Compasses from the Sine of 90 *gr.* unto the Sine of 30 *gr.* 12 *m.* the Height of the Pole above the Plane, and I find the same extent to reach in the Line of Tangents from 77 *gr.* 25 *m.* unto 66 *gr.* 4 *m.* for the distance belonging to the hour of 11; and from the Tangent of 62 *gr.* 25 *m.* to 43 *gr.* 55 *m.* for the Hour of 12, as when I found the distance of the Substylar from the Meridian: And so for the rest of the Arks of the Plane between the Substylar and the Hour-circles, as in the Table.

These

These Arks being thus found, will serve to draw the Hour-lines on either Side of this Plane: But supposing it to be the upper side,

1. I draw the Horizontal Line C N, serving for the Meridian and Hour of 12.



2. In this Line I make choice of a Center at C, and thence describe an occult Ark of a Circle representing the Plane proposed.

3. I find a Chord of 43 *gr.* 55 *m.* the distance of the Substylar from the Meridian, and inscribe it into this Circle from N unto A, according as I find the proper Meridian P V to fall in the Fundamental Diagram, and there I draw the Line C A serving for the Substylar.

 4. The

4. The Subſtylar being drawn, I may inſcribe the Chords of the Ark of the Plane from the Subſtylar, and draw the Hour-lines, and ſet up th Style, as in the former Planes.

CHAP. IX.

To draw the Hour-lines in a Polar Declining Plane.

THoſe Planes wherein a Line may be drawn parallel to the Axis of th World are called Polar Planes, becauſe that Line pointeth unto th Poles; and theſe Planes are always parallel to ſome one of the Hour-ci cles. If they be parallel to the Hour of 6, they are called Direct Pol Planes; if to the Hour of 12, they are called Meridian Planes; and bo theſe are deſcribed before: if to any other of the Hour-circles, they a then called by the name of Polar Declining Planes, becauſe of their i clining to the Pole, and declining from the Vertical.

Theſe kind of Planes may be known in this ſort: Firſt, conſider t Inclination of the Plane to the Horizon, which in theſe parts of the Wor muſt always be Northward, and more than the Latitude of the Pla Then find the Declination from the Vertical. Theſe two being know if the proportion hold,

As the Sine of 90 gr.
to the Co-ſine of the Declination:
So the Tangent of the Declination,
to the Tangent of the Latitude,
it is then a Polar Declining Plane; otherwiſe not.

For example: In our Latitude of 51 *gr.* 30 *m.* a Plane is propo declining from the Vertical 65 *gr.* 40 *m.* and inclining Northward 71 51 *m.* the upper Face being open to the South-eaſt, and the lower to North-weſt. If I number thoſe 65 *gr.* 40 *m.* in the Horizon of the Fu damental Diagram from E unto Q, and draw the Line H C Q, it ſh repreſent the Horizontal Line of the Plane: then croſſing it at Ri Angles with the Plane B Z D drawn through the Zenith, I num 71 *gr.* 51 *m.* for the Inclination from D unto R, and there draw Circle H R Q; this Circle ſo drawn ſhall repreſent the Plane propoſ and becauſe it alſo paſſeth through the Pole, it is therefore a Polar Pla

But for farther trial, I extend the Compasses from the Sine of *90 gr.* to the Sine of 24 *gr.* 20 *m.* the Complement of the Declination, and I find the same Extent to reach from the Tangent of 71 *gr.* 51 *m.* the Inclination proposed, unto the Tangent of 51 *gr.* 30 *m.* which is the true Latitude of the Place; and therefore it is a Polar Plane.

Again, I number the Inclination of 71 *gr.* 51 *m.* in the Circle B Z D from Z unto M, so this point M will fall at the meeting of B Z D with the Equator, and being 90 *gr.* from the Plane at R, it shall be the Pole of this Plane; and a Circle drawn through M and P will be the proper Meridian of this Plane. This Meridian M P here falling on the Hour of 8, doth give M P Z, the Angle of the Inclination of Meridians, to be 4 Hours, or 60 Degrees; then crossing the Plane at the point P, it shews that the Substylar should be C P, and be placed at the Hour of 8. But because P is the Pole, and C P the Axis of the World wherein all the Hour-circles do meet, and so there would be no distinction between the Axis, the Substylar, and the Hour-lines, I now suppose the Plane in a parallel to the Circle H R Q, according to the distance that I would have between the Axis of the Style and the Substylar, then will the Style be parallel to the Plane, as appears in the Fundamental Diagram.

Here then the Style will be parallel to the Plane, and the Hour-lines parallel one to the other, as in the Meridian and Direct Polar Planes. Yet that we may better know how to draw the Hour-lines, and where to place the Style, we are to consider,

1. *The Ark of the Plane between the Horizon and the Pole.*

In a Meridian Plane, the Ark between the Horizon and the Pole, which represents the Ark between the Horizon and the Hour-lines, is always equal to the Latitude of the Place; in a direct Polar it is an Ark of 90 *gr.* In these Declining Polars it is greater than the Latitude, and yet less than 90 *gr.* This Ark is here represented by P Q, and may be known by resolving the Triangle Q N P, or P R Z.

As the Sine of 90 gr.
to the Co-sine of the Latitude:
So the Sine of the Declination,
to the Co-sine of the Ark between the Horizon and the Pole.

Extend the Compasses from the Sine of *90 gr*, unto the Sine of 38 *gr.* 30 *m.*

30 *m.* the Complement of the Latitude, the same extent will reach from the Sine of 65 *gr.* 40 *m.* the Declination proposed, unto the Sine of 34 *gr.* 34 *m.* whose Complement is 55 *gr.* 26 *m.* the Ark of the Plane required between the Horizon and the Pole.

Or, *As the Co-sine of Inclination to the Horizon,*
to the Sine of 90 gr.
So the Co-tangent of the Declination,
to the Tangent of the Ark between the Horizon and the Pole.

And so extending the Compasses from the Sine of 18 *gr.* 9 *m.* the Complement of the Inclination to the Tangent of 24 *gr.* 20 *m.* the Complement of the Declination, the same extent doth reach from the Sine of 90 *gr.* unto the Tangent of 55 *gr.* 26 *m.* And such is Q P the Ark of the Plane between the Horizon and the Pole, the measure of the Angle Q C I between the Horizontal Line and the Substylar.

2. *The Inclination of the Meridian of the Plane to the Meridian of the Place.*

The Substylar in a Direct Polar Plane is always the same with the Hour-line of 12; in a Meridian Plane it is the same with the Hour-line of 6; in these Declining Polars it must be placed between 12 and 6, according to the Inclination of the Meridian of the Plane to the Meridian of the Place, which is here represented by M P Z, the Complement of the Angle R P Z, and thus known.

As the Sine of 90 g.
to the Sine of the Latitude:
So the Tangent of the Declination of the Plane,
to the Tangent of the Inclination of Meridians.

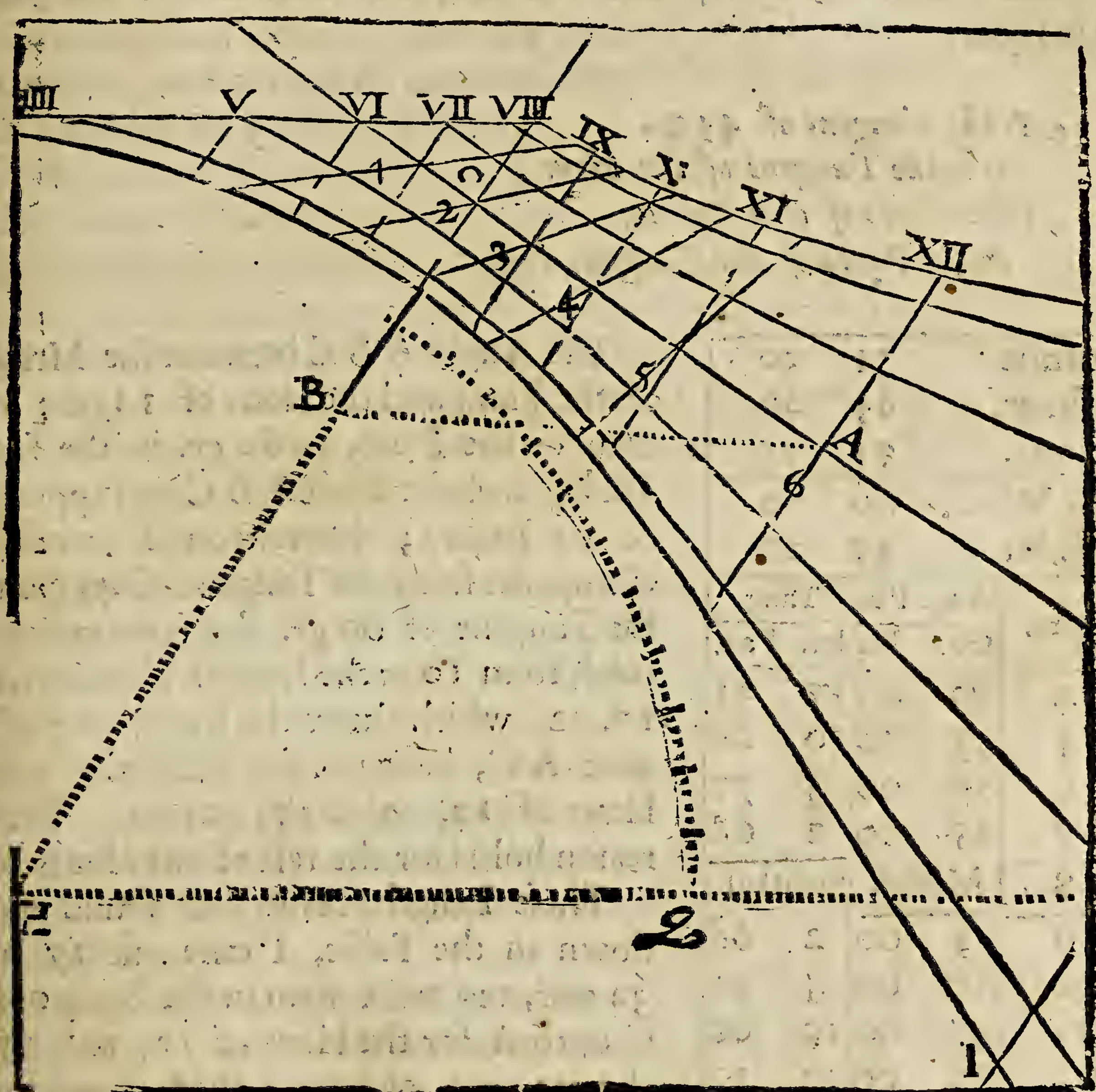
Extend the Compasses from the Sine of 90 *gr.* to the Sine of 51 *gr.* 30 *m.* the Latitude of the Place, the same extent will reach from the Tangent of 65 *gr.* 40 *m.* the Declination proposed, unto the Tangent of 60 *gr.* and such is the Angle of Inclination between the Meridian of the Place and the proper Meridian of the Plane, which resolved into Time, doth make four Hours; and so the Substylar must here be placed upon the Hour of 8 in the Morning.

Th

This Angle being known, the rest of the Angles at the Pole are easily gathered: For if the Hour of 12 be 60 *gr.* distant from the Meridian of the Plane, the Hour of 1 will be 75 *gr.* and the Hour of 11 will be 45 *gr.* distant, and the rest of the Hours, as in the Table following. Then coming to the Plane,

1. I draw an occult Horizontal Line H Q, wherein I make choice of a Center at H, and describe an occult Circle for the Horizon of the Plane.

2. I find a Chord of 55 *gr.* 26 *m.* and inscribe it into this Circle from Q unto B, according to the situation of the Plane; so the Line H B shall be the Meridian of the Plane, and therefore the Substylar; and the Line A C, crossing it at Right Angles, shall be the Equator.



3. I consider the length of the Plane, and how many Hours I am to draw upon it, that so I may proportion the Height of the Style; and I find by the Fundamental Diagram, and the former Table, that it will contain

contain all the Hours from Sun-rising until it be past 1 after Noon: and therefore the Meridian of the Plane falling on the Hour of 8 in the morning, there will be four Hours on the one side, and five on the other side of the Substylar. But in all Polar Planes the height of the Style above the Substylar must be equal to the distance of the third Hour from the Substylar, or about $\frac{4}{7}$ of the fourth Hour, or little more than $\frac{1}{4}$ of the fifth Hour, and thereupon I allow the height of this Style to be equal to C B, which you may suppose to be 10 Inches.

4. Because the Equator A C is a Tangent-line, in respect of the Radius B C, and the parts thereof are such as belong to the Angles between the Meridian of the Plane and the Hour-lines, which Angles are set down in the Table following, I may find the length of each several Tangent in this manner.

As the Tangent of 45 gr.
is to the Tangent of the Hour:
So the Parts of the Radius,
to the Parts of the Tangent-line.

Latitude	51	30
Declinat.	65	40
Inclinat.	71	51
Diff. Merid.	60	0
Dist. Subst.	55	20

Hours.	Ang. Po. Gr.	Ang. Po. M.	Tang. In.	Tang. Par.
4	60	00	17	31
5	45	00	10	00
6	50	00	5	77
7	45	00	2	68
8	Merid.		Subftyl.	
9	15	00	2	68
10	30	00	5	77
11	45	00	10	00
12	60	00	17	32
1	75	00	37	32
2	90	00	Infinit.	

The Angle A B C between the Meridian of the Plane and the Hour of 12, the Meridian of the Place, is 60 *gr.* in the former Table, and the Radius B C is supposed to be 10 Inches; whereupon I extend the Compasses from the Tangent of 45 *gr.* unto the Tangent of 60 *gr.* the same extent will reach from 10 in the Line of Numbers, unto 17.32, which shews the length of the Tangent A C, between the Substylar and the Hour of 12, to be 17, 32 *cent.* The like reason holds for the rest of the Hours.

These Lengths being thus found and set down in the Table, I take out 17 Inches 32 *cent*, and prick them in the Equator from C unto A for the Hour of 12, and 37 Inches 32 *cent.* and prick them down for the Hour of 1: And so the rest of the Hour-points.

6. This done, if I draw Right Lines through

through each of these Points, crossing the Equator at Right Angles, they shall be the Hour-lines required: And if I set the Style over the Substylar, so as the edge of it may be parallel to the Plane, and the height of it be 10 Inches, equal to the former Radius B C, it shall represent the Axis of the World, and be truly placed for casting of the Shadow upon the Hour-lines in this Declining Polar Plane.

CHAP. X.

To draw the Hour-lines in a Declining Inclining Plane.

IF a Plane shall decline from the prime Vertical, and incline to the Horizon, and yet not lie even with the Poles of the World, it is then called a Declining Inclining Plane.

Of these there are several sorts; for the Inclination being Northward, the Plane may fall between the Horizon and the Pole, as the Circle B M D in the Fundamental Diagram; or between the Zenith and the Pole, as B F D; or the Inclination may be Southward, and so be represented by B K D: it may also fall either below the Intersection of the Meridian and the Equator, or above it; and each of these hath two Faces, the upper toward the Zenith, and the lower toward the Nadir; wherein having the Latitude of the Place, with the Declination and Inclination of the Plane, we are further to consider,

1. The Ark of the Meridian between the Pole and the Plane.
2. The Inclination of the Plane to the Meridian.
3. The Ark of the Plane between the Horizon and the Meridian.
4. The Angle of Inclination between both Meridians.
5. The Height of the Pole above the Plane.
6. The distance of the Substylar from the Meridian.
7. The distances of each Hour-line from the Substylar.

And all these seven may be represented in the Fundamental Diagram, as in this Example.

In our Latitude of 51 *gr.* 30 *m.* a Plane is proposed declining from the Vertical 24 *gr.* 20 *m.* and inclining Northward 36 *gr.* the upper Face lying open to the South-west, the lower to the North-east. If I number these 24 *gr.* 20 *m.* in the Horizon from E to B, and there draw the Line B C D, it shall represent the Horizontal Line of the Plane: Then crossing it at Right Angles with the Plane H Z Q drawn through the Zenith, I number 36 *gr.* for the Inclination from Q unto M, and there draw the Circle B M D, croſſing the Meridian in the Point *a*; this Circle ſo drawn ſhall repreſent the Plane propoſed: and becauſe it doth not paſs through the Pole, is therefore no Polar, but an ordinary Declining Inclining Plane.

1. The Ark of the Meridian of the Place between the Pole and the Plane is here repreſented by P *a*, and may be found by reſolving the Triangle D N *a*, wherein the Angle at N is known to be a Right Angle, the Angle at D is the Angle of Inclination, the Side D N the Complement of the Declination; which being known,

> *As the Sine of* 90 gr.
> *to the Co-ſine of Declination:*
> *So the Tangent of Inclination to the Horizon,*
> *to the Tangent of the Ark of the Meridian between the Horizon and the Plane.*

Extend the Compaſſes from the Sine of 90 *gr.* unto the Sine of 65 *gr.* 40 *m.* the Complement of the Declination, the ſame extent will reach from the Tangent of 36 *gr.* the Inclination propoſed, unto the Tangent of 33 *gr.* 30 *m.* and ſuch is the Ark of the Meridian N *a* between the Horizon and the Plane. This Ark N *a* being compared with the Ark N P, which is the Elevation of the Pole above the Horizon, and is here ſuppoſed to be 51 *gr.* 30 *m.* the difference N *a* cometh to 18 *gr.* and ſuch is the Ark of the Meridian required between the Pole and the Plane.

2. The Inclination of the Plane to the Meridian is here repreſented by the Angle N *a* D, and may be found by that which we have given in the former Triangle D N *a*. For,

> *As the Sine of* 90 gr.
> *to the Sine of the Declination from the Vertical:*
> *So the Sine of Inclination to the Horizon,*
> *to the Co-ſine of Inclination of the Plane to the Meridian.*

Extend the Compaſſes from the Sine of 90 *gr.* unto the Sine of 24 *gr.* 20 *m.* the Declination of the Plane, the ſame extent will reach from the Sine of 36 *gr.* the Inclination given, unto the Co-ſine of 76 *gr.* and ſuch is N *a* D the Angle of Inclination between the Plane D *a* and N *a* the Meridian of the Place. Or,

As

As the Sine of the Ark of the Meridian between the Horizon and the Plane,
is to the Sine of 90 gr.
So the Co-tangent of the Declination,
to the Tangent of Inclination of the Plane to the Meridian.

Extend the Compasses from the Sine of 33 *gr.* 30 *m.* the Ark of the Meridian between the Horizon and the Plane, unto the Sine of 90 *gr.* the same extent will reach from the Tangent of 65 *gr.* 40 *m.* the Complement of the Declination, unto the Tangent of 67 *gr.* and such is the Inclination of the Plane to the Meridian, the same as before.

3. The Ark of the Plane between the Horizon and the Meridian is here represented by D *a*, and may also be found by that which we have given in the former Triangle D N *a*.

As the Co-sine of Inclination to the Horizon,
is to the Sine of 90 gr.
So the Co-tangent of the Declination,
to the Tangent of the Ark of the Plane from the Horizon to the Meridian.

Extend the Compasses from the Sine of 54 *gr.* the Complement of the Inclination of the Plane to the Horizon, unto the Sine of 90 *gr.* the same extent will reach from the Tangent of 65 *gr.* 40 *m.* the Complement of the Declination, unto the Tangent of 69 *gr.* 54 *m.* And such is D *a* the Ark of the Plane between the Horizon and the Meridian of the Place.

4. The Inclination of Meridians is here represented by the Angle *a b* P. For having drawn the proper Meridian *b* P *k*, or let down a Perpendicular P *b* from the Pole unto the Plane, this Perpendicular shall be the Meridian of the Plane, and we shall have another Triange *a b* P, wherein the Angle at *b* is a Right Angle because of the Perpendicular, the Angle at *a* is the Inclination of the Plane to the Meridian of the Place, and the Side P *a* is the Ark of the Meridian between the Pole and the Plane; which being known,

As the Co-sine of the Ark of the Meridian between the Pole and the Plane,
is to the Sine of 90 gr.

So the Co tangent of the Inclination of the Plane to the Meridian,
to the Tangent of Inclination of the Meridian of the Plane to the Meridian of the Place, that is, the Angle at the Pole between the two Meridians.

Extend the Compasses from the Sine of 72 *gr.* the Complement of the Ark P *a* between the Pole and the Plane, unto the Sine of 90 *gr.* the same extent will reach from the Tangent of 14 *gr.* the Complement of the Inclination of the Plane to the Meridian, unto the Tangent of 14 *gr.* 41 *m.* And such is the Angle *a* P *b* of Inclination between the Meridian of the Place and the proper Meridian of the Plane; which resolved into Time, doth make about 59 *m.* and so the Substylar must here be placed near the Hour of 1 after Noon.

5. The Height of the Pole above the Plane is here represented by P *b* the Ark of the proper Meridian between the Pole and the Plane, and may be found by that which we have given in the Triangle *a b* P. For,

As the Sine of 90 gr.
to the Sine of the Ark of the Meridian of the Place between the Pole and the Plane:
So the Sine of Inclination of the Plane to the Meridian,
to the Sine of the Height of the Pole above the Plane.

Extend the Compasses from the Sine of 90 *gr.* unto the Sine of 18 *gr.* the Ark P *a* of the Meridian of the Place from the Pole to the Plane, the same extent will reach from the Sine of *b a* P the Inclination of the Plane to the Meridian of the Place, unto the Sine of 17 *gr.* 26 *m.* Or,

As the Sine of 90 gr.
to the Co-sine of Inclination of Meridians:
So the Tangent of the Ark of the Meridian of the Place between the Pole and the Plane,
to the Tangent of the Height of the Pole above the Plane.

Extend the Compasses from the Sine of 90 *gr.* unto the Sine of 75 *gr.* 19 *m.* the Complement of *a* P *b* the Inclination of the two Meridians the same extent will reach from the Tangent of 18 *gr.* the Ark P *a* of the general Meridian between the Pole and the Plane, unto the Tangent of 17 *gr.* 26 *m.* And such is P *b* the Height of the Pole above the Plane

Plane; and such must be the Height of the Style above the Substylar.

6. This Distance of the Substylar from the Meridian of the Place is here represented by *a b* the Ark of the Plane between the two Meridians, and may be found by that which we had given at the first in the former Triangle *a b* P. For,

As the Sine of 90 gr.
to the Co-sine of the Inclination of the Plane to the Meridian:
So the Tangent of the Ark of the Meridian of the Place between the Pole and the Plane,
unto the Tangent of the Substylar from the Meridian of the Place.

Extend the Compasses from the Sine of 90 *gr.* unto the Sine of 14 *gr.* the Complement of *b a* P the Inclination of the Plane to the Meridian, the same extent will reach from the Tangent of 28 *gr.* the Ark of the general Meridian between the Pole and the Plane, unto the Tangent of 4 *gr.* 30 *m.* And such is the Ark of the Plane between the two Meridians; and such must be the Distance from the Hour of 12 to the Substylar.

Latitude	51	30
Declinat.	24	20
Inclin. N.	36	0
Alt. Merid.	69	54
Diff. Merid.	14	41
Alt. Styl.	17	26
Dist. Subst.	4	30

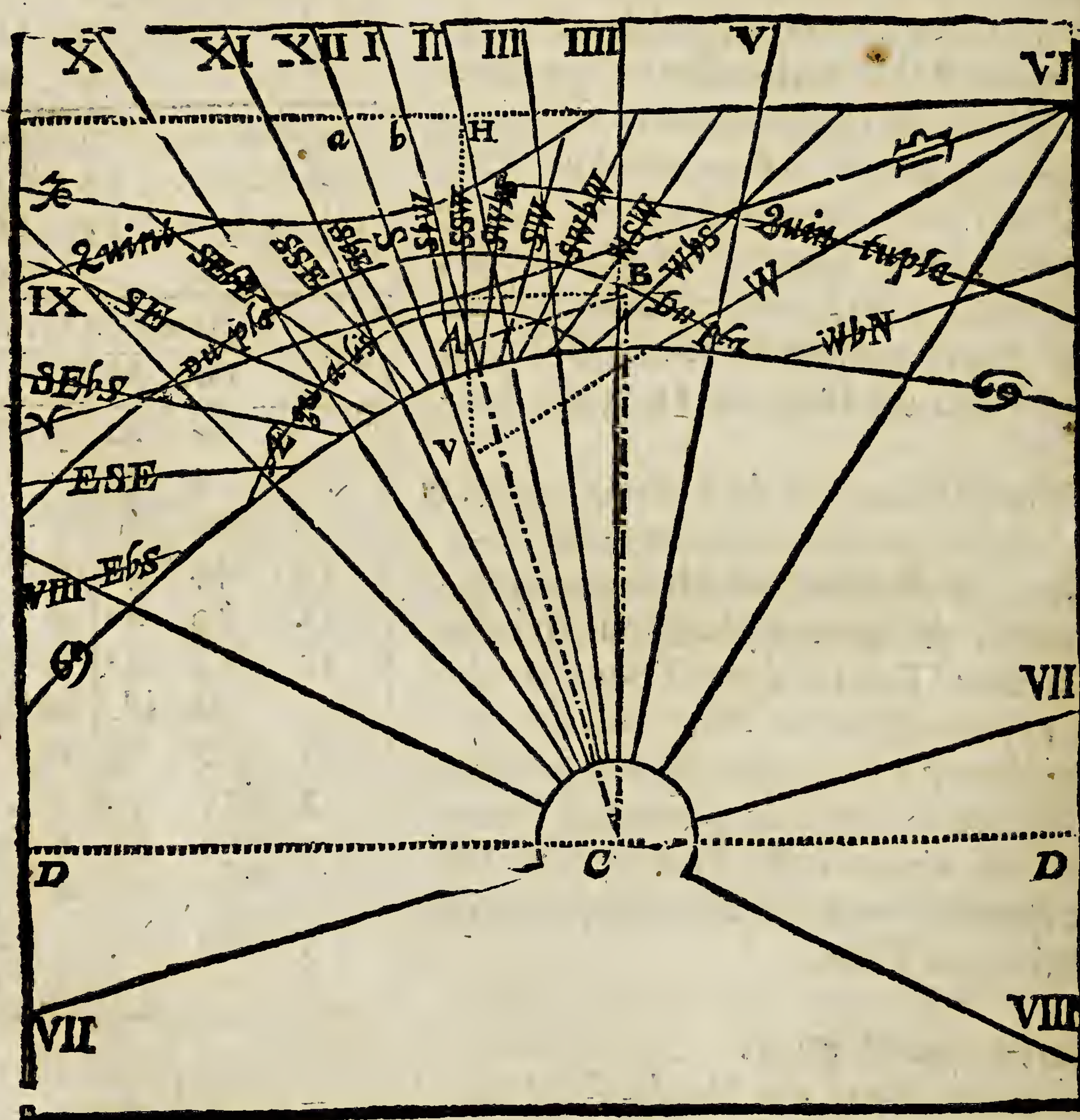
Hours.	Ang. Po.		Ar. Pla.	
	Gr.	M	Gr.	M.
7	89	41	88	57
8	74	41	47	35
9	59	41	27	9
10	44	41	16	31
11	29	41	9	41
12	14	41	4	30
	Merid.		Substyl.	
1	0	19	0	6
2	15	19	4	4
3	30	19	9	56
4	45	19	6	52
5	60	19	7	45
6	75	19	48	51

7. The Distances of the Hour-lines from the Substylar are here also represented by those Arks of the Plane which are intercepted between the proper Meridian and the Hour-circles: For in these Triangles, the Angle at *b* between the Plane and the proper Meridian is a Right Angle, the Side P *b* is the Height of the Pole above the Plane, and then the Angles at the Pole between the proper Meridian and the Hour-circles being gathered into a Table,

As the Sine of 90 gr.
to the Sine of the Height of the Pole above the Plane:
So the Tangent of the Angle at the Pole,
to the Tangent of the Hour-line from the Substylar.

Extend the Compasses from the Sine of 90 *gr.* unto the Sine of 17 *gr.* 26 *m.* the Height of the Pole above the Plane, the same extent will reach from the Tangents of 14 *gr.* 41 *m.* the Angle at the Pole belonging to the Hour of 12, unto the Tangent of 4 *gr.* 30 *m.* for the Ark of the Plane between the Substylar and the Hour of 12; and from the Tangent of 29 *gr.* 41 *m.* unto the Tangent of 9 *gr.* 41 *m.* for the Hour of 11: And so for the rest of the Arks of the Plane between the Substylar and the Hour-lines, as in the former Table.

These Arks being thus found, will serve for the drawing of the Hour-lines on either side of the Plane: but supposing it to be the upper side, I consider how the Lines do fall in the Fundamental Diagram, and accordingly,



1. I draw an occult Horizontal Line D D, wherein I make choice of the

e Center C, and thence draw an occult Circle for the Horizon of the lane.

2. I find a Chord of *69 gr. 54 m.* the Ark of the Plane between the orizon and the Meridian, and describe into this Circle from D unto and there draw the Line C *a* for the Hour of 12.

3. I find a Chord of *4 gr. 30 m.* the Ark of the Plane between the vo Meridians, and inscribe it into this Circle from *a* unto *b*, and there raw the Line C *b* for the Substylar.

4. The Substylar being drawn, I may inscribe the Chords of the Arks the Plane from the Substylar, and draw the Hour-lines, and set up the yle, as in the former Plane.

A second Example of a Plane falling between the Pole and the Zenith.

In like manner if in our Latitude a Plane e proposed declining from the Vertical 4 *gr.* 20 *m.* as before, but inclining to the lorizon 75 *gr.* 40 *m.* Northward, the uper Face being open to the South-west, the ower to the North-east, this Plane shall be ere represented by the Circle B F D, crosng the Meridian in the point *d*, between ie Pole and the Zenith, and the proper Median of this Plane, by the Perpendicular rk P *e*.

Then in this Triangle D N *d* knowing ie Side D N, the Complement of the Deination, with the Angle of Inclination to he Horizon at D, and the Right Angle at N, these former Canons will give N *d*, the Ark of the Meridian between the Horizon nd the Plane, to be 74 *gr.* 20 *m.* and therebre P D, the Ark of the Meridian between he Pole and the Plane, will be 22 *gr.* 50 *m.* he Angle D *d* N of the Inclination of the Plane to the Meridian will be found to be 6 *gr.* 29 *m.* and D *d* the Ark of the Plane etween the Horizon and the Meridian 3 *gr.* 36 *m.*

Latitude	51	30
Declinat.	24	20
Inclinat.	75	40
Alt. Merid.	83	36
Diff. Merid.	25	17
D.ft. Subft.	9	32
Alt. Styl.	20	50

Hours.	An. Po. Gr.	M	Ar. Pla. Gr.	M.
8	85	17	76	56
9	70	17	44	47
10	55	17	27	11
11	40	17	16	43
12	25	17	9	32
1	10	17	9	41
	Merid.		Substyl.	
2	4	43	1	40
3	19	43	7	16
4	34	43	13	50
5	49	43	22	46
6	64	43	37	0
7	79	43	62	5

Again,

Again, in the Triangle P *e d*, knowing the Side P *d*, the Ark of th Meridian between the Pole and the Plane, with the Angle of Incl natior to the Meridian at *d*, and the Right Angle at *e*, the Angle *d* P *e* of the In clination of the two Meridians will be found to be 25 *gr.* 17 *m.* and P the Height of the Pole above the Plane to be 20 *gr.* 50 *m.* and *d e* the di ſtance of the Subſtylar from the Meridian about 9 *gr.* 32 *m.*

Laſtly, having found the Height of the Pole above the Plane, and ga thered the Angles at the Pole, the Arks of the Plane from the Subſtylar to the Hour-lines will be as in the Table:

This done, if we conſider how the Lines do fall in the Fundamenta Diagram, we may there ſee how the North Pole is elevated above th lower Face, and the South Pole above the upper Face of the Plane, anc accordingly make choice of a Center, draw the Horizontal, the Meridi an, the Subſtylar, and the Hour-lines, and ſet up the Style, as in th other Planes.

Latitude	51	30
Declinat.	24	20
Inclinat.	14	20
Diſt. Merid.	13	27
Diſt. Subſt.	12	8
Alt. Styl.	64	0
Alt. Merid.	66	20

Hours.	Ang. Po.		Ar. Pla.	
	Gr.	M.	Gr.	M.
6	76	33	75	6
7	61	33	58	56
8	46	33	43	30
9	31	3[illegible]	28	55
10	16	33	14	58
11	1	33	1	25
	Merid.		Subſtyl.	
12	13	27	12	8
1	28	27	25	57
2	43	27	40	23
3	58	27	55	38
4	73	27	71	41
5	88	27	82	15

A third Example of a Plane inclining to the Southward.

If in our Latitude a Plane were propoſec declining from the Vertical 24 *gr.* 20 *m.* a before, but inclining to the Horizon 14 *gr* 20 *m.* Southward, the upper Face being oper to the North-eaſt, the lower to the South weſt, this Plane ſhall be there repreſentec by the Circle B K D, croſſing the Meridiar in the point *f*, between the Equator and th Horizon, and the proper Meridian of thi Plane, by the perpendicular Ark P *g*, le down from the Pole to the Plane, near th Hour of 11, at the North part of the Ho rizon, as may partly appear by the neareſ extent of the Compaſſes, if the Circle B K I were drawn round, and the two Letters and *g* ſupplied.

Then in the Triangle B S *f*, knowing th Side B S the Complement of the Declina tion, with the Angle of Inclination to th Horizon at B, at the Right Angle at S, w ma

ay find S *f* the Ark of the Meridian between the Horizon and the Plane
 be 13 *gr.* 6 *m.* And therefore P *f*, the Ark of the Meridian between
e Pole and the Plane to the Southward 115 *gr.* 24 *m.* but 64 *gr.* 36 *m.*
 the Northward, the Angle B *f* S, or D *f* N of the Inclination of the
lane to the Meridian will be found 84 *gr.* 9 *m.* and B *f* the Ark of the
lane between the Horizon and the Meridian 66 *gr.* 20 *m.*

Again, in the Triangle P *g f*, knowing the Side P *f* the Ark of the
eridian between the Pole and the Plane, with the Angle of Inclinati-
n to the Meridian at *f*, and the Right Angle at *g*, the Angle *f* P *g* of
e Inclination of the two Meridians will be found to be 13 *gr.* 27 *m.*
nd P *g* the Height of the Pole above the Plane, about 64 *gr.* and *f g* the
 ſtance of the Subſtylar from the Meridian 12 *gr.* 8 *m.*

Having found the Height of the Pole above the Plane, and gathered
e Angles at the Pole, the Arks of the Plane from the Subſtylar to the
our-lines will be found as in the Table.

This done, if we conſider how the Lines do fall in the Fundamen-
l Diagram, we may there ſee how the North Pole is elevated above the
pper Face, and the South Pole above the lower Face of this Plane; and
ccordingly make choice of the Center, draw the Horizontal, the Meri-
ian, the Subſtylar, and the Hour-lines, and ſet up the Style, as in the for-
er Planes.

CHAP. XI.

To deſcribe the Tropicks and other Circles of Declination in an Equinoctial Plane.

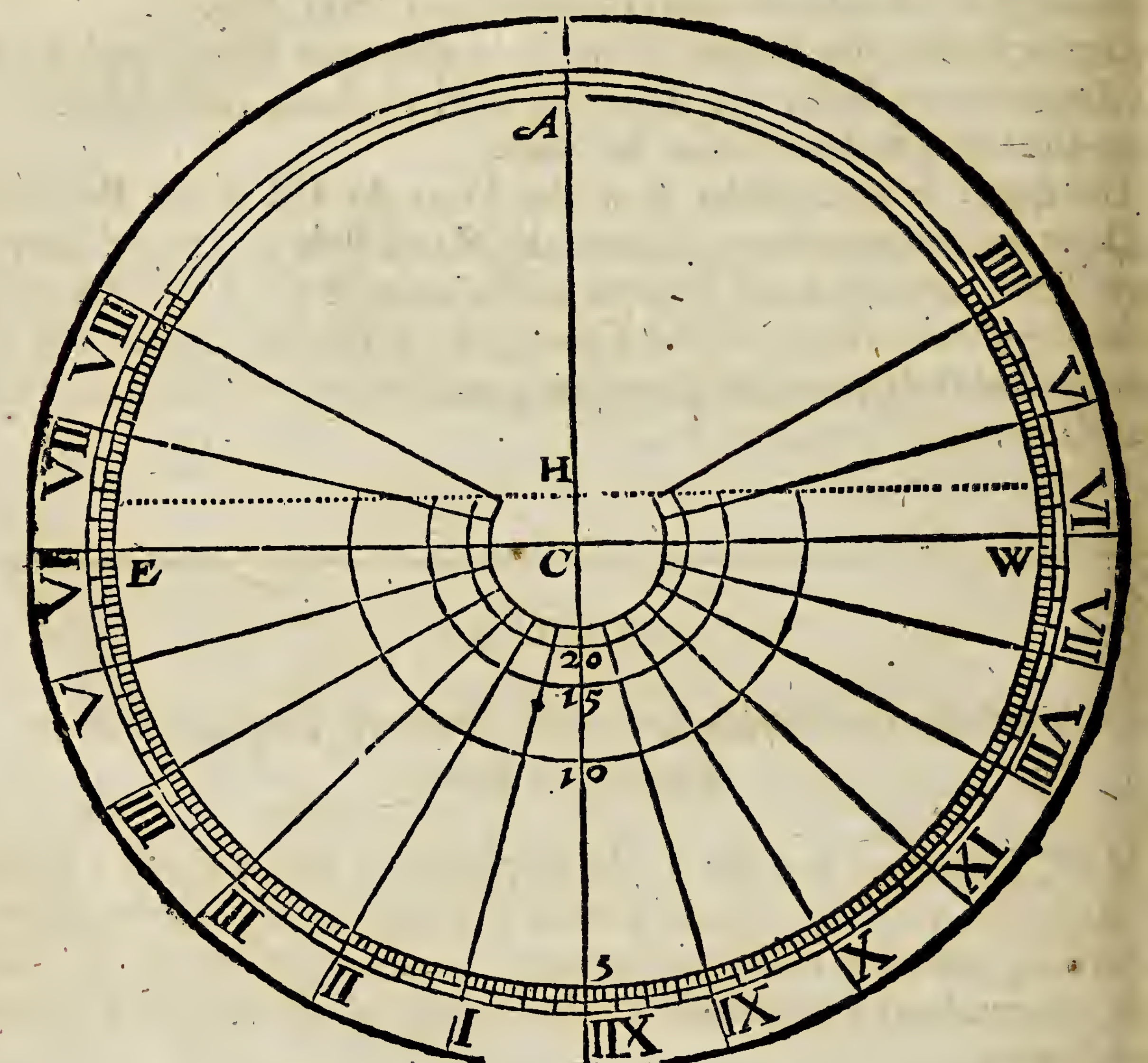
Such Circles as are parallel to the Equinoctial, and yet fall within the
Tropicks, may be deſcribed on any Plane by help of theſe Lines of
roportion, but after a different manner, according as the Style ſhall be
ther perpendicular or parallel to the Plane, or cut the Plane with Ob-
que Angles.

In an Equinoctial Plane, where the Style is perpendicular to the
lane, the Tropicks and other Circles of Declination will be perfect Cir-
es: Wherefore conſider the length of the Style in Inches and parts, and
e Declination of the Circle which you intend to deſcribe in Degrees
d Minutes, the proportion will hold,

As

As the Tangent of 45 gr.
to the Length of the Style:
So the Co-tangent of the Parallel,
to the Semidiameter of his Circle.

Suppose the Length of the Style above the Plane to be 10 Inches, an that it were required to find the Semidiameter of the Tropick, who Declination is known to be 23 *gr.* 30 *m.* Extend the Compasses fro the Tangent of 45 *gr.* unto the Tangent of 66 *gr.* 30 *m.* the same exte



will reach in the Line of Numbers from 10 unto 23, which shews Semidiameter of the Tropick to be 23 Inches. So if the Declinatio 20 *gr.* the Semidiamer will be 27 Inches 47 *cent.* if 15 *gr.* then 37. if 10 *gr.* then 56.71; if 5 *gr.* then 114. 305: and so in the rest.

Or if it were required to proportion the Style to the Plane,

As the Tangent of 45 gr.
to the Tangent of the Declination:
So the Semidiameter of the Plane,
to the Length of the Style.

As if the Semidiameter of the greateſt Parallel upon the Plane were but ſix Inches, and that Parallel ſhould be the fifth Degree of Declination; extend the Compaſſes from the Tangent of 45 *gr.* unto the Tangent of 5 *gr.* the ſame extent will reach in the Line of Numbers from 6. 00 unto about 0. 53, which ſhews that the length of the Style muſt be 53 parts of an Inch divided into 100: Then the length of the Style being known, the Semidiameter of the other Circles will be found as before.

I begin here with the fifth Parallel, and thence proceed unto the Tropick, becauſe the Shadow of the reſt near the Equinoctial would be over-long, and the Equinoctial it ſelf cannot be deſcribed. The Parallels of North Declination are to be ſet on the North Face, and the Parallels of South Declination on the South Face of the Plane. Neither need theſe Parallels to be drawn in full Circles, but onely to the Horizontal Line, which ſhall be deſcribed in *Chap.* 18.

Having by theſe means ſet up the Style to its true Height, and drawn the Circles of Declination, if we ſhall place the Plane ſo as it ſhall make an Angle with the Horizon equal to the Complement of the Latitude, and then turn it until the top of the Style caſt the Shadow upon the Parallel of Declination belonging to the Time, the Meridian of the Plane will ſhew the Meridian of the Place, and the Shadow of the Style the Hour of the day, without the help of a Magnetical Needle.

CHAP. XII.

To deſcribe the Tropicks and other Circles of Declination in a Polar Plane.

IN all Polar Planes, whether they be parallel to the Meridian, or to the Circles of the Hour of 6, or otherwiſe declining, the Equinoctial will be a Right Line, but the Tropicks and other Circles of Declination will be Sections Hyperbolical, and be thus deſcribed.

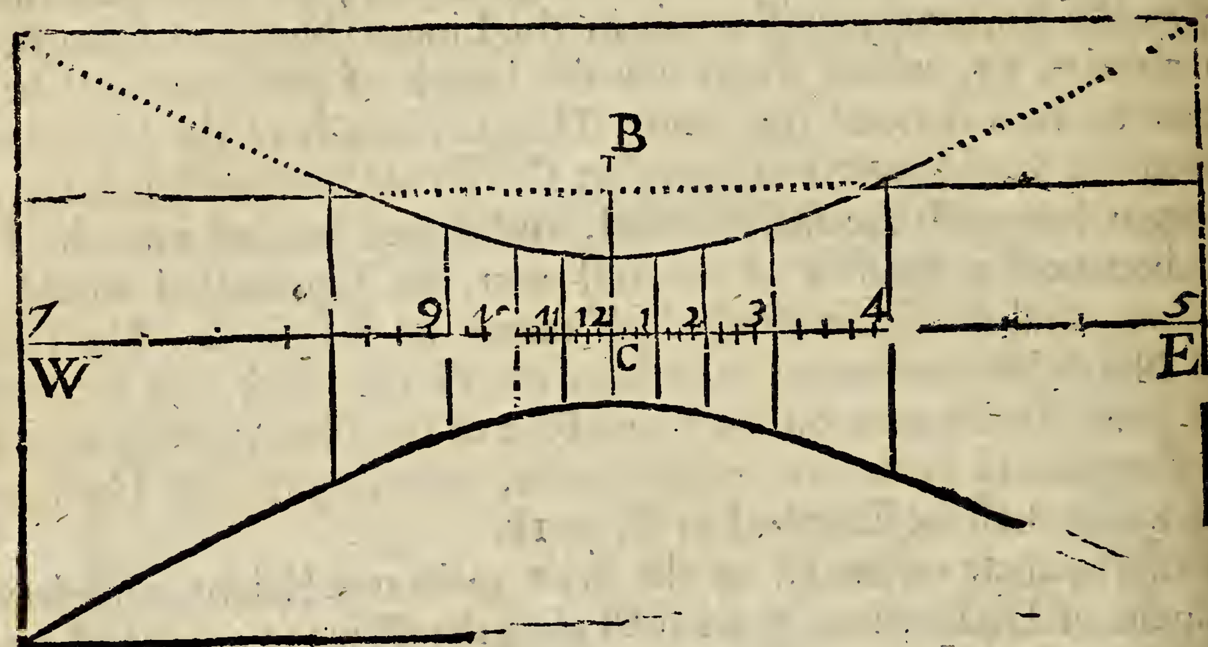
Conſider the length of the Style, the Declination of the Parallel, and

 the

the Angle at the Pole between the Subſtylar and the Hour-line, whereon you mean to deſcribe the Parallel.

If you would find where the Parallels do croſs the Subſtylar,

> *As the Tangent of* 45 gr.
> *to the Tangent of Declination* :
> *So is the Length of the Style,*
> *to the Diſtance of the Parallel from the Equinoctial.*

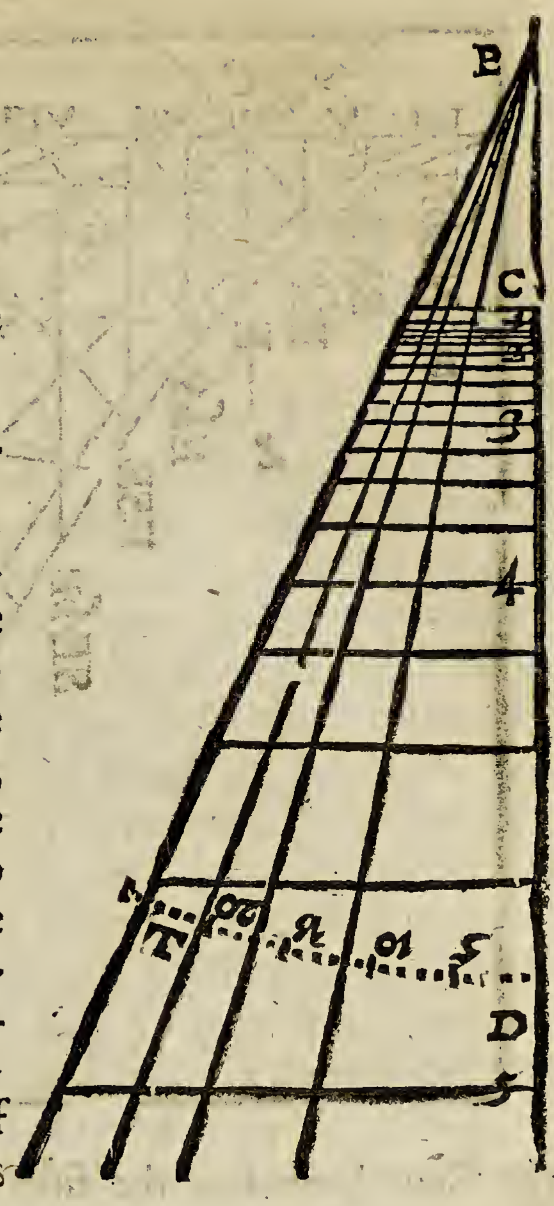


As in the Example of the Polar Plane, where the length of the Sty[le] B C was found to be 1 Inch *61 cent.* If you deſire to know the diſtan[ce] between the Equinoctial and the Tropick upon the Subſtylar Line, ex[tend] the Compaſſes from the Tangent of 45 *gr.* unto the Tangent [of] 23 *gr.* 30 *m.* the ſame extent will reach in the Line of Numbers fro[m] 1.61 unto 0.70; and therefore the diſtance required is 70 parts of a[n] Inch divided into 100. The like reaſon holdeth for all other Parall[els] of Declination croſſing the Subſtylar.

But if you would find where the Parallels do croſs any other of t[he] Hour-lines, firſt find the diſtance between the Axis of the Style and t[he] Hour-line, then the diſtance between the Equinoctial and the Parall[el]. Both theſe may be repreſented in this manner,

On the Center B, and any Semidiameter B D, deſcribe an occult A[rch] of a Circle, and therein inſcribe a Chord of 23 *gr.* 30 *m.* from D un[to] T, with ſuch other intermediate Declinations as you intend to deſcr[ibe] on the Plane; ſo the Line B D ſhall be the Equator, and B T the Tr[o-]

pi[ck]



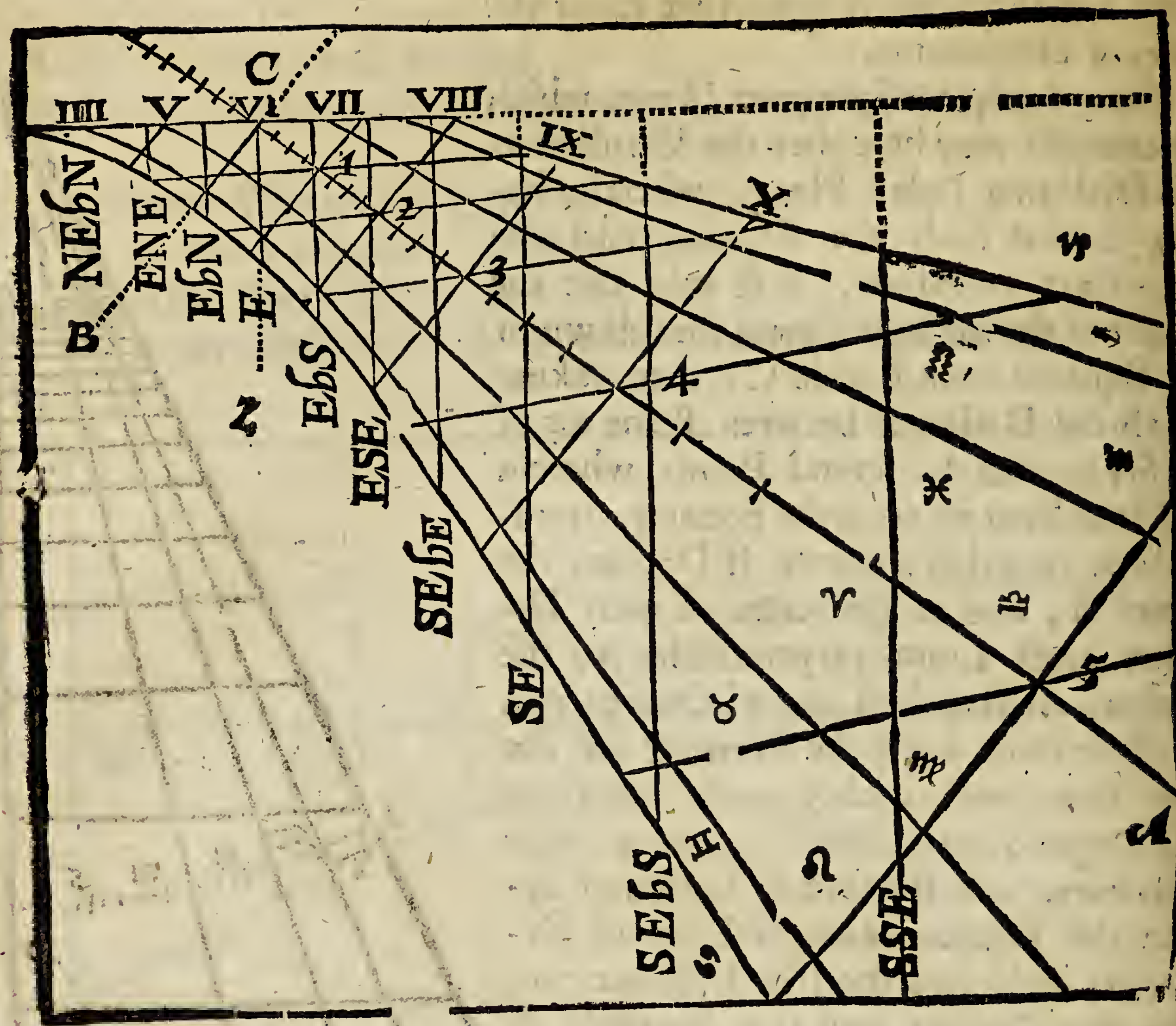
pick, and the other intermediate Lines the Lines of Declination.

That done, consider your Plane, which for example may be either the Meridian or the Declining Polar Plane; wherein having drawn both the Equator and the Hour-lines as before, first take out the Height of the Style, and prick that down in this Equator from B unto C; then taking out all the Distances between B the top of the Style, and the several Points wherein the Hour-lines do cross the Equator, transfer them into this Equator B D from the Center B, and at the terms of these Distances erect Lines perpendicular to the Equator, crossing the Lines of Declination, and note them with the Number of the Hour from whence they were taken: so these Perpendiculars shall represent those Hour-lines, and the several Distances between the Equator and the Lines of Declination shall give the like Distances between the Equator and the Parallels of Declination upon your Plane. Upon this ground it followeth,

To find the distance between the Axis and the Hour-lines.

As the Co-sine of the Hour from the Substylar,
is to the Sine of 90 gr.
So the length of the Style,
to the distance between the Axis and the Hour-line.

As if in the former Example of the Meridian Plane, where B C the height of the Style is supposed to be 10 Inches, it were required to find the distance between B to the top of the Style, and the point wherein the Hour of 11 in the Morning doth cross the Equator, which is here represented by B 5, because it is the fifth Hour from the Substylar, whose Angle at the Pole is 75 *gr*. Extend the Compasses from the Sine of 15 *gr*. the



the Complement of the fifth Hour from the Subſtylar, unto the Sine o 90 *gr.* the ſame extent will reach from 10,00 in the Line of Numbers unto 38.64; and therefore the diſtance B 5, between the Axis and th Hour-line, is 38 Inches and 64 *cent.* and may be called the Secant of th Hour. Then in the Rectangle B 5 T, having the Side B 5, and the An gle of Declination at B,

To find the diſtance between the Equinoctial and the Parallel.

As the Tangent of 45 gr.
to the Taagent of the Declination,
So the diſtance between the Axis and the Hour-line,
to the diſtance between the Equinoctial and the Parallel.

Extend the Compaſſes from the Tangent of 45 *gr.* unto the Tangen of 23 *gr.* 30 *m.* the Declination of the Tropick, ſo the ſame extent wi reac reach in the Line of Numbers from 38. 64 the diſtance between the Axis and the fifth Hour-line, unto 16, 80; and therefore the diſtance is 16 Inches and 80 *cent.* The like reaſon holdeth for all the reſt, which may be gathered, and ſet down in ſuch a Table as this which followeth.

Wherein I have ſet down the Diſtances for ſeveral Declinations, for 11 *gr.* 30 *m.* for 16 *gr.* 55 *m.* for 20 *gr.* 12 *m.* for 21 *gr.* 41 *m.* and for the Declination of the Tropick 23 *gr.* 30 *m.* which may be applied to the like Declinations in all Meridian and direct Polar Planes.

As in the former Example of the Polar Plane, where B C the height of the Style is found to be 1 Inch 61 *cent.* if it were required to find the diſtance between B the top of the Style, and the Points wherein the Hour-lines of 7 in the Morning or 5 Afternoon do croſs the Equator (which diſtances I called the Secants of thoſe Hours) either you may extend the Compaſſes from the Sine of 15 *gr.* the Complements of the Hour from the Subſtylar, unto the Sine of 90 *gr.* ſo the ſame extent will reach in the Line of Numbers from 1, 61 the length of the Style, unto 6. 21, according to the former Canon. Or elſe you may make uſe of the following Table, extending the Compaſſes in the Line of Numbers from 10. 00 the length of the Style in the Table, unto 1, 61 the length of the Style belonging to your Plane; ſo the ſame extent ſhall reach from 38.64, the Secant in the Table, unto 6. 21, and ſuch is your Secant required, the diſtance between the top of the Style and the point of Interſection, wherein the fifth Hour-line from the Subſtylar doth croſs the Equator.

Again, the ſame extent will reach from 16. 80 the diſtance in the Table belonging to the fifth Hour-line between the Equator and the Parallel of 13 *gr.* 30 *m.* declination, unto 2. 70 for the like diſtance upon your Plane; and ſo for the reſt, which may be gathered, and ſet down in a Table.

Hours.

Ho.	An.Po.	Tang.	Secant.	11 30	16 55	20 12	21 41	2 3
	Gr. M.	In. Pa.	In. Pa.	In. Pa.	In. Pa.	In. Pa.	In. Pa.	In. Pa.
0	0 0	0 0	10 00	2 3	3 4	3 68	3 98	4 35
	3 45	0 65	10 02	2 04	3 05	3 69	3 99	4 36
	7 30	1 32	10 09	2 05	3 07	3 71	4 01	4 39
	11 15	1 99	10 20	2 07	3 10	3 75	4 05	4 43
1	15 0	2 68	10 35	2 10	3 15	3 81	4 12	4 50
	18 45	3 39	10 56	2 15	3 21	3 99	4 20	4 59
	22 30	4 14	10 82	2 20	3 29	4 10	4 30	4 70
	26 15	4 93	11 15	2 36	3 39	4 24	4 45	4 85
2	30 0	5 77	11 55	2 34	3 51		4 60	5 02
	33 45	6 68	12 03	2 44	3 66	4 42	4 78	5 23
	37 30	7 67	12 60	2 56	3 83	4 64	5 02	5 48
	41 15	8 77	13 30	2 70	4 05	4 89	5 29	5 78
3	45 0	10 00	14 14	2 87	4 30	5 10	5 63	6 15
	48 45	11 40	15 17	3 08	4 62	5 58	6 03	6 00
	52 30	13 03	16 43	3 34	5 00	6 04	6 54	7 14
	56 15	14 97	18 00	3 66	5 48	6 62	7 15	7 83
4	60 0	17 32	20 00	4 07	6 08	7 36	7 95	8 70
	63 45	20 28	22 61	4 60	6 88	8 32	9 00	9 83
	67 30	24 14	26 13	5 31	7 95	9 61	10 39	11 36
	71 15	29 46	31 11	6 33	9 47	11 45	12 37	13 53
5	75 0	37 32	38 64	7 86	11 74	14 20	15 36	10 80
	78 45	50 27	51 26	10 43	15 60	18 89	60 38	22 28
	82 30	75 96	76 61	15 58	23 32	28 10	30 47	33 31
	86 15	152 57	152 90	31 10	46 44	56 26	60 81	66 48
6	90 0	Infin.	Infin.	Infin.	Infin.	Infin.	Infin.	Infin.

The Tangents and Secants in the third and fourth Columns of this Table are taken out of the Tables of the Natural Tangents and Secants, according to the Degrees and Minutes that are in the ſecond Column of this Table.

That done, and the Equator drawn as before, if you would draw the Tropicks in the Polar Plane, look into the Table, and take 70 *cent.* out of the Line of Inches, and prick them down in the Subſtylar, on either ſide of the Equator, and ſo 72 *cent.* on the firſt Hour, and 80 on the ſecond

Hours.		Ang. Po.		Tang.		Secan.		Trop;	
		Gr.	M.	In.	Pa.	In.	Pa.	In.	Pa.
12		0	0	0	0	1	61	0	70
11	1	15	0	0	43	1	63	0	72
10	2	30	0	0	93	1	85	0	80
9	3	45	0	1	61	2	27	0	99
8	4	60	0	2	79	3	22	1	40
7	5	75	0	6	00	6	21	2	70

ſecond Hour, and 2 Inches 70 *cent.* to the fifth Hour from the Subſtylar, and the reſt of theſe Diſtances on their ſeveral Hour-lines; and then draw a crooked Line through all theſe Points, ſo as it makes no Angles, the Line ſo drawn ſhall be the Tropick required. In like manner you may draw any other Parallel of Declination.

CHAP. XIII.

To deſcribe the Tropicks, and other Circles of Declination, in ſuch a Plane as is neither Equinoctial nor Polar.

IN Planes neither Equinoctial nor Polar, the Equator will be a Right Line, the Tropicks and other Parallels of Declination will be Conical Sections, ſome of them parabolical, ſome eliptical, but moſt of them hyperbolical.

To find the Points of Interſection of theſe Parallels with the Hour-lines, we are to conſider,

Firſt, The length of the Axis of the Style in Inches and parts of Inches.

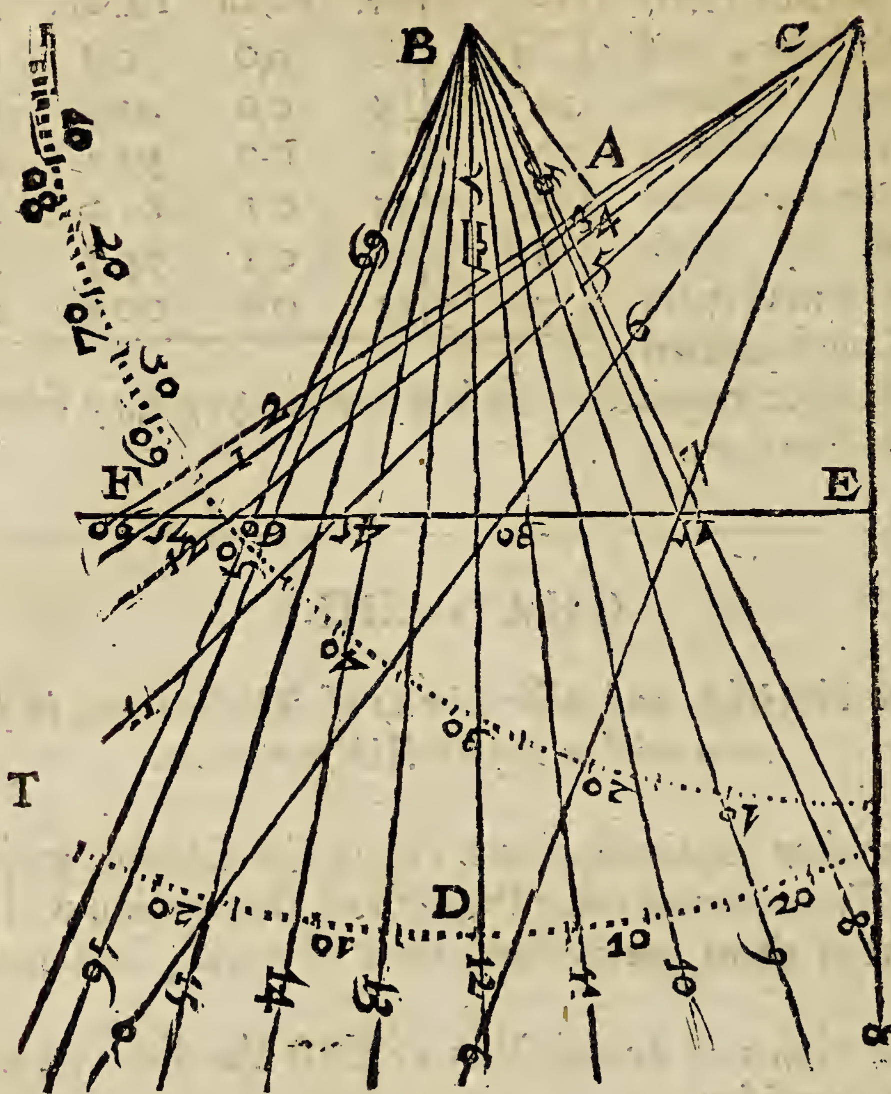
Secondly, The height of the Style above the Plane.

Thirdly, The Angles at the Pole between the proper Meridian and the Hour-circles.

Theſe being known, will help us to find, firſt, the Angle between the Axis and the Hour-lines on the Plane; and then the diſtance between the Center and the Parallels. Both theſe may be repreſented in this manner,

Let the Triangle A B C be made equal to the Style belonging to your Plane, A C the Subſtylar, B C the Axis of the Style, A B the length of the Style perpendicular to the Plane. Then having drawn the Line B D perpendicular to the Axis on the Center B, and any Semidiameter B D, deſcribe an occult Ark of a Circle, and therein inſcribe a Chord of 23 *gr.* 30 *m.* from D unto T, on either ſide of the Line, with ſuch other intermediate Declinations as you intend to deſcribe on the Plane; ſo the Per-

pendicular



pendicular B D ſhall be the Equator, and B T the Tropicks, and the other intermediate Lines the Parallels of Declination. Wherefore you may take out the diſtance C V from the Center to the Equator, and prick it down on the Subſtylar of your Plane from the Center at C unto V; ſo the Line drawn through ♈, perpendicular to your Subſtylar, ſhall be the Equator of your Plane.

That done, take the diſtance of each Hour-line between the Center and the Equator of your Plane, and prick them down in the Equator of this Figure, from the Center at C, noting the place where they croſs the Equator, with the Number belonging to the Hour, and drawing the Hour-lines from C, through the Lines of Declination.

Or, having the *Sector*, you may draw an occult Line C E, perpendicular to the Axis B C, and therein prick down the Tangent of the height of the Style above the Plane, from C unto E: Then draw the Line E F parallel to the Axis, croſſing the Subſtylar produced in the point F; this Line

Line E F will be the Line of Sines upon the Sector, and therein you may prick down the Sines of the Complement of the Angles at the Pole from E toward F, and draw the Hour-lines by those Points through the Lines of Declination; so the Angles at C, between the Axis B C and those Hour-lines, shall be the Angles between the Axis of your Style and the Hour-lines in your Plane; and the several Distances between the Point C and the Lines of Declination, shall give you the like Distances between the Center and the Parallels of Declination upon the Hour-lines in your Plane. Upon this ground it followeth,

1. *To proportion the Style unto the Plane.*

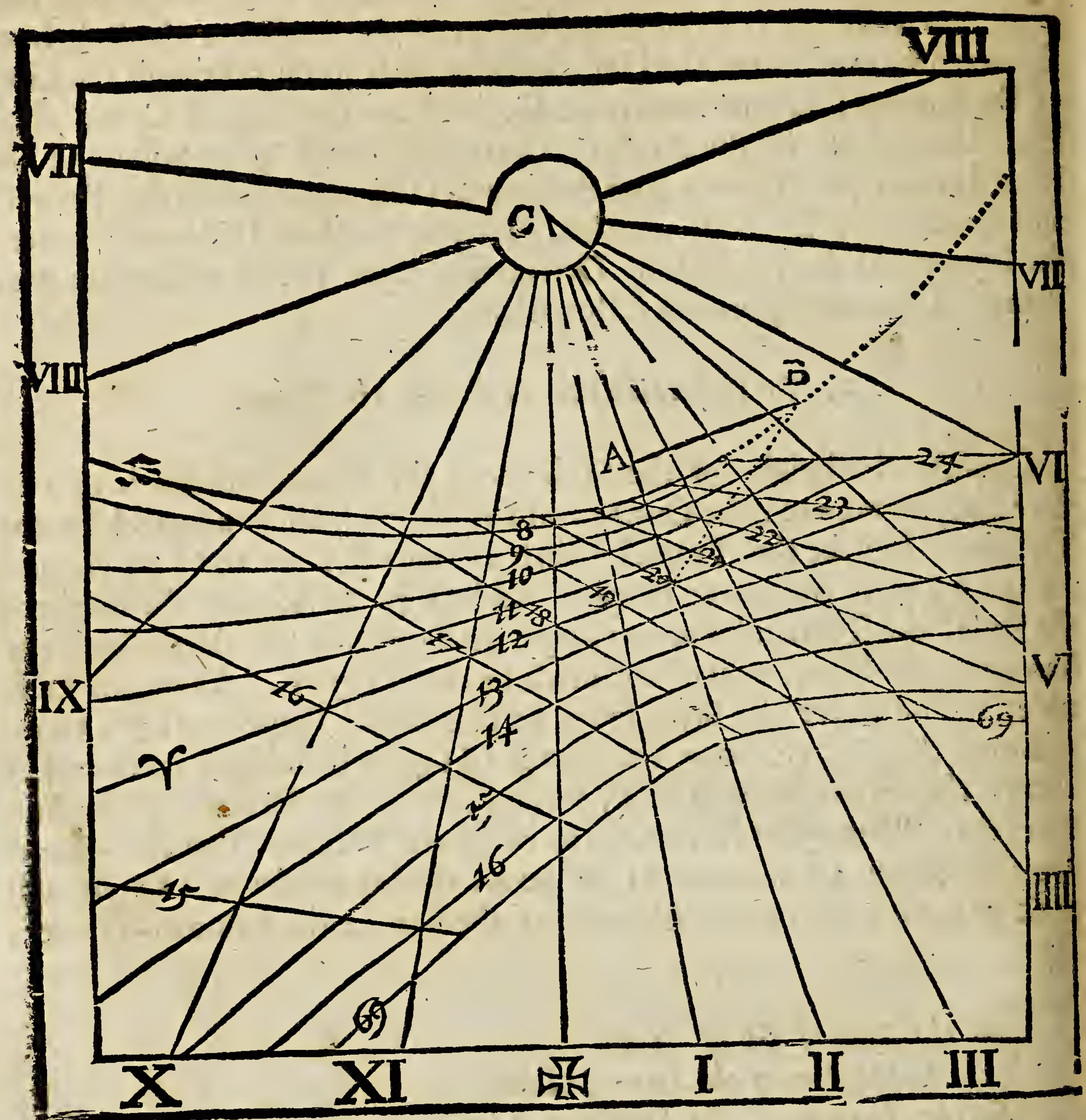
Consider the height of the Style above the Plane, and the length of the Substylar between the Center and the Place which you intend for the Tropick. If it be the Tropick which is farthest from the Center, add 113 *gr.* 30 *m.* if the nearer Tropick, add 66 *gr.* 30 *m.* unto the height of the Style, the Remainder unto 180 *gr.* shall give you the Altitude of the Sun above the Plane, when he cometh to that Tropick. As in our Latitude, the height of the Style above an Horizontal Plane is 51 *gr.* 30 *m.* add unto this 113 *gr.* 30 *m.* the sum is 165 *gr.* which being taken out of 180 *gr.* the remainder will be 15 *gr.* and such is the Altitude of the Sun above this Plane when he cometh to be in the Winter-Tropick: But if you add 66 *gr.* 30 *m.* unto 51 *gr.* 30 *m.* the remainder to 180 *gr.* will be 62 *gr.* And such is the Altitude of the Sun in the Summer-Tropick. Then,

As the Sine of 66 gr. 30 m.
to the Sine of the Suns Altitude:
So the Length of the Substylar Line,
to the Length of the Axis of the Style.

As in the first Examples of the Declining Vertical, where the height of the Style was found to be 34 *gr.* 33 *m.* and is here represented before, *pag.* 31. by the Angle B C ♋; add to this height 113 *gr.* 30 *m.* for the Angle C B ♋, the sum will be 148 *gr.* 3 *m.* and the remainder to 180 *gr.* will be 31 *gr.* 57 *m.* and such is the Angle B ♋ C of the Altitude of the Sun above the Plane, when he cometh to be in the Tropick of ♋, which is here the farthest Tropick from the Center.

Then supposing the length of the Substylar-line between the Center

 and



and the Place which is fit for the fartheſt Tropick, to be about 21 Inches
extend the Compaſſes from the Sine of *66 gr.* 30 *m.* unto the Sine of 31 *gr*
57 *m.* the ſame extent will reach in the Line of Numbers from 21 unt
12.11, and ſo the length of the Axis of the Style ſhould be 12 *inch.* 11 *cen*
Or it may ſuffice to make it juſt 12 Inches, as a more eaſie ground for th
reſt of the Work.

But if it were required to proportion the Style unto the Plane, ſo as
may caſt the Shadow to the full length of the Subſtylar-line at all times
the Year, you may then conſider the Sun in the Tropick, which is to be ſ
neareſt unto the Center, and add 66 *gr.* 30 *m.* unto 34 *gr.* 33 *m.* ſo th
remaind

remainder unto 180 *gr.* will be 78 *gr.* 57 *m.* And if you extend the Compaſſes from the Sine of 66 *gr.* 30 *m.* unto the Sine of 78 *gr.* 57 *m.* the ſame extent will reach in the Line of Numbers from 21 unto 22, 47 for the length of the Axis of the Style.

2. *Having the length of the Axis, and the height of the Style above the Plane, to find the length of the Sides of the Style.*

The Style of a Plane neither Equinoctial nor Polar, may be either a ſmall Rod of Iron ſet parallel to the Axis of the World, or perpendicular to the Plane, or elſe a thin Plate of Iron or Braſs, made in form of a Rectangle Triangle B A C, with the Baſe B C parallel to the Axis of the World, the Side A B perpendicular to the Plane, and the Side A C the ſame with the Subſtylar-line; wherein knowing B C, and the Angle B A C,

As the Sine of 90 gr.
to the Length of the Axis:
So the Sine of the Height of the Style,
to the Length of the Perpendicular Side:
And ſo the Co ſine of the Height of the Style,
to the Length of the Subſtylar ſide.

Thus in the former Example, the length of the Axis being ſuppoſed to be 12 Inches, and the height of the Style 34 *gr.* 33 *m.* Extend the Compaſſes from the Sine of 90 *gr.* (or elſe from the Sine of 5 *gr.* 45 *m.*) unto 12 in the Line of Numbers, the ſame extent will reach from the Sine of 34 *gr.* 33 *m.* unto 6.80 in the Line of Numbers, for the length of the perpendicular Side; and from the Sine of 55 *gr.* 27 *m.* unto 9.88 for the length of the Subſtylar ſide.

3. *To find the Diſtance between the Center and the Equator upon the Subſtylar Line.*

This is here repreſented by C ♈, and may be found by reſolving the Rectangle Triangle C B ♈.

As the Sine of the Height of the Style,
is to the Sine of 90 gr.
So the Length of the Axis,
To the Diſtance of the Equator from the Center.

 Extend

Extend the Compaſſes from the Sine of 55 *gr.* 27 *m.* unto the Sine of 90 *gr.* the ſame extent will reach in the Line of Numbers from 12 unto 14. 17. Wherefore if you take 14 *inch.* 57 *cent.* and pricking them down on your Subſtylar-line from C unto ♈, draw a Line through ♈, croſſing the Subſtylar at Right Angles, the Line ſo drawn ſhall be the Equator.

4. *To find the Angles contained between the Equator and the* ***Hour-lines*** *upon your Plane.*

Theſe Angles made by ♈ and the Hour-lines are Complements of thoſe which are at C, between B C the Axis and thoſe ſeveral Hour-lines, and depend upon the Angles at the Pole, between the proper Meridian and the Hour-circles.

As the Sine of 90 gr.
to Co-ſine the Angle at the Pole:
So the Co-tangent of the Height of the Style,
to the Tangent of the Angle between the Equator and the Hour-line:

In our Example the height of the Style is 34 *gr.* 33 *m.* and the proper Meridian falleth to be the ſame with the Circle of the ſecond Hour after Noon; whereupon the Angle at the Pole, between this proper Meridian, and the Circles of the Hour of 1 on the one ſide, and 3 on the other ſide, will be 15 *gr.* So between this Meridian and the Hour-circles of 10 and 4, the Angle will be 30 *gr.* *&c.* as in the Table.

Hour.		An. Po.		Ar. Pla.		An. Eqr		♈		♋		♑	
		Gr.	M	Gr.	M.	Gr.	M.	In.	P.	In.	P.	In.	P.
Subſtyl.		0	c	0	0	55	27	14	57	20	80	1	21
1	3	15	0	8	38	54	3c	14	74	21	36	11	25
12	4	30	0	18	8	51	3c	15	33	23	44	11	40
11	5	45	0	29	33	45	45	16	75	29	06	11	76
10	6	60	0	44	30	36	c	20	00	50	84	12	77
9	7	75	0	64	42	20	36	34	10	Infinit.		15	82
8	8	90	0	90	0	0	c	Infinit.				27	60

If then it be required to find the Angle which the Hour-line of 4 afte[r]noon doth make with the Plane of the Equator, that is the Angle C 4 B contained

ntained between the Hour-line C 4 and the Line B 4, drawn from the p of the Style unto the Intersection of the Hour-line of 4 with the quator.

Extend the Compasses from the Sine of 90 *gr.* unto the Sine of 60 *gr.* e Complement of the Angle at the Pole, the same extent will reach from e Tangent of 55 *gr.* 27 *m.* the Complement of the height of the Pole, nto the Tangent of 51 *gr.* 30 *m.* and such is the Angle C 4 B in the Diagram.

Or in Cross-work, if it were required to find the Angle C 9 B, look nto the Table for the Hour of 9, and there you shall find the Angle at he Pole to be 75 *gr.* and if you extend the Compasses from the Sine of 90 *gr.* unto the Tangent of 55 *gr.* 27 *m.* the same extent will reach from he Sine of 15 *gr.* the Complement of 75 *gr.* unto the Tangent of 20 *gr.* 36 *m.* and such is the Angle C 9 B, made at the Equator between the Line B 9, drawn from the top of the Style, and the Hour-line C 9, drawn from the Center. The like reason holdeth for the rest, which may be found and set down in a Table: Then may you either draw these Angles at C in the former Figure more perfectly, and thence finish your Work, or else proceed,

5. *To find the Distance between the Center and the Parallels of Declination.*

The Distances between the Center and the Parallels of Declination may be found by resolving the Triangles made by the Axis B C, the Lines of Declination, and the Hour-lines. For having the Angles at the Equator, and knowing the Declination of the Parallel, if the Parallel shall fall between the Equator and the Center, add the Declination unto the Angle at the Equator: or if it shall fall without the Equator, take the Declination out of the Angle at the Equator, so shall you have the Angle at the Parallel. Then,

As the Sine of the Angle at the Parallel,
to the Co-sine of the Declination:
So the length of the Axis of the Style,
to the Distance between the Center and the Parallel.

Thus in our Example, the Angle at the Equator belonging to the Hour of 4 after-noon was found before to be 51 *gr.* 30 *m.* if you would find the

the distance between the Center and the Equator, extend the Compass from the Sine of 51 *gr.* 30 *m.* unto the Sine of 90 *gr.* the Complemen of the Declination, the same extent will reach in the Line of Numbe from 12 unto 15. 33, and such is the distance upon the Hour-line of between the Center and the Equator.

If you would find the distance upon this Hour-line between the Cente and the inner Tropick, whose Declination is known to be 23 *gr.* 30 *m* add the Declination to the Angle at the Equator, so the Angle at the Pa rallel will be 75 *gr.* wherefore extend the Compasses from the Sine of 75 *g* unto the Sine of 66 *gr.* 30 *m.* the Complement of the Declination, th same extent will reach in the Line of Numbers from 12 unto 11. 40, an such is the length of the Hour-line of 4 between the Center and the Tro pick of ♑.

If you would find the distance upon this Hour-line between this Cen ter and the Tropick of ♋, which is here the farthest from the Cente take the Declination out of the Angle at the Equator, so the Angle at th Parallel will be 28 *gr.* wherefore extend the Compasses from the Sine 28 unto the Sine of 66 *gr.* 30 *m.* the same extent will reach in the Li of Numbers from 12 unto 23. 44, and such is the distance between th Center and Tropick of ♋ upon this Hour-line of 4. The like reaso holdeth for all the rest, which may be gathered and set down in a Tabl

That done, and the Equator drawn as before, if you would draw t Tropick of ♋, look into the Table, and there finding under the Ti C ♋ the distance of the Substylar between the Center and the Parallel ♋ to be 20 *inch.* 80 *cent.* take 20 *inch.* 80 *cent.* out of the Line of Inch and prick them down in the Substylar of your Plane from C unto ♋.

Or if either the Center fall without your Plane, or the extent be t large for your Compasses, you may prick down the difference betwe C ♈ and C ♋: As here the distance C ♈ between the Center and t Equator is 14. 57, the distance C ♋ 20. 80, the difference 6. 23. The fore taking ♈ 6 *inch.* 23 *cent.* prick them down on the Substylar from unto ♋, and you shall have the same Intersection of the Tropick a the Substylar as before: And the like reason holdeth for pricking dow of the rest of these Distances on their several Hour-lines.

Then having the Points of Intersection between the Hour-lines and t Parallel, you may joyn them all in a crooked Line, without making any Angles, the Line so drawn shall be the Tropick required. And aft this manner you may draw any other Parallel of Declination, wher you have Examples in most of the former Diagrams.

CHA

CHAP. XIV.

To describe the Parallels of the Sines in any of the former Planes.

THe Equator and the Tropicks before described do shew the Suns entrance into 4 of the Signs; the Equator into ♈ and ♎, the one Tropick into ♋, and the other into ♑: The rest of the intermediate Signs will be described in the same manner as the Tropicks, if first we know their Declination.

The manner of finding the Declination, not onely of the beginning of the Signs, but all other Points of the Ecliptick, is before set down in 2 *Prop. Astronomical*, by which you may find the Declination of the beginning of ♉, ♍, and ♏, ♓ to be 11 *gr.* 30 *m.* and of ♊, ♌, ♐ and ♒ to be 20 *gr.* 12 *m.* If then you inscribe the Chords of 11 *gr.* 30 *m.* and of 20 *gr.* 12 *m.* into the former Figure B D T, *pag.* 64. from D toward T, the Lines drawn from B through the Terms of those Chords shall be the Signs required.

And with these Declinations, the height of the Style, and the length of the Axis, you may find the Angles at the Parallel, and then the Distances between the Center and the Parallel, which being pricked down upon the several Hour-lines, shall give you the Points of Intersection, by which you may draw the Parallels of the Signs, as in the Figures belonging to the Polar Planes.

CHAP. XV.

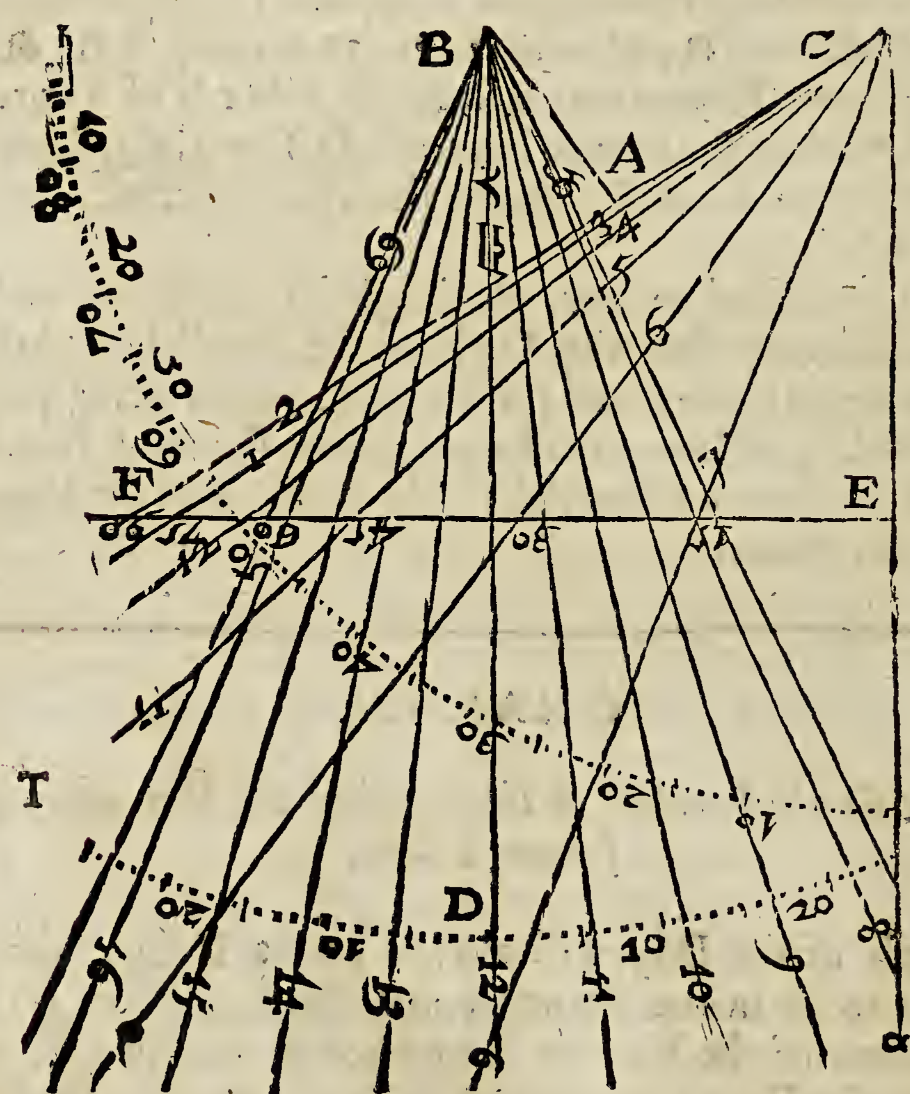
To describe the Parallels of the length of the Day in any of the former Planes.

THe length of the Day will always be 12 Hours, when the Sun cometh to be in the Equator, and this holdeth in all Latitudes: but at other times of the Year the same place of the Sun will not give the same length of the Day in another Latitude; wherefore the Latitude being known, we are first, *To find the Declination of the Sun agreeing to the length of the Day.*

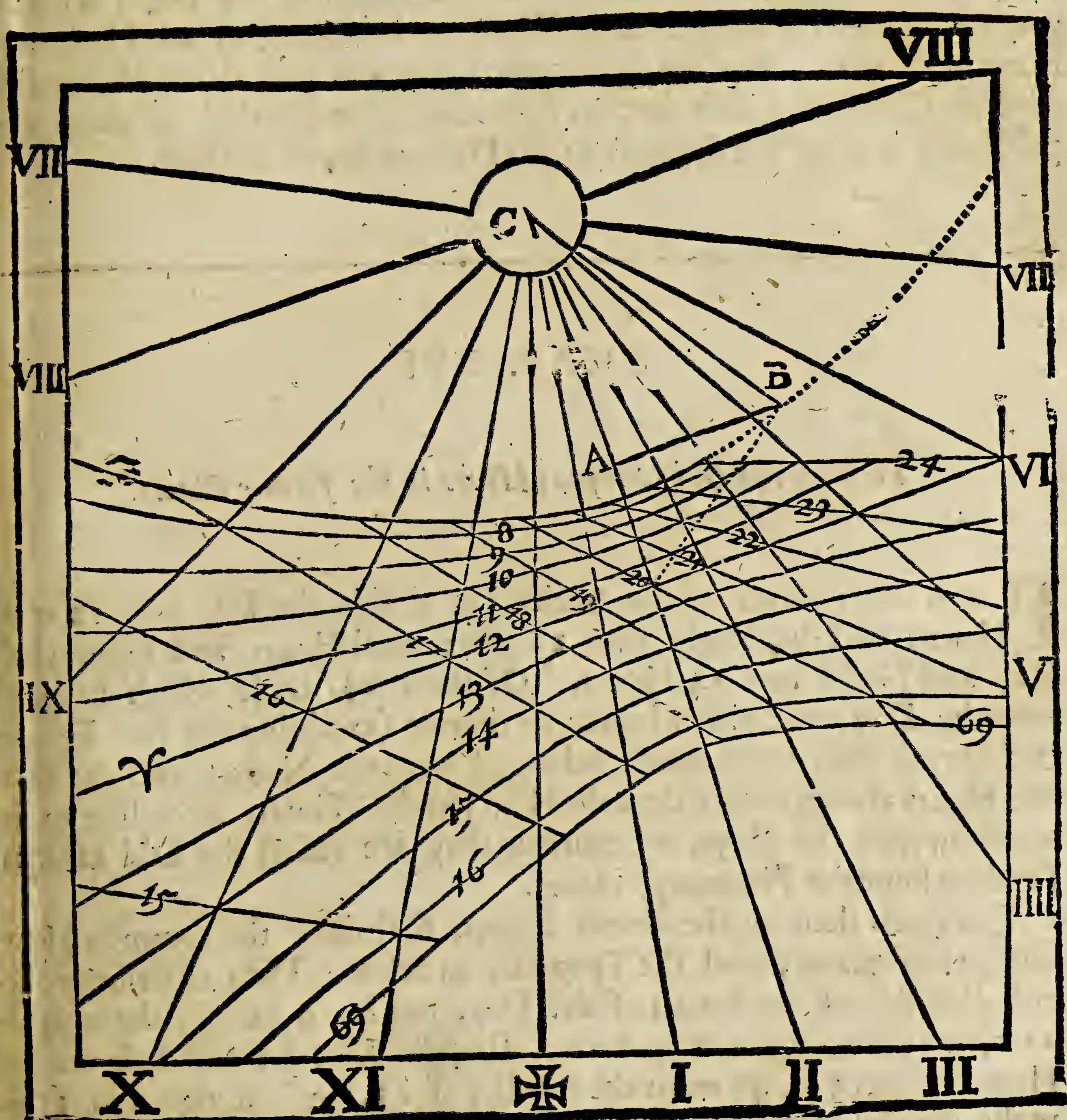
Consider the difference between the length of an Equinoctial day, an the day proposed, and turn the time into Degrees and Minutes.

As the Sine of 90 gr.
is to the Sine of half the difference:
So the Co-tangent of the Latitude,
to the Tangent of the Declination.

As if the length of the Day proposed were 15 Hours, the differen between this and an Equinoctial Day (whose length is always 12 Hour



would be three Hours(which make 45 *gr.*)and the half difference is 22 g 30 *m*. Wherefore extend the Compasses from the Sine of 90 *gr.* unto t Tangent of 38 *gr.* 30 *m.* the Complement of the Latitude, the same e t tent will reach from the Sine of 22 *gr.* 30 *m.* unto the Tangent of 16 *gr.* 55 *m.* for the Declination of the Sun at ſuch time as the length of the Day is either 9 or 15 Hours; and from the Sine of 30 *gr.* unto the Tangent of 21 *gr.* 40 *m.* for the Declination belonging to 8 or 16 Hours; and from the Sine of 15 *gr.* unto the Tangent of 11 *gr.* 38 *m.* for the Declination belonging to 10 or 14 Hours; and from the Sine of 7 *gr.* 30 *m.* unto the Tangent of 5 *gr.* 56 *m.* for the Declination of the Sun when the length of the Day is either 11 or 13 Hours.



If then you inſcribe the Chords of theſe Arks into the former Figure B D T, the Lines drawn from B through the Terms of theſe Arks ſhall

 be

be the Lines belonging to the Diurnal Arks, and the ſeveral Diſtances between them and the Point C, give the like Diſtances between the Center and the Parallels of the length of the Day upon the Hour-lines in your Plane.

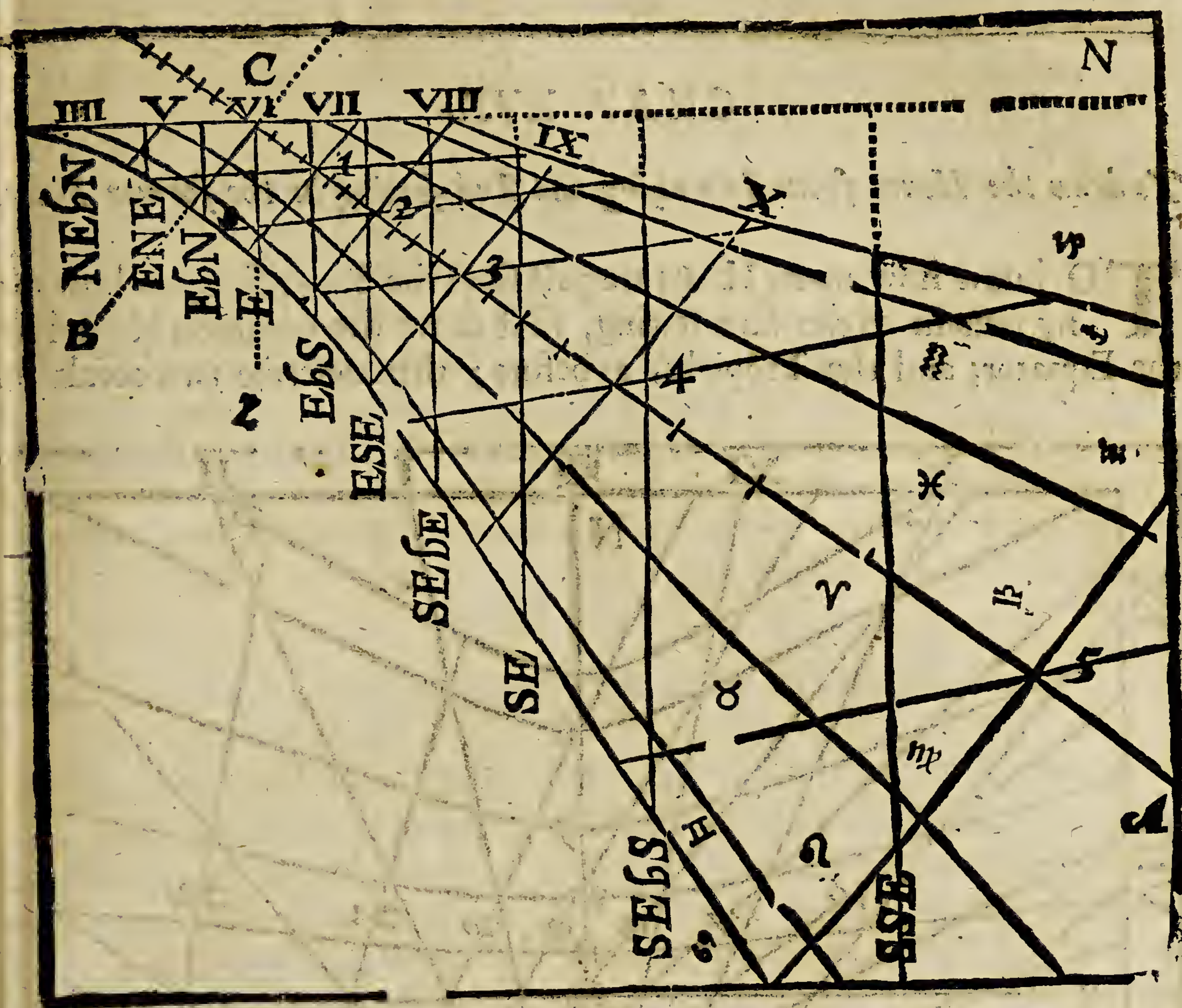
Or comparing theſe Angles of Declination with the Angles at the Equator, you may have the Angles at the Parallel, and then find the Diſtances between the Center and the Parallel, which being pricked down upon the ſeveral Hour-lines, ſhall give you the Points of Interſection, by which you may draw the Parallels of the length of the Day, whereof you have another Example in the Diagram belonging to an Horizontal Plane in *Chap.* 4. And by the ſame reaſon you may draw the Parallels of thoſe Circles to which the Sun is Vertical, the Parallels of the principal Feaſt, or what elſe depends on the Declination of the Sun.

CHAP. XVI.

To draw the Old Unequal Hours in the former Planes.

IT was the manner of the Ancients to divide the Day into 12 equa Hours, and the Night into 12 other equal Hours, and ſo the whol Day and Night into 24 Hours. Of theſe 24, thoſe which belonge unto the Day were either longer or ſhorter (excepting the two Equinc ctial Days) than thoſe which belonged unto the Night; and the Sum mer Hours always longer than the Hours in the Winter, according to th lengthning of the Days, whereupon they are called the Old Unequ (and by ſome the Planetary) Hours.

To expreſs theſe in the former Planes, firſt draw the common Hou lines, the Equator, and the Tropicks, as before: Then deſcribe two o cult Parallels of the length of the Day, one for 9 Hours, the other f 15 Hours; for ſo you may draw a ſtraight Line for the firſt unequ Hour through 5 *ho.* 45 *m.* in the Parallel of 15, and through 8 *ho.* 15 in the Parallel of 9. This ſtraight Line ſhall paſs directly through 7 *h* 0 *m.* in the Equator, and ſo cut off a twelfth part of the Arks above t Horizon, both from theſe two Parallels and the Equator; and bei continu



continued unto the Tropicks, it ſhall alſo cut off about a twelfth part from them, and all the reſt of the Parallels of Declination, without any ſenſible errour.

In like manner you may draw the ſecond unequal Hour through 7 *hour* in the Parallel of 15, through 8 *hour* in the Equator, and through 9 *hour* in the Parallel of 9, and ſo in the reſt, as in this Table.

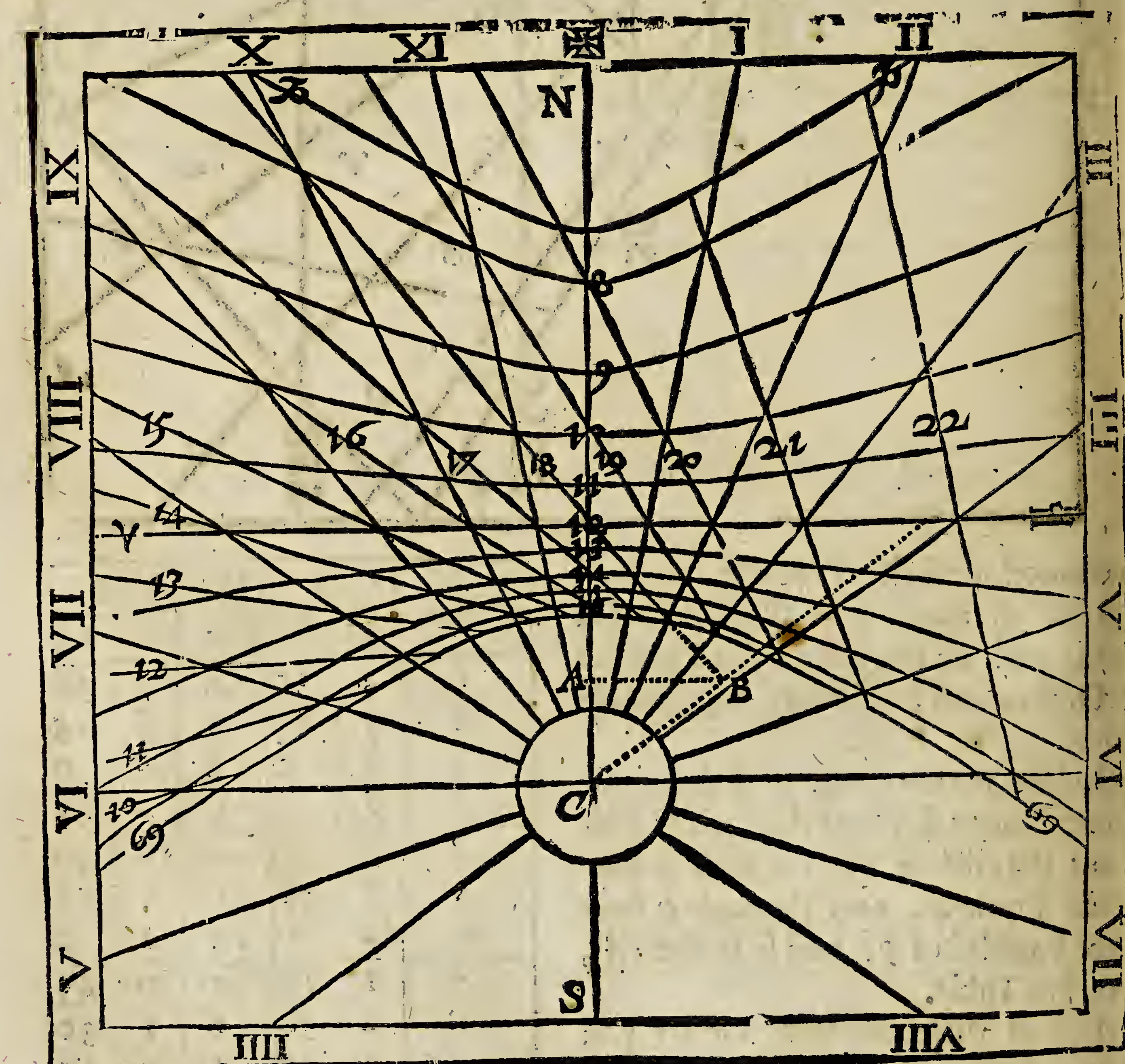
And of theſe unequal Hours you have a further Example in the Diagram belonging to the Polar Declining Plane.

Hours.	15		Ho.	9	
	Ho.	M.	Equ.	Ho.	M.
0	4	30	6	7	30
1	5	45	7	8	15
2	7	0	8	9	0
3	8	15	9	9	45
4	9	30	10	10	30
5	0	45	11	11	15
6	12	0	12	12	0
7	1	15	1	0	45
8	2	30	2	1	30
9	3	45	3	2	15
10	5	0	4	3	0
11	6	15	5	3	45
12	7	30	6	4	30

CHAP. XVII.

To draw the Hours from Sun-rising and Sun-setting in the former Planes.

TO know how many Hours are paſt ſince the Sun-riſing, or how many remain to the Sun-ſetting, firſt draw the common Hour-lines, the Equator, and the Tropicks, as before; then deſcribe two occult Pa-



rallels of the length of the Day, one for 8 Hours, and the other for 16 Hours: For ſo you may draw the firſt Hour from the Sun-riſing through the common Hours of 5 in the Parallel of 16, of 7 in the Equator, and f 9 in the Parallel of 8. In like manner, the ſecond Hour from Sun-iſing, through the common Hours of 6 in the Parallel of 16, of 8 in the Equator, and of 10 in the Parallel of 8. And ſo the reſt in their order.

The firſt Hour before Sun-ſetting, or the 23 Hour from the laſt Sun-etting, may be drawn in like ſort, through the common Hours of 3 after-noon in the Parallel of 8, of 5 in the Equator, and of 7 in the Parallel of 16. The ſecond Hour before Sun-ſetting, or the 22 Hour after the laſt Sun-ſetting, through the common Hours of 2 in the Parallel of 8, of 4 in the Equator, and of 6 in the Parallel of 16: And ſo the reſt in the like order, whereof you have another Example in the Diagram belonging to the Declining Vertical.

CHAP. XVIII.

To draw the Horizontal-line in the former Planes.

THe common Hour-lines do commonly depend on the ſhadow of the Axis; but the Parallels of the Signs, and of the length of the Day, the Hour-lines from Sun-riſing and Sun-ſetting, with many others, depend on the Shadow of the top of the Style, or ſome other Point in the Axis, which here ſignifieth the Center of the World, and is repreſented by the Point B. And theſe Lines ſo depending are then onely uſeful, when they fall between the two Tropicks, and within the Horizon.

There may be ſeveral Horizontal-lines drawn upon every Plane, as I ſhewed before in finding the Inclination of a Plane; but the proper Horizontal-line, which is here meant, muſt always be in the ſame Plane with B the top of the Style; ſo that in an Horizontal Plane there can be no ſuch Horizontal-line: but in all other Planes it may be found by applying the Horizontal Leg of the Sector unto the top of the Style, and then working as before; and the Interſection of this Line with the Meridian or Subſtylar-line may be found by Proportion.

1. *To find the Interſection of the Horizon with the Meridian in an Equinoctial Plane.*

As the Tangent of 45 gr.
to the Tangent of the Latitude:
So is the Height of the Style,
to the Diſtance between the Style and the Horizontal-line.

As

As in the Example of the former Equinoctial Plane, extend the Compasses from the Tangent of 45 *gr.* unto 51 *gr.* 30 *m.* the Tangent of the Latitude, the same extent will reach in the Line of Numbers from 51 the length of the Style, unto 66, and such is the Distance between the Style and the Horizontal-line: Wherefore I take 66 parts out of a Line of Inches, and prick them down in the Meridian-line from C unto H above the Style in the upper Face, but below the Style in the lower Face of the Plane; so a Right Line drawn through H, parallel to the Hour of 6, shall be the Horizontal-line.

2. *To find the Intersection of the Horizon with the Meridian in a Direct Polar Plane.*

As the Tangent of 45 gr.
to the Co-tangent of the Latitude:
So the length of the Style,
to the distance between the Style and the Horizontal-line.

As in the Example of the former Polar Plane, extend the Compasses from the Tangent of 45 *gr.* unto the Tangent of 38 *gr.* 30 *m.* the Complement of the Latitude, the same extent will reach in the Line of Numbers from 1. 61 the length of the Style, unto 1. 28, and such is the distance upon the Meridian between the Style and the Horizontal-line.

In all upright Planes, whether they be Direct, Vertical or Declining or Meridian Planes, the Horizontal-line must always be drawn through A the Foot of the Style, as may appear in the Examples before.

And generally, in all Planes whatsoever, the Horizontal-line must be drawn through the Intersection of the Equator with the Hour of 6. Or if that Intersection fall without the Plane, yet if any Arks of the length of the Day be drawn on the Plane, the Horizontal-line may be drawn through their Intersections with the Hours of the Suns rising or setting.

CHAP

CHAP. XIX.

To describe the Vertical Circles in the former Planes.

THe Vertical Circles, commonly called Azimuths, are Great Circles drawn through the Zenith, by which we may know in what part of the Heaven the Sun is, how far from the East or West, and how near unto the Meridian.

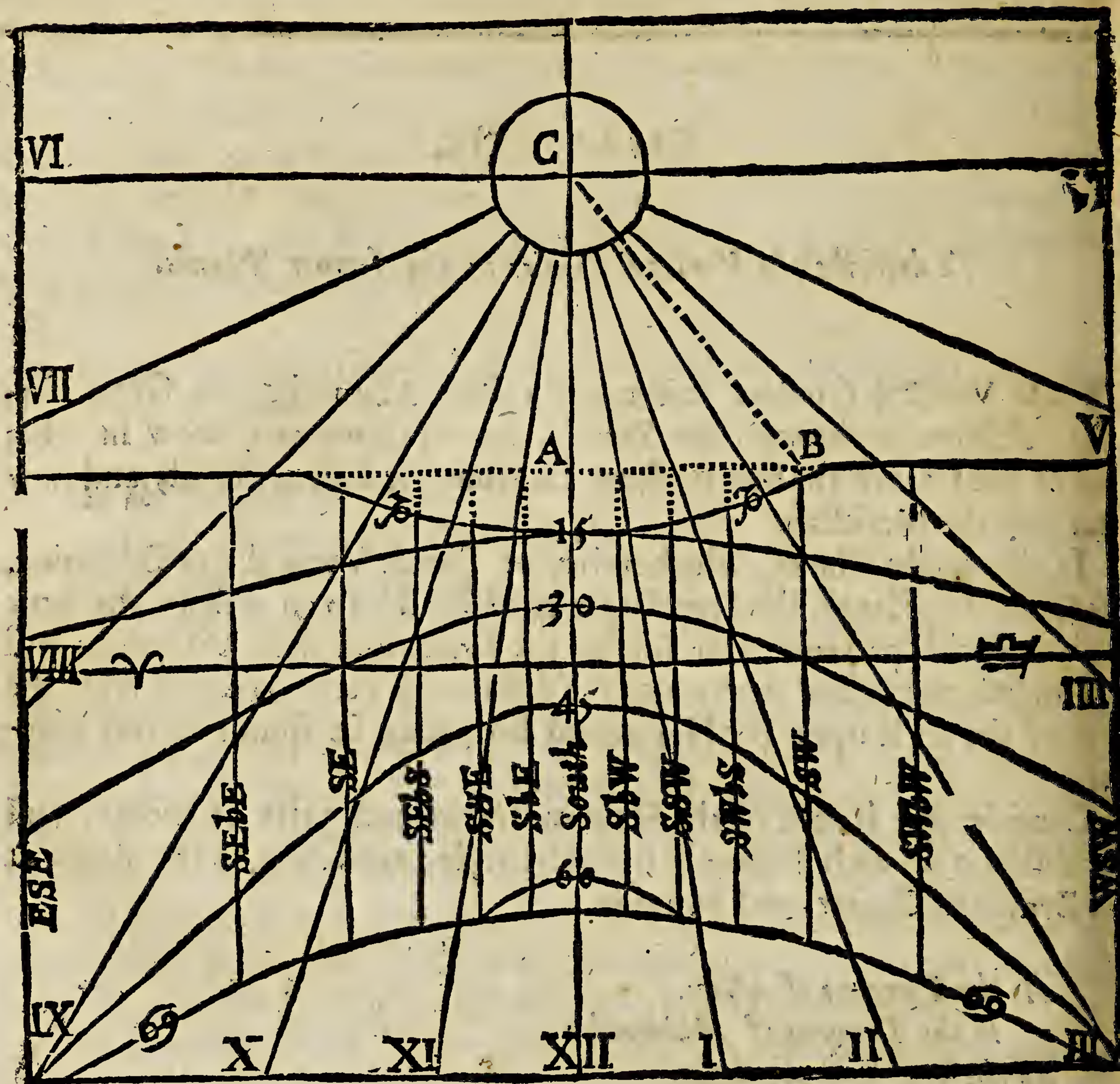
In all upright Planes, whether they be Direct Verticals, or Declining, or Meridian Planes, the Semidiameter of the Horizon will be the same with A B the perpendicular side of the Style, and these Azimuths will be Parallels one to the other, and the distance of each Azimuth from the Foot of the Style upon the Horizontal line, may be found in this manner.

Consider the length of the Style in Inches and parts of Inches, and the distance of each Azimuth from the Style, according to the Angle at the Zenith in Degrees and Minutes.

As the Tangent of 45 gr.
to the Tangent of Azimuth:
So the length of the Style,
to the length of the Horizontal-line between the Style and the Azimuth.

As if it were required to draw the common Azimuths on the South Face of the Vertical Plane before described, where A B the length of the Style may be supposed to be 10 Inches.

Here the Plane having no declination, the Style is in the Plane of the Meridian, and so pointeth directly into the South. The Point of *S b E* is 11 *gr.* 15 *m.* distant from the Style, and *S S E* 22 *gr.* 30 *m.* and the rest in their order: Wherefore extend the Compasses from the Tangent of 45 *gr.* unto 10 in the Line of Numbers, the same extent will reach from the Tangent of 11 *gr.* 15 *m.* unto



Azimuths.	An. Zen. Gr.	M.	Tang. In.	Pa.
South	0	0	0	0
S b E	11	15	1	99
S S E	22	30	4	14
S E b S	33	45	6	68
S E	45	0	10	00
S E b E	56	15	14	97
E S E	67	30	24	14
E b S	78	45	50	27
East.	90	0	Infinit.	

unto 1.99 in the Line of Numbers fo the length of the Tangent-line, betwee the Style and the Point *S b E*; an from the Tangent of 22 *gr.* 30 *m.* unt 4. 14 for *S S E*: And so for the rest as in this Table.

In like manner, in the first Exam ple of the Declining Plane, where th Style standeth according to the Declina tion 24 *gr.* 20 *m.* distant from the Sout toward the West, the next Point o

S b W

Azimuths.	An. Zen.		Tang.	
	Gr.	M.	In.	Pa.
S E b E	80	35	41	00
S E	69	20	18	03
S E b S	58	5	10	91
S S E	46	50	7	25
S b E	35	35	4	86
South	24	20	3	07
S b W	13	5	1	58
S S W	1	50	0	22
The	*Foot*	*of*	*the Style*	
S W b S	9	25	1	13
S W	20	40	2	57
S W b W	31	55	4	24
W S W	43	10	6	37
W b S	54	25	9	50
West	65	40	15	02
W b N	76	55	19	26
W N W	88	10	22	45

S b W is but **13** *gr.* **5** *m.* distant from the Style; and the second of *S S W* onely **1** *gr.* **50** *m.* and the third of *S W b S* is again *9 gr.* 25 *m.* and the rest in their order. Wherefore having before found the length of the Style to be *6* Inches 80 parts, extend the Compasses from the Tangent of 45 *gr.* unto 6. 80 parts in the Line of Numbers, the same extent will reach from the Tangent of 24 *gr.* 20 *m.* unto 3. 07 in the Line of Numbers, for the length of the Tangent-line between the Style and the South; and from the Tangent of **13** *gr.* 5 *m.* unto **1.** 58 for the Point of *S b W*: and so for the rest, as in this Table.

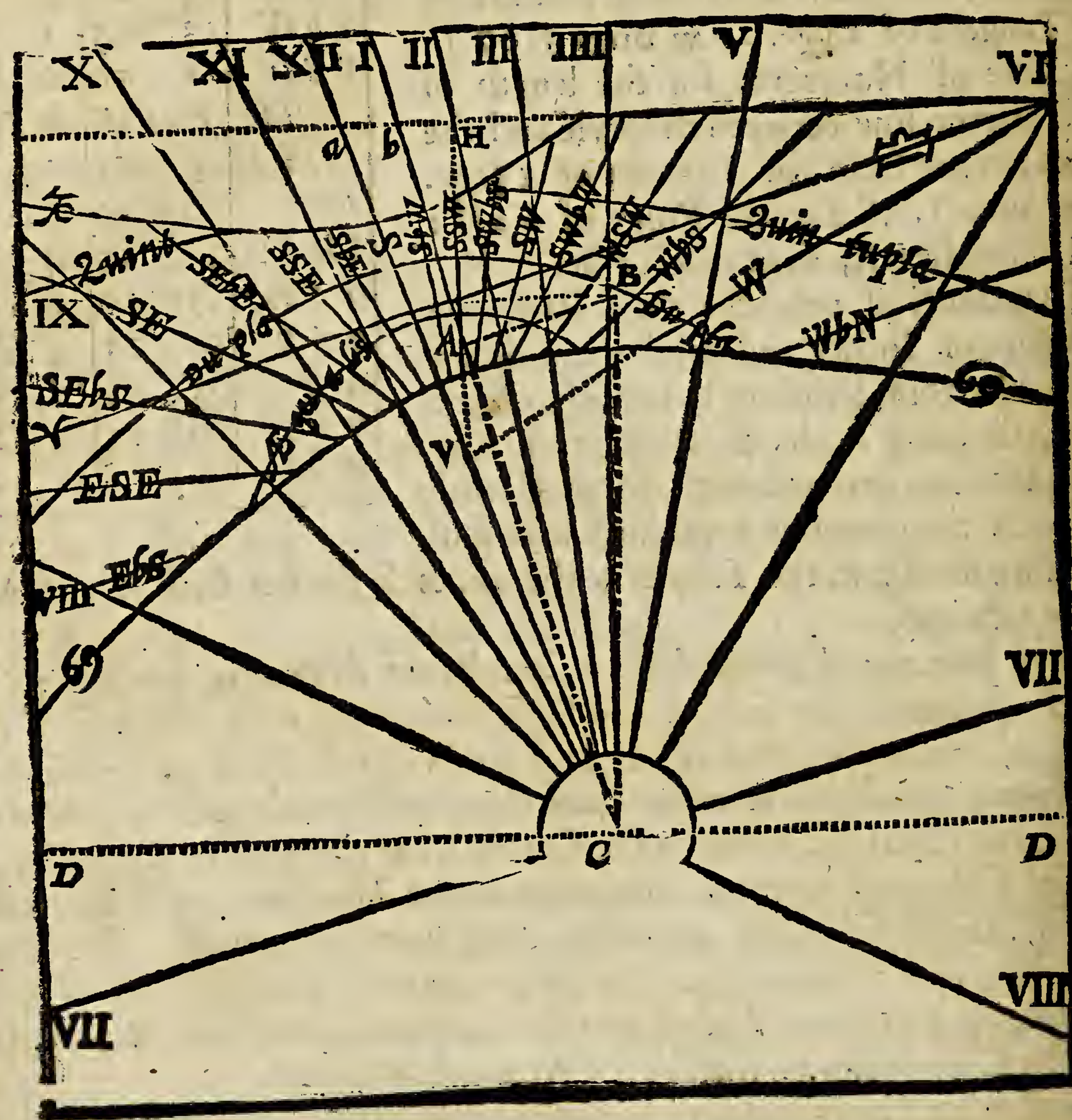
That done, if you take these Parts out of a Line of Inches, and prick them down in the Horizontal-line on either side of the Style, drawing Right Lines perpendicular to the Horizon through these Intersections, but so as they may be contained between the Horizontal and the Tropicks, the Lines so drawn shall be the Azimuths required.

In an Horizontal Plane these Azimuths are drawn more easily: For here the perpendicular side of the Style is the same with the Axis of the Horizon, and the Foot of the Style is the Vertical Point, in which all the Azimuth-lines do meet, as their Circles do in the Zenith: Wherefore let any Circle described on the Center A, at the Foot of the Style, be divided first into four parts, beginning at the Meridian; and then each quarter subdivided either into eight equal parts, according to the Points of the Mariners Compass, or into *90 gr.* according to the Astronomical division; if you draw Right Lines through the Center and these divisions, the Lines so drawn shall be the Azimuths required.

In all other Planes inclining to the Horizon, these Vertical Circles will meet in a Point; but that Vertical Point being more or less distant from the Foot of the Style, the Angles at this Point will be unequal.

1. To find the distance between the Foot of the Style and the Vertical Point.

The Vertical Point, wherein all the Vertical Lines do meet, will be always in the Meridian, directly under or over the top of the Style; and the Angle between the perpendicular side of the Style, and the Vertical line, will be equal to the Inclination of the Plane to the Horizon. Wherefore,



As the Tangent of 45 gr.
to the Tangent of the Inclination of the Plane:
So is the length of the Style,
to the distance between the Foot of the Style and the Vertical Poin

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Thus in the firſt Example of the Declining Inclining Plane, where the upper Face of the Plane looking South-weſt, the Declination was 24 *gr.* 20 *m.* the Inclination 36 *gr.* and you may ſuppoſe A B the length of the Style to be 6 Inches; if you extend the Compaſſes from the Tangent of 45 *gr.* unto the Tangent of 36 *gr.* the ſame extent will reach in the Line of Numbers from 6. 00 unto 4. 36, for the diſtance A V, between A the Foot of the Style and V the Vertical Point.

2. *To find the diſtance between the Foot of the Style and the Horizontal-line.*

As the Tangent of the Inclination of the Plane,
is to the Tangent of 45 gr.
So the length of the Style,
to the diſtance between the Foot of the Style and the Horizontal-line.

So the ſame extent of the Compaſſes as before will reach in the Line of Numbers from 6. 00 unto 8. 26 for the diſtance A H between the Foot of the Style and the Horizontal-line.

Then may you take 4 *inch.* 36 *cent.* and pricking them down from A the Foot of the Style, unto V the Vertical Point in the Meridian, draw the Line V A, which being produced, ſhall cut the Horizon in the Point H with Right Angles, and be that particular Azimuth which is perpendicular to the Plane.

Or, you may take 8 *inch.* 26 *cent.* and prick them down in the former Line V A, produced from A unto H, and ſo draw the Horizontal-line through H, perpendicular unto V H, which Horizontal-line being produced, will croſs the Equator in the ſame Point wherein the Equator croſſeth the Hour-line of 6, unleſs there be ſome former error.

3. *To find the Angles made by the Azimuth-lines at the Vertical Point.*

The Angles at the Zenith depend on the Declination of the Plane, as in our Example, where the Style ſtandeth according to the Declination 24 *gr.* 20 *m.* diſtant from the South toward the Weſt, the Azimuth of 10 *gr.* from the Meridian Eaſtward will be 34 *gr.* 20 *m.* the Azimuth of 10 *gr.* Weſtward will be onely 14 *gr.* 20 *m.* diſtant from the Style; and ſo the reſt in their order.

Or if you would rather describe the common Azimuths, the Point of *S b E* will be 35 *gr.* 35 *m.* the Point of *S b W* 13 *gr.* 5 *m.* distant from the Style; and so the rest in their order. Then,

As the Sine of 90 gr.
to the Co-sine of the Inclination of the Plane:
So the Tangent of the Angle at the Zenith,
to the Tangent of the Angle at the Vertical Point, between the Line drawn through the Foot of the Style, and the Azimuth required.

Azimuths.	Ang.Ze.		Ang.V	
	Gr.	M.	Gr.	M.
S E b E	80	35	78	25
S E	69	20	65	0
S E b S	58	5	52	25
S S E	46	50	40	40
S b E	35	35	30	3
South	24	20	20	5
S b W	13	5	10	39
S S W	1	50	1	29
	Style.		0	20
S W b S	9	25	7	38
S W	20	40	16	58
S W b W	31	55	26	45
W S W	43	10	37	11
W b S	54	25	48	30
West	65	40	60	48
W b N	76	55	73	58
W N W	88	10	87	42

Wherefore the Inclination of the Plane in our Example being 36 *gr.* extend the Compasses from the Sine of 90 *gr.* unto the Sine of 54 *gr.* the same extent shall reach in the Line of Tangents from 24 *gr.* 20 *m.* unto 20 *gr.* 5 *m.* for the Angle HV *a* at the Vertical Point, between the Line VH, drawn through A the Foot of the Style, and the South. Again, the same extent will reach from the Tangent of 13 *gr.* 5 *m.* unto 10 *gr.* 38 *m.* for the Angle belonging to *S b W*; and so for the rest, a in this Table.

These Angles being known, if on the Center V, at the Vertical Point, you describe an occult Circle, and therein inscribe the Chords of these Angles from the Line VH, and then draw Right Lines through the Vertical Point, and the Terms of those Chords, the Lines so drawn shall be the Azimuths required.

The like reason holdeth for the drawing of the Azimuths upon all other Inclining Planes, whereof you have another Example in the Diagram belonging to the Meridian Incliner, as before.

Or, for further satisfaction you may find where each Azimuth-line shall cross the Equator.

As the Sine of 90 gr.
to the Sine of the Latitude:

So the Tangent of the Azimuth from the Meridian,
to the Tangent of the Equator from the Meridian.

Extend the Compaſſes from the Sine of 90 *gr.* unto the Sine of our Latitude 51 *gr.* 30 *m.* the ſame Extent will reach in the Line of Tangents from 10 *gr.* unto 7 *gr.* 50 *m.* for the Interſection of the Equator with the Azimuth of 10 *gr.* from the Meridian. Again, the ſame extent will reach from 20 *gr.* unto 15 *gr.* 54 *m.* for the Azimuth of 20 *gr.* And ſo the reſt, as in theſe Tables.

Azim.		*Equat.*	
Gr.	M.	Gr.	M.
10	0	7	50
20	0	15	54
30	0	24	20
40	0	33	18
50	0	43	0
60	0	53	35
70	0	65	3
80	0	77	18
90	0	90	0

Azim.		*Equat.*	
Gr.	M.	Gr.	M.
11	15		51
22	30	18	58
33	45	27	36
45	0	38	2
56	15	49	30
67	30	62	6
78	45	75	44
90	0	90	0

By which you may ſee that the Azimuth 90 *gr.* diſtant from the Meridian, which is the Line of Eaſt and Weſt, will croſs the Equator at 90 *gr.* from the Meridian, in the ſame Point with the Horizontal-line and the Hour of 6: and that the Azimuth of 45 *gr.* will croſs the Equator at 38 *gr.* 2 *m.* from the Meridian; that is, the Line of S E will croſs the Equator at the Hour of 9 and 28 *m.* in the Morning, and the Line of S W at 2 *ho.* 32 *m.* in the Afternoon: And ſo for the reſt, whereby you may examine your former Work.

CHAP. XX.

To deſcribe the Parallels of the Horizon in the former Planes.

THe Parallels of the Horizon, commonly called Almicanters, or Parallels of Altitude (whereby we may know the Altitude of the Sun above the Horizon) have ſuch reſpect unto the Horizon, as the Parallels of Decli-

Declination unto the Equator, and so may be described in like manner.

In an Horizontal Plane these Parallels will be perfect Circles; wherefore knowing the length of the Style in Inches and parts, and the distance of the Parallels from the Horizon in Degrees and Minutes,

As the Tangent of 45 gr.
is to the length of the Style:
So the Co-tangent of the Parallel,
to the Semidiameter of his Circle.

Thus in the Example of the Horizontal Plane, if A B the length of the Style shall be 5 Inches, and that it were required to find the Semidiameter of the Parallel of 62 *gr.* extend the Compasses from the Tangent of 45 *gr.* unto 5. 00 in the Line of Numbers, the same extent will reach from the Tangent of 28 *gr.* the Complement of the Parallel, unto 2. 65: And if you describe a Circle on the Center A, to the Semidiameter of 2 *inch.* 65 *cent.* it shall be the Parallel required.

In all upright Planes, whether they be Direct Verticals, or Declining, or Meridian Planes, these Parallels will be Conical Sections, and may be drawn through their Points of Intersection with the Azimuth-lines, in the same manner as the Parallels of Declination through their Points of Intersection with the Hour-lines. To this end, you may first find the distance between the top of the Style and the Azimuth, and then the distance between the Horizon and the Parallel, both which may be represented in this manner.

On the Center B, and any Semidiameter B H, describe an occult Ark of a Circle, and therein inscribe the Chords of such Parallels of Altitude as you intend to draw on the Plane, (I have here put them for 15, 30, 45 and 60 *gr.*) then draw Right Lines through the Center and the Terms of those Chords, so the Line B H shall be the Horizon, and the rest the Lines of Altitude, according to their distance from the Horizon.

That done, consider your Plane, (which here for example is the South Face of our Vertical Plane) wherein having drawn both the Horizontal and Vertical Lines, as I shewed before, first take out A B the length of the Style, and prick that down in this Horizontal Line from B unto A, then take out all the distances between B the top of the Style and the several Points wherein the Vertical Lines do cross the Horizontal, transfer them into this Horizontal-line B H, from the Center B, and at the Terms of these distances erect Lines perpendicular to the Horizon, noting them

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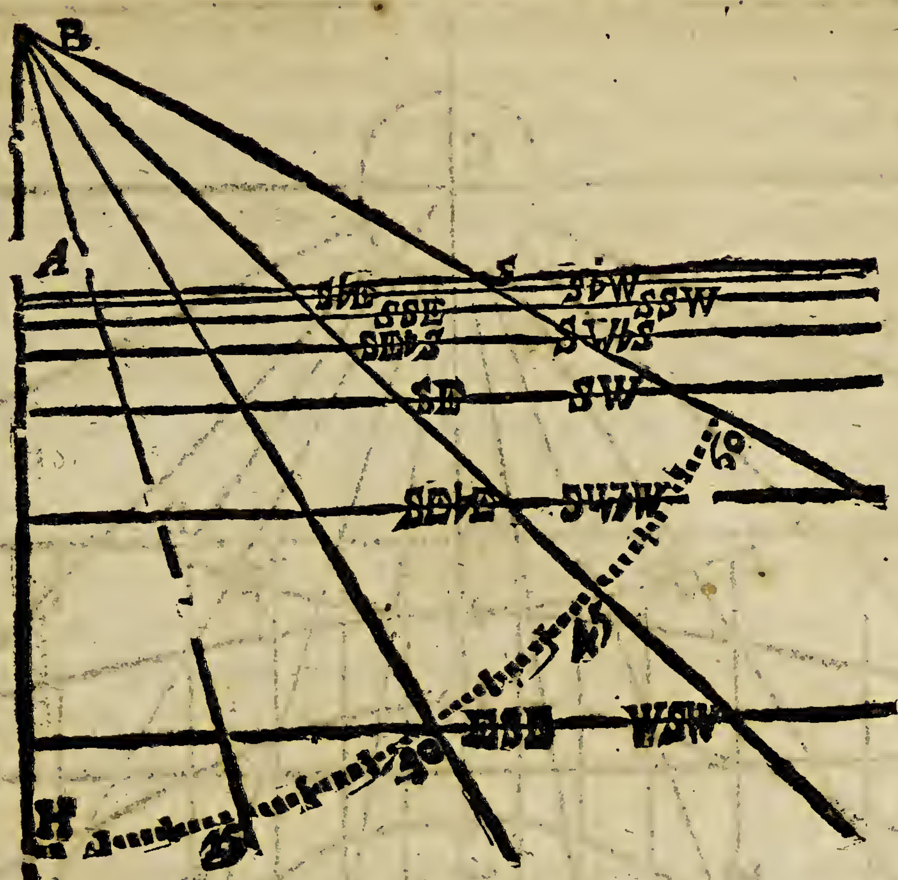


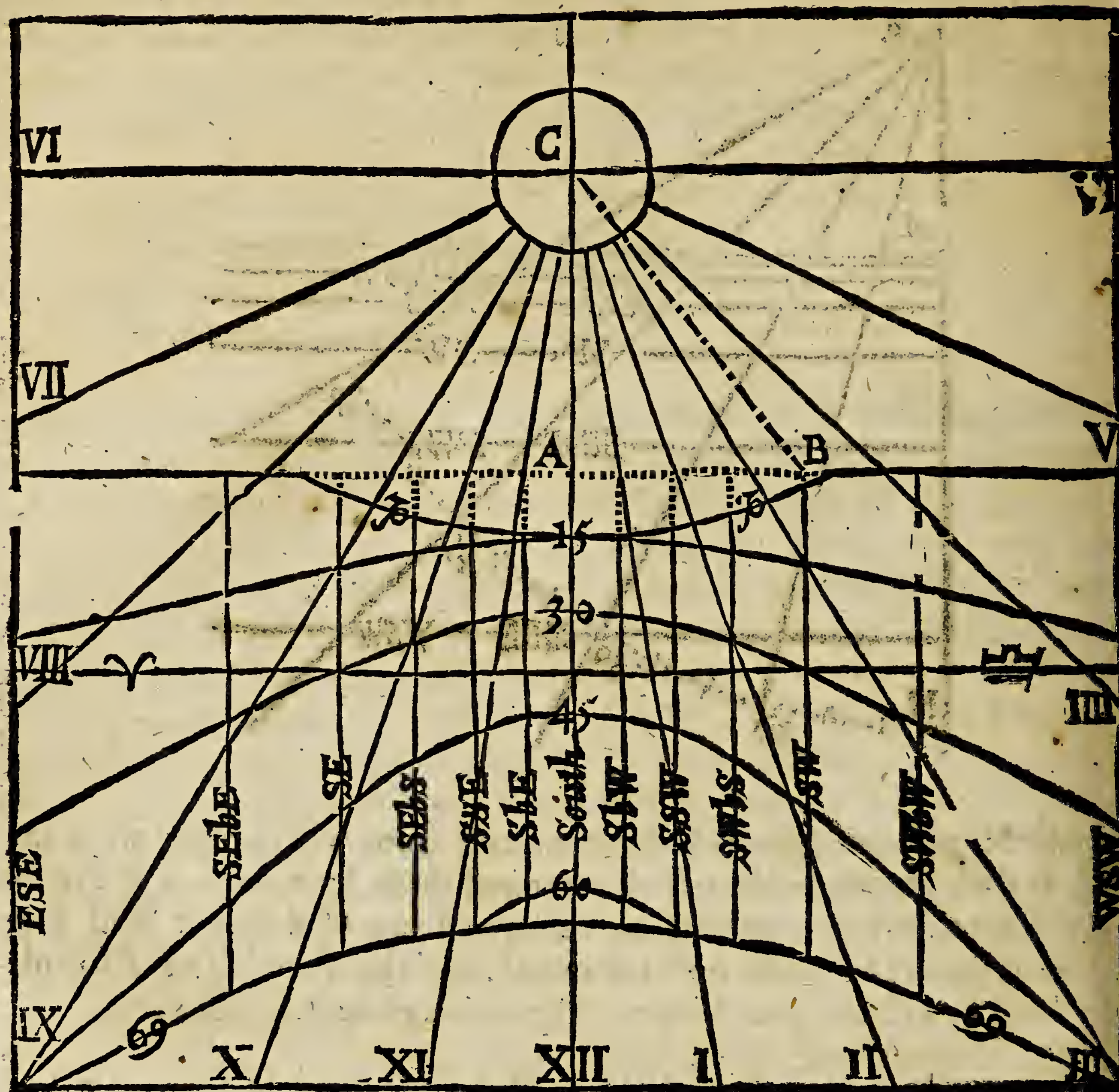
with the Number or Letter of the Azimuth from whence they were taken; so these Perpendiculars ſhall repreſent thoſe Azimuths, and the ſeveral diſtances between the Horizon and the Lines of Altitude ſhall give the like diſtances between the Horizontal and the Parallels of Altitude upon the Azimuths in your Plane. Upon this ground it followeth,

1. *To find the diſtance between the top of the Style, and the ſeveral Points wherein the Azimuths do croſs the Horizontal-line.*

Having drawn the Horizontal and Azimuth Lines as before, look into the Table by which you drew them, and there you ſhall have the Angles at the Zenith. Then,

As the Co ſine of the Angle at the Zenith,
is to the Sine of 90 *gr.*
So the length of the Style,
to the Diſtance required.

Azimuths.



Azi-muths.	Ang.Ze. Gr.	M.	Tangent In.	Pa.	Secant. In.	Pa.	Par. 15. In.	Pa.	Par. 30. In.	Pa.
South	0	0	0	0	10	00	2	68	5	77
S b E	11	15	1	99	10	20	2	73	5	90
S S E	22	30	4	14	10	82	2	90	6	14
S E b S	33	45	6	68	12	03	3	23	6	94
S E	45	0	10	00	14	14	3	80	8	16
S E b E	56	15	14	97	18	00	4	82	10	40
E S E	67	30	24	14	26	13	7	02	15	08
E b S	78	45	50	27	51	26	13	73	29	60
East.	90	0	Infinit.		Infinit.		Infinit.		Infinit.	

As in our Example of the Vertical Plane, where A B the length of the Style was supposed to be 10 Inches, extend the Compasses from the Sine of 78 *gr.* 45 *m.* (the Complement of 11 *gr.* 15 *m.* the Angle at the Zenith, belonging to *S b E* and *S b W*) unto the Sine of 90 *gr.* the same extent will reach from 10. 00 the length of the Style, unto 10. 20, for the distance between the top of the Style and the Intersection of the Azimuth *S b E* with the Horizontal line, which distance may be called the Secant of the Azimuth, and may serve for the drawing of the Parallel of 45 *gr.* from the Horizon. The like reason holdeth for the rest of these distances here represented in the Line B H.

2. *To find the distance between the Horizon and the Parallels.*

As the Tangent of 45 gr.
to the Tangent of the Parallel:
So the Secant of the Azimuth,
to the Distance required.

As if it were required to draw the Parallel of 15 *gr.* from the Horizon, upon this Vertical Plane; extend the Compasses from the Tangent of 45 *gr.* unto the Tangent of 15 *gr.* the same extent will reach in the Line of Numbers from 10. 00 the Secant of the South Azimuth, unto 2. 68, and therefore the distance between the Horizon and the Parallel of 15 *gr.* is 2 *inch.* 68 *cent.* upon the South Azimuth. Again, the same extent will reach from 10. 20 the Secant of *S b E*, unto 2. 73, for the like distance belonging to *S b E* and *S b W*: And so for the rest, which may be gathered and set down in the Table.

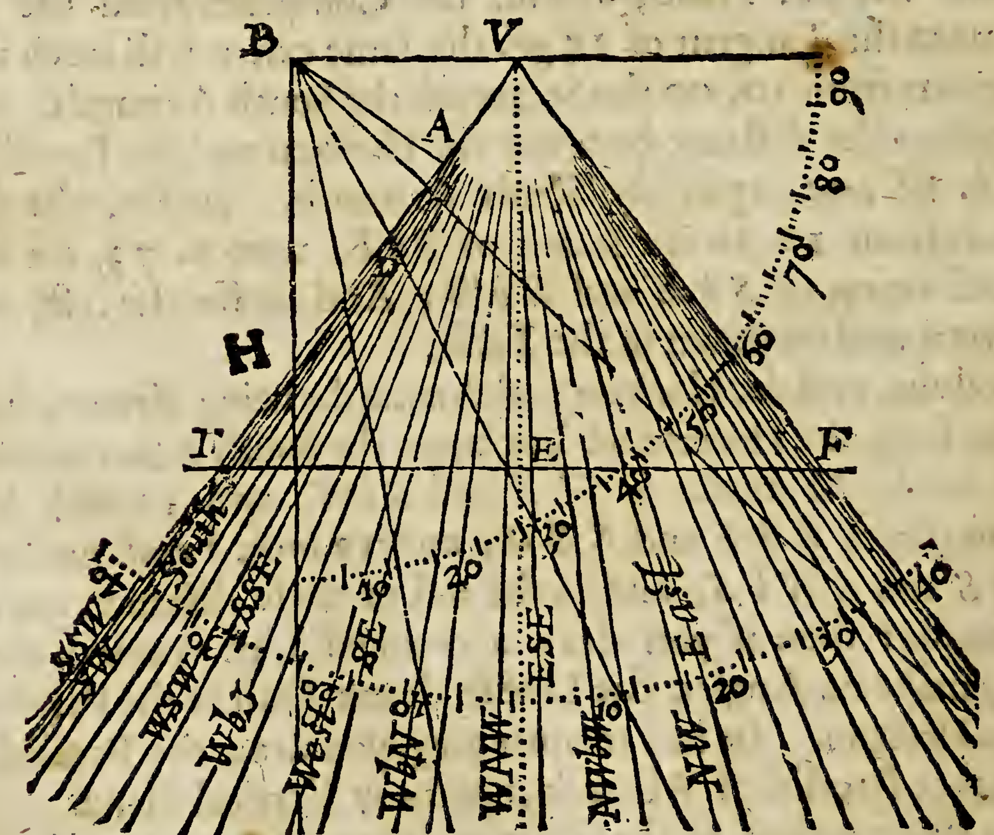
That done, and the Horizon and Azimuths being drawn, prick down 10 Inches from the Horizontal-line upon the South Azimuth, and 10 *inch.* 20 *cent.* on the Azimuths of *S b E* and *S b W*, and 10 *inch.* 82 *cent.* on the Azimuths of *S S E* and *S S W*, and 12 *inch.* 3 *cent.* on the Azimuths of *S E b S* and *S W b S*, and so the rest of these distances on their several Azimuths: then if you draw a crooked Line through these Points, that may make no Angles, the Line so drawn shall be the Parallel of 45 *gr.* from the Horizon. In like manner may you draw the Parallel of 15 *gr.* or any other Parallel of Altitude, upon any Vertical Plane.

If the Plane incline to the Horizon, after we have found the Vertical Point, and drawn the Horizontal-line, we are farther to find the length of the Axis of the Horizon, then the Angles betwixt this Axis and the

Azimuth-lines, and ſo the ſeveral diſtances between the Parallels and the Vertical Point, all which may be repreſented in this manner.

On the Center B, and any Semidiameter, deſcribe an occult Quadrant of a Circle, and therein inſcribe the Chords of ſuch Parallels of Altitude as you intend to draw on the Plane, drawing Right Lines through the Center and the Terms of theſe Chords, ſo the Line B H ſhall be the Horizon, and his Perpendicular B V the Axis of the Horizon, and the reſt the Lines of Altitude, according to their diſtance from the Horizon.

That done, conſider your Plane, which here, for example, is the firſt of of our three Declining Inclining Planes; wherein having drawn both the Horizontal and Vertical Lines, as I ſhewed before, firſt take out the Axis of the Horizon, which is the Line between B the top of the Style and V the Vertical Point, and prick that down in this Figure from B unto V; then take out both the Line V H, and all the reſt of the diſtances between V the Vertical Point and the ſeveral Points wherein the Vertica Lines do croſs the Horizontal Line of this Figure from the Point V, notin the place where they croſs the Horizontal-line, with the Number or Lette of the Azimuth from whence they were taken, and drawing the Azi muth-lines from V through the Lines of the Altitude.



Or having the Sector, you may draw an occult Line V E, perpendic lar to the Axis V B, and therein prick down the Tangent of the Comp

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ment of the Inclination of the Plane from V unto E: then draw the Line E F parallel to the Axis, croffing the Line V H produced in the Point F; fo this Line E F will be as the Line of Sines upon the Sector, and therein you may prick down the Sines of the Complement of the Angles at the Zenith from E towards F, and draw the Vertical-lines by thofe Points through the Lines of Altitude; fo the Angles at V, between the Axis V B and thofe Azimuth-lines, fhall be the Angles between the Axis of the Horizon and the Azimuth-lines on your Plane, and the feveral diftances between the Point V and the Lines of Altitude fhall give the like diftances between the Vertical Point and the Parallels of Altitude upon the Azimuths in your Plane. Upon this ground it followeth,

1. *To find the length of the Axis of the Horizon.*

The Vertical Point is always either directly over or under the top of the Style, and the diftance between them is that which I call the Axis of the Horizon, which may thus be found:

As the Co-fine of the Inclination,
to the Sine of 90 gr.
So the length of the Style,
to the length of the Axis of the Horizon.

For example, in the firft of the three Declining Inclining Planes, the Inclination to the Horizon is 36 *gr.* the length of the Style A B 6 Inches; extend the Compaffes from the Sine of 54 *gr.* the Complement of the Inclination, unto the Sine of 90 *gr.* the fame extent will reach in the Line of Numbers from 6. 00 unto 7. 42; and fuch is V B the length of the Axis required.

2. *To find the Angles contained between the Horizon and the Vertical Lines upon our Plane.*

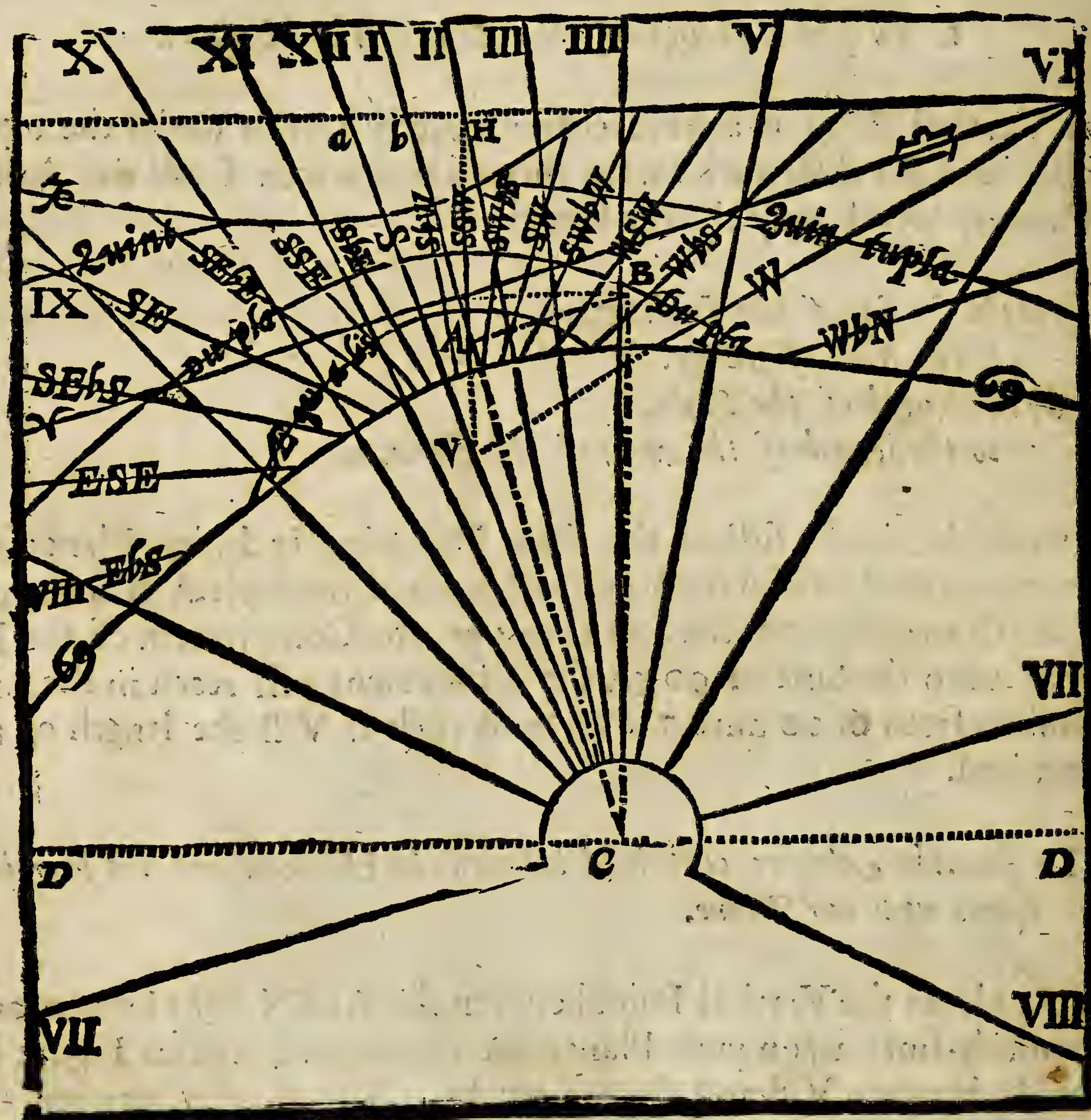
The Angles at the Vertical Point between the Axis of the Horizon and the Azimuth-lines upon your Plane, are reprefented in this Figure by thofe at V, between V B and the Azimuths. The Angles between the Horizon and the Azimuth-lines being Complements to the former, are reprefented either by thofe which are made by V E, or by B H, and the Azimuth-lines which are drawn from V.

 That

That you may find them, look into the Table by which you drew the Azimuth-lines, there shall you find the Angles at the Zenith. Then,

As the Sine of 90 gr.
to the Co-sine of the Angle at the Zenith:
So the Tangent of Inclination to the Horizon,
to the Tangent of the Angle between the Horizon and the Vertical Line.

In our example, where the Inclination to the Horizon is 36 *gr.* and the Angle at the Zenith, between the Azimuth at the Style and the Meridian,



is according to the Declination 24 *gr.* 20 *m.* extend the Compasses from the Sine of 90 *gr.* unto the Tangent of 30 *gr.* the same extent will reach from

:om the Sine of 65 *gr.* 40 *m.* the Complement of the Angle at the Zenith, nto the Tangent of 33 *gr.* 30 *m.* for the Angle contained between the Iorizon and the South part of the Meridian-line. Again, the fame ex- ent will reach from the Co-fine of 35 *gr.* 35 *m.* the Angle at the Zenith elonging to *S b E,* unto the Tangent of 30 *gr.* 3 *m.* for the Angle be- ween the Horizon and the Azimuth-line of *S b E.* The like reafon oldeth for the reft, which may be found and fet down in the Table.

Azimuths.	Ang.Ze.		Ang.V.		An. Ho.		Horiz		11	18	26	34	45	0
	Gr.	M.	Gr.	M.	Gr.	M.	In.	Pa.	In.	Pa.	In.	Pa.	In.	Pa.
East.	114	20	119	12	16	40	Infi-				38	60	11	05
E b S	103	5	106	2	9	20	nite.		210	24	22	40	9	00
E S E	91	50	92	16	1	20			41	98	15	57	7	60
S E b E	80	35	78	25	6	47	62	82	23	44	12	07	6	68
S E	69	20	65	0	14	23	29	87	16	79	10	12	6	00
S E b S	58	5	52	25	21	0	20	70	13	61	8	99	5	79
S S E	46	50	40	46	26	25	16	68	11	90	8	31	5	53
S b E	35	35	30	3	30	35	14	58	10	90	7	90	5	42
South	24	20	20	5	33	30	13	44	10	32	7	66	5	35
S b W	13	5	10	39	35	17	12	84	10	02	7	55	5	33
S S W	1	50	1	29	35	59	12	62	9	90	7	47	5	31
	Style.		0	0	36	0	12	62	9	90	7	47	5	31
S W b S	9	25	7	38	35	37	12	74	9	96	7	50	5	32
S W	20	40	16	58	34	12	13	20	10	20	7	59	5	34
S W b W	31	55	26	45	31	46	14	13	10	67	7	81	5	39
W S W	43	10	37	11	27	55	15	85	11	50	8	15	5	49
W b S	54	25	48	30	22	55	19	05	12	94	8	73	5	66
West	65	40	60	48	16	40	25	87	15	51	9	60	5	96
W b N	76	55	73	58	9	20	45	75	20	64	11	32	6	46
W N W	88	10	87	44	1	20	318	88	33	27	14	18	7	25
N W b W	99	25	101	35	6	47	Infi-		92	40	19	60	8	48
N W	10	40	115	00	14	23	nite.				31	44	10	30

Then may you either draw thefe Angles at V in the former Figure more perfectly, and thence finifh your Work, or elfe proceed.

3. *To find the distance between the Vertical Point, and the Parallel of the Horizon.*

These distances may be found by resolving the Triangles in the last Figure made by the Axis, the Lines of Altitude, and the Azimuth lines. For having the length of the Axis, and the Angle at the Horizon, if you add the distance of the Parallel from the Horizon, unto the Angle at the Horizon, you shall have the Angle at the Parallel. Then,

As the Sine of the Angle at the Parallel,
to the Co-sine of the Altitude:
So the length of the Axis,
to the distance between the Vertical Point and the Parallel.

Thus in our Example, if it were required to find the distance upon th Stylar Azimuth V H, between the Vertical Point and the Horizon, yo have the Rectangle Triangle V B H, wherein the Angle at the Horizo here represented by B H V is (equal to the Inclination of the Plane) *36 gr* and B V the Axis of the Horizon between the Plane and the top of th Style is *7 inch. 42 cent.* Wherefore extend the Compasses from the Si of *36 gr.* unto the Sine of *90 gr.* the Complement of the Altitude, th same extent will reach in the Line of Numbers from 7. 42 unto 12.62 and such is the distance of the Perpendicular Azimuth-line V H, betwee the Vertical Point and the Horizon.

In like manner, if you would find the distance upon the Meridia between the Vertical Point and the Horizon, extend the Compasses fro the Sine of 33 *gr.* 30 *m.* the Angle at the Horizon, to the Sine of 90 *g* the same extent will reach in the Line of Numbers from 7.42 unto 13.4 and such is V *a* the distance between the Vertical Point and the Horizo upon the Line of the South Azimuth, that is, upon the Meridian-line.

But if you would find the distance upon the Meridian between t Vertical Point and any other Parallel of the Horizon, as upon the Para lel of 26 *gr.* 34 *m.* then add these 26 *gr.* 34 *m.* unto 33 *gr.* 30 *m.* t Angle at the Horizon, so shall you have 60 *gr.* 4 *m.* for B D V the A gle at the Parallel. And if you extend the Compasses from the Sine of 60 *g* 4 *m.* unto the Sine of 63 *gr.* 26 *m.* the Complement of the Parallel fro the Horizon, the same extent will reach in the Line of Numbers fro 7. 42 the length of the Axis, unto 7. 66, and such is the distance V

betwe

between the Vertical Point and the Parallel of *26 gr. 34 m* upon the Meridian-line. The like reason holdeth for all the rest, which may be gathered and set down in the Table.

That done, and the Horizon drawn as before, if you would draw the Parallel of *26 gr. 34 m.* from the Horizon, look into the Table, and there finding under the Title of the Parallel of 26. 34 the distance on the South Azimuth-line to be 7. 66, take 7 *inch.* 66 *cent.* out of a Line of Inches, and prick them down on the Meridian of your Plane from the Vertical Point at V.

Or if either the Vertical Point fall without your Plane, or the extent at any time be too large for your Compasses, you may prick down the distance between the Horizon and the Parallel. As here the distance between the Vertical Point and the Parallel is 7.66, between the Vertical Point and the Horizon 13.44, the difference between them 5. 78 is the distance from the Horizon to the Parallel, which being pricked down upon the Meridian, shall give the same Intersection as before. And the like reason holdeth for the pricking down the rest of their distances on their several Azimuths.

Having the Point of Intersection between the Azimuths and the Parallel, you may joyn them all in a crooked Line, without making of Angles; the Line so drawn shall be the Parallel required. And upon this ground it followeth,

To describe such Parallels on the former Planes, as may shew the proportion of the Shadow unto the Gnomon.

The proportion of a Mans Shadow unto his Height, or other Shadow to his Gnomon, set perpendicular to the Horizon, may be shewed by Parallels to the Horizon, if they be drawn to a due Altitude, which may thus be found:

As the length of the Shadow,
to the length of the Gnomon:
So the Tangent of 45 gr.
to the Tangent of the Altitude.

As if it were required to find the Altitude of the Sun when the Shadow of a Man shall be decuple to his Height, extend the Compasses from 10 unto 1 in the Line of Numbers, the same extent will reach in the Tangent

gent of 45 *gr.* unto the Tangent of 5 *gr.* 42 *m.* which ſhews that when the Sun cometh to the Altitude of 5 *gr.* 42 *m.* your Shadow upon a level Ground will be ten times as much as your Height. In the ſame manner you may find, that at 7 *gr.* 7 *m.* of Altitude your Shadow will be a duple, at 9 *gr.* 27 *m.* ſextuple, at 11 *gr.* 18 *m.* quintuple, at 14 *gr.* 2 *m.* quadruple, at 18 *gr.* 26 *m.* triple, at 26 *gr.* 33 *m.* double to your Height, at 33 *gr.* 41 *m.* as 3 unto 2, at 36 *gr.* 52 *m.* as 2 unto 3, at 34 *gr.* 39 *m.* as 5 unto 4, at 45 *gr.* equal, at 51 *gr.* 20 *m.* as 4 unto 5, at 53 *gr.* 7 *m.* as 3 unto 4, at 56 *gr.* 19 *m.* as 4 unto 3, at 58 *gr.* 2 *m.* as 3 unto 5, at 63 *gr.* 26 *m.* as 1 unto 2, *&c.*

If then you draw a Parallel to the Horizon at 5 *gr.* 42 *m.* another at 7 *gr.* 7 *m.* and ſo the reſt, when the ſhadow of the Style falleth on the Parallel, you have the proportion, and thereby may you know the Shadow by the Height, and the Height by the Shadow, whereof you have an example *pag.* 8.

I might here proceed to ſhew the Deſcription of the Circles of Poſition, the Signs of the Zodiack in the Meridian, the Signs aſcending and deſcending, with ſuch other Gnomonical Concluſions: but theſe would prove ſuperfluous to ſuch as underſtand the Doctrine of the Sphere; and for others, that which is delivered may ſuffice for ordinary uſe, it being my intention not ſo much to explain the full uſe of Shadows, (whereof I have lately given a large example in another place) as the uſe of theſe Lines of Proportion, that were not extant heretofore.

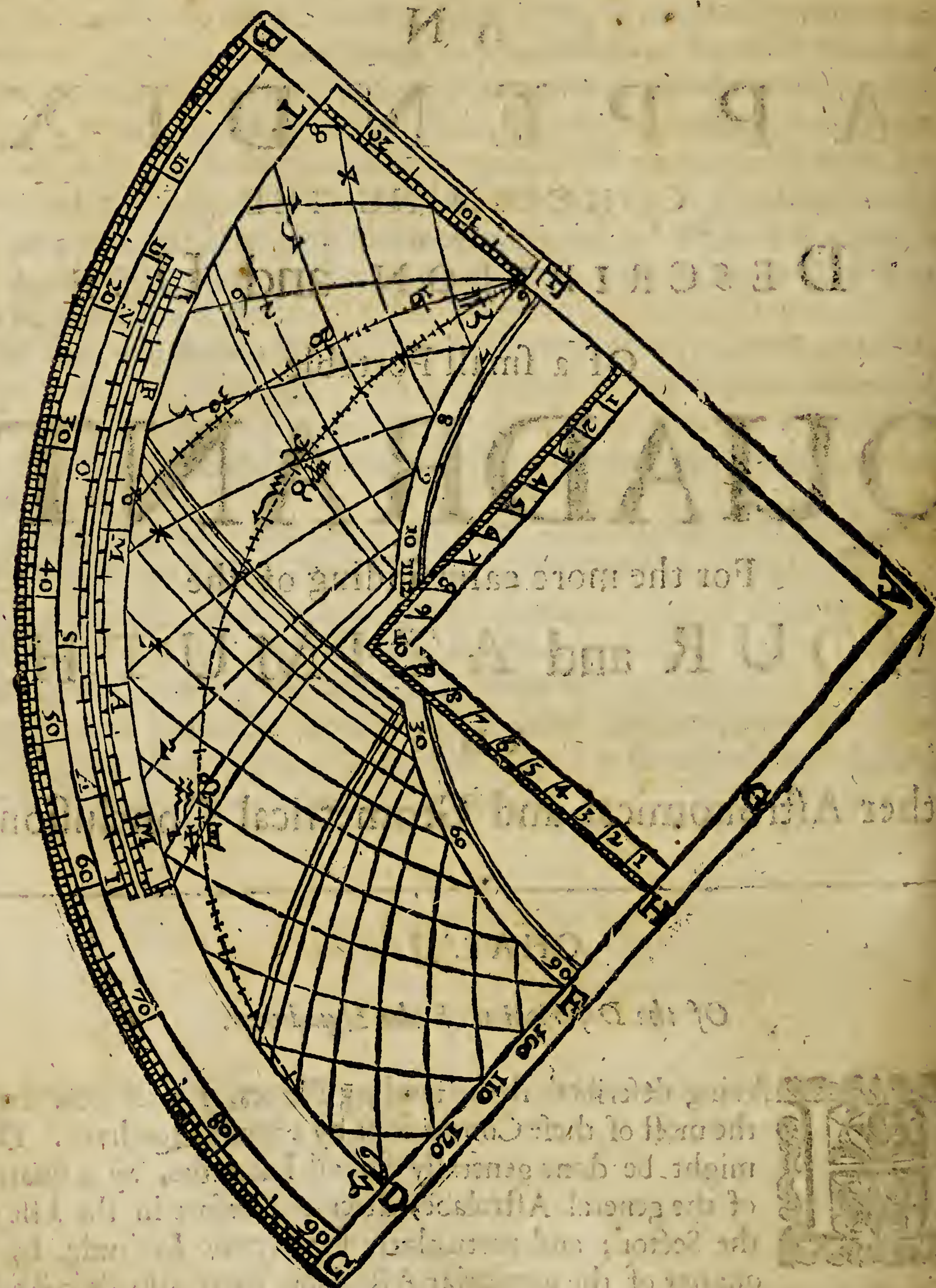
AN APPENDIX CONCERNING THE DESCRIPTION and USE Of a ſmall Portable QUADRANT, For the more eaſie finding of the HOUR and AZIMUTH, AND Other Aſtronomical and Geometrical Concluſions.

CHAP. I.

Of the Deſcription of the Quadrant.

HAving deſcribed theſe ſtanding Planes, I will now ſhew the moſt of theſe Concluſions by a ſmall Quadrant. This might be done generally for all Latitudes, by a quarter of the general Aſtrolabe, deſcribed before in the Uſe of the Sector; and particularly for any one Latitude, by a quarter of the particular Aſtrolabe, there alſo deſcribed; which if it be a Foot Semidiameter, may ſhew the Azimuth unto a Degree, and the time of the Day unto a Minute: But for ordinary uſe this ſmaller Quadrant may ſuffice, which may be made portable in this manner.

1. Upon



1. Upon the Center A, and Semidiameter A B, deſcribe the Ark B C
the ſame Semidiameter will ſet off 60 *gr.* and the half of that will
30 *gr.* which being added to the former 60 *gr.* will make the Ark B C

be 90 *gr.* the fourth part of the whole Circle, and thence comes the name of a Quadrant.

2. Leaving some little space for the Inscription of the Months and Days, on the same Center A, and Semidiameter A T, describe the Ark T D, which shall serve for either Tropick.

3. Divide the Line A T in the Point E, in such proportion, as that A T being 10000, A E may be 6556, and there draw another Line E F, which shall serve for the Equator; or A E being 10000, let E T be 5253.

4. Divide A F the Semidiameter of the Equator in the Point G, so as A F being 10000, the Line A G may be 4343: and on the Center G, and Semidiameter G D, describe the Ark E D, which shall serve for a fourth part of the Ecliptick.

5. This part of the Ecliptick may be divided into three Signs, and each Sign into 30 *gr.* by a Table of Right Ascensions, made as followeth.

A Table of Right Ascensions.

Gra.	♈		♉		♊	
	Gr.	M.	Gr.	M.	Gr.	M.
0	0	0	27	54	57	48
5	4	35	32	42	63	3
10	9	11	37	35	68	21
15	13	48	42	31	73	43
20	18	27	47	33	79	7
25	23	9	52	38	34	32
30	27	54	57	48	90	0

As the Right Ascension of the first Point of ♉ being 27 *gr.* 54 *m.* you may lay a Ruler to the Center A and 27 *gr.* 54 *m.* in the Quadrant B C, the Point where the Ruler crosseth the Ecliptick shall be the first Point of ♉. In like manner, the Right Ascension of the first Point of ♊ being 57 *gr.* 48 *m.* if you lay a Ruler to the Center A and 57 *gr.* 48 *m.* in the Quadrant, the Point where the Ruler crosseth the Ecliptick shall be the first Point of ♊: And so for the rest. But the Lines of distinction between Sign and Sign may be best drawn from the Center G.

 6. The

Gr.	Parts.
1	176
2	355
3	537
4	723
5	913
6	1106
7	1302
8	1503
9	1708
10	1917
11	2130
12	2348
13	2571
14	2799
15	3032
16	3290
17	3514
18	3763
19	4019
20	4281
21	4550
22	4825
23	5104
Trop.	5258

6. The Line E T between the Equator and the Tropick, which I call the Line of Declination, may be divided into 23 *gr.* ½ out of this Table. For let A E the Semidiameter of the Equator be 10000, the diſtance between the Equator and 10 *gr.* of Declination may be 1917 more; between the Equator and 20 *gr.* 4281; the diſtance of the Tropick from the Equator 5252.

7. You may put in the moſt of the principal Stars between the Equator and the Tropick of ♋, by their Declination from the Equator, and Right Aſcenſion from the next Equinoctial Point. As the Declination of the *Wing of Pegaſus* being 13 *gr.* 7 *m.* the Right Aſcenſion 358 *gr.* 34 *m.* from the firſt Point of ♈, or 1 *gr.* 26 *m.* ſhort of it. If you draw an occult Parallel through 13 *gr.* 7 *m.* of Declination, and then lay the Ruler to the Center A, and 1 *gr.* 26 *m.* in the Quadrant B C, the Point where the Ruler croſſeth the Parallel ſhall be the Place for the *Wing of Pegaſus*, to which you may ſet the name and the time when he cometh to the South at midnight in this manner; *W. Peg.* * 23 *Ho.* 54 *M.* And ſo for the reſt of theſe five, or any other Stars.

				Ho.	M.	R.	Aſc.	Dec.	M.
Pegaſus Wing	*	*March*	8	23	56	1	06	13	17
Arcturus	*	*October*	14	14	00	30	07	21	8
Lions Heart	*	*Auguſt*	7	9	50	32	28	13	42
Bulls Eye	*	*May*	16	4	18	64	18	15	46
Vultures Heart	*	*January*	1	19	35	66	26	8	3

8. There being ſpace ſufficient between the Equator and the Center you may there deſcribe the Quadrat, and divide each of the two Side[s] fartheſt from the Center A into 100 parts; ſo ſhall the Quadrant be prepared generally for any Latitude.

But before you draw the particular Lines, you are to fit four Table[s] under your Latitude. Firſt

First, a Table of Meridian Altitudes, for division of the Circle of Days and Months, which may be thus made. Consider the Latitude of the Place, and the Declination of the Sun for each Day of the Year. If the Latitude and Declination be alike, both North, or both South, add the Declination to the Complement of the Latitude; if they be unlike, one North, and the other South, subtract the Declination from the Complement of the Latitude, the Remainder will be the Meridian Altitude belonging unto the Day.

Thus in our Latitude of 51 *gr.* 30 *m.* Northward, whose Complement is 38 *gr.* 30 *m.* the Declination upon the tenth day of *June* will be 23 *gr.* 30 *m.* Northward; wherefore I add 23 *gr.* 30 *m.* unto 38 *gr.* 30 *m.* the sum of both is 62 *gr.* for the Meridian Altitude at the tenth of *June.* The Declination upon the tenth of *December* will be 23 *gr.* 30 *m.* Southward, wherefore I take these 23 *gr.* 30 *m.* out of 38 *gr.* 30 *m.* there will remain 15 *gr.* for the Meridian Altitude at the tenth of *December*; and in this manner you may find the Meridian Altitude for each Day of the Year, and set them down in a Table.

A Table of the Meridian Altitudes.

Dies.	0		5		10		15		20		25		30	
Months.	Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M	Gr.	M	Gr.	M.	Gr.	M.
January	16	31	17	24	18	26	19	37	20	57	22	24	23	58
February	24	17	25	59	27	45	29	35	31	29	33	25		
March	34	35	36	33	38	32	40	30	42	27	44	22	46	15
April	46	37	48	26	50	11	51	50	53	25	54	53	56	15
May	56	15	57	29	58	35	59	33	60	22	61	2	61	31
June	61	36	61	54	62	0	61	58	61	45	61	22	60	49
July	60	49	60	6	59	14	58	13	57	4	55	48	54	24
August	54	7	52	36	50	59	49	17	47	31	45	41	43	26
September	43	26	41	30	39	33	37	36	35	38	33	41	31	46
October	31	46	29	53	28	3	26	16	24	35	22	59	21	29
Novemb.	21	12	19	51	18	39	17	36	16	43	16	0	15	28
December	15	28	15	5	15	0	15	2	15	17	15	44	16	22

The Table being made, you may inscribe the Months, and Days of each

each Month into your Quadrant, in the space left below the Tropick. For, lay the Ruler unto the Center A, and 16 *gr.* 31 *m.* in the Quadrant B C, there may you draw a Line for the end of *December* and beginning of *January*; then laying your Ruler to the Center A, and 24 *gr.* 17 *m.* in the Quadrant, there draw the end of *January* and beginning of *February*, and so the rest, which may be noted with J, F, M, A, M, J, *&c.* the first Letters of each Month, and will here fall between 15 *gr.* and 62 *gr.*

The second Table which you are to fit, may serve for the drawing and dividing of the Horizon. For drawing of the Horizon,

> *As the Co-tangent of the Latitude,*
> *to the Tangent of the greatest Declination:*
> *So the Sine of* 90 gr.
> *to the Sine of the Intersection where the Horizon shall cross the Tropick.*

So in our Latitude of 51 *gr.* 30 *m.* we shall find the Horizon to cut the Tropick in 33 *gr.* 9 *m.* wherefore if you lay the Ruler to the Center A, and 33 *gr.* 9 *m.* in the Quadrant, the Point where the Ruler crosseth the Tropick shall be the Point where the Horizon crosseth the Tropick. And if you find a Point at H in the Line A C, whereon setting the Compasses, you may bring the Point at E and this Point in the Tropick both into a Circle, the Point H shall be the Center, and the Ark so drawn shall be the Horizon.

Then for the division of this Horizon,

> *As the Sine of* 90 gr.
> *to the Sine of the Latitude:*
> *So the Tangent of the Horizon,*
> *to the Tangent of the Ark in the Quadrant, which shall divide the Horizon.*

So in our Latitude of 51 *gr.* 30 *m.* we shall find 7 *gr.* 52 *m.* belonging to 10 *gr.* in the Horizon, and 15 *gr.* 54 *m.* belonging to 20 *gr.* And so the rest, as in this Table.

Ho.	Gr.M.	Ho.	Gr.M.	Ho.	Gr.M.	Ho.	Gr.M.	Ho.	Gr.M.	Ho.	Gr.M.
0	0 0	15	11 51	30	24 19	45	38 2	60	53 35	75	71 5
	0 47		12 39		25 11		39 1		54 41		72 19
	1 34		13 27		26 4		40 0		55 48		73 33
	2 21		14 16		26 57		41 0		56 56		74 48
	3 8		15 4		27 50		42 0		58 4		76 3
5	3 55	20	15 54	35	28 43	50	43 0	65	59 13	80	77 18
	4 42		16 43		29 37		44 1		60 22		78 33
	5 29		17 33		30 32		45 3		61 31		79 49
	6 17		18 22		31 27		46 5		62 41		81 5
	7 4		19 12		32 22		47 8		62 52		82 21
10	7 52	25	20 2	40	33 18	55	48 11	70	65 3	85	83 37
	8 39		20 53		34 14		49 14		66 15		84 53
	9 27		21 44		35 10		50 19		67 27		86 10
	10 14		22 36		36 7		51 24		68 39		87 26
	11 2		23 27		37 4		52 29		69 52		88 43
15	11 51	30	24 19	45	38 2	60	53 35	75	71 5	00	90 0

Wherefore you may lay the Ruler to the Center A, and 7 *gr.* 52 *m.* in the Quadrant B C, the Point where the Ruler crosseth the Horizon shall be 10 *gr.* in the Horizon; and so for the rest: But the Lines of distinction between each fifth Degree will be best drawn from the Center H.

The third Table for drawing of the Hour-lines must be a Table of the Altitude of the Sun above the Horizon at every Hour, especially when he cometh to the Equator, the Tropicks, and some other intermediate Declinations.

If the Sun be in the Equator, and so have no Declination,

As the Sine of 90 gr.
to the Co-sine of the Latitude:
So the Co-sine of the Hour from the Meridian,
to the Sine of the Altitude.

Thus in our Latitude of 51 *gr.* 30 *m.* at six Hours from the Meridian the Sun will have no Altitude, at five the Altitude will be 9 *gr.* 17 *m.* at four

four 18 *gr.* 8 *m.* at three 26 *gr.* 7 *m.* at two 32 *gr.* 37 *m.* at one 36 *gr.* 58 *m.* at Noon it will be 38 *gr.* 30 *m.* equal to the Complement of the Latitude.

If the Sun have Declination, the Meridian Altitude will be found as before, for the Table of Days and Months.

If the Hour proposed be six in the Morning or six at Night,

As the Sine of 90 gr.
to the Sine of the Latitude:
So the Sine of the Declination,
to the Sine of the Altitude.

Thus in our Latitude the Declination of the Sun being 23 *gr.* 30 *m.* the Altitude will be found to be 18 *gr.* 11 *m.* the Declination being 11 *gr.* 30 *m.* the Altitude will be 9 *gr.*

If the Hour proposed be neither twelve nor six,

As the Co-sine of the Hour from the Meridian,
to the Sine of 90 gr.
So the Tangent of the Latitude,
to the Tangent of a fourth Ark.

So in our Latitude, and one Hour from the Meridian, this fourth Ark will be found to be 52 *gr.* 28 *m.* at two 55 *gr.* 26 *m.* at three 60 *gr.* 39 *m.* at four 68 *gr.* 22 *m.* and at five Hours from the Meridian 78 *gr.* 22 *m.*

Then consider the Declination of the Sun, and the Hour proposed; if the Latitude and Declination be both alike, as with us in North Latitude, North Declination, and the Hour fall between Noon and six, take the Declinationout of the fourth Ark, the remainder shall be your fifth Ark.

But if either the Hour fall between six and midnight, or the Latitude and Declination shall be unlike, add the Declination unto the fourth Ark, and the sum of both shall be your fifth Ark: or if the sum shall exceed 90 *gr.* you may take the Complement unto 180 *gr.* This fifth Ark being known,

As the Sine of the fourth Ark,
to the Sine of the Latitude:
So the Co-sine of the fifth Ark,
to the Sine of the Altitude.

Thus

Thus in our Latitude of 51 *gr.* 30 *m.* Northward, the Sun having 23 *gr.* 30 *m.* of North Declination, if it ſhall be required to find the Altitude of the Sun for ſeven in the Morning: here (becauſe the Latitude and Declination are both alike to the Northward, and the Hour propoſed falleth between Noon and ſix) you may take 23 *gr.* 30 *m.* the Ark of the Declination, out of 78 *gr.* 22 *m.* the fourth Ark belonging to the fifth Hour from the Meridian, ſo there will remain 54 *gr.* 52 *m.* for your fifth Ark: Then working according to the Canon, you ſhall find,

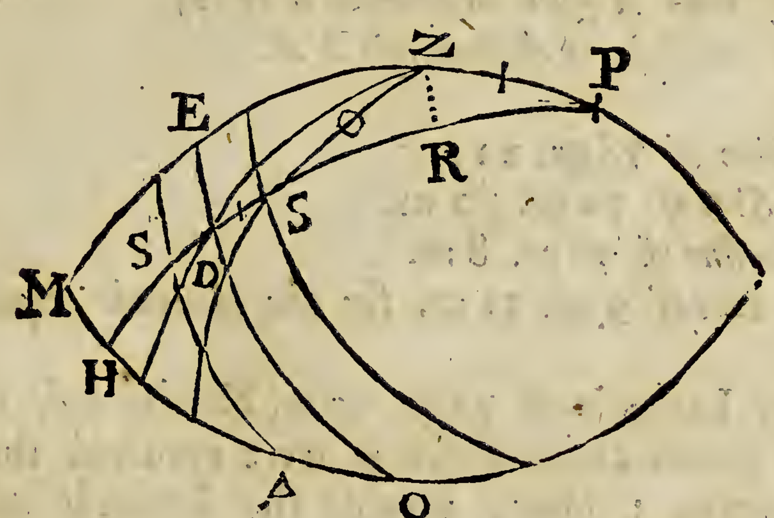
As the Sine of 78 gr. 22 m. *your fourth Ark,*
to the Sine of 51 gr. 30 m. *for the Latitude:*
So the Sine of 35 gr. 8 m. *the Complement of your fifth Ark,*
to the Sine of 27 gr. 17 m. *the Altitude required.*

If in the ſame Latitude and Declination it were required to find the Altitude for five in the Morning, here the Hour falling between ſix and Midnight, if you add 23 *gr.* 30 *m.* unto 78 *gr.* 22 *m.* the ſum will be 101 *gr.* 52 *m.* and the Complement to 180 *gr.* will be 78 *gr.* 8 *m.* for your fifth Ark. Wherefore,

In Rectangulo O D H,

Ut O E Radius,
ad E M Co-tan. Lat.
Ita O D Coſi. Horæ.
ad D H Tan. D H.

Cujus æqualis eſt P R, cujus Compl. D R, novis dr. arcus quartus.



Conferatur Arcus D H cum Arcu Declinationis D S, ita dabitur Arcus H S, cujus Compl. est S R & prius dr. Arcus quintus. Unde erit,

Ut Cosi. P R,	*Hoc est, Ut Sin. D R,*
ad Cosi. P Z:	*ad Sin. E Z:*
Ita Cosi. S R,	*Ita Sin. H S,*
ad Cosi. S Z.	*ad Sin. A S.*

Hinc forte præstabit vocare H S Arcum quintum, ita secunda operatio instituetur per solos sinus.

Vel si libet subtractionem sinus quarti Arcus evitare, inveniatur Angulus O H D quod fieri potest variis modis. Nam,

1. *Ut Radius,*	2. *Ut Sin. D H,*
ad Sin. Ang. O:	*ad Sin O:*
Ita Cosi. Lat. O D,	*Ita Sin. D O,*
ad Cosi. An. O H D.	*ad Sin. H.*

3. *Ut Sin. D H,*	4. *Ut Sin. D R,*
ad Tan. D O:	*ad Sin. E Z;*
Ita Radius,	*Ita Rad.*
ad Tan. Ang. H.	*ad Sin. H.*

Invento utcunque Angulo ad H, erit in Rectangulo H A S.

Ut Sinus Recti Anguli H A S,
ad Sinum Arcus quinti H S:
Ita Sinus Anguli ad Horiz. S H A,
Ad Sin. Solaris Altitudinis S A.

As the Sine of 78 gr. 22 m.
to the Sine of 51 gr. 30 m.
So the Co-sine of 78 gr. 8 m.
to the Sine of 9 gr. 32 m. *for the Altitude required.*

If in the same Latitude of 51 *gr.* 30 *m.* Northward, the Sun having 23 *gr.* 30 *m.* of South Declination, it were required the Altitude for nine in the Morning: Here, because the Latitude and Declination are

are unlike, the one North, and the other South, you may add 23 *gr.* 30 *m.* the Ark of Declination, unto 60 *gr.* 39 *m.* the fourth Ark belonging to the third Hour from the Meridian; so shall you have 84 *gr.* 9 *m.* for your fifth Ark. Wherefore,

As the Sine of 60 gr. 39 m.
to the Sine of 51 gr. 30 m.
So the Co-sine of 84 gr. 9 *m.*
to the Sine of 5 gr. 15 m. *for the Altitude required.*

And so by one or other of these means you may find the Altitude of the Sun for any Point of the Ecliptick at all Hours of the Day, and set them down in such a Table as this.

A Table for the Altitude of the Sun in the beginning of each Sign at all Hours of the Day, calculated for 51 gr. 30 m. *of North Latitude.*

Hours.		♋		♊ ♌		♉ ♍		♈ ♎		♓ ♏		♒ ♐		♑	
		Gr.	M.	Gr.	M.	Gr.	M	Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M.
12		62	0	58	42	50	0	38	30	27	1	18	18	15	0
11	1	59	43	56	34	48	12	36	58	25	40	17	6	13	52
10	2	53	45	50	55	43	12	32	37	21	51	13	38	10	30
9	3	45	42	43	6	36	0	26	7	15	58	8	12	5	15
8	4	36	41	34	13	27	31	18	8	8	33	1	15		
7	5	27	17	24	56	18	18	9	17	0	6				
6	6	18	11	15	40	9	0	0	0						
5	7	9	32	6	50									11	37
4	8	1	32											21	40

Lastly, You may find what Declination the Sun hath when he riseth or setteth at any Hour.

As the Sine of 90 gr.
to the Sine of the Hour from six:
So the Co-tangent of the Latitude,
to the Tangent of the Declination.

And so in the Latitude of 51 *gr.* 30 *m.* you shall find that when the Sun riseth, either at five in the Summer, or seven in the Winter, his Declination is 11 *gr.* 37 *m.* when he riseth at four in the Summer, or eight in the Winter, his Declination is 21 *gr.* 40 *m.* which may be also set down in the Table.

That done, you may there see, that in this Latitude the Meridian Altitude of the Sun in the beginning of ♋ is 62 *gr.* in ♊ 58 *gr.* 42 *m.* in ♉ 50 *gr.* in ♈ 38 *gr.* 30 *m.* &c. But the beginning of ♋ and ♑ is represented by the Tropick T D, drawn at 23 *gr.* 30 *m.* of Declination, and the beginning of ♈ and ♎, by the Equator E F. If you draw an occult Parallel between the Equator and the Tropick, at 11 *gr.* 30 *m.* of Declination, it shall represent the beginning of ♉, ♍, ♏, and ♓, if you draw another occult Parallel through 20 *gr.* 12 *m.* of Declination, it shall represent the beginning of ♊, ♌, ♐, and ♒.

Then you may lay a Ruler to the Center A, and 62 *gr.* in the Quadrant B C, and note the Point where it crosseth the Tropick of ♋; then move the Ruler to 58 *gr.* 52 *m.* and note where it crosseth the Parallel of ♊; then to 50 *gr.* and note where it crosseth the Parallel of ♉; and again to 38 *gr.* 30 *m.* noting where it crosseth the Equator: so the Line drawn through these Points shall shew the Hour of 12 in the Summer, while the Sun is in ♈, ♉, ♊, ♋, ♌, or ♍. In like manner, if you lay the Ruler to the Center A, and 27 *gr.* in the Quadrant, and note the Point where it crosseth the Parallel of ♓; then move it to 18 *gr.* 18 *m.* and note where it crosseth the Parallel of ♒; and again to 15 *gr.* noting where it crosseth the Tropick of ♑; the Line drawn through these Points shall shew the Hoor of 12 in the Winter, while the Sun is in ♎, ♏, ♐, ♑, ♒, and ♓; so may you draw the rest of these Hour-lines: only that of 7, from the Meridian in the Summer, and 5 in the Winter, will cross the Line of Declination at 11 *gr.* 37 *m.* and that of 8 in the Summer, and 4 in the Winter, at 21 *gr.* 40 *m.*

The fourth Table for drawing of the Azimuth-lines must likewise be fitted for the Altitude of the Sun above the Horizon at every Azimuth, especially

especially when he cometh to the Equator, the Tropicks, and some other intermediate Declination.

If the Sun be in the Equator, and so have no Declination,

As the Sine of 90 gr.
to the Co-sine of the Azimuth from the Meridian:
So the Co-tangent of the Latitude,
to the Tangent of the Altitude at the Equator.

Thus in our Latitude of 51 *gr.* 30 *m.* at 90 *gr.* from the Meridian the Sun will have no Altitude; at 80 *gr.* the Altitude will be 7 *gr.* 52 *m.* at 70 *gr.* it will be 15 *gr.* 30 *m.* at 60 *gr.* it will be 21 *gr.* 41 *m.*

If the Sun have Declination, the Meridian Altitude will be easily found as before, for the Table for Days and Months. And for all other Azimuths,

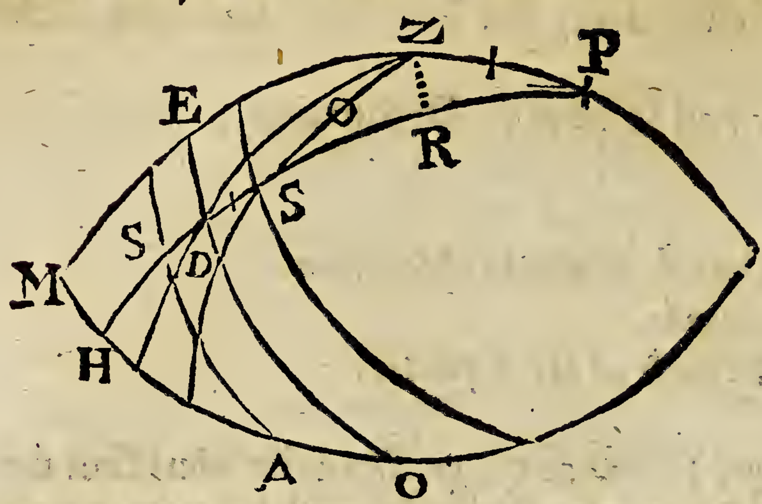
As the Sine of the Latitude,
to the Sine of the Declination:
So the Co-sine of the Altitude at the Equator,
to the Sine of a fourth Ark.

When the Latitude and Declination are both alike in all Azimuths from the prime Vertical unto the Meridian, add this fourth Ark unto the Ark of Altitude at the Equator.

When the Latitude and Declination are both alike, and the Azimuth more than 90 *gr.* distant from the Meridian, take the Altitude at the Equator out of this fourth Ark.

When the Latitude and Declination are unlike, take this fourth Ark out of the Ark of Altitude at the Equator, so shall you have the Altitude of the Sun belonging to the Azimuth.

Thus in our Latitude of 51 *gr.* 30 *m.* Northward, if it were required to find the Altitude of the Sun in the Azimuth of 60 *gr.* from the Meridian, when the Declination is 23 *gr.* 30 *m.* Northward, you may find the Altitude at the Equator belonging to this Azimuth to be 21 *gr.* 41 *m.* by the former Canon; and by this last Canon you may find the fourth Ark to be 28 *gr.* 15 *m.* Then because the Latitude and Declination are both alike to the Northward, if you add them both together, you shall have 49 *gr.* 56 *m.* for the Altitude required.



O M 90. 60.
M E Comp. Lat.
O A Com. Azim.
A B Alt. Æqua.

E Z Lat.
Z B Com. *A B*.
D S Deelin.
S B Arc. 4.

Tables for the Altitude of the Sun in the beginning of each Sign for every tenth Azimuth.

Lat. 50 *Gr.* 00 *M.*

	Merid.	10	20	30	40	50	60	70	80	90
♋	63 30	63 14	62 23	60 54	58 42	55 32	51 25	46 2	31 17	31 22
♊	60 12	59 54	59 0	57 23	55 1	51 43	47 18	41 40	34 47	26 48
♉	51 31	51 9	0 3	48 10	45 23	41 34	36 38	30 30	23 12	15 5
♈	40 0	39 34	38 15	36 0	32 44	28 20	22 45	60 0	8 17	0 0
♓	28 30	28 0	26 27	23 50	20 5	15 6	8 52	1 30	6 38	
♒	19 48	19 14	17 31	14 37	10 27	4 57	1 43	9 40	18 13	
♑	16 30	14 54	14 7	11 6	6 46	1 5	6 58	14 2	22 43	

Lat, 51 *Gr.*

	Merid.	10	20	30	40	50	60	70	80	90
♋	62 30	62 14	61 22	59 54	57 40	54 35	50 27	45 8	38 33	30 53
♊	59 12	58 54	57 59	56 23	54 0	50 43	46 22	41 51	34 6	26 23
♉	50 30	50 7	49 3	47 11	44 25	40 40	35 47	29 48	22 43	14 52
♈	39 0	38 34	37 16	35 3	31 49	27 30	22 2	15 29	8 0	0 0
♓	27 30	27 1	25 29	22 55	9 13	14 20	8 17	1 10	6 43	
♒	18 48	18 14	16 33	13 43	9 38	4 17	2 18	9 53	18 6	
♑	15 30	14 54	13 10	10 12	5 58	0 25	6 23	14 10	22 33	

Lat. 52 *Gr.*

	Merid.	10	20	30	40	50	60	70	80	90
♋	61 30	61 14	60 22	58 52	56 38	53 33	49 29	44 14	37 58	30 24
♊	58 12	58 54	56 28	56 22	53 0	49 43	44 25	40 0	33 28	26 0
♉	49 30	49 9	48 3	46 11	43 26	39 44	34 58	29 6	22 15	14 40
♈	38 0	37 35	36 17	34 5	30 54	26 40	21 20	14 57	7 44	0 0
♓	26 30	26 1	24 31	22 0	18 22	13 26	17 42	0 48	6 46	
♒	17 48	17 16	15 36	12 48	8 49	3 37	2 45	10 6	18 0	
♑	14 30	14 56	12 12	9 18	5 10	0 13	6 49	14 19	22 30	

If the Declination had been 23 *gr.* 30 *m.* to the Southward, you should then have taken this fourth Ark out of the Ark at the Equator; which because it cannot here be done, it is a sign that the Sun is not then above the Horizon: But if you take the Ark at the Equator out of this fourth Ark, you shall have 6 *gr.* 34 *m.* for the Altitude of the Sun when he is in the Azimuth of 60 *gr.* from the North, and 120 *gr.* from the South part of the Meridian. The like reason holdeth for the rest of these Altitudes, which may be gathered, and set down in a Table.

Lastly, when the Sun riseth or setteth upon any Azimuth, to find his Declination.

As the Sine of 90 gr.
to the Cosine of the Latitude:
So the Co-sine of the Azimuth from the Meridian,
to the Sine of the Declination.

A Table for the Altitude of the Sun in the beginning of each Sign for every tenth Azimuth, in 51 gr. 30 m. *of North Latitude.*

Azimuths.	♋		♊ ♌		♉ ♍		♈ ♎		♓ ♏		♒ ♐		♑	
	Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M	Gr.	M	Gr.	M.	Gr.	M.
0	62	0	58	42	50	0	38	30	27	0	18	18	15	0
10	61	43	58	24	49	38	38	4	26	30	17	45	14	25
20	60	51	57	28	48	33	36	46	25	09	16	5	12	41
30	59	52	55	52	46	40	34	34	22	27	13	15	9	45
40	57	20	53	29	43	55	31	21	18	48	9	14	5	34
50	54	3	50	12	40	11	27	5	13	58	3	57	0	6
60	49	56	45	53	35	23	21	41	8	0				
70	44	40	40	25	29	27	15	13	1	0				
80	38	11	33	46	21	29	7	52						
90	30	38	26	10	14	25	0	0						
100	22	27	18	2	6	45							6	12
110	14	14	9	58									12	18
120	6	34	2	30									18	8

And thus in our Latitude of 51 *gr.* 30 *m.* when the Azimuth is 80 *gr.* from

from the Meridian, the Declination will be found to be *6 gr.* 12 *m.* if the Azimuth be 70 *gr.* the Declination will be found 12 *gr.* 18 *m.* if 60 *gr.* then 18 *gr.* 8 *m.* And so for the rest, which may be also set down in the Table.

That done, if you would draw the Line of East or West, which is 90 *gr.* from the Meridian, lay the Ruler to the Center A, and 30 *gr.* 38 *m.* numbred in the Quadrant from C toward B, and note the Point where it crosseth the Tropick of ♋; then move the Ruler to 26 *gr.* 10 *m.* and note where it crosseth the Parallel of ♊; then to 14 *gr.* 45 *m.* and note where it crosseth the Parallel of ♉; then to 0 *gr.* 0 *m.* and you shall find it to cross the Equator in the Point F: so a Line drawn through these Points shall shew the Azimuth belonging to East and West. The like reason holdeth for all the rest.

These Lines being thus drawn, if you set two Sights upon the Line A C, and hang a Thred and Plummet on the Center A, with a Bead upon the Thred, the Fore-side of the Quadrant shall be fully finished.

On the Back-side of the Quadrant you may place the Nocturnal described before in the Use of the Sector, which consisteth of two parts.

The one is an Hour-Plane, divided equally according to the 24 Hours of the Day, and each hour into Quarters, or Minutes, as the Plane will bear. The Center represents the North Pole; the Line drawn through the Center from XII to XII stands for the Meridian, and the lower XII stands for the Hour of XII at midnight.

The other part is a Rundle for such Stars as are near the North Pole, together with the twelve Months, and the Days of each Month, fitted to the Right Ascension of the Sun and Stars, in this manner,

First, consider where the Sun will be at the beginning of the 5, 10, 15, 20, 25, 30, and, if you will, every day of each Month, and find the Right Ascension belonging to the place of the Sun, as I shewed before.

For example: The Sun at midnight, the last of *December*, or beginning of *January*, will be *communibus annis* about 20 *gr.* 40 *m.* of ♑, whose Right Ascension is 292 *gr.* 20 *m.* At midnight, the last of *January*, or beginning of *February*, he will be about 22 *gr.* 12 *m.* of ♒, whose Right Ascension is 324 *gr.* 35 *m.* and so the rest, which may be set down in a Table.

That done, consider the Longitude and Latitude of the Stars, and thereby find their Right Ascension and Declination as I shewed before, and set them down in a Table. These Tables thus made, let the uppermost part

part of the Rundle be made even with the innermoſt Circle of the Hour-Plane, and a convenient ſpace allowed to contain the diviſions for the Days, and names of the Months. Then lay the Center of this Rundle upon the Center of ſome other Circle divided into *360 gr.* and by the Center and *292 gr 20 m.* in that Circle, draw a Line for the beginning of *January*: In like manner, by the Center and *324 gr. 35 m.* draw a Line for the end of *January* and beginning of *February*; and ſo the reſt of the Days of each Month.

For the Inſcription of the Stars, let one of the Lines from the Center, as that at the beginning of *July*, or rather let a movable Index be divided from the Center toward the inward Circle of the Months into *40 gr.* more or leſs, which may be done for ſpeed equally, but for exactneſs in ſuch manner as the Semidiameter of the General Aſtrolabe was divided before in the Uſe of the Sector. So laying the Index to the Right Aſcenſion in the outward Circle, you may prick down the Stars by their Declination in the Index.

For example: If the Right Aſcenſion of the Pole-ſtar be *6 gr. 28 m.* and his Declination *87 gr. 20 m.* having ſet the Center of the Index both to the Center of the Rundle and of the other Circle, turn the Index to *6 gr. 28 m.* in that outward Circle, and prick down the Star by *87 gr. 20 m.* in the edge of the Index, that is, at the diſtance of *2 gr. 40 m.* from the Pole. The like reaſon holdeth for the reſt of the Stars, which may be diſtinguiſhed according to their Magnitudes, and then be reduced into their Forms, as in the Example. So the Quadrant will be fitted both for Day and Night.

CHAP. II.

Of the Uſe of the Quadrant, in taking the Altitude of the Sun, Moon, and Stars.

THe Quadrant is the fourth part of a Circle divided equally into *90 gr.* and here numbred by 10, 20, 30, *&c.* unto *90 gr.* each Degree being ſubdivided into 4.

Lift up the Center of the Quadrant ſo as the Thred with the Plummet may play eaſily by the Side of it, and the Sun-beams may paſs through both the Sights; ſo ſhall the Degrees cut by the Thred ſhew what is the Altitude at the time of obſervation, as may appear by this Example.

Upon the 14 day of *April*, about Noon, the Sun-beams passing through both the Sights, the Thred fell upon 51 *gr.* 20 *m.* and this was the true Meridian Altitude of the Sun for that day, in this our Latitude of 51 *gr.* 30 *m.* for which this Quadrant was made.

Again, towards three of the Clock in the afternoon the Thred fell upon 38 *gr.* 40 *m.* and such was the Suns Altitude at that time.

CHAP. III.

Of the ECLIPTICK.

1. *The Place of the Sun being given, to find his Right Ascension.*

THe Ecliptick is here represented by the Ark figured with the Characters of the 12 Signs, ♈, ♉, ♊, *&c.* each Sign being divided unequally into 30 *gr.* and they are to be reckoned from the Character of the Sign.

Let the Thred be laid on the place of the Sun in the Ecliptick, and the Degrees which it cutteth in the Quadrant shall be the Right Ascension required.

As if the place of the Sun given be the fourth Degree of ♊, the Thred laid on this Degree shall cut 62 *gr.* in the Quadrant, which is the Right Ascension required.

But if the place of the Sun given be more than 90 *gr.* from the beginning of ♈, there must be more than 90 *gr.* allowed to the Right Ascension; for this Instrument is but a Quadrant. And so if the Sun be in 26 *gr.* of ♋, you shall find the Thred to fall in the same place, and yet the Right Ascension to be 118 *gr.*

2. *The Right Ascension of the Sun being given, to find his Place in the Ecliptick.*

Let the Thred be laid on the Right Ascension in the Quadrant, and it shall cross the place of the Sun in the Ecliptick, as may appear in the former Example.

CHAP. IV.

Of the Line of Declination.

1. *The Place of the Sun being given, to find his Declination.*

THe Line of Declination is here drawn from the Center to the beginning of the Quadrant, and divided from the beginning of ♈ downward into 23 *gr.* 30 *m.*

Let the Thred be laid, and the Bead set on the Place of the Sun in the Ecliptick; then move the Thred to the Line of Declination, and there the Bead shall fall upon the Degrees of the Declination required.

As if the place of the Sun given be the fourth Degree of ♊, the Bead first set to this place, and then moved to the Line of Declination, shall there shew the Declination of the Sun at that time to be 21 *gr.* from the Equator.

2. *The Declination of the Sun being given, to find his place in the Ecliptick.*

Let the Thred and Bead be first laid to the Declination, and then moved to the Ecliptick.

As if the Declination be 21 *gr.* the Bead first set to this Declination, and then moved to the Ecliptick, shall there shew the fourth of ♊, the fourth of ♐, the 26 of ♋, and the 26 of ♑; and which of these four is the place of the Sun, may appear by the Quarter of the Year.

CHAP. V.

Of the Circle of Months and Days.

THis Circle is here represented by the Ark figured with these Letters, J, F, M, A, M, *&c.* signifying the Months *January*, *February*, *March*, *April*, *&c.* each Month being divided unequally, according to the number of the Days that are therein.

A Table for the Inscription of the Months in the Nocturnal.

Dies.	0		5		10		15		20		25		30	
Menſ.	Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M	Gr.	M.	Gr.	M.
January	292	20	297	46	303	7	308	21	313	30	318	36	323	36
February	324	35	329	28	334	16	339	1	343	42	348	21		
March	351	17	355	52	0	26	4	58	9	30	14	2	18	34
April	19	30	24	4	28	42	33	23	38	5	42	52	47	42
May	47	42	52	35	57	32	62	34	67	39	72	45	77	52
June	78	55	84	5	89	17	94	28	99	39	104	48	109	55
July	109	55	115	0	120	0	124	58	129	54	134	45	139	30
Auguſt	140	27	145	9	149	48	154	25	159	0	163	32	168	0
Septemb.	168	57	173	26	177	56	182	26	186	56	191	28	196	5
October	196	5	200	45	205	25	210	12	215	3	220	0	225	0
Novemb.	226	2	231	10	236	23	241	40	247	2	252	30	258	2
Decemb.	258	2	263	35	269	8	274	42	280	16	285	46	291	15

1. *The Day of the Month being given, to find the Altitude of the Sun at Noon.*

Let the Thred be laid to the Day of the Month, and the Degrees which it cutteth in the Quadrant ſhall be the Meridian Altitude required.

As if the Day given be the 15 of *May*, the Thred laid on this day ſhall cut 59 *gr.* 30 *m.* in the Quadrant, which is the Meridian Altitude required.

2. *The Meridian Altitude being given, to find the Day of the Month.*

The Thred being ſet to the Meridian Altitude, doth alſo fall on the day of the Month.

As if the Altitude at Noon be 59 *gr.* 30 *m.* the Thred being ſet to his Altitude, doth fall on the 15 of *May*, and the 9 of *July*; and which of theſe two is the true day, may be known by the quarter of the year, or by another days Obſervation. For if the Altitude prove greater, the Thred will fall on the 16 day of *May*, and the 8 of *July*; or if it prove leſſer, the

the Thred will fall on the 14 of *May*, and the 10 of *July*; whereby the question is fully answered.

CHAP. VI.

Of the Hour-lines.

THat Ark which is drawn upon the Center of the Quadrant by the beginning of Declination, doth here represent the Equator: That Ark which is drawn by 23 *gr.* 30 *m.* of Declination, and is next above the Circle of Months and Days, representeth the Tropicks: Those Lines which are between the Equator and the Tropicks, being undivided, and numbred at the Equator by 6, 7, 8, 9, 10, 11, 12; at the Tropick by 1, 2, 3, 4, *&c.* do represent the Hour-circles: That which is drawn from 12 in the Equator to the middle of *June*, representeth the Hour of 12 at Noon in the Summer; and those which are drawn with it to the right hand, are for the Hours of the Day in the Summer, and the Hours of the Night in the Winter: That which is drawn from 12 in the Equator, to the middle of *December*, representeth the Hour of 12 in the Winter; and those which are drawn with it to the left hand, are for the Hours of the Day in the Winter, and the Hours of the Night in the Summer; and of both these, that which is drawn from 11 to 1 serves for 11 in the forenoon, and 1 in the afternoon; that which is drawn from 10 to 2, serves for 10 in the forenoon, and 2 in the afternoon: for the Sun on the same day is about the same height two Hours before Noon, as two Hours after Noon. The like reason holdeth for the rest of the Hours.

1. *The Day of the Month, or the Height at Noon being known, to find the Place of the Sun in the Ecliptick.*

The Thred being laid to the day of the Month, or the height at Noon, (for one gives the other by the former Proposition) mark where it crosseth the Hour of 12, and set the Bead to that Intersection; then move the Thred till the Bead fall on the Ecliptick, and it shall fall on the place of the Sun.

As if the day given be the 15 of *May*, or the Meridian Altitude 59 *gr.* 30 *m.* lay the Thred accordingly, and put the Bead to the Intersection of the Thred with the Hour of 12; then move the Thred till the Bead fall on the Ecliptick, and it shall there shew the fourth of ♊, the fourth of ♐, the 26 of ♋, and the 26 of ♑; and which of these is the place of the Sun, may appear by the Quarter of the Year, or another days observation.

2. *The place of the Sun in the Ecliptick being known, to find the Day of the Month.*

Let the Thred and Bead be first laid on the place of the Sun in the Ecliptick, and then moved to the Line of 12.

As if the place of the Sun given be the fourth of ♊, the Bead being laid to this Degree, and then moved to the Hour of 12 in the Summer, the Thred will fall on the 15 day of *May*, and the 9 of *July*; or if it be moved to the Hour of 12 in the Winter, the Thred will fall on the 6 of *January*, and the 16 of *November*: which of these is the day of the Month required, may appear by the Quarter of the Year.

In this and the former Propositions you have two ways to rectifie the Bead, by the place of the Sun, and by the day of the Month: the better way is by the place of the Sun; for in the other, the Leap-year may breed some small difference.

There is yet a third way: For the Seamen having a Table for the Declination on each day of the year, may set the Bead thereto in the Line of Declination.

3. *The Hour of the Day being given, to find the Altitude of the Sun above the Horizon.*

The Bead being set for the time by either of the three ways, let the Thred be moved from the Hour of 12 toward the Line of Declination, till the Bead fall on the Hour given; and the Degrees which it cuts in the Quadrant, shall shew the Altitude of the Sun at that time.

As if the time given be the 10 of *April*, the Sun being then in the beginning of ♉, the Bead being rectified, you shall find the Height at Noon 50 *gr.* 0 *m.* at 11 in the morning 48 *gr.* 12 *m.* at 10 but 43 *gr.* 12 *m.* at 9 but 36 *gr.* at 8 but 27 *gr.* 30 *m.* at 7 but 18 *gr.* 18 *m.* at 6 but 9 *gr.* at 5 it meeteth with the Line of Declination, and hath no Altitude at all, and therefore you may think it did rise much about that Hour.

Then if you move the Thred again from the Line of Declination toward the Hour of 12, you shall find that the Sun is 8 *gr.* 33 *m.* below

th

the Horizon at 4 in the morning, and near 16 *gr.* at 3, and 21 *gr.* 51 *m.* at 2, and 25 *gr.* 40 *m.* at 1, and 27 *gr.* at midnight.

4. *The Altitude of the Sun being given, to find the Hour of the Day.*

The Altitude being obſerved as before, let the Bead be ſet for the time, then bring the Thred to the Altitude, ſo the Bead ſhall ſhew the Hour of the day.

As if the 10 of *April*, having ſet the Bead for the time, you ſhall find by the Quadrant the Altitude to be 36 *gr.* the Bead at the ſame time will fall upon the Hour-line of 9 and 3; wherefore the Hour is 9 in the fore-noon, or 3 in the afternoon. If the Altitude be near 40 *gr.* you ſhall find the Bead at the ſame time to fall half way between the Hour-line of 9 and 3, and the Hour-line of 10 and 2; wherefore it muſt be either half an Hour paſt 9 in the morning, or half an Hour paſt 2 in the after-noon; and which of theſe is the true time of the day, may be ſoon known by a ſecond Obſervation: for if the Sun riſe higher, it is the fore-noon; if it become lower, it is the afternoon.

5. *The Hour of the Night being given, to find how much the Sun is below the Horizon.*

The Sun is always ſo much below the Horizon at any Hour of the Night, as his oppoſite Point is above the Horizon at the like Hour of the Day; and therefore the Bead being ſet, if the queſtion be made of any Hour of the Night in the Summer, then move it to the like Hour of the Day in the Winter; if of any Hour of the Night in Winter, then move it to the like Hour of the Day in Summer: ſo the Degrees which the Thred cutteth in the Quadrant ſhall ſhew how much the Sun is below the Horizon at that time.

As if it be required to know how much the Sun is below the Horizon the 10 of *April* at 4 of the Clock in the Morning, the Bead being ſet to his place according to the time in the Summer-hours, bring it to 4 of the Clock in the afternoon in the Winter-hours, and ſo ſhall you find the Thred to cut 8 *gr.* and about 30 *m.* in the Quadrant; and ſo much is the Sun below the Horizon at that time.

6. The

6. *The Depression of the Sun supposed, to give the Hour of the Night with us, or the Hour of the Day to our Antipodes.*

Here also, because the Sun is so much above the Horizon at all Hours of the day, as his opposite point is below the Horizon at the like Hour of the Night; therefore first set the Bead according to the time, then bring the Thred to the Degree of the Suns Depression below the Horizon, so shall the Bead fall on the contrary Hour-lines, and there shew the Hour of the Night in regard of us, which is the like Hour of the Day to our *Antipodes.*

As if the 10 of *April*, the Sun being then in the beginning of ♉; and by supposition 8 *gr.* 30 *m.* below the Horizon in the East, it be required to know what time of the Night it is; first, set the Bead according to the Day in the Summer-hours, then bring the Thred to 8 *gr.* 30 *m.* in the Quadrant, so shall the Bead fall among the Winter-hours on the Line of 4 of the Clock in the afternoon: wherefore to our *Antipodes* it is 4 of the Clock in their afternoon, and to us it is then 4 of the Clock in the morning.

7. *The time of the Year, or the place of the Sun being given, to find the beginning of Day-break, and end of Twilight.*

This Proposition differeth little from the former: for the Day is said to begin to break when the Sun cometh to be but 18 *gr.* below our Horizon in the East; and Twilight to end, when it is gotten 18 *gr.* below the Horizon in the West: Wherefore let the Bead be set for the time, and then bring the Thred to 18 *gr.* in the Quadrant, so shall the Bead fall on the contrary Hour-lines, and there shew the Hour of Twilight, as before.

So if it be required to know at what time the Day begins to break on the 10 of *April*, the Sun being then in the beginning of ♉; first, set the Bead according to the time in the Summer-hours, and then bring the Thred to 18 *gr.* in the Quadrant, so shall the Bead fall among the Winter-hours a little more than a quarter before 3 in the morning; and that is the time when the Day begins to break upon the 10 of *April.*

CHAP. VII.

Of the Horizon.

THe Horizon is here repreſented by the Ark drawn from the beginning of Declination towards the end of *February*, divided unequally, and numbred by 10, 20, 30, 40, *&c.*

1. *The Day of the Month, or the Place of the Sun being known, to find the Amplitude of the Suns Riſing and Setting.*

Let the Bead rectified for the time be brought to the Horizon, and there it ſhall ſhew the Amplitude required.

As if the day given be the 15 of *May*, the Sun being in the fourth Degree of ♊, the Bead rectified and brought to the Horizon, ſhall there fall on 35 *gr.* 8 *m.* ſuch is the Amplitude of the Suns Riſing from the Eaſt, and of his ſetting from the Weſt; which Amplitude is always North when the Sun is in the Northern Signs, and when he is in the Southern Signs always Southward.

2. *The Day of the Month, or the Place of the Sun being given, to find the Aſcenſional Difference.*

Let the Bead rectified for the time be brought to the Horizon, ſo the Degrees cut by the Thred in the Quadrant ſhall ſhew the difference of Aſcenſions.

As if the day given be the 15 of *May*, the Sun being in the fourth Degree of ♊, let the Bead be rectified and brought to the Horizon; ſo ſhall the Thred in the Quadrant ſhew the Aſcenſional difference to be 28 *gr.* and about 50 *m.*

Upon the Aſcenſional difference depends this Corollary.

To find the Hour of the Riſing and Setting of the Sun, and thereby the length of the Day and Night.

The time of the Suns Riſing may be gueſſed at by the 3 of the laſt Chapter: but here by the Aſcenſional difference it may be better found, and that to a minute of time. For if the Aſcenſional Difference be converted into time, allowing an Hour for 15 *gr.* and 4 Minutes of an Hour for each Degree, it ſheweth how long the Sun riſeth before ſix of the Clock in the Summer, and after ſix in the Winter.

As if the day given be the 15 of *May*, the Sun being in the fourth of ♊, and his Aſcenſional Difference found as before 28 *gr.* 50 *m.* this converted into time, maketh 1 *ho.* and ſomewhat more than 55 *m.* of an Hour: wherefore the Sun at that time, in regard it was Summer, roſe 1 *ho.* and full 55 *m.* before 6 of the clock; and ſo having the quantity of the Semidiurnal Ark, the length of the Day and Night need not be unknown.

CHAP. VIII.

Of the Five Stars.

I Might have put in more Stars, but theſe may ſuffice for the finding of the Hour of the Night at all times of the Year: And firſt I make choice of *Ala Pegaſi*, a Star in the extremity of the ***Wing of Pegaſus***, in regard it wants but 6 minutes of time of the beginning of ♈; but becauſe it is but of the ſecond magnitude, and not always to be ſeen, I made choice of four more, one for each quarter of the Ecliptick, as ***Oculus*** ♉, ***The Bulls Eye***, whoſe Right Aſcenſion converted into time, is 4 *ho.* 15 *m.* then of *Cor* ♌, ***The Lions Heart***, whoſe Right Aſcenſion is 9 *ho.* 48 *m.* next of ***Arcturus***, whoſe Right Aſcenſion is 13 *ho.* 58 *m.* and laſtly of ***Aquila***, or ***The Vultures Heart***, whoſe Right Aſcenſion is 19 *ho.* 33 *m.* Theſe five Stars have all of them Northern Declination; and if any others, ſome of theſe will be ſeen at all times of the Year. The uſe of them is,

The Altitude of any of theſe five Stars being known, to find the Hour of the Night.

Firſt, put the Bead to the Star which you intend to obſerve, take his Altitude, and find how many Hours he is from the Meridian by the fourth *Prop.* of the ſixth *Chap.* then out of the Right Aſcenſion of the Star, take the Right Aſcenſion of the Sun converted into Hours, and mark the difference; for this difference being added to the obſerved Hour of the Star from the Meridian, ſhall ſhew how many Hours the Sun is gone from the Meridian, which is in effect the Hour of the Night.

A

As if the 15 of *May*, the Sun being in the fourth of ♊, I should set the Bead to *Arcturus*, and observing his Altitude, should find him to be in the West about 52 *gr.* high, and the Bead to fall on the Hour-line of 2 afternoon, the Hour would be 11 *ho.* 50 *m.* past noon, or 10 *m.* short of midnight.

For, 62 *gr.* the Right Ascension of the Sun, converted into time, makes 4 *ho.* 8 *m.* which if we take out of 13 *ho.* 58 *m.* the Right Ascension of *Arcturus*, the difference will be 9 *ho.* 50 *m.* and this being added to 2 *ho.* the observed distance of *Arcturus* from the Meridian, shews the Hour of the Night to be 11 *ho.* 50 *m.* Another Example will make all more plain.

If the 9 of *July*, the Sun being then in 26 *gr.* of ♋, I should set the Bead of *Oculus* ♉, and observing his Altitude, should find him to be in the East about 12 *gr.* high, and the Bead to fall on the Hour-line of 6 before Noon, which is 18 *ho.* past the Meridian, the Hour of the Night would be better than a quarter past 2 of the clock in the morning.

For, 118 *gr.* the Right Ascension of the Sun, converted into time, make 7 *ho.* 52 *m.* this taken out of 4 *ho.* 15 *min.* the Right Ascension of *Oculus* ♉, adding a whole Circle, (for otherwise there could be no subtraction) the difference will be 20 *ho.* 23 *m.* and this being added to 18 *ho.* which was the observed distance of *Oculus* ♉ from the Meridian, shews that the Sun (abating 24 *ho.* for the whole Circle) is 14 *ho.* 23 *m.* past the Meridian, and therefore 23 *m.* past 2 of the clock in the morning.

If the *Nocturnal* be placed on the back side of the Quadrant, you may avoid this Equation of Right Ascensions. For knowing the time of the Year when the Star will be in the South at midnight, you may bring that time to the Hour observed, then will the Day of the Month wherein you made the Observation point at the Hour of the Night required.

As in the first Example, where, on the 15 of *May*, the Bead set to *Arcturus* fell on the Hour-line of 2 afternoon, because *Arcturus* will be in the South the 14 of *October* complete at midnight, you may place the 14 of *October* at the Hour of 2, so the 15 of *May* will point to 11 *ho.* 50 *m.*

In the second Example, where the 9 of *July* the Bead set to the *Bulls Eye* fell on the Hour-line of 6 before Noon, because the *Bulls Eye* will be in the South the 16 of *May* complete at midnight, you may turn the 16 of *May* to the Hour of 6, and so you shall find the 9 of *July* to point 2 *ho.* 23 *m.* as before.

CHAP. IX.

Of the Azimuth-lines.

THose Lines which are drawn between the Equator and the Tropicks, on that side of the Quadrant which is nearest unto the Sights, and are numbred by 10, 20, 30, *&c.* do represent the Azimuths; the uttermost towards the left hand representeth the Meridian; that which is numbred with 10, the tenth Azimuth from the Meridian; and that which is numbred with 20, the twentieth: and so the rest. Those Lines which are drawn from the Equator to the left hand, do shew the Azimuth in the Summer; and those other to the right hand do shew the same in the Winter. The Use of them is:

1. *The Azimuth whereon the Sun beareth from us being known, to find the Altitude of the Sun above the Horizon.*

First, let the Bead be set for the time, as in the former Chapter; then move the Thred until the Bead fall on the Azimuth: so the Degrees which the Thred cutteth in the Quadrant shall shew the Altitude of the Sun at that time. Where you are to observe, That seeing the Azimuths are drawn on the right side of the Quadrant, you are also to begin to number the Degrees of the Suns Altitude from the right hand toward the left: As if the Sights had been set on the Line A B, and you had turned your right hand towards the Sun in observing of his Altitude, contrary to our practice in the former Chapter.

As if the time given were the 2 of *August*, when the Sun hath about 15 *gr.* of North Declination, you may set the Bead for the time, so you shall find the Height at Noon, when the Sun is in the South, to be 53 *gr.* 30 *m.* when he is 10 *gr.* from the South 53 *gr.* 10 *m.* when 20 *gr.* then about 52 *gr.* 8 *m.* when 30 *gr.* then 50 *gr.* 20 *m.* when 40 *gr.* then 47 *gr.* 48 *m.* when 50 *gr.* then 44 *gr.* 12 *m.* when 60 *gr.* then 39 *gr.* 35 *m.* when 70 *gr.* then 33 *gr.* 50 *m.* when 80 *gr.* then 27 *gr.* when he is in the East or West 90 *gr.* from the Meridian, then is the height near 19 *gr.* 20 *m.* when he comes to be 100 *gr.* then 11 *gr.* 15 *m.* when 110 *gr.* then 3 *gr.* 20 *m.* and before he cometh to the Azimuth of 120 *gr.* he hath no Altitude. For the Sun having 15 *gr.* of North Declination, will rise and set at 114 *gr.* 34 *m.* from the Meridian.

2. *The*

2. *The Altitude of the Sun being given, to find on what Azimuth he beareth from us.*

Let the Bead be ſet for the time, and the Altitude obſerved as before; then bring the Thred to the Complement of that Altitude, ſo the Bead ſhall ſhew the Azimuth required.

As if the ſecond of *Auguſt*, having ſet the Bead for the time, you ſhall find the Altitude of the Sun to be 19 *gr.* 20 *m.* remove the Thred unto 70 *gr.* 40 *m.* the Complement of the Altitude; or, which is all one, to 19 *gr.* 20 *m.* from the right hand toward the left, and the Bead will fall on the Line of 90 *gr.* from the Meridian; and therefore the Point whereon the Sun beareth from us is one of theſe two, either due Eaſt, or due Weſt: And which of theſe is the true Point of the Compaſs, may be ſoon known by a ſecond Obſervation; for if the Sun riſe higher, it is the forenoon; if it be lower, it is the afternoon.

By knowing the Azimuth or Point of the Compaſs whereon the Sun beareth from us, it is eaſie to find,

A Meridian Line, and thereby
The Coaſting of the Countrey,
The Site of a Building,
The Variation of the Compaſs.

As if the ſecond of *Auguſt* in the afternoon I ſhould find by the Height of the Sun that he bears from me 60 *gr.* from the Meridian toward the Weſt; then there being 90 *gr.* belonging to each quarter, the Weſt will be 30 *gr.* to the right hand; the Eaſt is oppoſite to the Weſt, the North and South lie equally between them.

CHAP. X.

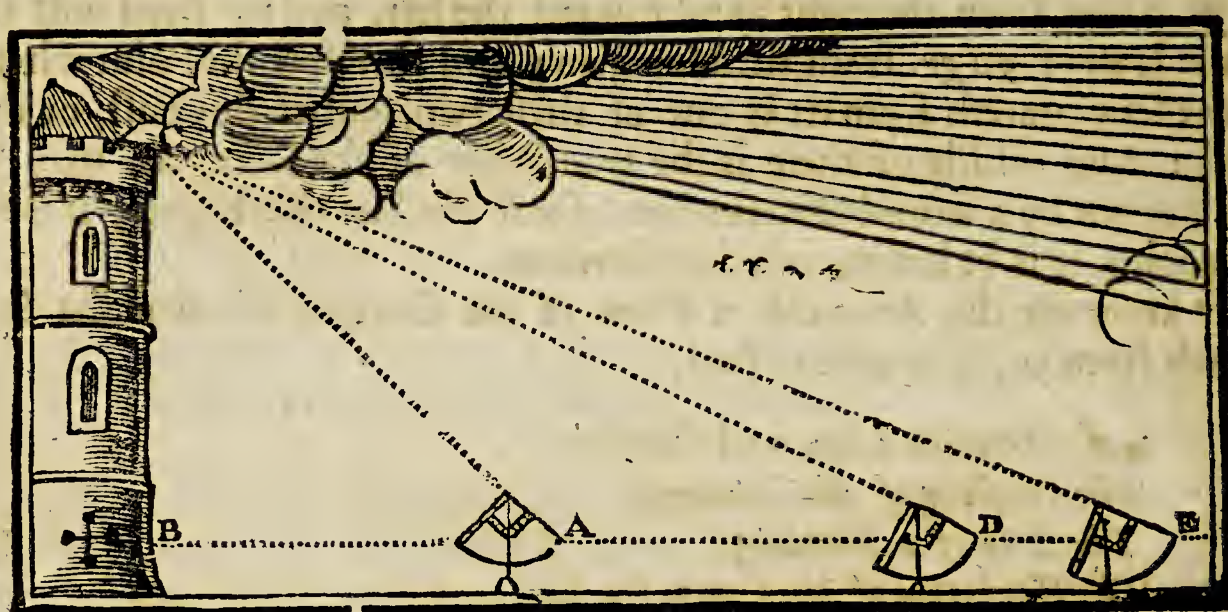
Of the Quadrat.

THe Quadrat hath two Sides divided; the other two Sides next the Center may be ſuppoſed to be divided each of them into 100 equal parts: That which is next the Horizontal line contains the parts of Right Shadow; the other next the Sights, the parts of Contrary Shadow. The Uſe of the Quadrat is,

1. *Any*

1. *Any Point being given, to find whether it be level with the Eye.*

Lift up the Center of the Quadrant, ſo as the Thred with the Plummet may play eaſily by the Side of it: then look through the Sights to the Place given; for now if the Thred ſhall fall on A B the Horizontal-line, then is the Place given level with the Eye: But if it ſhall fall within the ſaid Line on any of the Diviſions, then it is higher; if without, then it is lower than the level of the Eye.



2. *To find an Height above the Level of the Eye, or a Diſtance at one Obſervation.*

Look through the Sights to the Place, going nearer or farther from it, till the Thred fall on 100 parts in the Quadrat of 45 *gr.* in the Quadrant, ſo ſhall the Height of the Place above the Level of the Eye, be equal to the Diſtance between the Place and the Eye.

If the Thred fall on 50 parts of a Right Shadow, the Height is but half the Diſtance: If it fall on 25, it is a quarter of the Diſtance; if on 75, it is three quarters of the Diſtance. For as oft as the Thred falleth on the parts of Right Shadow,

As 100, to the Parts on which the Thred falleth:
So is the Diſtance, to the Height required.

And on the contrary,

As the Parts cut by the Thred are to 100:
So the Height, unto the Diſtance.

But

But when the Thred shall fall on the parts of Contrary Shadow, if it fall on 50 parts, the Height is double unto the Distance; if on 25, it is four times as much as the Distance. For as oft as the Thred falleth on the parts of Contrary Shadow,

As the Parts cut by the Thred, are unto 100:
So is the Distance, unto the Height.

And on the contrary,

As 100, *are unto the Parts cut by the Thred*:
So is the Height, unto the Distance.

And what is here said of the Height and Distance, the same may be understood of the Height and Shadow.

3. *To find a Height or a Distance at two Observations.*

As if the Place which is to be measured might not otherwise be approached, and yet it were required to find the Height B C and the Distance: First, I make choice of a Station at A, where the Thred may fall on 100 parts in the Quadrat, and 45 *gr.* in the Quadrant, the Distance A B will be equal to the Height B C: then if I go farther in a direct Line with the former Distance, and make choice of a second Station at D, where the Thred may fall on 50 parts of Right Shadow, the Distance B D should be double to the Height B C; wherefore I may measure the difference between the two Stations A and D, and this difference D A will be equal both to the Distance A B, and the Height B C.

Or if I cannot make choice of such Stations, I take such as I may, one at D, where the Thred falleth at 50 parts of Right Shadow; the second at E, where it falleth on 40 parts; and supposing the Height B C to be 100, I find that,

As 50 *Parts are unto* 100, *the Side of the Quadrat*:
So 100 *the supposed Height, unto* 200 *the Distance* D B.
And as 40 *Parts at the second Station, unto* 100:
So 100 *the supposed Height, unto* 250 *the Distance* B E.

Wherefore the difference between the Stations D and E should seem to be 50; and then if in the measuring of it I should find it to be either more or less, the Proportion will hold, As from the supposed Difference,

to

to the meaſured difference; ſo from Height to Height, and from Diſtance to Diſtance.

As if the difference between the two Stations D and E being meaſured, were found to be 30,

As 50 *the ſuppoſed difference, unto* 30 *the true difference*:
So 100 *the ſuppoſed Height, unto* 60 *the true Height*:
And 200 *the ſuppoſed Diſtance, unto* 120 *the true Diſtance*:
And 250 *at the ſecond Station, unto* 150 *the Diſtance* BE.

The like reaſon holdeth in all other Examples of this kind. And if an Index with Sights were fitted to turn upon the Center, it might then ſerve by the ſame reaſon for the finding of all other Diſtances.

FINIS.

A SECOND

APPENDIX

CONCERNING THE

DESCRIPTION and USES

OF ANOTHER

QUADRANT,

Fitted for Daily Practice;

For finding the

HOUR and AZIMUTH,

AND

Other things of the *Suns Course*, in reference to the *Horizon*; with New Lines, ſerving to the fore-mentioned, and other Purpoſes, more accurately.

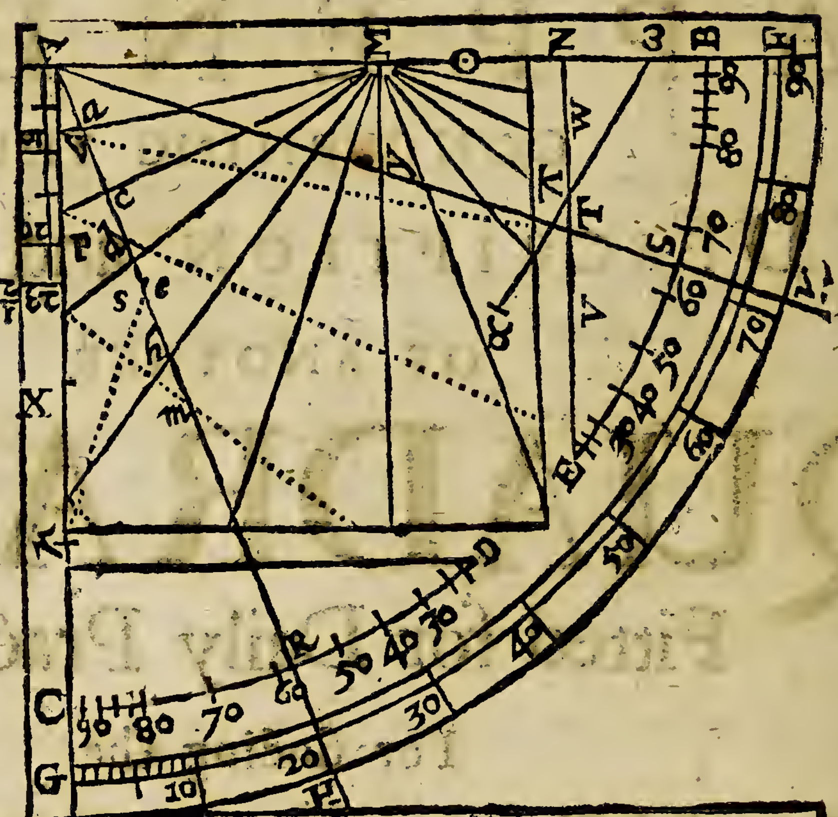
Invented by Mr: *SAM. FOSTER*, ſometime Profeſſor of *Aſtronomy* in *Greſham-College*.

The Deſcription of the QUADRANT.

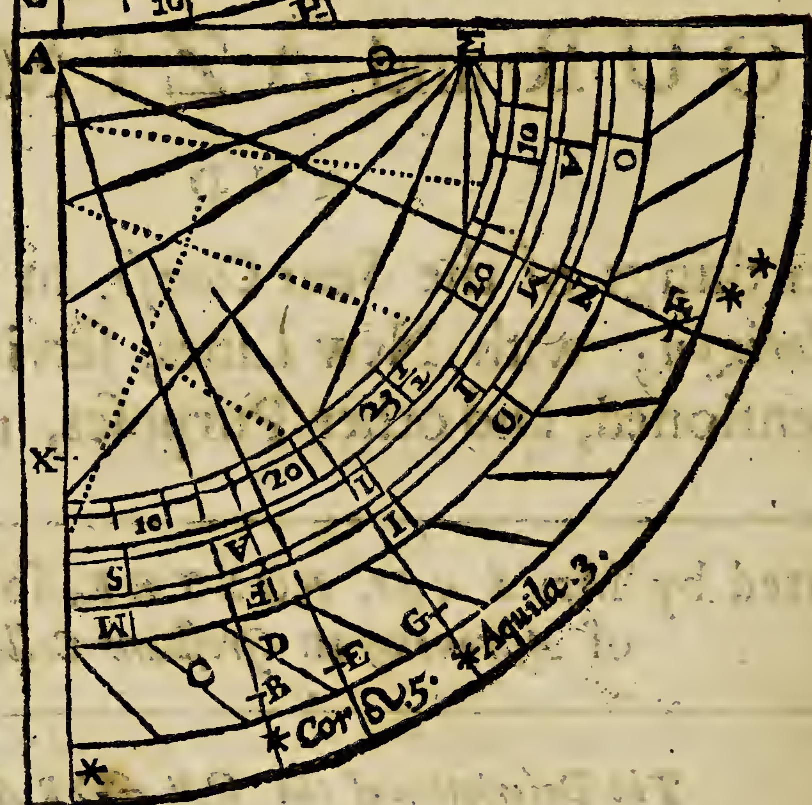
COncerning the Making and Uſe of this Quadrant, you are to underſtand, That the Hour and Azimuth-lines are like thoſe that are ſeen upon the former Quadrant, and the Uſes are (moſt part) the ſame; and therefore we lightly paſs them over, as is ſeen in the

 ſecond

The Foreside of the QUADRANT.



The Backside of the QUADRANT.



ſecond Propoſition: But the diſtance between the Equinoctial and the Tropicks is here ſhortned, to the end that more room might be gained above, for the better placing, and the more exact dividing of the Equinoctials, which in ſmall Inſtruments may be divided to each ſecond Degree; and in larger, to each ſingle Degree.

If it be required to make theſe yet larger, then may the forementioned Azimuth-lines be left quite out: for the uſe of them, as they are here deſcribed, is of ſmall moment, very hardly making good the Suns Coaſt to one entire Degree; and for ſerious Practice, the new Lines added are far more ſufficient. If this be granted, then may the Equinoctial ſtand below, by which means they ſhall become large enough, even in ſmall Inſtruments. Eſpecially this may moſt fairly be done, if the Hour-lines be reverted, by changing the Places of the Equinoctial and Tropicks; that is, if the Equinoctial Altitudes be inſerted below, on the Circle neareſt the Limb, and the Tropical Altitudes above, in the Circle neareſt to the Center. Thus becoming more large, they will ſupply all intended purpoſes very well.

There is no Scheme given of this change now mentioned, nor of the Vulgar Hours and Azimuths, becauſe thoſe Lines are well enough known already, and this mutation is eaſie to be underſtood: But for the inſcribing of the new additional Lines, take theſe following Tables and Directions.

How to inſcribe the Additional Lines upon the QUADRANT.

I. *For the Lines on the Foreſide.*

THe two Equinoctials D C and E B, the one for the Hours, the other for the Azimuths, are to be divided from the equal Limb by help of the following Table; and are beſt to be numbred from the cloſeſt parts of them to the wideſt, as is done in the Figure.

A Table of Equinoctial Altitudes, both for Hours and Azimuths, which are to divide the Equinoctial.

Degrees upon the Equinoctial. Deg.	Degrees upon the Equal Limb for the Hours. D.	M.	Azimuths D.	M.	Degrees upon the Equinoctial. Deg.	Degrees upon the Equal Limb for the Hours. D.	M	Azimuths D.	M.
2	1	15	1	36	48	27	33	30	35
4	2	29	3	11	50	28	29	31	21
6	3	44	4	45	52	29	23	32	4
8	4	58	6	19	54	30	14	32	46
10	6	12	7	52	56	31	4	33	24
12	7	27	9	23	58	31	52	34	0
14	8	39	10	54	60	32	37	34	34
16	9	53	12	22	62	33	21	35	5
18	11	6	13	49	64	34	1	35	34
20	12	18	15	13	66	34	40	36	0
22	13	29	16	36	68	35	15	36	25
24	14	40	17	56	70	35	48	36	47
26	15	50	19	13	72	36	18	37	6
28	17	0	20	29	74	36	45	37	24
30	18	8	21	41	76	37	10	37	40
32	19	16	22	51	78	37	31	37	53
34	20	22	23	59	80	37	49	38	4
36	21	28	25	4	82	38	3	38	14
38	22	32	26	5	84	38	15	38	21
40	23	35	27	5	86	38	23	38	26
42	24	37	28	2	88	38	28	38	29
44	25	37	28	55	90	38	30	38	30
46	26	36	29	47					

The two Equinoctials being divided,

1. Make E N parallel to A C, and O D parallel to A B.

2. Make

2. Make E N a Tangent of 45 *deg.* or *Radius*; then shall A M be the Co-tangent of the Latitude, *viz.* in our Example 38 *deg.* 30 *m.*

3. Make N A *Radius*; then shall A P be the Tangent of 23 *deg.* 30 *m.* and the Line A P to be so divided into 23 ½ Parts.

4. Making A M equal to the Co-sine of the Latitude, A ⊙ shall be the Sine of the Latitude.

5. Make A X equal to A M on both sides.

6. Make *α ω* perpendicular to A *α* 60; and A *α* equal to A N.

7. A r is the half Tangent of 75 *deg.*

8. M A being made the Co-secant of the Latitude, find the *Radius* thereunto belonging, which *Radius* make a Tangent of 45 *deg.* then are the Hour-points upon the Side A C the respective Tangents of 15, 30, 45, and 60 *deg.*

9. Draw r s from the middle Point of M A, and draw the fifth Hour from M parallel to r s: And all the rest of the Hour-lines must be drawn from their several Points to M as their Center. The Line of 6 is drawn from the Center M, perpendicular to A B, or parallel to A C: And all the other Hours beyond 6 may be transferred by a Bevel Square.

10. The double Square of 23 ½ equal *gr.* is done thus:

Add 20 *gr.* to the Equinoctial Altitude; insert the Sum, and the Equinoctial Altitude: Divide the intercepted Arks into 20 equal parts, to which add 3 and a half of the same parts. This is to be set both ways from the Equinoctial, upward and downward, which the inserted Tables will help you to do.

II. *For the Lines on the Back side.*

1. On the Back-side, let the Points M and ⊙ change Places (or be set contrary to what they are on the Fore-side) and then all the other Work (for the manner of it) is the same as on the Foreside.

2. For the reverted Hours, take every Hour-point (upon A C) from A, and turn it twice upon the 6 a clock Line from M, through which Points (and their Correspondents on the Line A C) draw the reverted Hours.

3. The Scales for the Suns Declination, and Months, are inserted from such Tables as are common.

4. The Limb for the Slope-hours may be about a seventh or eighth part of the *Radius*; and the Marginal Divisions numbred 1, 2, 3, 4, 5, 6, *&c.* for Stars, must be put in by that Scale of Declinations according to which you put in the Hours and Azimuths: And the Stars may be such as in the following

following Table, or such other as any shall design to use; but those were conceived by the Author to be as select as any, they being (one or more of them) always in view, and fit for observation.

N	Names.	R.Asc.		Declin.		M
5	*Extra ala Peg.*	0	26	13	27	2
6	*Cauda Leonis*	3	28	16	25	1
5	*Cor Leonis*	16	8	13	27	1
3	*Os Pegasi*	18	58	8	14	3
3	*Aquila*	33	9	8	14	2
1	*Procyon*	34	43	6	3	1
2	*Dex. Hum. Orion*	47	49	7	24	1
4	*Cap. Ophiu.*	50	2	12	52	3
2	*Med. Nex. Coll. Serp.*	63	58	7	24	2
9	*Lucida Pleiad.*	64	0	23	3	4
8	*Arcturus*	74	53	20	58	1
7	*Cornu ♈ prec.*	78	3	17	40	4

☞ THe *Construction of this Quadrant, as it is thus metamorphosed, was communicated by* Edward Page, *living at the Sign of the* Sugar-loaf *in* Hosier-lane, *who maketh this and all other Mathematical Instruments.*

IF other Quadrants were thought complete in use, this will be found much more copious: For it serveth not onely to find the Hour of the Day by the Sun, of the Night by the Stars, and what else belongs to their Risings, Settings, Amplitudes, *&c.* but is very well fitted also to describe all the most usual sorts of standing Dials; that is, all that are upright, or else reclining or inclining to the full East and West; which two sorts will furnish many kinds of such Bodies as are regularly formed. These are here performed by very easie and familiar ways of working. The Nocturnal for the Hour by the Stars, is more expedient in this than in other Quadrants: For in judging of Time onely by the Appulse of the Stars to the Meridian, and finding that Meridian too onely by a rude conjecture from the North-Star, an errour of a quarter or half an Hour is easily unawares committed. This cannot be so here, if any ordinary care be had in taking the Stars Altitude. For this purpcſe there are twelve ſelect Stars inſerted, all of them of North Declination, lying between the Equinoctial and the Tropick of *Cancer* ; and in ſuch difference of Right Aſcenſions, as that one or other of them will be always in ſuch convenient place of the Heavens, as from whence the Hour may very fully be collected every Night throughout the whole Year. Since therefore they are ſo convenient for uſe, there would be a little more diligence uſed to come to the knowledge of them in the Heavens, that due Obſervations may be made whenſoever any of them ſhall be in view. If any deſire that other Stars (ſuch as are better known to them) ſhould be inſerted, they may have their deſire eaſily fulfilled : Onely they muſt take care, that the Stars be ſuch as fall between the Tropicks in the Heavens, and chiefly between the Equinoctial and North Tropick, becauſe ſuch Stars are longeſt in view, and their Hours beſt found.——The Propoſitions that are here ſet down, might have been encreaſed both in number and in variety of performance, if perplexity had been affected ; but ſuch of them, and ſuch ways of effecting them, are here pitched upon, as ſeemed moſt conducible for daily uſe. And for the ſame reaſon it is, that the ſeveral Lines upon the Quadrant are denoted by Letters onely, that by ſuch brevity all unneceſſary circumlocution might be taken off, which, by impoſition of Names to each of them, could not ſo eaſily have been avoided.

If the former Quadrant have heretofore found good acceptance, becauſe it is of ſome good uſe, I doubt not but a greater proportion of thanks will be given from the Ingenious, for making publick this larger Improvement of this Inſtrument.

The USES of the QUADRANT.

I. *To find the Suns Declination.*

LAy the Thred to the Day of the Month upon the back-ſide of the Quadrant, and it will ſhew you the Declination of the Sun in that unequal Scale, which is numbred with twice 3. If your Day fall in the upper Scale of Months, (which may be called the Summer-ſcale) then is the Declination North : If it fall in the lower (or Winter) Scale, the Declination is South from the Equinoctial.

Thus upon *April* 20. you ſhall find the Sun to decline 15*gr.* Northward ; and *January* 30. it declines about 14*gr.* 30*m.* Southward.

¶ The

¶ The contrary Work is eaſie; by aſſigning the Suns Declination, to know on what Day of the Month the ſame ſhall be. For the Thred may be laid to the Declination in two Places, in both which it will croſs the two half years, ſhewing two ſeveral days on which the Sun ſhall have ſo much Declination North; and two days more, on which it ſhall have that Declination Southward. It will be eaſie to diſtinguiſh which of theſe days ſerves your purpoſe, by the two Seaſons of the Year, unto which the two Scales of Months do anſwer.

II. *To rectifie the Bead for Obſervation of Hour or Azimuth; and to perform thoſe things that are done by the uſual Lines upon the Quadrant.*

Having found the Suns Declination for your Day, you muſt count the ſame upon the double equal Scale which is on the fore-ſide of the Quadrant, namely, from the middle of it towards the right hand, if the Declination be North, or towards the left hand if it be South. The Thred being laid thereto, you muſt move the Bead till it fall juſtly upon the Hour of 12, ſo ſhall it be ſet right for the intended uſes of that day As,

1. *For the Hour.* If you obſerve the Suns Altitude (by letting the Sun-beams to ſhine through the Sights, and the Plummet to hang at full liberty cloſe to the Plane of your Quadrant) the Bead will ſhew the Hour, if you have reſpect to the time of the Year: That is, If the Suns Declination be North, the Bead ſhews the time of the day among the Summer-hours, thoſe which ſpread from the Equinoctial towards the right hand If the Sun decline South, the time muſt be accounted in the croſs Lines which are the Winter-hours. And in this Obſervation you ſhall ſee the Thred to cut (in the equal Limb) the Suns Altitude above the Horizon ——Thus at *London*, if the ☉ decline 15 *gr.* Northward, and the Altitude were 9 $\frac{1}{12}$ *gr.* the Hour would be about a quarter before 6 in the Morning, or a quarter paſt 6 in the Evening. But if the Sun had the ſame Declination Southward, and the ſame Altitude alſo, then would the time be half an Hour paſt 8 in the morning, or half an Hour paſt 3 in the evening. The former of theſe times is ſhewed by the Bead among the Summer-hours, the latter among the Winter-hours.

2. *For the Azimuth.* If the Suns Altitude be numbred the contrary way in the equal Limb, and the Thred be laid thereto, the Bead will then ſhew the Azimuth of the Sun, if you account it according to the time of th

the Year; that is, among the Summer-azimuths when the Sun hath North-declination, and among the Winter-azimuths when the Sun declines South. The Summer-azimuths are thoſe that ſpread from the Equinoctial towards the left hand; the other croſſing them are the Winter-azimuths.——Thus if the Suns Declination were 8 *gr.* Northward, and the Altitude 18 *gr.* the Azimuth would be 80 *gr.* from the South: But if the Sun had 8 *gr.* of South Declination, and 18 *gr.* Altitude, the Azimuth would be 50 *gr.* from the South here at *London.* This way may ſerve for groſs works, when the Azimuth is required onely within one or two whole Degrees. You ſhall find it done more accurately, and for better purpoſes, in the thirteenth following.

3. *For the Aſcenſional Difference.* The Bead being rectified as before, and applied to the left ſide of the Quadrant, gives the Aſcenſional Difference, or the time of Sun-riſing and ſetting, before or after 6 a clock, among thoſe Hours and Quarters which interſect each other upon the ſame left ſide of the Quadrant, if you count them agreeable to the time of the Year: And from the Bead to the Line of 12, rightly taken, according to your time of Summer and Winter, gives the Semidiurnal Ark of the Sun, or half the Days length:——As alſo, from the Bead to the other Line of 12, which ſerves for the contrary time of the Year, gives the Seminocturnal Ark, or half the length of the Night.——Thus if the Suns Declination were $14\frac{1}{2}$ *gr.* the Aſcenſional Difference would be 1 Hour and $\frac{1}{4}$ of an Hour: And if the ſaid Declination were North, then the Sun riſeth that day $\frac{1}{4}$ of an Hour before 5, ſetteth $\frac{1}{4}$ after 7. The Semidiurnal Ark (from the Bead to the Summer 12) is $7\frac{1}{4}$ Hours. The Seminocturnal Ark (from the Bead to the Winter 12) is $4\frac{3}{4}$ Hours. Theſe doubled make the day $14\frac{1}{2}$ Hours long; the night $9\frac{1}{2}$ long.

4. *For the Amplitude.* The Bead applied to the right ſide of the Quadrant, gives the Amplitude of Sun-riſing and ſetting in all varieties: Namely; From the Bead to that South-azimuth which is proper to the ſeaſon of the Year, is the Amplitude from South; as alſo, to the contrary South-azimuth, gives the Amplitude from North: ſhewing how many Degrees of the Horizon the Sun riſeth and ſetteth any day from the juſt South or North. So from the Bead to the Eaſt and Weſt-azimuth (which is the ninetieth Azimuth) gives the Amplitude from Eaſt or Weſt.——Thus if the ☉ decline $14\frac{1}{2}$ *gr.* the Amplitude is here $23\frac{1}{2}$ *gr.* almoſt. If the Declination be North, then is this Amplitude from Eaſt and Weſt towards the North $23\frac{1}{2}$ Degrees. The Amplitude from the North it ſelf is then $66\frac{1}{2}$ *gr.* From the South point of the Horizon it is $113\frac{1}{2}$ *gr.*

You may easily (in such manner) account it for South Declinations of the ☉.

V. *To find when Twilight begins in the Morning, and ends at Evening; which Moments are the two utmost Terms of Dark Night.*

AFter the Bead is rectified for your Day, the Thred laid to 18 *gr.* in the equal Limb, will shew the Hour or part required. Only here remember to take your Hour aright: Namely, in Winter time look among the Summer-hours, where it is that the Bead resteth; for that is the Morning or Evening Hour of Twilight: So in Summer time you must look among the Winter-hours ——Thus when the Sun declines 11 $\frac{1}{10}$ *gr.* Southward, the Twilight begins at *London* at 5 in the morning, and ends at 7 a clock at night, as the Bead shews among the Summer-hours: But if that Declination were North, the Twilight would begin at $\frac{1}{4}$ of an Hour before 3 in the morning, and end at $\frac{1}{4}$ after 9 at night.——The Suns depression 18 *gr.* under the Horizon, is the usual Term whereon to begin and end the Twilight. You may as well do this to any Degree of Light, as to 12 or 13 Degrees depression; at which time in the morning all things begin to be visible, and the Light to be of some use. As if the Sun decline 3 $\frac{1}{2}$ *gr.* Southward, if you set the Bead thereto, and then lay the Thred at 12 *gr.* in the equal Limb, you shall see the Bead (among the Summer-hours) fall upon 5 in the morning, and 7 at night; so that at 5, and till 7, there is a reasonable degree of Light. Or if in Summer the ☉ had declined 7 $\frac{1}{2}$ *gr.* Northward, the said degree of Light would begin at 4 in the morning, and end at 8 in the evening.———Near the longest days you shall find no Twilight at all, according to 18 Degrees depression of ☉ under the Horizon; for then the Bead will fall beyond the Winter 12 a clock Line.

¶ These are the chief Uses of the Hour and Azimuth-lines, as they are here, and in all Quadrants commonly inserted. There are other things, concerning the Suns place in the Ecliptick, the Suns Declination, the Suns Right Ascension: Namely,—How by having any one of these to find out the rest.—These are here omitted, as matters onely of curiosity, being of no further use in this Instrument, than that they may be known: Yet if any should desire them, they may have a Scale of the 12 Signs inscribed on the back side, by help of which, the fore-named requisites may be attained.

The Particulars that follow are most aimed at, (as being more then them, and more accurate) and therefore the precedent things are thus briefly passed over.

III. *To find the Suns Ascensional Difference*, &c.

COunt the Declination in the equal Limb from F to K: The Thred there laid gives B S the Ascensional difference :— The said Ascensional difference gives the times of Sun-rising and setting before and after *6*, with the lengths of Day and Night.—The same may be done for all Stars whose Declinations are known.

¶ So by having the Ascensional difference, you may find the Suns Declination thereunto belonging.

Here at *London*, if the Declinntion be 20 *gr.* the Ascensional difference is 27 *gr.* 14 *m.* that is 1 *ho.* 49 *m.* And if this Declination be North, the Sun riseth 1 *ho.* 49 *m.* before *6*, and setteth so much after *6*: that is, it riseth 11 *m.* after 4 in the morning, and setteth 49 *m.* after 7 a clock at night: And the time of setting being doubled, gives 15 *ho.* 38 *m.* for the days length: The time of rising being doubled, gives 8 *ho.* 22 *m.* for the length of the night. But if the Declination had been South, the Sun should rise 1 *ho.* 49 *m.* after *6*, (that is, at 7, 49 *m.*) and should set 1 *ho.* 49 *m.* before *6*, (that is, at 4, and 11 *m.*) and the day would be 8 *ho.* 22 *m.* long; the night, 15 *ho.* 38 *m.*

IV. *To find the Suns Amplitude*, &c.

COunt the Declination in the equal Limb from G to H; The Thred there laid gives C R for the Amplitude.—The same may be done for Stars whose Declinations are known.

¶ So by having the Amplitude, you may find the Declination: For if the Amplitude be counted from C to R, the Thred laid at R gives the Declination G H.

At *London*, if the Declination be 20 *gr.* the Amplitude is 32 *gr.* 20 *m.* from the East and West Points of the Horizon.

V. *Having the Declination of any upright Plane, to find the Elevation of the Style*, &c.

LAy the Thred to the Planes Declination, counted from D to R: so will G H be the Elevation.

 ¶ So

¶ So by having the Elevation G H, you may find D R the Declination.

If an upright Plane (here) decline 20 *gr.* the Styles Elevation will be 35 *gr.* 48 *m.*

VI. *To find the Deflection, &c.*

COunt the Declination from B to S: The Thred there laid gives F K the Deflexion.

¶ So by having F K the Deflexion, you may find B S the Planes Declination.

If a Plane declining 20 *gr.* the Deflexion is 15 *gr.* 13 *m.*

VII. *To find the Difference of Longitude, &c.*

1. COunt the Elevation from F to K: E S is the Difference of Longitude.

2. Count the Deflexion from G to H: C R is the Difference of Longitude.

¶ By the contrary Works, having the Difference of Longitude, you may find the Elevation and Deflexion.

A Plane declining 20 *gr.* hath 25 *gr.* difference of Longitude.

VIII. *To make an Horizontal Dial.*

1. COunt the Hour from E to S; the Thred laid at S gives F K. Then count G H equal to F K; the Thred at K laid gives D R, the ſpace of that Hour from 12.

2. Count the Hour from C to R, and by help of the Thred you ſhall have G H. Then count F K equal to G H; the Thred laid at R, gives B S for the ſpace of that Hour from 12.

3. With a pair of Compaſſes take the Hour from C to R, and ſet it from B to S: B S is the Space or Angle of that Hour from 12.

4. Take with your Compaſſes the Hour from E to S, and ſet it from D to R: So the number D R ſhews how many Degrees that Hour muſt be from 12.

By all theſe ways (here at *London*) the third Hour will be found about 38 *gr.* from 12. The reſt will be in like manner found according to their true quantities.

IX. *To find what Angle any Hour-circle maketh with the Horizon, or any Azimuth makes with the Equinoctial.*

LEt the number of the Hour-circle (or Azimuth) from South, be counted from C to R ; the Thred laid at R will cut the equal Limb in H, and F H will be the Angle required.

¶ By the Angle known, it will be easie, by the contrary Work, to find the Hour (or Azimuth) to which that Angle belongeth.

The third Hour (or 45 Azimuth) makes with theHorizon (or with the Equinoctial) an Angle of *36 gr. 55 m.* here at *London.*

X. *To find what Ark of any Hour-circle is intercepted between the Equinoctial (or any Parallel) and the Horizon.*

COunt the number of the Hour-circle from South, from E to S, or, if it be above 90,from E to B,and back again to S : So F K in the equal Limb will be the Ark required, between the Equinoctial and Horizon.

The Ark intercepted between any Parallel and the Horizon, may hence also be found.—If the Declination of the Parallel be North, and the Hour be between 12 and 6, add the Declination to the Ark found by the former Work : In other Hours beyond 6 subtract the former Ark out of the Declination, the result will be the Ark required. Upon the Hour of 6 it self, the Declination of the Parallels is the Ark intercepted.—If the Declination be South, subtract it out of the Ark found before, (namely, the Ark intercepted between the Equinoctial and Horizon)what remains is the Ark intercepted between that Parallel and the Horizon.

Thus at *London*, the Ark of the third Hour intercepted between the Equinoctial and Horizon is 29 *gr.* 21 *m.*—And if the Declination be 18 *gr.* North,the Ark intercepted between that Parallel and the Horizon is 47 *gr.* 21 *m.*—If the Parallel be 18 *gr.* South,the Ark will be 11 *gr.* 21 *m.*

¶ The first Work will also shew what Ark of any Azimuth from South is intercepted between the Horizon and Equinoctial, if in stead of the Hour-circle from South, you use the Azimuth from South. This intercepted Ark is the Equinoctial Altitude of that Azimuth.

So in the 45 Azimuth from South, the Equinoctial is 29 *gr.* 21 *m.* high. In the 135 Azimuth from South, the Equinoctials depression under the Horizon is 29 *gr.* 21 *m.*

This is made use of afterwards.

XI. *How*

XI. *How high the Sun shall be upon any Azimuth, and in any Declination.*

THe Azimuth is best numbred from the South: And this Proposition (with most of those that follow) is done by help of Compasses.

¶ If the Sun be in the Equinoctial, the first Work of the last Proposition gets the Equinoctial Altitude or Depression, by counting the Azimuth from E to S, whereby the Ark F K will be found. This Ark (if the Azimuth be less than 90) is the Altitude; if more than 90, it is the Depression.

But if the Sun have Declination, then first lay the Thred from F towards K, according to that Declination, and take the least distance from the Point B to your Thred, and keep this extent. Then,

¶ If the Suns Declination be South, count your Azimuth from E to S, and lay the Thred there, which will cut the Line E N in T: Set one Foot of the former extent in T, and turn the other about toward the Side A B, applying the Thred to the remotest distance of that Circuit: The Thred so laid will give the Altitude required, if you count the Degrees from F. Thus the Sun declining South 11 *gr.* 30 *m.* will have 16 ½ *gr.* 30 *m.* of Altitude in the 45 Azimuth.

¶ If the Suns Declination be North, and the Azimuth less than 90 from South, count your Azimuth from E to S, and lay the Thred at it, and let it cut E N in T: Then set one Foot of your former Extent in T, and with the other Foot turned about, lay the Thred at the remotest distance from T towards the Side A C: The Thred so lying, shews from F in the equal Limb the Altitude required. Thus if the Sun decline 11 *gr.* 30 *m.* North, his Altitude upon the 45 Azimuth will be 42 ⅓ *gr.*—But if the Azimuth be more than 90, count from B to S, the excess above 90; and applying the Thred thereto, see what Degrees of the equal Limb the Thred cuts from F. Count that number of Degrees from 60 (in the equal Limb) forwards, towards 70, 80, 90, and lay the Thred there, which suppose to cut the Line $\alpha\ \omega$ in π. Set your Compasses (keeping still their first extent) upon π, and turn the other Foot towards the Side A C, laying the Thred at the remotest turn. If now, to the Thred so laid, you number the Degrees in the equal Limb from 60, the same shall be the Altitude required. Thus if the Sun decline 11 *gr.* 30 *m.* North, and the Azimuth be 101 *gr.* 15 *m.* from the South, the Altitude must be 5 *gr.* 45 *m.* in our Latitude of 51 *gr.* 30 *m.*

Another

Another way for this Proposition.

BY the first Work in this 11th, get the Equinoctial Altitude or Depression for your Azimuth: Then lay the Thred at E, and in C D, from D, count the said Altitude or Depression; from which Number, or Point, take the least distance to the Side A C. Enter this length between the Side A C and the Thred, keeping one Foot upon the Line A C, and removing it thereon too and fro, till the other Foot turned about may justly touch the Thred: Then keeping your Compasses there set, remove the Thred from G toward H, according to the Suns Declination, and take the least distance from your former standing to the Thred. This length measured in the Scale C D (so as one Foot standing upon the Scale, the other turned about may justly touch the Side A C) shews an Ark, which,

If the Suns Declination be South, must be subtracted from
If the Suns Declination be North, and the Azimuth less than 90, must be added to
} the Azimuths Equinoctial Altitude.

If the Suns Declination be North, and the Azimuth more than 90, the Azimuths Equinoctial Depression must be taken out of this Ark.

The result is the Altitude looked for.

Thus if the Azimuth be 70 / 110 from South,

the Equinoctial Altitude / Depression will be 25 $\frac{1}{4}$ *gr.*

The Ark found will be 14 $\frac{1}{4}$. Then,

If the ☉ decline 11 $\frac{1}{2}$ South, the Altitude upon the 70 Azimuth will be 1 Degree.

If the ☉ decline 11 $\frac{1}{2}$ North, the Altitude upon the 70 Azimuth will be 29 $\frac{1}{2}$ Degrees.

If the Suns Declination were 20 *gr.* North, that forementioned Ark would be 25 *gr.* whence taking 15 $\frac{1}{4}$, there remains 9 $\frac{1}{4}$ for the Altitude of the Sun upon the 110 Azimuth from South, at that Declination of 20 *gr.* North.

¶ By this Work may a Table of Altitudes be made, by which the former Azimuth-lines upon the Quadrant may be inserted.

XII. *To*

XII. *To find how high the Sun shall be at any Hour, and in any Declination.*

FIrst, find the intercepted Ark of your Hour Hour, between the Parallel of Declination and the Horizon, by the tenth.

Secondly, Find what Angle your Hour circle maketh with the Horizon, by the ninth.

Thirdly, Count that Angle from C towards D, and from thence take the least distance to the side A C: Measure this length upon the side A C (from A) and there set your Compasses: Then keeping that station of your Compasses, lay the Thred to the intercepted Ark, counted in the equal Limb from G, and take the least distance from your standing to the Thred. Set one Foot of this length in the Scale C D, so as that the other being turned about may touch the side A C; so shall that Foot in the Scale C D give the Degrees of Altitude required, if you number them from C.

Let the Hour be 3 from Noon: The intercepted Ark between the Equinoctial and Horizon will be 29 *gr.* 22 *m.* And if the Sun decline $\begin{matrix}\text{North}\\ \text{South}\end{matrix}$ 11 $\frac{1}{3}$ *gr.* the intercepted Arks will be $\begin{matrix}40.\ 52\\ 17.\ 52\end{matrix}$ And the Angle of the third Hour with the Horizon is 63 *gr.* 53. So that the Altitude for $\begin{matrix}\text{North}\\ \text{South}\end{matrix}$ Declination of 11 $\frac{1}{2}$ *gr.* will be $\begin{matrix}36\\ 16\end{matrix}$ Degrees.

¶ By this Work you may make a Table of the Suns Altitudes upon any Parallel of Declination: And by those Altitudes you may insert those Summer and Winter-hours which are upon the Quadrant.

XIII. *To find the Suns Azimuth.*

FIrst, Lay the Thred to the Suns Declination, counted in the equal Limb from F to K, and take the least distance from the Point B to the Thred, and keep your Compasses at that extent: Then count the Suns Altitude in the equal Limb from F, and lay the Thred to it. This being done,

¶ If the Sun decline South, keep one Foot of your Compasses always upon the Line E N, beyond the Thred, towards E, and remove it still upon that Line, till the other Foot being turned about may touch the Thred precisely. Observe then where the Foot of your Compass standeth upon the Line E N; suppose at V: Bring the Thred to V, and it shews (from E) the Azimuth from the South.

¶ If

¶ If the Sun decline North, keep one Foot of your former extent, upon the Line E N, on this side the Thred towards N, and remove it still upon that line, until the Foot that is turned about do touch upon the Thred. And observe where your Compass Foot then standeth, upon the line E N (suppose it stand at W) Lay the Thred at W, and it will cut the Scale E B; the parts whereof, from E to the Thred, are the Azimuth from South.

But if it so fall out in North Declinations, that when the Thred is laid to the Altitude, you cannot find room upon the line E N, whereon to set your Compasses so as to keep the conditions before required; then work in this manner: Add always 30 degrees to the Suns Altitude, and lay the Thred at that compound Altitude, numbred in the equal limb from F. To the Thred so laid, enter the former extent of your Compasses between the Thred and the line α ω, keeping one Foot always upon that line. And look where the Foot of your Compasses resteth upon that line, suppose at π. Take then the length from π to α, and set it upon the line N E (from N towards E) and to the point where it rests, apply the Thred, observing what parts it cuts upon the Scale from B. The number of those parts gives the quantity of the Azimuth above 90 from the South. Or the parts cut from E, give the Azimuth from the North.

¶ If the Sun decline not at all, but is in the Equinoctial, then the sole Altitude from F to K (by help of the Thred thereto applied) gives E S the Azimuth from South.

If the Altitude of the Sun be 21 $\frac{2}{3}$ in the Equinoctial, the Azimuth from South is 60 degrees.

If the Sun decline South 5 *gr.* and the Altitude were 15 $\frac{1}{4}$ *gr.* the Azimuth would be found 60 *gr.*

If the Sun decline North 20 *gr.* and the Altitude were 50, the Azimuth would be 50 *gr.*

If the Sun decline North 20 *gr.* and the Altitude were 9 $\frac{1}{4}$ *gr.* the Azimuth would be 310 *gr.* from the South.

¶ If you suppose the Sun to have no Altitude and do work by these by these Rules, you shall find the Suns Amplitude, Ortive and Occasive, from the South. As if the Sun decline 20 *gr.* North, you will find 123 *gr.* 20 *m.* for the Amplitude from the South.

XIV. *To find the Hour of the Day by the Sun.*

COunt the Suns Altitude in the equal limb from F, and to the Thred there laid, take the leaſt Diſtance from the point B, and keep this Diſtance.

Then count the Suns Declination (which is had eaſily by the firſt Propoſition:) from F in the equal limb, and apply the Thred to it. Then further,

¶ If the Declination be South, ſet one Foot of your former extent, upon the line E N (always on that ſide the Thred on which E ſtandeth from it) and remove it thereon, till the other (turned about) may juſtly touch the Thred A K. Suppoſe (in ſo doing) the Compaſs Foot ſtayeth at V. The Thred applied to the point V, will cut the hour from Noon, if you count the intercepted parts upon E B, from E.—Thus if the Sun decline 20 degrees South, and the Altitude were 13 *gr.* 50 *m.* the hour at *London* would be 10 or 2.

¶ If the Declination be North, ſet one Foot of your former extent upon the ſide A C, removing it thereon to and fro, till the other Foot turned about, will only touch the Thred. When it is ſo fitted, let that Foot upon the ſide A C, keep its ſtation, and from thence extend the other Foot to the Suns Declination counted in the Scale A P. This laſt extent muſt be applied to the line N E from N: and where it ſtays, lay the Thred. So the parts cut upon the Scale E B, will give the hour.———But this muſt be done with caution. For if that Foot that kept its ſtation, ſtood from A, beyond the Suns Declination in the Scale A P, then the intercepted Ark from E to the Thred, gives the hour from Noon. But if the fore-named Foot ſtood between A and the Declination, then the whole Ark E B 90, with the Ark from B back again to the Thred (theſe two put together) give the hour from Noon.

Thus if the Sun decline 15 *gr.* Northward, and be 21 *gr.* high, the hour is 7 before or 5 afternoon. Or if the Altitude were $\frac{2}{3}$ *gr.* the hour muſt have been 5 in the morning, or 7 in the evening: namely, 90 and 15 degrees from Noon.

XV. *On*

XV. *On an upright declining Plane, to find the Angle between* 12 *and* 6.

COunt the Planes Declination from C towards D: From that point take the leaſt Diſtance to the ſide C A. Set that length from M to Y, upon the line M Y. The Thred laid at Y gives G K for the Angle between 12 and 6.

Or count the Declination of the Plane from B towards E, and lay the Thred at it. The Thred will cut N E. Take from N to the interſection, and apply it to M Y; the Thred put to Y gives G K, as before.

If a Plane decline 20 *gr.* this Angle will be 66 $\frac{1}{4}$ at *London.*

XVI. *To find the Declination of a Plane.*

FIrſt, draw an Horizontal line upon your Plane (which you may do by your Quadrant.) Then apply one ſide of the Quadrant to that line, ſo as the limb may be toward the Sun, and the Plane of the Quadrant may lie Horizontally flat. Thirdly, having a looſe Thred and Plummet, you muſt hold that Thred cloſe by the edge of the limb (letting the Plummet hang down at liberty) till the ſhadow of the Thred paſſeth directly through the Quadrants Center. Which done, you ſhall ſee what degrees of the limb the ſhadow cuts from that ſide of the Quadrant which is perpendicular to the Horizontal line. This is called the Horizontal Diſtance. At the ſame moment of time, obſerve the Suns Altitude. By this Altitude you may get the Suns Azimuth from South, by the thirteenth.

After this Preparation, take diligent notice, whether the ſhadow of the Thred fall betwixt the South, and the perpendicular ſide of the Quadrant. Or whether the ſame ſhadow fall ſo, as to leave both the South and the ſaid perpendicular ſide (both of them) upon one coaſt of the ſhadow.

In the firſt caſe you muſt add the Horizontal Diſtance to the Azimuth. In the latter caſe, you muſt ſubtract the leſſer out of the greater. The reſult (whether it be Sum or Difference) gives the Planes Declination from the South.

Note here in the ſecond caſe. That if the Horizontal Diſtance be greater than the Azimuth, then doth the Plane decline to that coaſt (Eaſt or Weſt) which is contrary to the coaſt on which the Sun ſtood from the South. This falleth out very frequently.

Note also in the first case: That if the Sum of the Horizontal Distance and Azimuth do exceed 180*gr.* then the Planes Declination from South is contrary to that coast whereon the Sun stood. And it is found by subtracting the fore-mentioned Sum out of 360 degrees. This happens more seldom; that is, only upon some North Planes; and on them, only then, when the Suns Azimuth is more than 90 from the South; and the Horizontal Distance more than is the Azimuth from the North.

Examples are here omitted for Brevities sake. Only add this; that if the Planes Declination from South be above 90*gr.* you must subduct it out of 180, and the remainder is the Declination from the North. —By this accounting from North and South, you may always make that your Plane decline not above 90. And as when it declines nothing, it is a full South or North Plane; so if it decline just 90, it is then a full East or West Plane.

XVII. *How to draw any upright declining Dial.*

FIrst, draw a Perpendicular or Plumb-line A B, and cross it at right Angles with the Horizontal line B C, and make B A equal to A O in your Quadrant.

2. Upon the equal limb of your Quadrant, count the Planes Declination (from North to South) from G, and there keep the Thred: which will cut some of those Lines that are drawn within the upper Square.

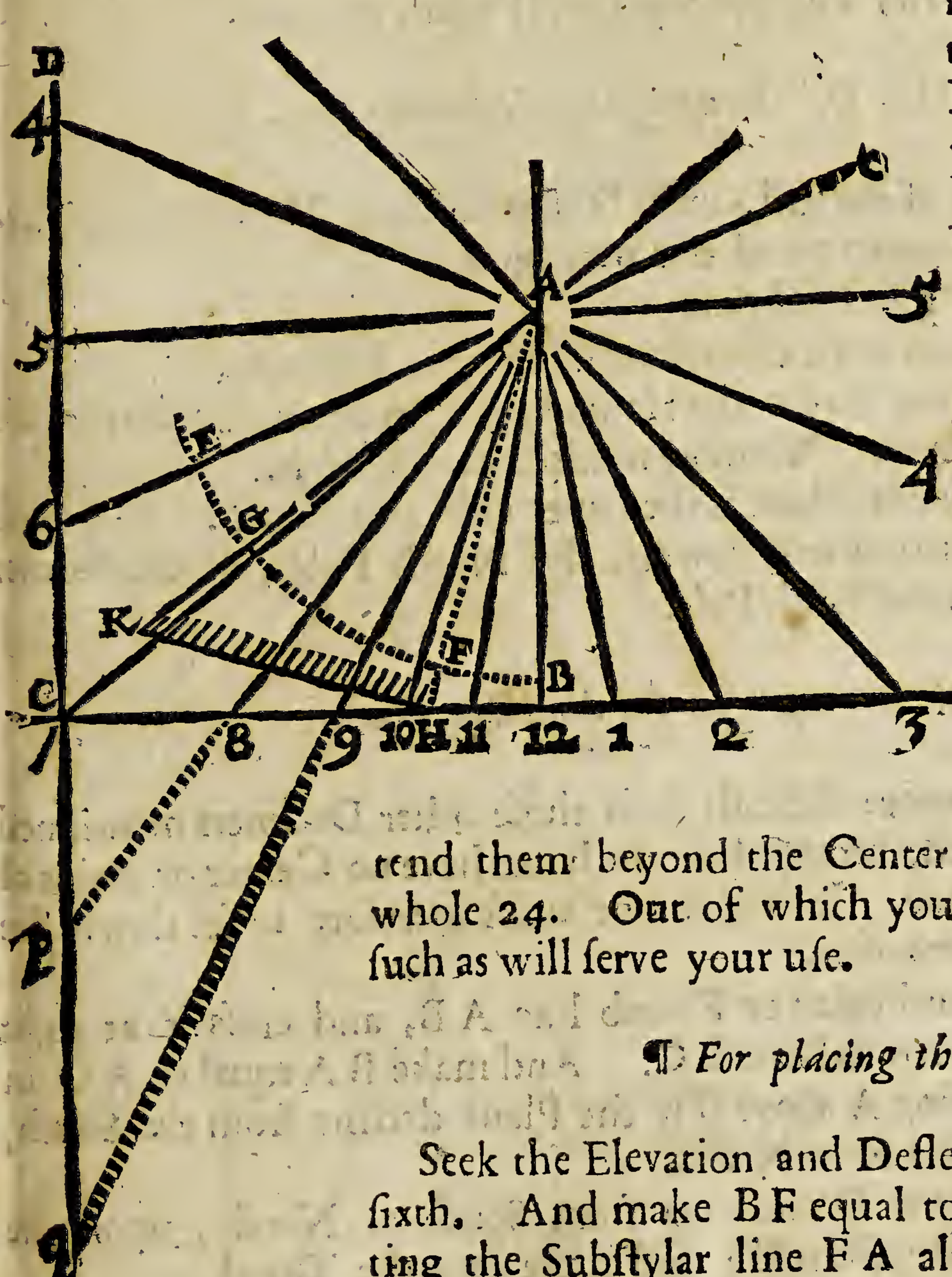
3. Observe first, those Intersections which the prickt lines make with the Thred at *b*, *d*, *m*, —— Take then the Length from A, the Center of the Quadrant to *b*, and set it here upon the Horizontal Line from B to 1, (always on that side of B, which looks to the same coast whereunto the Plane declineth.) So take from the Quadrants Center A, to the second prickt Lines Intersection with the Thred, at *d*; and set it here from B to 2. So likewise the third A *m*, must be set from B to 3.

4. Observe again all such Intersections as are made with the Thred, by the rest of those lines whose common Concurrence is in the point M, namely, at *a*, *c*, *e*, *h*: and take their several lengths from the Quadrants Center A, and prick them here down on the other side B (contrary to the coast of Declination) namely, at 11, 10, 9, 8. Then for the next line upon the Quadrant (which doth not, but would intersect the Thred, if it were drawn out far enough) observe where the Thred cuts the

extra-

extravagant line *r s*, namely in *s*: and take from A to *s*, and turn that Length twice from B, so shall it design the point 7. Afterwards at the point 7, draw the infinite line C D parallel to B A. Also set off the hour of 6, on that side B which is contrary to the coast of Declination; namely, from B to E, according as the Angle between 12 and 6 shall be found by the fifteenth.

Declination 28 deg. *S. East.*



5. Draw all the Hour-lines from A, the Center of your Dial, through the points 3, 2, 1, 12, 11, 10, 9, 8, 7, in such wise, that as many as well can, may cut the line D C, as is here done, in *p* and *q*.

6. Make *6*, 5, equal to *6*,7: and *6*,4, equal to *6*, *p*, and *6*, 3, equal to *6*, *q*; and draw the rest of the hours *A* 5, *A* 4, *A* 3. Thus you may get 12 hours, and if you extend them beyond the Center, you shall have the whole 24. Out of which you may make choice of such as will serve your use.

¶ *For placing the Style.*

Seek the Elevation and Deflection by the fifth and sixth. And make B F equal to the Deflection; setting the Substylar line F A always on that side 12, which is contrary to the coast of the Planes Declination, Make also F G equal to the Elevation. So F A G will be the pattern of the Style.

Or

Or the Thred lying ſtill at the Planes Declination upon the Quadrant as it did, Take the leaſt Diſtance from the point X to the Thred, and ſet that Length from B to H, and draw A H for the Subſtylar. Then making A H K a right Angle, take the leaſt Diſtance from M to the Thred, and make H K equal to this Diſtance: So is K A H the pattern of your Style.

¶ *In all Dials*, The Style muſt ſtand juſt over the Subſtylar, elevated ſo much above it, as the Elevation (before found) cometh to.

In South upright Decliners the Center of the Dial is above (as in the former figure) and the Style points downward. But in North Decliners, the Center muſt be low, and the Style muſt point upward.

XVIII. *Of the upright full South-Dial.*

THe Declination of the full South-Dial is nothing. Whence it is, That The Angle between 12 and 6 is 90 degrees.

The Line of 12 is the Subſtylar.

The Styles Elevation is the Complement of your Latitude.

The way of pricking down the Hours is in a manner the ſame with that before for Decliners. No more needs to be ſaid of it.

The Erect full North Plane is the ſame with this South. Only the Style of this points upwards toward the North Pole, as the former downwards towards the South Pole.

XIX. *Of upright far declining Planes.*

THeſe Dials are more difficult than thoſe other Decliners mentioned in the ſeventeenth, becauſe here the hours have no Center or Point of meeting upon the Plane. It will not be amiſs therefore to ſet down the whole work in all parts of it.

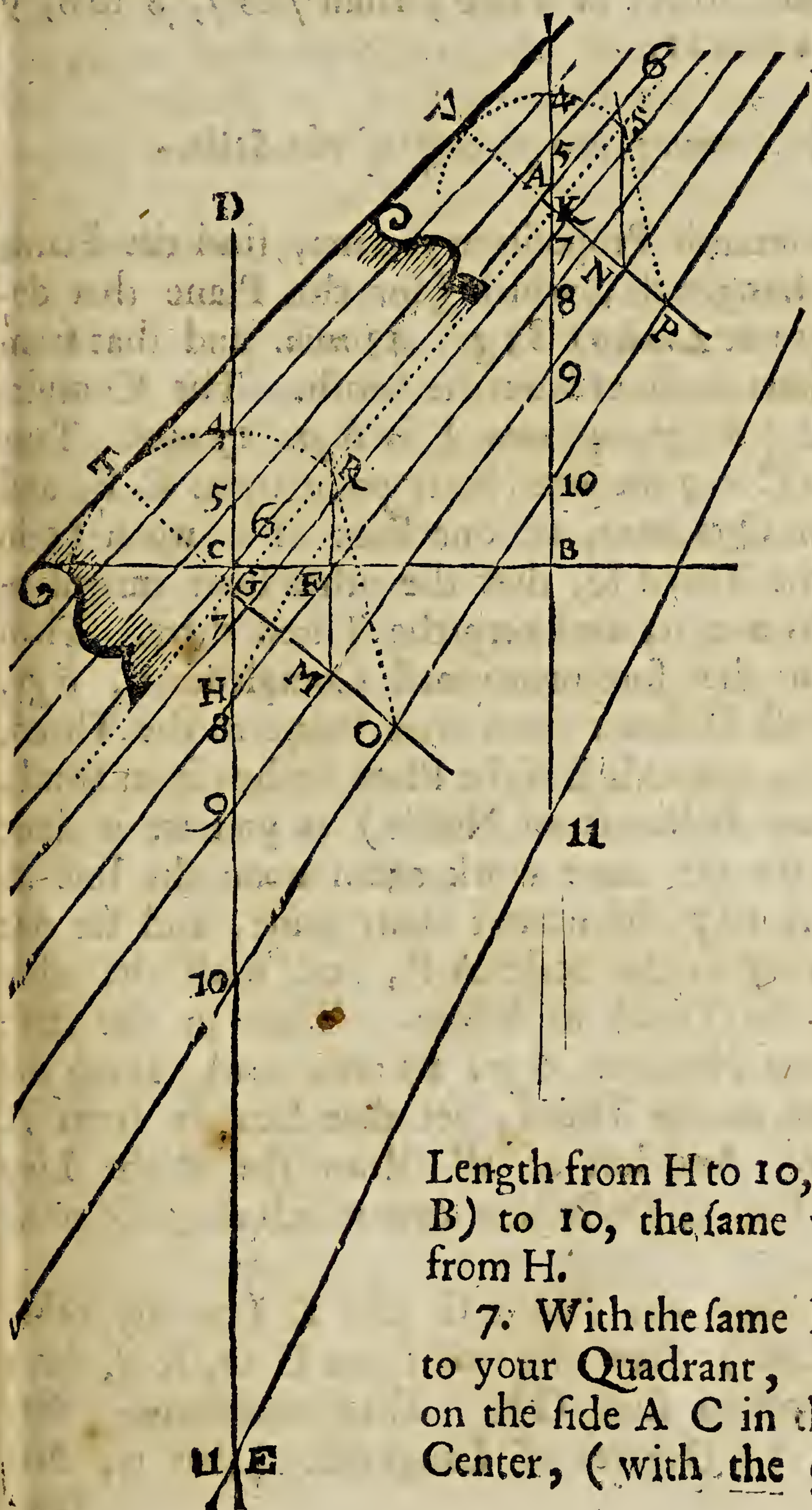
1. Draw a Perpendicular or Plumb-line A B, and croſs it at right Angles with the Horizontal line B C. And make B A equal to A O in your Quadrant, ſetting A above B if the Plane decline from the South, or below B if it decline from North.

2. Count the Planes Declination from South or North, upon the limb of your Quadrant, from G; and there keep the Thred.

3. Among thoſe Lines on the Quadrant (whoſe common Concurrence is at M) obſerve that Interſection which is made by the ſixth Hour from the Quadrants Center with the Thred: Take the length from

from the same Center to that Interfection, and prick it down here from B to C (and on that side B which looketh toward the South, if the Plane decline from South; or toward the North, if the Plane decline from North.) And draw out the Line C D E parallel to B A.

4 Observe again upon that Quadrant the Interfection which the second line from the Center makes with the Thred and take the length from the Center of the Quadrant thereunto and prick it down towards C, namely from B to F.



An upright Plane declining 82 degrees *from* South *Eastward.*

5. Take the Lengths from the Center of your Quadrant to every hour point upon the side A C: and prick them all down there, from C to 7 and 5, from C to 8 and 4, from C to 9, 10. And Lastly, Take from the Center of your Quadrant to the point *r*, and turn that Length twice from C: this double Length will reach from C to 11, at E.

6. Lay a Ruler to A and F, and transfer the point F unto H in the line C E. Then take the Length from H to 10, and set it from A (towards B) to 10, the same way from A that 10 stands from H.

7. With the same Length H 10 or A 10, go to your Quadrant, and setting one Foot of it on the side A C in the fourth point from the Center, (with the other turned about) lay the

the Thred at the remoteſt Diſtance, and keep it there.

8. From every point on the ſide A C of your Quadrant, take the leaſt Diſtances to the Thred ſo laid; ſetting them down from A to 7 and 5, from A to 8 and 4, from A to 9: A 10 was put on before. Then the leaſt Diſtance from *r* to the Thred being twice turned from A towards B, will give the Length from A to 11.

9. For the finiſhing then of the hours you have no more to do, but draw right lines through each couple of correſpondent points, namely, from 4 to 4, 5 to 5; from C to A, or 6 to 6; from 7 to 7, 8 to 8, 9 to 9, 10 to 10, and from 11 to 11.

¶ *Concerning the forming and placing of the Stile.*

10. BY the precedent ſeventh Propoſition you may find the Planes Difference of Longitude, which (for this Plane that declines 82 *gr.*) will be (here at *London*) 83 *gr.* 43 *min.* and that from the South, becauſe the Plane declines from the South. The Complement of which Longitude (83 *gr.* 43 *min.*) is 6 *gr.* 17 *min.* Take then firſt, the Length from C to 7 the next hour point upon C E, and carrying that extent to your Quadrant, ſet one Foot of it upon 15 in the Scale A P: and lay the Thred ſo, that the other Foot turned about may juſt touch or paſs over it, and keep the Thred there. Then (in the Scale A P) count the fore-mentioned Complement, 6 *gr.* 17 *min.* and taking the leaſt Diſtance from that Point to the Thred, ſet it from 6 a Clock at C, towards E if the Plane decline from South, (or towards D if the Plane decline from North) as you ſee it done here, at G. Secondly, do the ſame work again upon the line A B; That is, take from A to 7 the neareſt Hour point, and ſet one Foot of that extent upon 15 in the Scale A P, and with the other Foot turned about, lay the Thred as before. Then in the ſame Scale A P, count the ſame Number 6 *gr.* 17 *min.* and taking the leaſt Diſtance from thence to the Thred, ſet that Length from A to K, anſwering to C G. And laſt of all, draw the Right Line G K. This ſhall be the Line of Deflection over which the Stile muſt ſtand.

11. Furthermore, Through the Points G and K (or any other two points of the ſame Line) draw the two Lines G O, K P, both perpendicular to the Deflection Line G K. Then conſidering, that every Hour comprehends 15 Degrees of Longitude (that is, that from

from C to 7 is 15, and from 7 to 8 is 15, *&c.*) and ſince that C G is 6 *gr.* 17 *min.* If C G be taken out of C 7 which is 15 *gr.* there will remain G 7, 8 *gr.* 43 *min.* To which, if you add from 7 to 9, which is two hours or 30 degrees, the Sum will be 38 *gr.* 43 *min.* whoſe Complement is 51 *gr.* 17 *min.* If now you make the Angles G M R, and K N S, each 51 *gr.* 17 *min.* they will cut the Deflection Line G K, in R and S. And if further, to the Radius G R you deſcribe the Ark R T; and to the Radius K S you deſcribe the Ark R T; and to the Radius K S you deſcribe the Ark S V; and draw the Line T V, a Tangent to both theſe Arks, the *Trapezium* G T K V ſhall be the pattern of your Stile. In placing which, you muſt be careful that theſe perpendicular Lengths G T and K V (perpendicular I ſay to T V the fiducial Edge) be juſtly placed upon the two aſſumed points at G and K.—Or having found G 7 to be 8 *gr.* 43 *min.* you may add to it from 7 to 10, which is (three hours or) 45 degrees. The Sum will be 53 *gr.* 43 *min.* whoſe Complement is 36 *gr.* 17 *min.* If now from the points O and P (where the ſaid hour of 10 cuts the two fore-mentioned Perpendiculars G O and K P) you make the Angles G O R and K P S, each equal to 36 *gr.* 17 *min.* they will cut the Deflection Line G K in the ſame two points R and S. After which you may proceed to make the pattern of your Stile, as before.

¶ 1. Note, That in performing the fifth Section of this Propoſition, inſtead of taking thoſe Hour points from the Center of your Quadrant upon A C the ſide for your Quadrant (if thoſe Diſtances ſhould be too great of your Plane) you may lay the Thred any where upon the Quadrant, and inſtead of taking from the Center to the fore-named Points, you may take the leaſt Diſtances from the ſaid Points to the Thred, ſeverally, and ſet them down from C to 7 and 5, and from C to 8 and 4, and ſo to 9, 10; and for 11, you muſt take from the Point *r* to the Thred, and ſet it twice from C; by which means they will be all of leſs Diſtance from C. And then all the work is to be continued, as is before preſcribed. ——Or if the ſaid Diſtances ſhould be too little, you may double, triple, or, *&c.* to make them greater.

¶ 2. Note again, That in Decliners from the North, that Difference of Longitude which you find by the ſeventh, is to be reckoned from the North, and ſo the Complement of it is to be accounted from C (or 6 a Clock) towards D. And that the wideſt part of the hours in theſe North Planes muſt point upwards, and the cloſeſt parts downwards; contrary to what is expreſſed here in this Plane, which hath its Declination from the South.

¶ 3. Note laſtly, that this Direction here given for enlarging the Hours in far Decliners, may eaſily be applied to ſuch Direct or Horizontal Dials (as are mentioned in the 26. following) upon which the Pole hath but ſmall Elevation. For the Dial (or only ſome chief Hours of it) being deſcribed in its natural ſtreightneſs, may be enlarged by the ſame means that this leaſt was. Which will not be hard to do, but would be tedious here to run over again.

XX. *Of full East and West upright Dials.*

THeſe are more eaſie than the former ſort were. For having drawn the Plumb-line A B, and aſſumed the Point A for the Hour of 6; go to your Quadrant, and take from the Center of it to all the Hour-points upon the ſide A C; and prick the firſt of them down in the Line A B, from A to 5 and 7, the ſecond from A to 4 and 8, the third from A to 3, the fourth from A to 2; and for the fifth, take from the Center of your Quadrant to the Point *r*, and ſet that Length twice from A, ſo it ſhall limit out the point 1. —Having theſe points, draw Lines through them, all parallel one to the other, and all pointing up to the North; namely, ſo as to make the acute Angles B A C equal to the Complement of your Latitude.

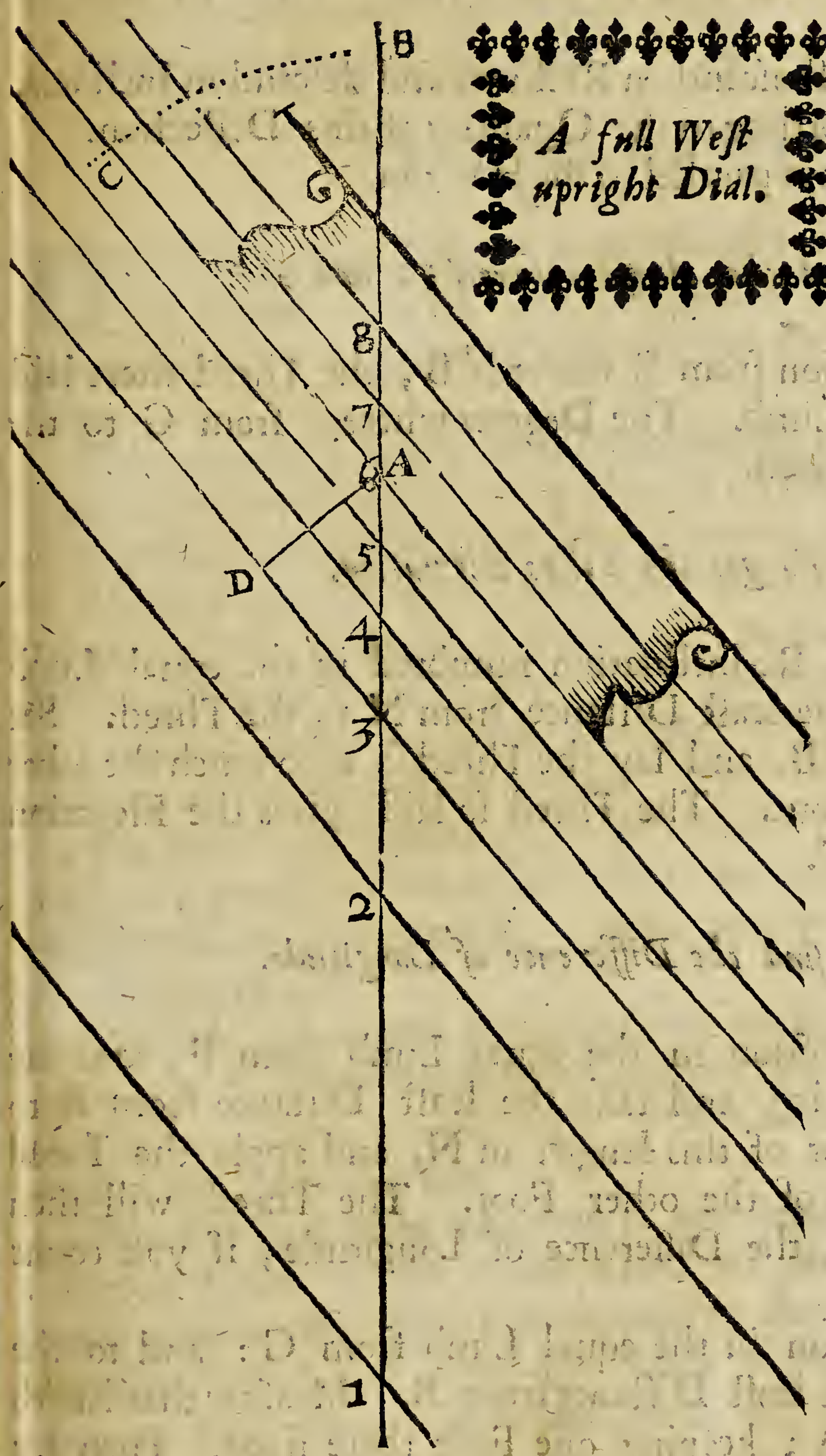
¶ *For the Stile.*

IT muſt always ſtand over the Line of 6 a Clock, parallel to it, and diſtant every where from it according to the Length of A D. Which Length is ſoon found, by drawing A D perpendicular to the Hour-Lines, cutting the third hour from 6, in D. By which Line you may make the pattern of your Stile. For the fiducial Edge lies parallel to the Line of 6, A C, and at the Diſtance of that Line A D.

1. Note here too, that if your Lengths from the Quadrants Center to the Hour-points be too long, you may ſhorten them by laying the Thred upon the Quadrant, according as your Conve-

nience

nience shall direct, and taking the least Distances from those Hour-Points to the Thred; and so pricking them on from A to 6, to 5, 4, 3, *&c.* as was before mentioned in the first Note upon the former Proposition. —Or if they be too little they may be doubled, *&c.* as is there expressed.



A full West upright Dial.

¶ 2. Note further, that what is here done for describing these East and West Dials, may be applied to the Direct Polar Plane. Only remember that you are not tied (if the Polar) to make the Hours to any set Angle with the Line B A, but they are best at right Angles; for then the Line A B may be taken for, and placed as the Horizontal Line of the said Plane; all the Hours lying as vertical Lines unto it. And also the Line of 6 here must be taken (in the Direct Polar) for the Line of 12, and the rest of the Hours are to be drawn alike on both sides 12: nothing in substance differing from these East and West Planes.

XXI. *In East and West Re-incliners, to get the Deflection.*

COunt the Re-inclination from D towards C. Take the least Distance from thence to the side A C. Set that Length from M to Y, and lay the Thred at Y. The Degrees F K will give the Deflection.

The Substylar Line must ascend in Recliners and descend in Incliners, from the Line of 12, according to the Quantity of this Deflection.

The Line of 12 lies always parallel to the Horizon.

XXII. *To find the Angle between 12 and 6.*

COunt the Re-inclination from E towards B, the Thred there laid will cut the equal Limb. The Degrees whereof from G to the Thred, are the Angle required.

XXIII. *To get the Stiles Elevation.*

LAy the Thred to the Re-inclination numbred in the equal Limb from F, and take the least Distance from N to the Thred. Set one Foot of that length in B, and lay the Thred so as to touch the other Foot when it is turned about. The Thred so laid, gives the Elevation in the equal Limb, from F.

XXIV. *To find the Difference of Longitude.*

1. COunt the Deflection in the equal Limb from F, and lay the Thred to it; and take the least Distance from B to the Thred. Put one Foot of this length in N, and apply the Thred to the remotest Distance of the other Foot. The Thred will then shew in the equal Limb, the Difference of Longitude, if you count from F.

2. Count the Deflection in the equal Limb from G: and to the Thred there laid, take the least Distance from B. Measure that length upon the side A B from A; keeping one Foot there fixed. Then lay the Thred to the Planes Re-inclination counted also from F in the equal Limb, and take the least Distance from your standing to the Thred. Set one Foot of this length in B, applying the Thred to the other Foot

Foot turned about. The Thred ſo laid, gives the Difference of Longitude in the equal Limb, from G.

Thus if an Eaſt or Weſt Plane re-incline, here at *London*, 30 degrees, it will have in

Deflection————	47 *deg.*	26 *m.*
Angle from 12 to 6———	55	26
Elevation————	23	02
Difference of Longitude—	70	14

XXV. *How to draw the Dial.*

UPon the Back-ſide of your Quadrant, in the upper part of it, you have Lines drawn altogether like thoſe on the Fore-ſide placed near the Quadrants Center, the uſe of which was ſhewed before.

The manner of work in this Propoſition is in moſt things ſuitable to that in the ſeventeenth, and will need no other direction.

Only for placing the Lines, Take notice, that

The line of 12 in theſe Eaſt and Weſt Re-incliners, lieth always parallel to the Horizontal line of the Plane. So that if we ſuppoſe the former Figure of the ſeventeenth to repreſent one of theſe Dials, then A B muſt be conceived to lie Horizontal, and B C Vertical. All other works will be like to thoſe in the ſeventeenth.

The Stile in Recliners pointeth upward, and the Subſtilar and the hour of 6 do aſcend above the line of 12, ſo much as the Deflection and Angle from 12 to 6 come to. The Center of the Dial is on the South end of the line of 12.

The Stile in Incliners pointeth downward, and the Subſtilar and the hour of 6 do deſcend below the line of 12, ſo much as the Deflection and Angle from 12 to 6 come unto. The Center of the Dial is on the North end of the 12 a clock line.

Theſe things being obſerved, you muſt count the Re-inclination of your Plane in the equal Limb on the Back-ſide from the left hand toward the right, according as the Figures are ſet: and there lay the Thred and keep it. Then obſerve how it cuts the Lines next to the Center, and proceed in all things as in the ſeventeenth before.

¶ Note, That you may find the Inclination of a Plane by applying one ſide of your Quadrant to the Planes Vertical line: for ſo the Thred will

will cut the quantity of Inclination in the degrees of the equal limb being numbred from that side of the Quadrant which toucheth the Plane. ——And for finding the Reclination, you may lay a Ruler to the Vertical line of the reclining face, and take the Inclination of the under-side of that Ruler. That Inclination will be the same with the Reclination.

Note also, that this here delivered for East and West Re-incliners, is intended chiefly for drawing hours upon those kinds of Planes when you meet with them upon Bodies cut regularly. For otherwise you will hardly ever find any such just Plane upon a fixed Building.

Lastly, for a Scale of Chords, which here, and in some of the precedent Precepts is required, you may make use of the equal limb of your Quadrant.

XXVI. *To make an Horizontal Dial to any Latitude.*

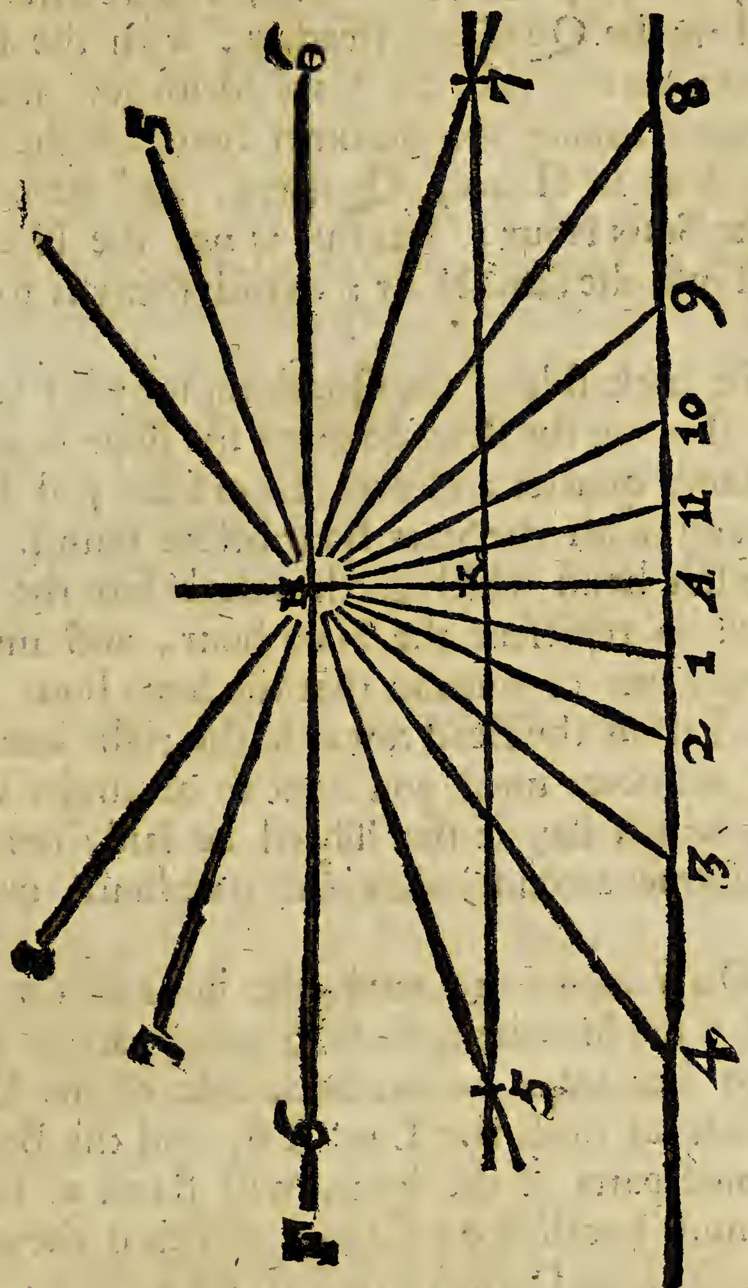
FIrst, draw the right Line B C, and erect the Perpendicular A H. Then take from the Center (on either side of your Quadrant) to the third hour upon the side A C; and make A H equal thereto. And draw F H parallel to B C; and the line 5 K 7 also just in the midst of them. ——After this lay the Thred to the Latitude of the place counted in the equal limb: and take from every point of the side A C, the least distance to the Thred, and set each of them down both ways, namely, from A to 4 and 8, from A to 3 and 9, to 2 and 10, and from A to 1 and 11. Then take from the point *r* upon the side A C, to the Thred, and set that length from K to 5 and 7 both ways. —— You have now nothing more to do, but only from H to draw the Hour-lines to all the fore-named points: so the draught is easily finished.

The Stile must stand upon the Line of 12, and is to be elevated according to the Planes Latitude: as the manner is in all Horizontal Dials.

¶ The use of this Proposition is to draw all Dials in any Latitude for any direct re-inclining Plane. For, the Re-inclination compared (in North Re-incliners) with the Poles Elevation: or (in South direct Re-incliners) with the Equinoctials Altitude, will easily give the Planes Latitude: in the former the Difference was the Elevation it self: in the latter, the Complement of the Poles Ele-

vation

vation. ——And this Propoſition, with the ſeventeenth for upright Planes; the twentieth for upright Eaſt and Weſt, and ſo alſo for Polar Planes on which the Pole hath no Elevation: the twenty fifth



for Eaſt and Weſt Re-incliners: the eighteenth for full North and South erect, will furniſh you with ways to draw Dials upon ſuch regular Bodies, whoſe Planes have any ſuch of the fore-mentioned Aſpects.

XXVII. *To find the Hour of the Night by the Stars.*

THE Stars upon the Quadrant (one or other of them) will always be in a convenient place of the Heavens: that is, of two or more

more hours Diſtance from the Meridian. ——— Having then made choice of that Star that is fitteſt, look what number is annexed to the name of it. Seek that number in the left margin of the fore-ſide of your Quadrant, cloſe by the Hour-lines, and rectifie the Bead to it. ——Then hold up the Quadrant ſteadily, with the ſights levelled to the Star, as if you were to take the Stars Altitude: and you ſhall find the Bead to ſhew (among the Summer hours of the Quadrant) the Motion of the Star in Hours, Quarters, and parts of a Quarter. This is called the Stars Hour; but this is not the Hour of the Night till it be turned into the Suns Hour: which thing is to be done in this manner.

Look upon the back-ſide of the Quadrant for your Star, and lay the Thred upon it; ſlipping the Bead down to the ſlope hours below, till it ſtand upon the ſame quarter and part (from ſome juſt hour on the left hand of the Bead) with the Stars hour before found. Then note the ſaid hour on the left hand which goeth next before the Bead, for that muſt be ſuppoſed to repreſent the Stars hour, and muſt therefore be called by the ſame name or number that the Stars hour was. And the following hours (from the Bead towards the right hand) muſt ſucceſſively take their numbers until you come to be under the day of your Month. Unto which day if the Thred be laid, the Bead will (by keeping of your former account) ſhew the true hour, quarter, and part of the Night.

Example I. On *January* the 20*th.* the hour of *Cor Leonis* was obſerved Eaſtward of the Meridian, to be 9 and $\frac{1}{3}$ part of a quarter. The Thred laid upon that Star, on the back-ſide of the Quadrant, will croſs the ſlope hours as doth the Line A B, and the Bead put down to the fore-mentioned parts of the hour, will ſtand at the point B. So that the hour C muſt be called 9 a Clock, which is the obſerved hour of the Star. Then the Line D muſt be called 10 a clock: and the Thred being put to *January* 20. (taken in the lower circular Line of Months) will lie in the line A E; and the Bead at E ſhews the time of the Night to be paſt (the line D, that is paſt) 10 a clock about $\frac{1}{2}$ and $\frac{1}{3}$ part of a quarter, which is 15 and 5 *min.* or 20*min.* paſt 10 at Night. ——But if this Obſervation had been made upon the ſecond day of *November*: then the Thred laid upon the day given in the lower Circle of Months, *November* 2, would lie in the line A F: and the Bead would be upon the full Hour-line that paſſeth through F, which would be 4 a Clock in the morning. For if the line C be 9, the line D is 10, the next line is 11, and so forward till your account fall upon F: which must be 4 a clock past (12 or) Midnight.

Example II. Upon the 8 of *August*, the Star *Aquila* was seen on the West side of the Meridian, and the hour of it was found 3 and ½ an hour and ½ a quarter. The Thred therefore being laid upon that Star would be as the line A G, and the Bead (rectified to the ½ hour and ½ quarter) would stand at the point G. So that the next Hour-line on the left hand of G, must be called 3 a clock, and the line F must be 8 a clock. Then, the Thred being removed to the day of your Month (*August* 8, in the upper circular line of Months) will lie in the line A B; and the Bead at B will shew the Hour of the Night (if you keep your former account) to be ¼ and half past 1 a clock. For if F be 8 a clock (as is before expressed) then the last hour of the limb is 11, the first is 12, the second 1; beyond which the Bead B is about 22 *m.* of an hour. Therefore the hour of the Night is 1 a clock 22 *min.*

By these Examples the manner of the work will sufficiently appear in all cases.

The Use of the Altimetrick Scale.

THE Scale on the Fore-side of the Quadrant next to the equal Limb is here called the Altimetrick Scale. It is numbred by 1, 2, 3, *&c.* to 10, 20, 30, *&c.* to 100. Each of which numbers are best supposed to be 100 fold, *viz.* 100, 200, *&c.* to 1000, 2000, *&c.* to 10000: and all the lesser parts estimated accordingly. ——The ground on which you stand to make your mensuration, is also supposed to be a just Level.

I. *To find any Height at one Observation.*

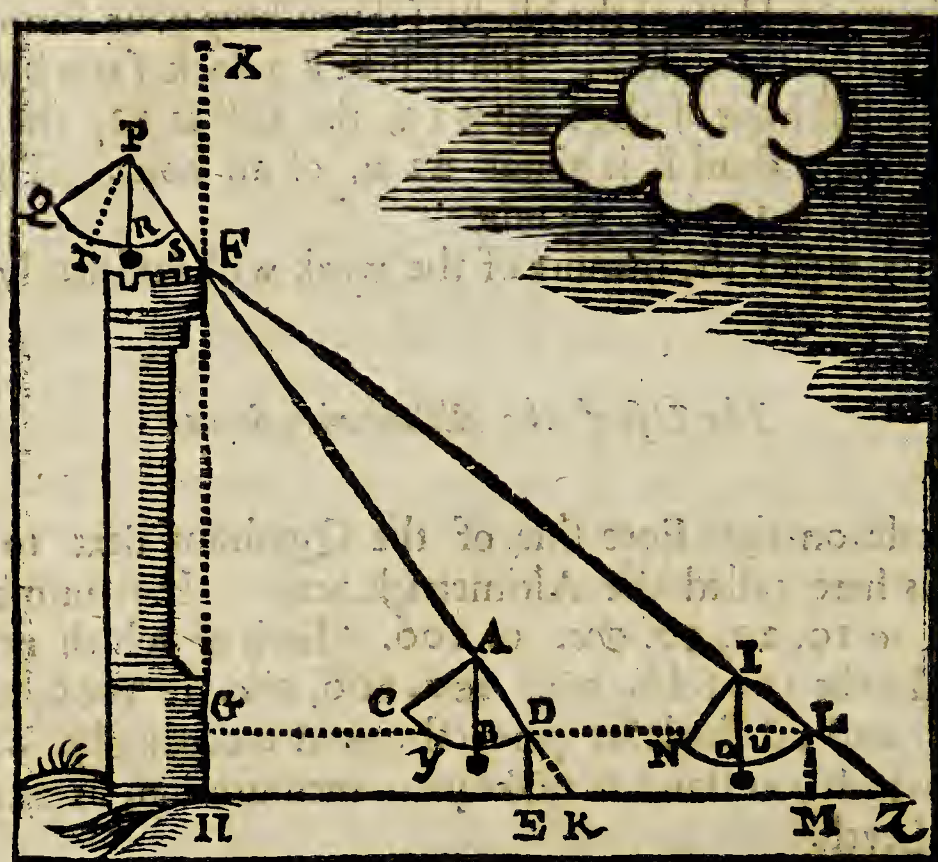
LEt your Station be at E; and the sights D A directed to the point F: the Thred A B cuts off the parts C B in the measuring Scale: which parts must be remembred. —Then measure from your Station E, to the point H, which is just under F. And always in this case multiply this distance E H by the fore-named parts of C B, and from the Product cut off three figures toward the right hand. The Remainder is the Altitude G F. To which you must add H G, or D E, the height from your eye at D to your foot at E.

 Thus

Thus if the Thred A B ſhould cut off C B 1500 parts, and the Diſtance E were 59 feet, the height G F would be 88. 5000 or 78 $\frac{1}{3}$ feet.

II. *To find part of an Altitude.*

LEt the length of F X be only required. Standing then at E, you may find the Altitude G F. Keep ſtill the ſame Standing at E, and find the Altitude G X by the laſt Precedent. So G F taken from G X, gives F X required.



III. *Standing upon a known Height to find a Diſtance.*

LEt the height F H be known, and the Diſtance H K be required. Order your ſtanding ſo, that the two ſights P, S, the point F, and the Diſtance K, may all appear in one right line. Then look what degrees the Plummet cuts off in the equal limb from Q. Count the ſame number in the ſame limb from S; and there lay the Thred, as P T. Note then what parts it cuts upon the meaſuring Scale from Q to T. Multiply thoſe parts into F H the known Altitude: and from the Product cut off three Figures the Remainder or Quotient is the Diſtance H K.

Thus

Thus if the Thred P R ſhould cut off Q R in the equal limb, 56 ⅓ degrees, the ſame counted the other way from S to T in the equal limb, and the Thred laid thereto would give *667* in the meaſuring Scale. Then F G being 88 ½ feet and G H (ſuppoſe) five feet, F H muſt be 93 ½ feet. This multiplied into *667*, makes *62364*: from whence cutting away the three right hand figures, there remains *62.364* or *62* ⅓ feet for the Diſtance H K.

IV. *To find part of a Diſtance.*

IF the Diſtance of K from Z were required. Firſt, find H K, then H Z, by the third precedent: their Difference is K Z. If K Z were a Trench, you might from the Tower F, find the bredth of it without any approach unto it.

V. *To find a Height at two Obſervations.*

IF F H were to be meaſured, and the way from E to H were unpaſſable, ſo that the Diſtance of E from H could not be meaſured. You muſt in this caſe make two Obſervations. For which purpoſe, take your firſt ſtation at E, and direct the ſights D, A, to the point F: noting what parts the Thred cuts upon the equal limb from C to B. Then go backwards in a right line, to a competent Diſtance, as to M; and there making a ſecond ſtation, obſerve (as before) what degrees the Thred cuts upon the equal limb from N to O: (the two ſights L, I, being juſtly directed to the point F.) Then count theſe two Arks in the equal limb from the contrary ſide of the Quadrant, namely from D to Y, and from L to *u* and applying the Thred thereto, look what parts it cuts from the meaſuring Scale at Y and V. Take the leſſer number of parts out of the greater, noting the Difference. Meaſure alſo the Diſtance of your two ſtations, namely, from E to M, and add three ciphers to that meaſure; This laſt number muſt (in this kind of work) be divided always by the fore-noted Difference: and the Quotient will give the Altitude of F above G.

Example.

Let the firſt Obſervation cut off 38 ⅔ *gr.* in the equal limb. The ſecond 56 ⅓ *gr.* Count the firſt Ark from D to Y: the Thred there laid gives 1250 in the meaſuring Scale. The ſecond ſo counted from L to V, gives *667*: The Difference of theſe two is 583. Let the Diſtance

ſtance of the ſtations meaſured from E to M, be 51. 60 feet. This number, with three ciphers added, is 5160000. Which divided by 583 (the former Difference) gives in the Quotient 88.50 or 88 $\frac{1}{3}$ feet for the Height G F. And if G H be 5 foot more: The whole Heighth H F will be 93 $\frac{1}{2}$ feet.

¶ Note, That in theſe Menſurations, the point G is ſuppoſed to to ſtand in the ſame Level with the corner of your Quadrant D and L. So that G H, D E, L M are all of one Heighth. And note too, that the two ſtationary points are E and M, namely, thoſe which are juſt under the corners D and L.

FINIS.

THE GENERAL USE OF THE CANON, AND Tables of LOGARITHMS.

LOgarithmetick is a Logical kind of Arithmetick, or artificial uſe of Numbers invented for the eaſe of the Calculation, wherein each Number is fitted with an Artificial, and theſe Artificial Numbers ſo ordered, that what is produced by Multiplication of natural Numbers, the ſame may be effected by the Addition of theſe their Artificial Numbers; what they perform by Diviſion, the ſame is here done by Subtraction: and ſo the hardeſt part of Calculation avoided by an eaſie Proſthaphæreſis.

All this ſhall be made plain by applying that to theſe Artificial Numbers, which I have ſet down before, for the uſe of my Lines of Numbers, Sines and Tangents in the Uſe of the Sector and Croſs-Staff. Wherein the Reader is to obſerve, that what is to be wrought by round Numbers only, is beſt done by Mr. *Brigges* his Logarithms, but the Aſtronomical part concerning Arks and Angles, by my Canon of Artificial Sines and Tangents.

CHAP.

CHAP. I.

Concerning the Use of the Line of Numbers, I have set down ten general Propositions in the first Book of the Use of the Cross-Staff, Chap. VI. *and those may be applied to the Table of Logarithms.*

PROP. I.

To multiply one Number by another.

THis is the sixth Proposition of the ten; but I begin with the easiest, add the Logarithm of the Multiplicator, to the Logarithm of the Multiplicand, the Sum of both shall be the Logarithm of the Product.

As when when we multiply 25 by 30, the Product is	750
So here, add the Logarithm of 25, *viz.*	1.39794001
To the Logarithm of 30	1.47712125
The Sum of both will be	2.87506126

And this is the Logarithm of 750.

In like manner, if we multiply 10 by 10, the Product is 100. If 100 by 10, the Product is 1000 so here

The Logarithm of 10 being	1.00000000
The Logarithm of 100 shall be	2.00000000
1000	3.00000000
10000	4.00000000
100000	5.00000000

And so forward: All intermediate Numbers which have intermediate Logarithms.

If we multiply 101 by 10, the Product is 1010; of 102 by 10, the Product is 1020: so here

The Logarithm of 10, *viz.*	1.00000000
Added to the Logarithm of 101	2.00432137
Gives the Logarithm of 1010	3.00432137
The same Logarithm of 10	1.00000000
Added to the Logarithm of 102	2.00860017
Gives the Logarithm of 1020	3.00860017

The

The Difference being only in the first Figure, and that is always less by one than the number of Places, in the Number given. As when we find the Logarithm to be 2,00860017 the first Figure 2 is Characteristical, *i. e.* the Index, shewing that the whole number 102 belonging to this Logarithm, consists of three places. If the Logarithm had been 1,00860017, the whole Number must have been 10.2 consisting of two places, and the rest a Fraction $\frac{2}{10}$.

If the Logarithm were 0.00860017 the Number belonging to it would be 1. 02. 1. 1 and $\frac{2}{100}$. And this is one of the reasons why the Differences were omitted in the first hundred Logarithms. All those Logarithms may be found afterwards under a larger Index.

Again, if we multiply 201 by 5, the Product is 1005: so here: If we add the Logarithm of 5 unto the Logarithm of 201, the Sum of both shall be the Logarithm of 1005, and the Sum of the Logarithms of 5 and 203 shall be the Logarithm of 1015. Thus the most part of the Table may be continued beyond 1000.

PROP. II.

To divide one Number by another.

SUbtract the Logarithm of the Divisor, out of the Logarithm of the Dividend, the Remainder shall be the Logarithm of the Quotient.

As when we divide 750 by 25, the Quotient is 30	so here
From the Logarithm of 750, *viz.*	2.87506126
Subtract the Logarithm of 25	1.39794001
There remains the Logarithm of 30	1.47712125

In like manner, when we divide 11 by 4, the Quotient is $2\frac{3}{4}$, so

here the Logarithm of 4, *viz.*	0.60205999
Taken from the Logarithm of 11	1.04139269
Leaves the Logarithm of $2\frac{3}{4}$	0.43933270

Wherefore, if it were required to find the Logarithm of a whole Number with a Fraction annexed (as of $2\frac{3}{4}$) we might first reduce it into an improper Fraction of $\frac{11}{4}$ (or rather of $\frac{275}{100}$) and then subtract as before.

If it were required to find the Logarithm of a single Fraction, as of $\frac{4}{15}$, we may subtract as before: But this Fraction being less than 1, then the Logarithm must be less than 0, and therefore noted with — a defective sign.

So the Logarithm of $\frac{11}{4}$ or $2\frac{3}{4}$ is + 0.43933270
And the Logarithm of $\frac{4}{11}$ — 0.43933270

PROP. III.

To find the Square of a Number.

HAlf the Logarithm of the Number given is the full Logarithm of the Square Root.

So the Logarithm of 144 being 2.15836249
The half thereof is 1.07918124
the Logarithm of 12, and such is the Square Root of 144.

Then by conversion, having extracted the Square Root, we may soon find the Logarithm.

As the Logarithm of 10.0000 being 1.00000000
The Logarithm of the Square Root 316227, is 0.50000000
And for the Root of that 177827 0.25000000

PROP. IV.

To find the Cubique Root of a Number.

THe third part of the Logarithm of the Number given, is the full Logarithm of the *Cubique Root.*

So the Logarithm of 125 is 2.09691001
And $\frac{1}{3}$ the Logarithm of 5 0.69897000

By the same reason we may find the Biquidrate Root, by dividing the Logarithm of the Number given by 4: the solid Root, by dividing by 5, and so forward.

And by conversion, having extracted the Root, we may soon find the Logarithm.

As the Logarithm of 10.000, *&c.* is 1.00000000
The Logarithm of the Cubique Root, 21544 0.33333333
The Logarithm of 100.000, *&c.* 2.00000000
The Logarithm of the Cubique Root 4641 0.66666666
Then multiplying the Square and Cubique Roots one by another, we may produce infinite other Numbers, and have all their Logarithms.

PROP.

PROP. V.

Three Numbers being given, to find a fourth Proportional.

THis Golden Rule the most useful of all others may be wrought several ways, as it appears by this Example:

As 12 *unto* 24 : *so* 4 *to a fourth number.*

The ordinary way in Arithmetick is by Multiplication and Division. For first they multiply the second into the third, and then divide the Product by the first Number given. As here, multiplying 24 by 4, the Product is 96, then dividing 96 by 12, the Quotient will be 8, the fourth number here required. I. Tactus 2 & 3 divisus per 1.

According to this way we add the Logarithms of the second and third, and subtract the Logarithms of the first, so that which remaineth shall be the Logarithm of the fourth Number required.

Thus the Logarithm of the first Number	12 is	1.07918125
The Logarithm of the second	24	1 38021124
The Logarithm of the third	4	0.60205999
The Sum of the second and third Logarithms		1.98227123
Subtract the first, and there remaineth		0 90338998

And this is the Logarithm of 8, the fourth Proportional.

A second way in Arithmetick is by Division and Multiplication. For where the second Number is greater than the first, they may divide the second by the first, and then multiply the third by the Quotient. As here, dividing 24 by 12, the Quotient is 2: then multiplying 4 by 2, the Product will be 8. II. Quotiens 2 per 1 divisi multiplicatus in tertiū.

According to this way we take the Logarithm of the first out of the Logarithm of the second, and then add the difference to the Logarithm of the third. So the Sum of this Addition shall be the Logarithm of the fourth required.

Thus the Logarithm of the first Number	12 is	1.07918125
The Logarithm of the second	24	1.38021124
The Difference between the increasing		30102999
Added to the Logarithm of	4	0.60205999
Gives the Logarithm of	8	0,90308998

III. A third way in Arithmetick is by Division and Division, for where the second Number is less than the first, they may divide the first by the second, and then again divide the third by the Quotient. As here, dividing 12 by 4, the Quotient is 3: then dividing 24 by 3, the Quotient is 8.

Quotiens 1 per 1, fit divisor 3.

According to this way we take the Logarithm of the second out of the Logarithm of the first, and then take the Difference out of the Logarithm of the third: so that which remaineth shall be the Logarithm of the fourth Number required.

Thus the Logarithm of the first Number	12 is	1.07918125
The Logarithm of the second	4	0.60205999
The Difference decreasing		47712126
Subtracted from the Logarithm of	24	1.38021124
Gives the Logarithm of	8	0.90308999

These two latter ways by Difference of Logarithms, may be considered as the same. Though there be some difference between them, yet that may easily be reconciled, if we have regard to the nature of the question. For three numbers being given in direct proportion, if the second be greater than the first, the fourth must be greater than the third: If the second be less than the first, the fourth must be less than the third, and their Logarithms accordingly. But in reciprocal proportion, considering the first and second numbers to be of one denomination, we are to observe the contrary.

If we desire to turn Subtraction into Addition, we may take the Logarithm which is to be subtracted out of the Radius, and add the Complement. So the Sum of this Addition, the Radius being subtracted, shall give the required Logarithm as before.

Thus in the last Example: where subtracting the Difference 4.7712126 out of 1.38021124, the Logarithm of 24, we found the Remainder to be 0.90308998 the Logarithm of 8.

The Radius being	10.00000000
The Logarithms to be subtracted	0.47712126
The Complement to the Radius is	9.52287874
This added to the Logarithm of 24	1.38021124
Gives us a compound Logarithm	10.90308998

From

From this, if we ſubtract the Radius, (that is, if we cancel the firſt figure to the left hand) the reſt is 0.90308998
the Logarithm of 8, the fourth Proportional, as before.

By help of this fourth Proportional we may come ſomewhat near to find a Logarithm for a number of 6 places.

As if it were required to find a Logarithm for this number 868624, the Table will afford us Logarithms for a leſſer and a greater number; and then the intermediate may be found by the part proportional in this manner.

Here we have the Logarithm of	868	2.93851973
And the Logarithm of the next following	869	2.93901978
And the tabular Difference between them		50005

If the Index be fitted to the number of places,

The Logarithm of	868000	ſhall be	5.93851973
And the Logarithm of	869000		5.93901978
The Difference being	1000		50005

Then taking 868000 out of 868624, (the number given) the third Difference will be 624. And having theſe three Differences the Proportion will hold.

As	1000	*unto*	50005	
So	624	*unto*	31203	the part propor-

tional to be added to the leſſer Logarithm 5.93851973
ſo ſhall we have 5.93883176 for the Logarithm required.

In like manner, having a Logarithm given, we may find the value of it in a number of ſix places.

As if the Logarithm given were 3.93883182
and it were required to find the Number to which it belongeth:

This Logarithm is not to be found in the Table; but changing the Index and making it 2.93883182

The next leſſer Logarithm of 868 is	2.93851973
And the tabular Difference following	50005
And the proper Difference	31209

As the tabular Difference	50005	*unto*	100000
So the proper Difference	31209	*unto*	62411

the part proportional to be joyned to the end of the former number 868:

so shall we have 86862411 for the value of this Logarithm. But the Index of the Logarithm being 3, the Number required must consist of four places, *viz.* 8686, and the rest a Fraction of $\frac{24}{100}$.

This I say is somewhat near the Truth. For this number here proposed 868624 is the Square of 932.

The true Logarithm of the Root	932 is	2.96941591
The true Logarithm of the Square 868624		5.93883182

PROP. VI.

Three Numbers being given, to find a fourth in a duplicated Proportion.

IN Questions that hold in a duplicated Proportion between Lines and Superficies, the Logarithms for Lines given may be doubled, the Logarithms for Lines required may be halfed, and then the work will be the same as in the first part of the former Proposition.

Suppose, the Diameter being 14, the content of the Circle was 154, the Diameter being 28, what may the content be?

Here the Question concerns both Lines and Superficies, I double the Logarithms of the two Lines given, and then work as before in this manner:

The Logarithm of	14 is	1.14612803
The Logarithm of	28	1.44715803
The same again		1.44715803
The Logarithm of	154	2.18752072
The Sum of these last		5.08183678
Subtract the double of the first		2.29225606
There remains the Logarithm of 616		2.78958072

And such is the content of the Circle here required.

Suppose the content of a Circle being 154, the Diameter of it was 14; the content being 616, what may the Diameter be?

Here being one Line given, and one Line required, I double the Logarithm of the Line given, and then working as before, the half of the remainder shall be the Logarithm of the Line required.

Thus

Thus the Logarithm of 154 is	2.18752072
The Logarithm of 616	2.78958072
The Logarithm of 14	1.14612803
The same again	1.14612803
The Sum of these three last	5.08183678
Subtract the Logarithm of the first	2.18752072
The Remainder will be	2.80431606
The half thereof is	1.44715803

The Logarithm of 28 the Diameter required.

Or according to the second manner of operation, the difference between the Logarithms of Lines given may be doubled; the difference between the Logarithms of the content given may be halfed, and then the work will be the same as in the latter part of the former Proposition.

So in the first Question, where the Diameters were given and the content required.

The Logarithm of 14 is	1.14612803
The Logarithm of 28	1.44715803
The Difference increasing	30103000
The double of this Difference	60206000
Added to the Logarithm of 154	2.18752072
Gives the Logarithm of 616	2.78958072

In the second Question, where the content of both the Circles was known, and the Diameter of the one required.

The Logarithm of 154	2.18752072
The Logarithm of 616	2.78958072
The Difference increasing	60206000
The half of this Difference	30103000
Added to the Logarithm of 14	1.14612803
Gives the Logarithm of 28	1.44715803

PROP.

PROP. VII.

Three Numbers being given, to find a fourth in a triplicated Proportion.

IN Queſtions concerning Proportion between Lines and Solids, the Logarithms for lines given may be tripled; the Logarithms for lines required may be divided into three parts, and then the work will be the ſame, as in the firſt way for the Rule of Three.

Suppoſe the Diameter of an Iron Bullet, being four inches, the weight of it was nine pound, the Diameter being eight inches, what may the weight be?

The Logarithm of	4	is	0.60205999
The Logarithm of	8		0.90308999
The Triple of it			2.70926997
The Logarithm of	9		0.95424251
The Sum of theſe laſt			3.66351247
Subtract the Triple of the firſt Logarithm			1.80617997
There remains the Logarithm of 72			1.85733251

And ſuch is the Weight required.

Suppoſe the Weight of an Iron Bullet being nine pound, the Diameter was four inches; the Weight being ſeventy two pound, what may the Diameter be?

The Logarithm of	9	is	0.95424251
The Logarithm of	72		1.85733250
The Logarithm of	4		0.60205999
The double of this again			1.20411998
The ſum of theſe laſt			3.66351247
The firſt Logarithm ſubtracted, there remains			2.70926996
The third part thereof is			0.90308999

The Logarithm of 8, and ſuch is the Diameter required.

Or

Or according unto the ſecond manner of operation in the Rule of Three, the Difference between Logarithms of lines given may be tripled; the Difference between the Logarithms of the Solidity or Weight given may be divided into three parts.

So in the firſt Queſtion, where the Diameters were known, and the Weight required.

The Logarithm of	4	is	0.60205999
The Logarithm of	8		0.90308999
The Difference increaſing			30103000
The triple of this Difference			90309000
Added to the Logarithm of	9		0.95434254
Gives the Logarithm of	72		1.85733251

In the ſecond Queſtion, where the Weight was known, and the Diameter required.

The Logarithm of	9	is	0.95424251
The Logarithm of	71		1.85733250
The Difference increaſing			90308999
The third part of this Difference			30102990
Added to the Logarithm of	4		0.60205999
Gives the Logarithm of	8		0.90308998

PROP. VIII.

Having two Numbers given, to find a third in continual Proportion, a fourth, a fifth, a ſixth, and ſo forward.

ACcording to the firſt way in the Rule of Three, we may ſubtract the Logarithm of the firſt number, out of double the Logarithm of the ſecond, the remainder ſhall be the Logarithm of the third, then ſubtracting the Logarithm of the firſt Number again, out of the Logarithms of the ſecond and third, that is, out of triple the Logarithm of the ſecond, the remainder ſhall be the Logarithm of the fourth, and ſo forward.

As,

As, when we say: *As* 1 *unto* 2, *so* 2 *unto* 4, *and* 4 *unto* 8, *and* 8 *unto* 16, &c. Because the first Number is 1, there is no need of Division, but only to multiply 2 the second Number into it self, the Product gives the third Proportional Number to be 4: then multiplying 2 into 4, the fourth Proportional is 8: and multiplying 2 into 8, the fifth Proportional is 16; and so forward. So here the Logarithm of the first number being 1, there is no need of Subtraction.

But finding the Logarithm of 2 to be	0.30102999
The double gives the Logarithm of 4	0.60235999
The Triple gives the Logarithm of 8	0.90308999
The Quadruple give the Logarithm of 16	1.20411998

and so forward *in infinitum*.

In all other numbers that begin not with 1, we may either subtract the Logarithm of the first Number or add the Complement unto the Radius.

As when the Numbers given are 100 and 108.

The Logarithm of the first Number 100 is	2.00000000
The Logarithm of the second 108	2.03342276
From the double of this second Logarithm	4.06684752
Subtract the first Logarithm, there remains	2.06684752

the Logarithm of $116\frac{64}{}$ the third Proportional.

Again, subtract the first Logarithm	2.00000000
Out of the Sum of the Logarithms of	2.03342376
The second Number and the third Proportional	2.06684752
There remains the Logarithm	2.09927128

answering unto $125\frac{671}{}$ the fourth Number in continual Proportion.

According to the second manner of Operation we may take the Difference between the Logarithms of the two Numbers given; so this Difference applied to the Logarithm of the second Number, shall give the Logarithm of the third Proportional: the same Difference applied to Logarithm of the third Proportional, shall give the Logarithm of the fourth Proportional, or the double of this Difference applied to the Logarithm of the first Number, shall give the Logarithm of the third Proportional: the treble of this Difference applied to the Logarithm of the first Number, shall give the Logarithm of the fourth Proportional; and so forward.

As in the former Example where the two Numbers given were 100 and 108, suppose 100 increasing to 108, and so yearly in continual Proportion after the Rate of 8 in the 100, and that it were required to find what this 100 would grow unto by the end of 20 years.

The

The Logarithm of the first Number 100 is		2.00000000
The Logarithm of the second	108	2.03342376
The yearly difference increasing		3342376
Added to the Logarithm of the second, gives		2.06682752

the Logarithm of 116 $\frac{64}{}$ for the third Proportional; And such is the increase at the end of the second year.

Again, the same yearly Difference added to the Logarithm of the third Proportional, gives 2.10025128

the Logarithm of 125 $\frac{971}{}$ for the fourth Proportional, and the increase at the end of the third year, and so the rest.

But because the Question is only of the 20th. year without knowing the rest, we may multiply the former yearly Difference 3342376

By 20: so the Difference of 20 years		66847520
Added to the Logarithm of the first Number	100, *viz.*	2.00000000
Gives the Logarithm of	466 $\frac{096}{}$	2.66847520

that is 466 *l.* 1 *s.* 11 *d. fere*, the Sum that 100 would grow unto by the end of 20 years at the rate proposed.

In like manner if the two first Numbers given were 108 and 100: Suppose 108 decreasing to the 100, and so yearly in continual proportion and that it were required to find what 100 would decrease unto by the end of 20 years: Or (which is all one) suppose 100 to be due 20 years hence, and that it were required to find the worth thereof in ready money according to the former rate.

The Logarithm of the first Number	108 is	2.03342376
The Logarithm of the second	100	2.00000000
The Differences for the year decreasing		3342376
Taken from the Logarithm of 100 leaves		1.96657624

the Logarithm of 92 $\frac{592}{}$ for the third Proportional, and such is the present worth of 100 *l.* due at the years end.

The same difference subtracted once more leaves 1.93315248

The Logarithm of 85 $\frac{734}{}$ for the fourth Proportional, and the present worth of 100 *l.* due at the end of two years.

The same Difference multiplied by 20 makes 66847520

And subtracted from the Logarithm of 100, leaves 1.33152480

the Logarithm of 21 $\frac{4548}{}$ that is 21 *l.* 9 *s.* 1 *d.* and such is the present

 worth

worth of 100 *l.* due at the end of 20 years; So that this present worth being taken forth of the 100 *l.* principal debt, there remains 78*l.* 10*s.* 1 1*d.* for the present worth of the continued gain that may be made either of the loan of 100 *l.* or of 8 *l.* Annuity after 20 years according to the former rate.

If a Leafe of 100 *l.* by the year, or such other yearly Pension were to continue for 20 years, and that it were required to find the worth thereof in ready money. This might be found upon the same ground of continual proportion, and that several ways.

1. It appeareth before, that 100 *l.* due at the years end is worth but $92^{\underline{592}}$ in ready money: If it be due at the end of two years, the present worth is 85 *l.* $^{\underline{733}}$: then adding these two together, we have 178 *l.* $^{\underline{326}}$ for the present worth of 100 *l.* Annuity for two years, and so forward.

2. It appeareth before that the present worth of 8 *l.* Annuity for 20 years is 78 *l.* 5452: and then it follows by proportion.

As an Annuity of	8 *l.* 0000	0.90308999
Is to the worth thereof	78.5452	1.89511953
		9.9202954
So an Annuity of	100.0000	2.00000000
Unto the worth thereof	981.8147	2.99202954

3. As the yearly Loan of 100 *l.* includes an Annuity of 8 *l.* So there is a Sum equivalent to 100 *l.* Annuity.

This Sum equivalent may be diminished according to the Number of years as before: to the Complement of the Sum diminished to the Sum equivalent shall be the present worth of the Annuity.

As the yearly gain of	8	0.90308999
To the Loan of	100	2.00000000
So an Annuity of	100	2.00000000
To the Sum equivalent	1250	3.09691001

Then

Then for diminishing of this Sum equivalent, we may multiply the former yearly Difference 3342376
By 20, so the Difference for 20 years 66847520
Taken from the Logarithm of 1250 3.09691001
There remains the Logarithm of 268.1853 2.42843481
Whose Complement to 1250 is 981.8147, that is 981 *l.* 16 *s.* 3 *d. ob.* and such is the present worth of 100 *l.* Annuity for 20 years, at the rate of 8 in the 100 *per annum.*

The like reason holdeth for any other rate and time proposed.

PROP. IX.

Having two extreme Numbers given, to find a mean Proportional between them.

ADd the Logarithms of the two extreme Numbers: the one half of the Sum shall be the Logarithm of the mean Proportional.

As if the two extreme Numbers given were 8 and 32.

The Logarithm of 8 is	0.90308999
The Logarithm of 32	1.50514998
The Sum of both Logarithms	2.40823997
The half of this Sum is	1.20411998

The Logarithms of 16: and such is the mean Proportional here required.

PROP. X.

Having two extreme Numbers given, to find two mean Proportionals between them.

IN the ordinary way of Arithmetick we commonly multiply the greater Extreme by the Square of the lesser, so the Cubique Root of the Product shall be the lesser mean: then multiplying the lesser Mean into the greater Extreme, the Square Root of the Product shall be the greater Mean Proportional: Or having found the lesser Mean, we may find the other Mean by continual Proportion.

Accordingly we may add the Logarithm of the greater Extreme, to double the Logarithm of the lesser, so the third part of the Sum shall be the

the Logarithm of the leſſer Mean. Then adding this Logarithm of the leſſer Mean, to the Logarithm of the greater Extreme, the one half of the Sum ſhall be the Logarithm of the greater Mean Proportional.

As if the two extreme Numbers given were 8 and 27.

Add to the Logarithm of 8, *viz.*	0.90308999
The ſame again	0.90308999
And the Logarithm of 27	1.43754374
The Sum of theſe will be	3.23754374
The third part of this Sum is	1.07918125

the Logarithm of 12 the leſſer Mean Proportional.

Add to this Logarithm of the leſſer Mean	1.07918123
The Logarithm of the greater Extreme	1.43136736
The Sum of both Logarithm will be	2.51054501
And the half of this Sum is	2.15527250

The Logarithm of 18, the greater of the two Mean Proportionals here required.

Or according to the ſecond manner of Operation in the Rule of Three, (which is the work that I always follow in the line of Numbers) we may take the Difference between the Logarithms of the two extreme Numbers, and divide this Difference into three equal parts, ſo the Sum of the Logarithm of the leſſer Extreme and $\frac{1}{3}$ part, ſhall be the Logarithm of the leſſer Mean: the Sum of this Logarithm of the leſſer Mean and the ſame $\frac{1}{3}$ part, ſhall be the Logarithm of the Greater Mean Proportional.

So the Logarithm of 8 being	0.9030900
The Logarithm of 27	1.4313637
The Difference between them	5282737
The third part of this Difference	1760912
Added to the Logarithm of 8 gives	1.0791812

the Logarithm of 12 the leſſer Mean.

The ſame added to the Logarithm of 12, gives 1.2552725
the Logarithm of 18 the Greater Mean Proportional.

And

And by the ſame reaſon, if it were required to find three Mean Proportionals, we might divide the former Difference into four equal parts and ſo forward.

As if it were required to find the firſt of eleven Mean Proportionals between 100 and 108. Or (which is all one) ſuppoſing 100 *l.* increaſing in continual Proportion, ſo as that by the end of 12 months it came to 108 *l.* and that it were required to find what this 100 *l.* did grow unto by the end of the firſt Month.

The Logarithm of the firſt Extreme	100 is	2.0000000
The Logarithm of the ſecond	108	2.0334237
The yearly Difference between them		334237
The 12. part or monthly Difference		27853
Added to the Logarithm of	100 gives	2.0027053

The Logarithm of 100.64340301 the firſt of the eleven Mean Proportionals: and the growth required.

Then having theſe two, 100 and 100.64340301, together with 108, the laſt of twelve, the other Intermediate may be found by continual Proportion as before.

This Explication of my ten former Propoſitions may ſerve for the frugal Uſe of the Table of Logarithms. Thoſe which require more may have recourſe to that Treatiſe which is mentioned before in the Front of the Table.

CHAP.

CHAP. II.

Concerning the Use of the Lines of Sines and Tangents in the resolving of Spherical Triangles.

Concerning the Use of the *Lines* of *Sines* and *Tangents* I have shewed in general, in the seventh and eighth Chapters of the first Book of the *Cross-staff*, how they might serve for the Resolution of all Spherical Triangles. More particularly in the Use of my *Sector*, Chap. 5. I reduced that which is commonly required in a Spherical Triangle into 28 Cases. And for these they may be all resolved by my Tables of Artificial Sines and Tangents without the help of Secants or versed Sines.

This manner of the work will be always such as in the ordinary Rule of Three, For, here we have three Numbers given, whereby to find a fourth Proportional. And therefore either we may add the Logarithms of the second and third, and subtract the Logarithm of the first:

Or we may take the Difference between the Logarithms of the first and second, and apply that Difference to the Logarithm of the third.

The first of these ways is best for the resolution of right angled Triangles where the Radius, *viz.* 10.000000 is one of the three Numbers given, but the second way by Differences is more convenient for the rest.

The like manner of work may be observed when we are to consider the Sines or Tangents of Degrees, Minutes and Seconds. For the Seconds, not expressed in the Canon, will be found by the Part Proportional: as I will shew in the Examples following.

1. If it were required to find the Sine of 51 *gr.* 32 *min.* 15 *sec.* I should find,

The Sine of 51 *deg.* 32 *m.*	is	9.8937452
The Sine of 51 *deg.* 33 *m.*		9.8938455
The Tabular Difference between them		1003

Then the Difference between 32 *m.* and 33 *m.* being 60 Seconds, the Proportion will hold,

As

As 60 Seconds	*unto*	1003	
So 15	*unto*	251	the part Proportio-

nal to be added unto the Sine 51 *deg.* 32 *min.*
So ſhall we have 9.8937703, for the Sine of 51 *deg.* 32 *min.* 15 *ſec.*

2. If it were required to find the Degrees, Minutes and Seconds belonging to this Tangent 10 0999782
I ſhould find by the Canon that this is ſomewhat more then the Tangent of 51 *deg.* 32 *min.* 10.0999134
Leſs than the Tangent of 52 *deg.* 33 *min.* 10.1001728

The tabular Difference between theſe is 2594
And the proper Difference is 648
between the leſſer of theſe Tangents, and the Tangent given: therefore,

As 2594	*unto*	60 *Seconds.*	
So 648	*unto*	15	And ſo, I find

this to be the Tangent of 51 *deg.* 32 *min.* 15 *ſec.*

3. If it were required to find the Sine belonging to this Tangent 10.0999782, I ſhould find the Ark to be ſomewhat more than 51 *gr.* 31 *min.* and the Sine correſpondent ſomewhat more than 9.8937452, then taking out the Differences as before, I find, that

As the tabular Difference of Tangent	2594	3.4139700
Is to the proper Difference	648	2.8119750
		6023950
So the tabular Difference of Sines	1003	3.0013009
To the Part proportional	251	2.3989059
This Part proportional added unto the former Sine		9.8937452

gives 98937703 for the Sine required.

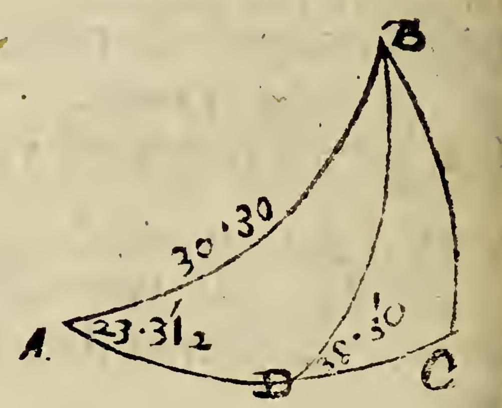
Theſe Premiſſes conſidered, I come to the 28 Caſes before-mentioned, wherein I ſet down a Canon and an Example for each Caſe, and theſe for the moſt part the ſame which I uſed before.

Thoſe which have no further uſe but of Degrees and Minutes, may take that Sine or Tangent which they find to be next in the Canon, and neglect the Seconds.

In

In a RECTANGLE TRIANGLE.

1. To find a Side by knowing the Base and the Angle opposite to the enquired Side.



As in the Rectangle Triangle A C B, wherein A stands for the Equinoctial point; A B, an Ark of the Ecliptick representing the Longitude of the Sun in the beginninf of ♉, B C an Ark of the Declination og the Sun from the Equator, and A C an Ark of the Equator representing the Right Ascension of the Sun in B: Knowing the Base A B to be 30 *gr.* and the Angle B A C 23 *gr.* 31 *min.* 30 *sec.* if it were required to find the Side B C.

	d.	m.	sec.	
As the Radius, the Sine of	90	00	00	10.0000000
Is to the Sine of the Base	30	00	00	9.6989700
So the Sine of the opposite Angle	23	31	30	9.6011352
To the Sine of the Side required	11	30	43	19.3001052

And so writing the Sine 9.6001052 in a Paper by itself and holding to the Sine of the Base in the Canon 1 *gr.* 2. 3. 4. 5. and so forward, it would be no long work to write the Sum in a Column by itself, and so find the Declination for each Degree and Minute of the Ecliptick.

2. To find a Side by knowing the Base and the other Side.

As in the Rectangle A C B having A B 30 *gr.* and B C 11 *gr.* 30 *m.* 43 *sec.* to find the Side A C.

As the Cosine of the Side given	11	30	43	9.9911740
Is to the Radius	90	00	00	10.0000000
So the Cosine of the Base	30	00	00	9.9375306
To the Cosine of the Side required	27	52	43	9.9463566

3. To

3. To find a Side by knowing the two Obliqus Angles.

As in the Rectangle A C B having C A B for the first Angle 23 *gr.* 31 *min.* 30 *sec.* and A B C for the second 69 *gr.* 20 *m.* 35 *sec.* to find the side A C.

As the Sine of the next Angle	23	31	30	9.6011352
Is to the Radius	90	00	00	10.0000000
So the Cosine of the opposite Angle	69	20	35	9.5474918
To the Cosine of the Side required	27	53	43	9.9463566

4. To find the Base by knowing both the Sides.

As in the Rectangle A C B, having A C 27 *gr.* 53 *m.* 43 *sec.* and B C, 11 *gr.* 30 *m.* 43 *sec.* to find the Base A B.

As the Radius	90	00	00	10.0000000
To the Cosine of the one Side	27	53	43	9.9463566
So the Cosine of the other Side	11	30	43	9.9911640
To the Cosine of ~~the Base~~	30	00	00	9.9375306

5. To find the Base by knowing one Side and the Angle opposite to that Side.

As if in the former Triangle A C B we draw B D and Ark of the Horizon for the Latitude of 51 *gr.* 30 *min.* reputing the Amplitude of the Suns Rising from the East, we shall have two Triangles more, one Rectangle B C D, the other Obliquadrangled A B D, and so in the Rectangle D C B, having B C 11 *gr.* 30 *m.* 43 *sec.* and B D C 38 *gr.* 30 *min.* if it were required to find the Base D B.

As the Sine of the Angle	38	30	00	9.7941495
To the Sine of the Side	11	30	43	9.3001052
So is the Radius	90	00	00	10.0000000
To the Sine of the Base	18	41	56	9.5059556

6. To

6. To find an Angle by knowing the other Oblique Angle, and the Side opposite to the Angle required.

As in the Rectangle A C B, having B A C 23 *gr.* 31 *min.* 30 *sec.* and A C 27 *gr.* 53 *min.* 43 *sec.* to find the Angle A B C.

As the Radius	90	00	00	10 0000000
To the Sine of the Angle given	23	31	30	9.6011352
So the Cosine of the Side	27	53	43	9.9463566
To the Cosine of the Angle required	69	20	35	19.5474918

7. To find an Angle by knowing the other Oblique Angle, and the Side opposite to the Angle given.

As in the Rectangle A C B, having B A C 23 *gr.* 31 *min.* 30 *sec.* and B C 11 *deg.* 30 *min.* 43 *sec.* to find the Angle A B C.

As the Cosine of the Side	11	30	43	9.9911740
To the Cosine of the Angle given	23	31	30	9.9623153
So is the Radius	90	00	00	10.0000000
To the Sine of the Angle required	69	20	35	9.9711413

8. To find an Angle by knowing the Base, and the Side opposite to the Angle required.

As in the Rectangle B C D, having B D 18 *gr.* 41 *m.* 56 *sec.* and B C 11 *gr.* 30 *min.* 43 *sec.* to find the Angle B D C.

As the Sine of the Base	18	41	56	9.5050000
Is to the Radius	90	00	00	10.0000000
So the Sine of the opposite Side	11	30	43	9.3001052
To the Sine of the Angle	38	30	00	9.7941495

These eight Propositions have been wrought by Sines alone; the eight following require joynt help of Tangents.

9. *To*

9. *To find a Side, by knowing the other Side, and the Angle opposite to the Side required.*

As in the Rectangle A C B, having A C 27 *gr.* 53 *min.* 43 *sec.* and B A C 23 *gr.* 31 *min.* 30 *sec.* to find the Side B C.

As the Radius	90	00	00	10.0000000
To the Sine of the Side given	27	53	43	9.6701112
So the Tangent of the opposite Angle	23	31	30	9.6388199
To the Tangent of the Side required	11	30	43	19.3089311

10. *To find a Side by knowing the other Side, and the Angle next the Side required.*

As in the Rectangle B C D, having B C 11 *gr.* 30 *min.* 43 *sec.* and B D C 38 *gr.* 30 *min.* to find D C.

As the Tangent of the Angle	38	30	00	9.9006052
To the Tangent of the Side given	11	30	47	9.3089311
So the Radius	90	00	00	10.0000000
To the Sine of the Side required	14	50	11	9.4083259

11. *To find a Side by knowing the Base and the Angle next the Side required.*

As in the Rectangle A C B, having A B 30 *gr.* 00 *min.* and B A C 23 *gr.* 31 *m.* 30 *sec.* to find the Side A C.

As the Radius	90	00	00	10.0000000
To the Cosine of the Angle	23	31	30	9.9623153
So the Tangent of the Base	30	00	00	9.7614393
To the Tangent of the Side required	27	53	43	19.7237546

12. *To find the Base by knowing both the Oblique Angles.*

As in the Rectangle A C B, having B A C 23 *gr.* 31 *min.* 30 *sec.* A B C 69 *gr.* 20 *m.* 35 *sec.* to find the Base A B.

As the Tangent of the one Angle	23	31	30	9.6388199
To the Cotangent of the other	69	20	35	9.5763505
So the Radius	90	00	00	10.0000000
To the Cosine of the Base	30	00	00	9.9375306

13. *To find the Base by knowing one of the Sides and the Angle next that Side.*

As in the Rectangle A C B, having A C 27 *gr.* 53 *min.* 43 *sec.* and B A C 23 *gr.* 31 *min.* 30 *sec.* to find the Base A B.

As the Cosine of the Angle	23	31	30	9.9623153
Is to the Radius	90	00	00	10.0000000
So the Tangent of the Side	27	53	43	9.7237547
To the Tangent of the Base	30	00	00	9.7614394

14. *To find an Angle by knowing both the Sides.*

As in the Rectangle A C B, having A C 27 *gr.* 53 *min.* 43 *sec.* and B C 11 *gr.* 30 *min.* 43 *sec.* to find the Angle A B C.

As the Sine of the next Side	11	30	43	9.3001052
Is to the Radius	90	00	00	10.0000000
So the Tangent of the opposite Side	27	53	43	9.7237547
To the Tangent of the Angle	69	20	35	10.4236495

15. *To*

15. *To find an Angle by knowing the Base, and the Side next the Angle required.*

As in the Rectangle B C D, having B D 18 *gr.* 41 *m.* 56 *sec.* and B C 11 *gr.* 30 *m.* 43 *sec.* to find the Angle B D C.

As the Tangent of the Base	18	41	56	9.5295063
To the Tangent of the Side	11	30	43	9.3089311
So is the Radius	90	00	00	10.0000000
To the Cosine of the Angle	53	00	46	9.7794248

16. *To find an Angle by knowing the Base and the other Oblique Angle.*

As in the Rectangle A C B, having the Base A B 30 *gr.* and B A C 23 *gr.* 31 *m.* 30 *sec.* to find the Angle B A C.

As the Cosine of the Base	30	00	00	9.9370000
Is to the Radius	90	00	00	10.0000000
So the Cotangent of the Angle given	23	31	30	10.3601801
To the Tangent of the Angle required	69	20	35	10.4236495

These 16 Cases are all that can fall out in a Rectangle Triangle. Those which follow do hold in any Spherical Triangle whatsoever.

In any *SPHERICAL TRIANGLE* whatsoever.

17. *To find a Side opposite to an Angle given, by knowing one Side and two Angles, the one opposite to the Side given, the other to the Side required.*

As in the Triangle A B D, having A B 30 *gr.* B D C 38 *gr.* 30 *m.* and B A D 23 *gr.* 31 *m.* 30 *sec.* to find the Side B D, which here representeth the Amplitude.

As the Sine of the next Angle	38	30	00	9.7941495
To the Sine of his opposite Side	30	00	00	9.6989700
				951795
So the Sine of the opposite Angle	23	31	30	9.6011352
To the Sine of the Side required	18	41	56	9.5059557

Or

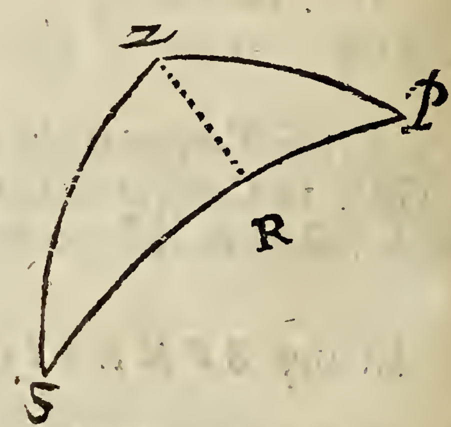
Or changing the Site of the two middle Terms.

As the Sine of the next Angle	38	30	00	9.7941495
To the Sine of the opposite Angle	23	31	30	9.6011352
				1930143
So the Sine of the Side given	30	00	00	9.6989700
To the Sine of the Side required	18	41	56	9.5059557

And so writing this Difference 1930143 in a Paper by itself, and holding it to the Sine of the Side in the Canon 1 *gr.* 2, 3, 4, 5, and so forward, it would be no long work to subtract, and write the Remainder in a Column by itself, and so find the Amplitude for each Degree and Minute of the Ecliptick.

Or instead of subtracting this Difference, we might first take the same out of the Radius, and then add the Complement as I shewed before, in the general explication of the Rule of Three.

18. *To find an Angle opposite to a Side given, by knowing one Angle and two Sides, the one opposite to the Angle given, the other to the Angle required.*



As in the Triangle Z P S representing the Zenith, Pole, and Sun: where Z P is the Complement of the Latitude, P S the Complement of the Declination, Z S the Complement of the Suns Altitude, P Z S the Azimuth, Z P S the hour of the day from the Meridian, and P S Z the Angle of the Suns Position in regard of the Pole and Zenith; having P Z S, 130 *gr.* 3 *min.* 11 *sec.* P S 70 *gr.* and Z S 40 *gr.* to find the Angle Z P S.

As the Sine of the next Side	70	00	00	9.9729858
Is to the Sine of his opposite Angle	130	03	11	9.8839153
				890705
So the Sine of the opposite Side	40	00	00	9.8080675
To the Sine of the Angle required	31	34	26	9.7189970

19. *To find an Angle by knowing the three Sides.*

As in the Triangle Z P S, having Z P 38 *gr.* 30 *min.* P S 70 *gr.* and Z S 40 *gr.* to find the Angle Z P S, subtending the Base Z S.

A

As the Rectangle conteined under the Sines of the Sides, is to the Square of the Radius:

So the Rectangle conteined under the Sines of the Half-Sum of the three Sides, and the Difference between this Half-Sum and the Base,

To the Square of the Cosine of half the Angle required.

The Base subtended is	40 *gr.*	00 *m.*
The two Sides including the Angle	38	30
	70	00
The Sum of the three Sides	148	30
The Half-Sum of these three	74	15
The Difference between this and the Base	34	15

Here for the Square of Radius we take 20.0000000, to this we add 9.9833805 the Sine of 34 *gr.* 15 *min.* and 9.7503579 the Sine of 34 *gr.* 15 *min.* which make 39.7337384.

Then for the Rectangle of the Sides, we add 9.7941495 the Sine of 38 *gr.* 30 *min.* and 9.9729858, the Sine of 70 *gr.* which make 19.7671353. This we take out of 39 7337384, and there remains for the Logarithm of the Square 19.9666031, the half thereof 9.9833015 we find to be the Cosine of 15 *gr.* 47 *min.* 13 *sec.* And so the whole Angle required is 31 *gr.* 34 *min.* 26 *sec.*

Or for such Numbers as are to be subtracted, we may take them out of the Radius, and write down their Complements, and then add them together with the rest, the manner of the work in either way will be such as followeth.

40 *gr.*	00 *m.*					
38	30	— 9.7941495				2058505
70	00	9.9729858				270142
148	30	19.7671353				
74	15	9.9831805				9.9833805
34	15	9.7503579				9.7503579
		20.0000000				
		39.7337384				
		19.9666031	*gr.*	*m.*	*sec.*	19.9666031
		9.9833015	15	47	13	9.9833015
			31	34	26	

In

In the like manner we may find the Angle P Z S to be 130 *gr.* 3 *m.* 11 *ſec.* and the Angle Z S P 30 *gr.* 28 *m.* 11 *ſec.*

20. *To find a Side by knowing the three Angles.*

If for either of the Angles next the Side required, we take the Complement to 180 *gr.* theſe Angles will be turned into Sides, and the Sides into Angles. Then may the work be the ſame as in the former Propoſition.

As in the Triangle Z P S, knowing the Angle Z P S to be 31 *gr.* 34 *m.* 26 *ſec.* P Z S 130 *gr.* 3 *m.* 11 *ſec.* and Z S P 30 *gr.* 28 *m.* 11 *ſec.* if it were required to find the Side Z S oppoſite to the Angle Z P S, I would take 130 *gr.* 3 *m.* 11 *ſec.* out of 180 *gr.* the Remainder will be 49 56 49.

Then, as if I had a Triangle of three known Sides, one of 31 *gr.* 34 *m.* 26 *ſec.* another of 30 *gr.* 28 *m.* 11 *ſec.* and the third of 49 *gr.* 56 *m.* 49 *ſec.* I would ſeek the Angle oppoſite to the firſt of theſe Sides by the laſt Propoſition.

So the Angle which is thus found would be the Side which is here required.

Thus here the Angle oppoſite is	31	34	26	
The leſſer of the next Angles	30	28	11	9.7050790
The Complement of the other	49	56	49	9.8839153
The Sum of theſe three	111	59	26	
The Half Sum	55	59	43	9.9185490
The Diff. from the opp. Angle	25	25	17	9.6164170
The Sum of double the Radius and				20.0000000
The Sines of Half Sum and Difference is				39.5349660
Take hence the Sines of the next Angles				19.5889943
There remains for the Square				19.9459717
The half whereof is				9.9729858

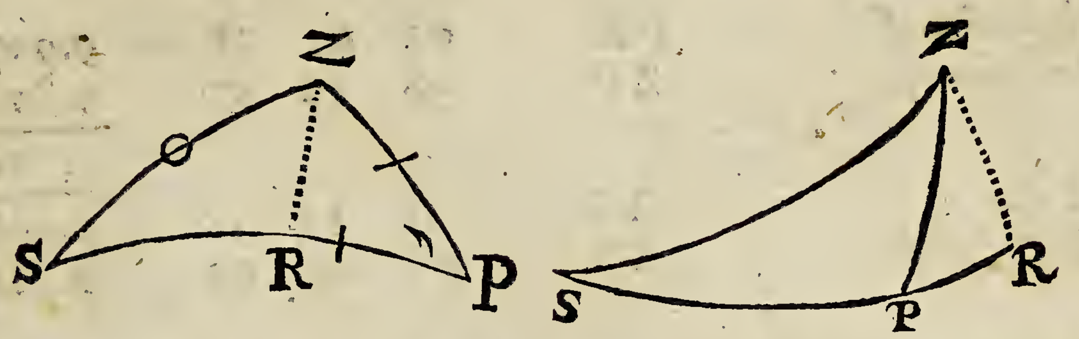
the Coſine of 20 *gr.* 00 *m.* and ſo the Side required, 40 *gr.* 00 *m.*

The other Sides may be found in the ſame ſort; but when we know either three Sides and one Angle, or three Angles and one Side, the reſt may be found more readily by the 17 or 18 Propoſition.

21. *To*

21. *To find a Side by having the other two Sides and the Angle comprehended.*

This and the Proportion following are beſt reſolved by reducing the oblique-angle Triangles given, into two Rectangles.



As in the Triangle Z P S, having Z P 38 *gr.* 30 *m.* P S 70 *gr.* 00 *m.* and Z P S 31 *gr.* 34 *m.* 26 *ſec.* to find the Side Z S.

In that we have Z P and Z P S, we may ſuppoſe a Perpendicular Z R to be let down from the Angle at Z upon the greater Side P S: So if Z P S the Angle given be leſs than 90 *gr.* it will fall within the Triangle; if more than 90 *gr.* it will fall without the Triangle, upon the Side produced, and divide the Triangle given into two Rectangles Z R S and Z R P. Wherein

1. We may find the quantity of this Perpendicular by the firſt Propoſition of Spherical Triangles.

2. We may find the Side P R either by the ſecond or tenth, or rather by the eleventh Propoſition: which Side P R will give the Side R S.

3. Having Z R and R S, we may find the Baſe Z S, by the fourth Propoſition, as I ſhew in the uſe of the Sector.

But here for variety I will ſhew how the ſame may be done at two Operations, both in this and the reſt of the Caſes following, without knowing the quantity of the Perpendicular.

As

1. *As the Radius or Sine of*	ZRP	90	00	00	10.0000000
To the Cosine of the Angle	ZPR	31	34	26	9.9304233
So the Tangent of the Side	ZP	38	30	00	9.9006052
To the Tangent of the Ark	PR	34	07	30	19.8310275
2. *As the Cosine of*	PR	03	07	30	9.9179342
To the Cosine of	ZP	38	30	00	9.8935443
					243899
So the Cosine of	RS	35	52	30	9.9086438
To the Cosine of	ZS	40	00	00	9.8842539

22. *To find a Side by knowing the other two Sides and one Angle next the Side required.*

As in the Triangle ZPS, having ZP, 38 *gr.* 30 *m.* and ZS 40 *gr.* 00 *m.* and ZPS, 31 *gr.* 34 *m.* 26 *sec.* to find the Side PS.

1. Find the Ark PR by the eleventh Propofition as before.

2. *As the Cosine of*	PZ	38	30	00	9.8935443
To the Cosine of	PR	34	07	30	9.9179342
					243899
So the Cosine of	ZS	40	00	00	9.8842539
To the Cosine of	SR	35	52	30	9.9086438

23. *To find a Side by knowing one Side and the two Angles next the second Side.*

As in the Triangle ZPS, having ZP 38 *gr.* 30 *m.* ZPS 31 *gr.* 34 *m.* 26 *sec.* and ZSP 30 *gr.* 28 *m.* 11 *sec.* to find the Side PS.

1. Find the Ark PR as before.

2. *As the Tangent of*	ZSP	30	28	11	9.7696236
To the Tangent of	ZRS	31	34	26	9.7885746
					189510
So the Sine of	PR	34	07	30	9.7489617
To the Sine of	SR	35	52	30	9.7679127

24. *To*

24. *To find a Side by knowing two Angles and the Side inclosed by them.*

As in the Triangle ZPS, having ZP 38 *gr.* 30 *m.* ZPS 31 *gr.* 34 *m.* 26 *sec.* and PZS 130 *gr.* 3 *m.* 11 *sec.* to find the Side ZS.

1. *As the Cosine of*	PZ	38	30	00	9.8935443
Is to the Radius		90	00	00	10.0000000
So the Cotangent of	ZPS	31	34	26	10.2114253
To the Tangent of	PZR	64	18	50	10.3178810
2. *As the Cosine of*	SZR	65	44	22	9.6137228
To the Cosine of	PZR	64	18	50	9.6369311
					232083
So the Tangent of	PZ	38	30	00	9.9006052
To the Tangent of	ZS	40	00	00	9.9238135

25. *To find an Angle by knowing the other two Angles and the Side inclosed by them.*

As in the Triangle ZPS having ZP 38 *gr.* 30 *m* ZPS 31 *gr.* 34 *m.* 26 *sec.* and PZS 130 *gr.* 3 *m.* 11 *sec.* to find the Angle ZSP.

1. Find the Angle PZR by the sixteenth Proposition as before.

2. *As the Sine of*	PZR	64	18	50	9.9548122
To the Sine of	SZR	65	44	21	9.9598453
					50331
So the Cosine of	ZPS	31	34	26	9.9304223
To the Cosine of	ZSP	30	28	11	9.9354554

26. *To find an Angle by knowing the other two Angles and one Side next the Angle required.*

As in the Triangle ZPS, having ZP 38 *gr.* 30 *m.* ZPS 31 *gr.* 34 *m.* 26 *sec*, and ZSP 30 *gr.* 28 *m.* 11 *sec.* to find the Angle PZS.

 1. Find

1. Find the Angle P Z R as before.

2. *As the Cosine*	Z P S	31	34	26	9.9304223
To the Cosine of	Z S P	30	28	11	9.9354554
					50331
So the Sine of	P Z R	64	18	50	9.9548122
To the Sine of	S Z R	64	44	21	9.9598453

27. *To find an Angle by knowing two Sides and the Angles contained by them.*

As in the Triangle Z P S, having Z P 38*gr.* 30*m.* P S 70*gr.* and Z P S 31*gr.* 34*m.* 26*sec.* to find the Angle Z S P.

1. Find the Ark P R as before.

2. *As the Sine of*	S R	35	52	30	9.7679127
To the Sine of	P R	34	07	30	9.7489617
					189510
So the Tangent of	Z P S	31	34	26	9.7885746
To the Tangent of	Z S P	30	28	11	9.7696236

28. *To find an Angle by knowing two next Sides, and one of the other Angles.*

As in the Triangle Z P S having Z P 38*gr.* 30*m.* Z S 40*gr.* and Z P S 31*gr.* 34*m.* 26*sec.* to find the Angle P Z S.

1. Find the Angle P Z R as before.

2. *As the Tangent of*	Z S	40	00	00	9.9238135
To the Tangent of	Z P	38	03	00	9.9006052
					232083
So the Cosine of	P Z R	64	18	50	9.6369311
To the Cosine of	S Z R	65	44	21	9.6137228

These

These 28 Cases are those which I set down in the use of the Sector, and all that are commonly required in a Spherical Triangle. I will here add two more, to shew how that which is found before by the 22, 23, 26, and 28. Propositions may sometimes be found more easily, *viz.*

29. *To find a Side, by knowing the other two Sides, and their opposite Angles.*

As in the Triangle Z P S, having P S 70 *gr.* and P Z S 130 *gr.* 3 *m.* 11 *sec.* together with Z S 40 *gr.* and Z P S 31 *gr.* 34 *m.* 26 *sec.* to find the third Side Z P.

As the Sine of half the Difference of the Angles given,
To the Sine of half the Sum of those Angles:
So the Tangent of half the Difference of the Sides given,
To the Tangent of half the Side required.

30. *To find an Angle by knowing the other two Angles, and their opposite Sides.*

As in the Triangle Z P S, having the former parts P S, P Z S, Z S, and Z P S, to find the third Angle Z S P.

As the Sine of half the Difference of the Sides given,
To the Sine of half the Sum of those Sides:
So the Tangent of half the Difference of the Angles given,
To the Cotangent of half the Angle required.

CHAP.

CHAP. III.

Concerning the joynt Use of the Lines of Numbers, Sines and Tangents.

CONcerning the joynt Uſe of the Lines of Numbers, Sines and Tangents, I ſhewed how they might ſerve for the Reſolution of Right-lined Triangles, whereof I ſet down five Propoſitions in the nineth Chapter of the firſt Book of the *Croſs-ſtaff.* And theſe alſo may be applied to the Table and Canon of Logarithms.

The Sides of theſe Triangles are meaſured by abſolute Numbers, and ſo repreſented by Logarithms.

The Angles are meaſured by degrees and minutes, and ſo to be found by Sines and Tangents in the Canon.

PROP. I.

Having three Angles and one Side, to find the other two Sides.

IF it be a Rectangle Triangle, wherein one Side about the right Angle being known, it were required only to find the other, this might be readily done by Sines and Tangents. As in the Rectangle A I B, knowing the Angle B A I to be 43 *gr.* 20 *m.* and the Side A I to be 244, if it were required to find the other Side A I.

As the Radius (the Tangent of)		45 *gr.*	00 *m.*	10.0000000
Is to the Tangent of the Angle		43	30	9.9749195
So is the Side given	A I	244	000	2.3873898
To the Side required	B I	230	000	12.3621093

But where both the other Sides are required, it is beſt done by Logarithms and Sines. As in the ſame Rectangle A I B, having the three Angles and the Side A I, to find both B I and A B.

As

As the Sine of the opposite Angle	ABI	46	40	9.8617575
Is to the Side given	AI	244	000	2.3873898
				7.4743676
So the Sine of the second Angle	BAI	43	20	9.8364770
To his opposite Side	BI	230	222	2.3621093
And the Sine of the third Angle	AIB	90	00	10.0000000
To his opposite Side	AB	335	141	2.5256323

The like holdeth alſo in Oblique-angled Triangles.

As in the Triangle A B D (which I propoſed formerly as an example for the finding the Diſtances) where knowing the Diſtance between A and D, to be 100 paces; the Angle B A C to be 43 *gr.* 20 *m.* the Angle B D A 122, or the outward Angle B D C 58 *gr.* and conſequently the Angle A B D oppoſite to A D the Side given, to be 140 *gr.* 40 *m.* it was required to find the Diſtances A B and D B.

As the Sine of the opposite Angle	ABD	14	40	9.4034554
Is to the Side given	AD	100	000	2.0000000
				7.4034554
So the Sine of the second Angle	ADB	58	00	9.9284204
To his opposite Side	AB	334	917	2.5249650
And the Sine of the third Angle	DAB	43	26	9.8364770
To his opposite Side	DB	271	012	2.4330216

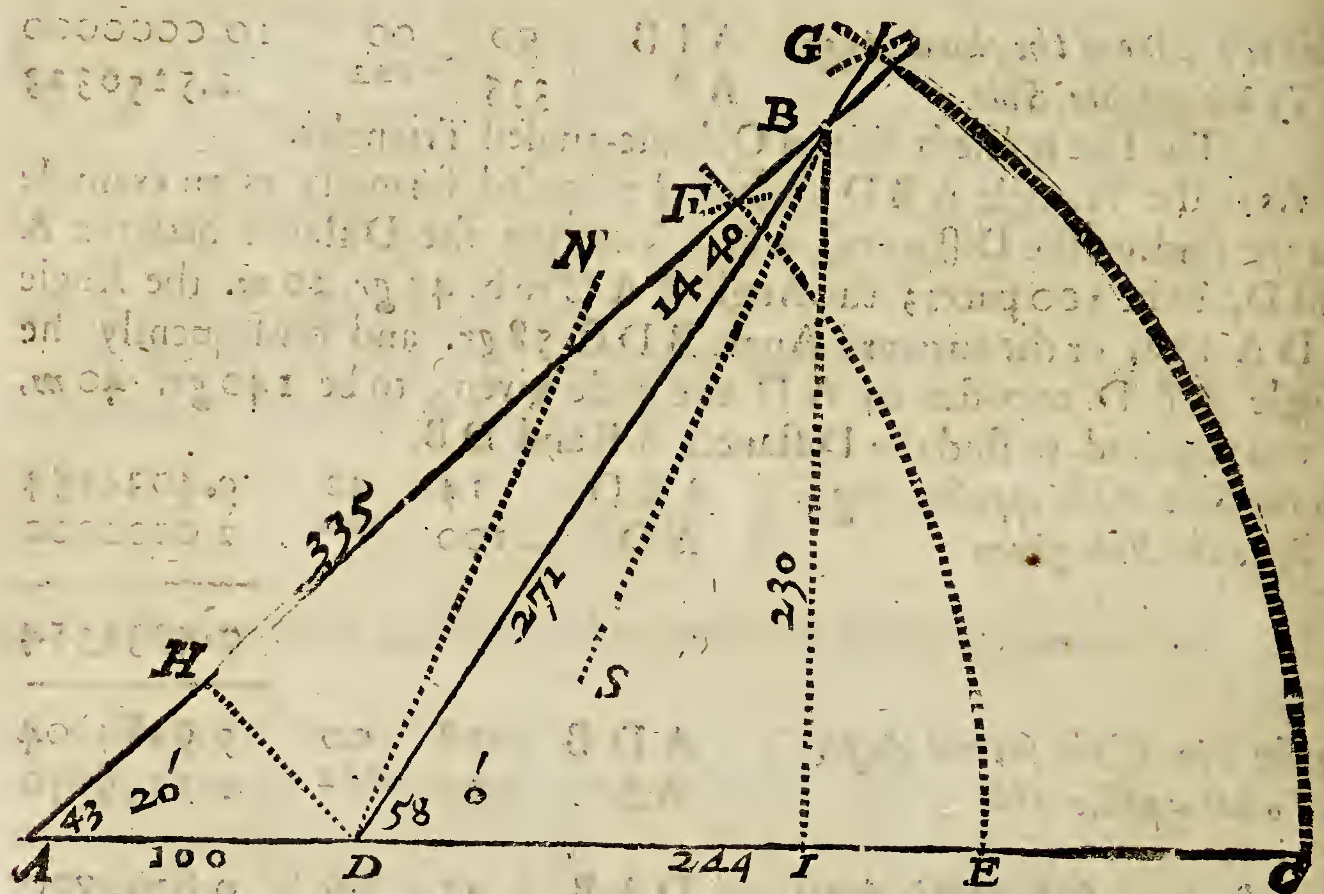
PROP. II.

Having two Sides and one Angle oppoſite to either of thoſe Sides, to find the other two Angles and the third Side.

As in the Triangle A B D, having the two Sides A B 335 paces; and A D 100 paces, and knowing the Angle A D B which is oppoſite to the Side A B, to be 122 *gr.* or the outward Angle B D C to be 58 *gr.* if it were required to find the other two Angles at A and B, and the third Side B D, I may firſt find an Angle A B D oppoſite to the other known Side A D.

As

As the opposite Side	A B	335	000	2.5250448
To the Sine of the Angle given	A D B	58	00	9.9284204
				7.4033756
So is the next Side	A D	100	000	2.0000000
To the Sine of his opposite Angle	A B D	14	59 ⅚	9.4033756



Then knowing these two Angles at D and B, I take the inward Angle A B D 14 *gr.* 59 *m.* 50 *sec.* out of the outward Angle B D C 58 *gr.* 00 *m.* and so find the third Angle B A D, to be 43 *gr.* 20 *m.* 10 *sec.* So having three Angles and two Sides I may well find the third Side B D by the former proportion.

As the Sine of the first Angle	A D B	58	00	9.9284204
Is to his opposite Side	A B	335	000	2.5250448
				7.4033756
So the Sine of the last Angle	D A B	43	20 ⅙	9.8365033
To his opposite Side	D B	271	120	2.4331277

PROP.

PROP. III.

Having two Sides, and the Angle between them, to find the other two Angles and the third Side.

IF the Angle conteined between the two Sides given be a right Angle, the other two Angles will be found readily by Tangents and Logarithms. As in the Rectangle A I B having the Side A I 244, and the Side I B, to find the Angles at A and B.

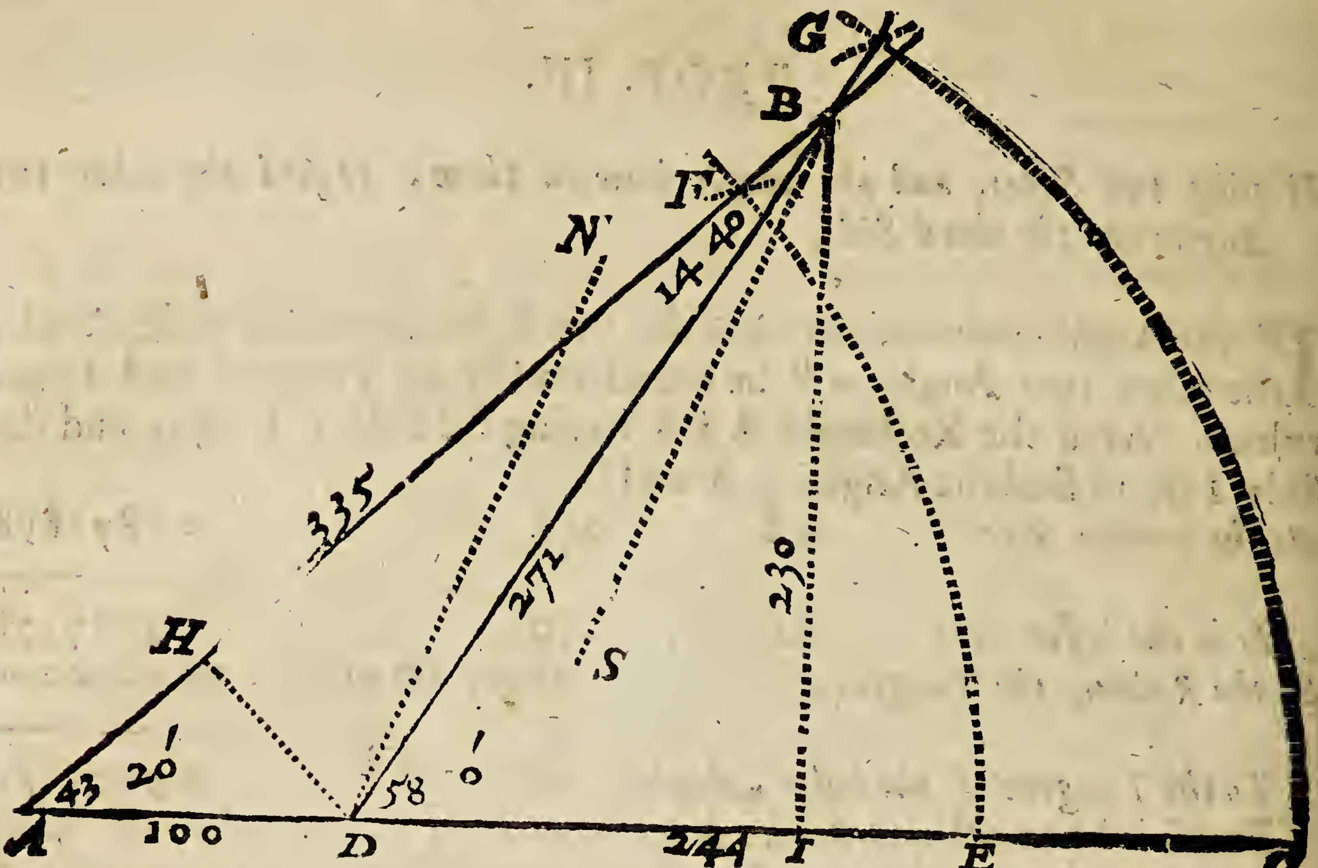
As the greater Side	A I	244	2.3873898
Is to the lesser Side	I B	230	2.3617278
So the Radius, the Tangent of		45 *gr.* 00 *m.*	10.0000000
To the Tangent of the lesser Angle		43 18½	9.9743380

But if it be an oblique Angle that is conteined between the two Sides given, the Triangle may be reduced into two Rectangle Triangles, and then resolved as before.

As in the Triangle A D B, having the Sides A B 335, A C 100, and the Angle B A D 43 20′, to find the Angles at B and D, and the third Side B D. First, I would suppose a Perpendicular D H to be let down from D, the end of the lesser Side, upon the greater Side A B: so shall I have two Rectangled Triangles D H A and D H B. And in the Rectangle A H D, the Angle at A being 43 20′, the other Angle A D H will be 46 40′ by Complement, and with these Angles and the Side A D, I may find both A H and D H by the first Proportion. Then taking A H out of A B, there remains H B for the Side of the Rectangle D H B, and therefore with this Side H B and the other D H, I may find the Angle at B, by the former part of this Proportion. And with this Angle and the Perpendicular D H, I may find the third Side D B, by the first Proposition.

Or having two Sides and the Angle between them, we may find the other two Angles without letting down any Perpendicular, in this manner.

As the Sum of the two Sides given,
Is to the Difference of these Sides:
So the Tangent of half the Sum of the two opposite Angles,
To the Tangent of half of the Difference between those Angles.



So here having the Side	A B	335	
and the other Side	A D	100	
The Sum of these Sides is		435	2.6384892
and the difference of these Sides		235	2.3719678
			2674214
The Angle conteined B A D is		43 20′	
The Sum of the two opposite Angles		136 40	
The Half Sum of these Angles		68 20	10.4009092
And by Proportion and half difference		53 40 $\frac{1}{3}$	10.1334878

This half Sum and half Difference make 12 20 $\frac{1}{3}$ the greater Angle and the Difference between them 14 19 $\frac{2}{3}$ the lesser Angle.

PROP. IV.

Having three Sides, to find the three Angles.

Let one of the three Sides given be the Base (but rather the greater Side) that the Perpendicular may fall within the Triangle. Then gather the Sum and the Difference of the two Sides, and the Proportion will hold.

As the Base of the Triangle,
To the Sum of the Sides:
So the Difference of the Sides,
To the alternate Base.

This

This alternate Base being taken forth of the true Base, if we let down a Perpendicular from the opposite Angle, it shall fall upon the middle of the Remainder. As in the Triangle A D B.

The lesser Side	A D	100	
The other Side	B D	271	
The Base of the Triangle	A B	335	2.5250448
The Sum of the Sides		371	2.5693739
			443291
The Difference between these Sides		171	2.2329961
And so the alternate Base is		189 $\frac{376}{}$	2.2773252
This taken out of 335 leaves		145 $\frac{624}{}$	
The half whereof is		72 812.	

And such is the Segment A H, the Distance between the Angle at A, and the Perpendicular D H. So that having drawn this Perpendicular, we have two Rectangle Triangles D H A and D H B, in which having two Sides, and the right Angle, we may find the other Angles by the second Proportion.

These four Propositions may suffice for the Resolution of the Sides and Angles in all right-lined Triangles.

PROP. V.

Having the Base and Perpendicular in a right-lined Triangle, to find the superficial Content.

THe Perpendicular may be found by one or other of the former Propositions, and that being known we may find the superficial Content. As in the Triangle A D B, having the Base A B 335, and the Perpendicular D H 68545.

As the Number of	2	0.3010700
To the Perpendicular	68.545	1.8359757
		1.5349457
So the Base	335	2.5250448
To the Content	11481 $\frac{221}{}$	4.0599905

Or if we would find the Content without knowing the Perpendicular, we may put two or more Operations into one, as in the Proportion following.

PROP. VI.

Having two Sides of a right-lined Triangle, and the Angle between them, to find the Content.

ADd the Sine of the Angle, and the Logarithms of both the Sides, from the Sum of these subtract —10.3010300, so the Remainder shall be the Logarithm of the Content.

As in the Triangle A D B, having the Sides A B 335, A D 100, and the Angle B A D 43 *gr.* 20 *m.*

The Sine of the Angle		43 *gr.* 20 *m.* is	9.8364770
The Logarithm of the Side	A B	335	2.5250448
The Logarithm of the Side	A D	100	2.0000000
The Sum of these make			14.3615218
From which subtract the solemn Logarithm			10.3010300
The Remainder will be			4.0604918

The Logarithm of 11494 the Content required.

PROP. VII.

Having three Angles, and one Side of a right-lined Triangle, to find the Content.

ADd the double of the Logarithm of the Side given, and the Sines of the two next Angles: from the Sum of these subtract the Sum of 10.3010300, and the Sine of the opposite Angle, so the Remainder shall be the Logarithm of the Content.

As in the Triangle A D B supposing the Angles B A C to be 34 *gr.* 20 *m.* BDA 122 *gr.* 00 *m.* A B D 14 *gr.* 40 *m.* and the Side A D to the 100 parts.

The Logarithm of the Side	A C	100 is	2.0000000
The same again			2.0000000
The Sine of the Angle	B A C	43 *gr.* 20 *m.*	9.8364770
The Sine of the Angle	B D A	58 00	9.9284204
The Sum of these four make			23.7648974
Again, if we add the solemn Logarithm			10.3010300
To the Sine of the opposite Angle		14 *gr.* 40 *m.*	9.4034554
The Sum of both will make			19.7044854
Which subtracted from 23.7648974 leave			4.0604120

The Logarithm of 11492 the Content required.

PROP.

PROP. VIII.

Having the three Sides of a right-lined Triangle, to find the Content.

First, set down the three Sides, the Sum of them, and the Half-Sum. Then from this Half-Sum subtract each Side severally and note the Differences. That done, add the Logarithms of the Half-Sum, and these Differences, the half thereof shall be the Logarithm of the Content.

Thus in the Triangle A D B, the three Sides are	A B	335	
	D B	271	
	A D	100	
The Sum of these Sides is		706	
The Half-Sum		352	3.5477747
The Difference from	A B	18	1.2552725
The Difference from	D B	82	1.9138138
The Difference from	A D	253	2.4031205
The Sum of their Logarithms			8.1199815
And the half thereof is			4.0599907

The Logarithm of 11481^{223} the Content required.

PROP. IX.

Having the three Sides of a right-lined Triangle, to find the Perpendicular.

As in the former Triangle A D B, to find the Perpendicular D H. First, find the Content of the Triangle by the former Proportion, then may the Perpendicular be found by the converse of the fifth Proposition.

As the Base of the Triangle	335	2.5250448
To the Superficial Content	11481^{22}	4.0599907
		1.5349459
So always the Number of	2	0.3010300
To the Perpendicular	68^{242}	1.8359759

PROP.

PROP. X.

Having the Semidiameter of a Circle, to find the Chord for any Ark proposed.

AS if in protracting the former Triangle A D B, it were required to find the length of a Chord of 43 *gr.* 20 *m.* agreeing to the Semidiameter A E, which we suppose to be three inches. This might be done by the first Proportion, for if the Chord were drawn from E to F we should have a Triangle E A F of three Angles and two Sides known. But, more generally comparing the Sine of 30 *gr.* with the Sine of half the Ark proposed, the Proportion will hold.

As the Sine of the Semi-radius	30 *gr.* 00 *m.*	9.6989700
To the Semidiameter	3 000	0.4771212
		9.2218488
So the Sine of half the Ark	21 *gr.* 40 *m.*	9.5672689
To the Chord required	2 215	0.3454201

So that having drawn the Line A E, and described an occult Ark of a Circle upon the Center A, and Semidiameter A E, at the Distance of three inches, if we take out two inches, and 215 parts of 1000, and inscribe them into that Ark from E to F, the line A F shall make the Angle F A E to be 43 *gr.* 20 *m.* as was required.

Thus having applied that to the Canon and Tables of Logarithms which I had set down before for the general Use of the Lines of Numbers, Sines and Tangents, it may appear sufficiently, that, if we observe the Rules of Proportion set forth by others, and work by these Tables, we may use Addition instead of their Multiplication, and Subtraction instead of their Division, and so apply these general Rules to infinite particulars.

CHAP.

CHAP. IV.

Conteining some Use of right-lined Triangles in the Practice of FORTIFICATION.

IN the late manner of Fortification the ordinary Care is :
1. That the Angle of the Bulwark may be either a right Angle or near unto it.
2. That this Angle may be defended from the Flank and Cortin on either Side.
3. That the Lines of Defence may not exceed the reach of a Musket, which is said to be twelve score Yards, and those make 720 foot.
4. That the depth of the Flanks and the bredth of the Rampart be sufficient to resist battery ; and that may be about 100 foot at the ground.

Upon these considerations depend the rest of Lines and Angles : whereof I will set down some Propositions, beginning with that which may resolve the works of others.

PROP. I.

Having the Side of a Regular Fort, with the length of the Gorge, the Flank and the Face of the Bulwark, to find the rest of the Lines and Angles.

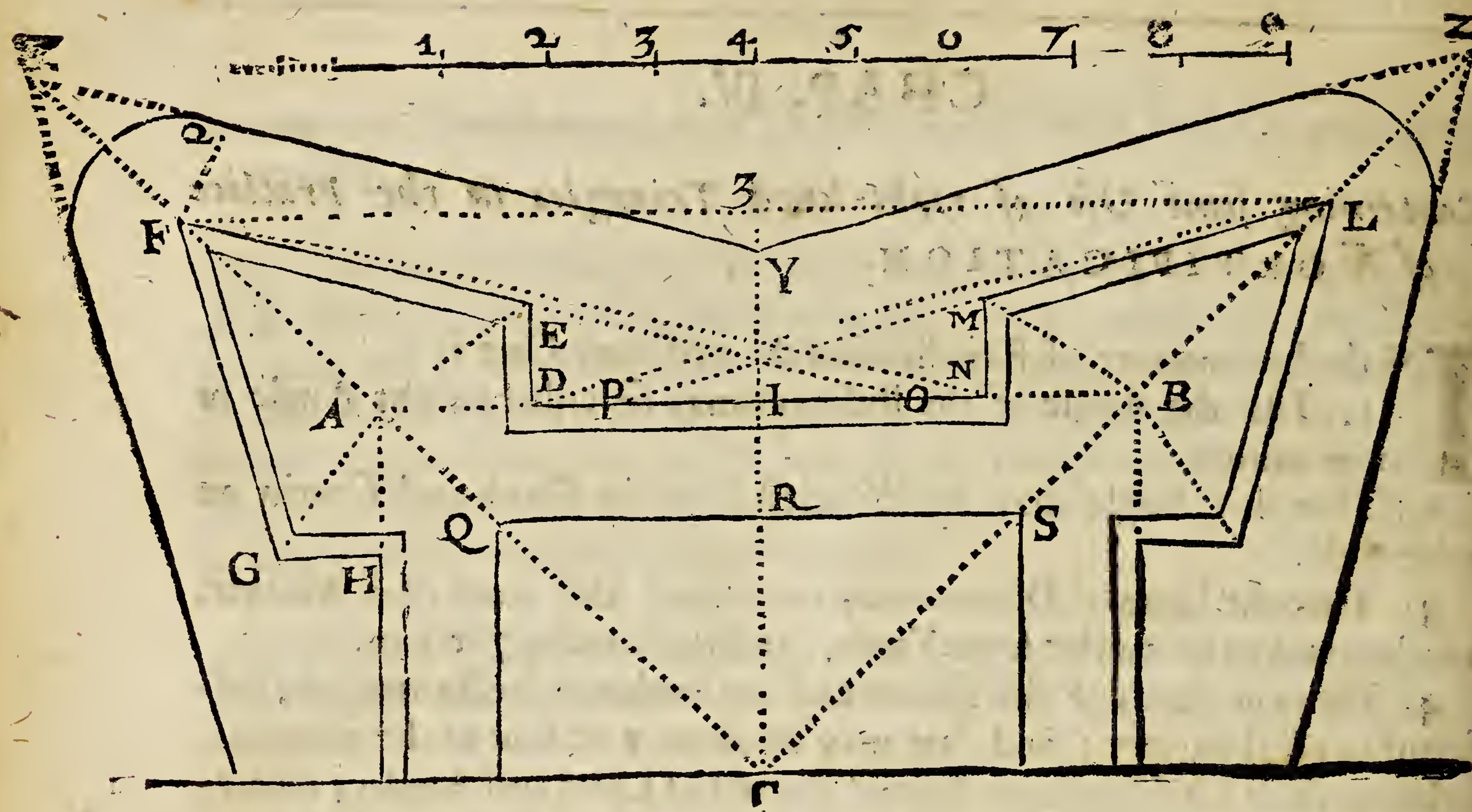
A Regular Fort is that, which is made with equal Sides and Angles, each Bulwark like unto other.

Suppose, that by observation or otherwise, we have found that in a square Fort, the Side was 700 foot, the Gorge 140, the Flank 100, and the Face 335 : In a Pentagonal, Hexagonal, Heptagonal, as in this Table.

		Quadr.	*Pentag.*	*Hexag*	*Heptag.*	*Octagon*
The Side	*AB*	700	800	900	950	1000
The Gorge	*AD*	140	180	190	200	230
The Flank	*DE*	100	120	140	150	158
The Face	*EF*	335	352	370	360	420

And that it were required to find the rest of the Lines, and the quantity of the Angles belonging to each Fort, beginning with the Quadrate.

First,



First, we may protract this Fort, by making a Square whose Side A B shall be 700 foot by the Scale: then take but 140 for the Gorge, and set them off from A unto D, and from A unto H; at D and H raise two Flanks perpendicular to the Sides of the Fort, and there prick down 100 from D unto E, and from H unto G. That done, take 335 out of the same Scale, and setting one foot of the Compasses in the point E, make an occult Ark of a Circle. Again, setting one foot of the Compasses in the point G, make another occult Ark, crossing the former in the point F; So the Lines E F, F G shall represent the Face of the Bulwark.

In like manner for the Bulwark at B, we may set off the Gorge from B unto N, *&c.* So have we divers Triangles, which may be resolved by the first three Propositions of right-lined Triangles: and the manner of it shall be so set down, as that Precept may be easily distinguished from the Example, and applied to any other, not only by this Canon and Table of Logarithms, but by the old Canon of Sines and Tangents, and by the Lines of Sines and Tangents both upon the Sector and the Cross staff.

1. In the Rectangle A D E, having the Sides A D, A E, we may find the Angles at A and E, and the third Side A E, by the former part of the third Proportion of Right-lined Triangles.

As

As the Gorge	A D	140	2.1461280
To the Flank	D E	100	2.0000000
So the Radius		90 00 *m.* 00 *sec.*	10.0000000
To the Tangent of	D A E	35 $32\frac{1}{4}$	9.8538720

Take the Angle DAE out of 90 *gr.* the Complement will give the Angle DEA; and then, having two Sides and three Angles, we may well find the third Side A E by the firſt Propoſition of the right-lined Triangles.

As the Sine of	D A E	35 $32\frac{1}{4}$	9.7643542
To the Side	D E	100	2.0000000
So the Sine of	A D E	90 00 *m.* 00 *sec.*	10.0000000
To the Side	A E	172^{041}	2.2356458

2. Becauſe the Fort is ſuppoſed to be ſure, the Angle H A D, muſt be 90 *gr.* and the half Angle C A D 45 *gr.* if we add this Angle C A D unto the Angle D A E and take the Sum out of 180 *gr.* the Remainder 99, $27\frac{1}{4}$ ſhall be the Angle E A F. Then in the Triangle E A F, having the Angle at A, and the two Sides F E, A E, we may find the other Angles at E and F, by the third Propoſition of right-lined Triangles.

As the Face	E F	335	2.5250448
To the Sine of	E A F	99 $27\frac{1}{4}$	9.9940502
			7.4690054
So the Line	A E	172^{042}	2.2356459
To the Sine of	A F E	30 $26\frac{1}{5}$	9.7046513

Add this Angle A F E to the Angle E A F, and take the Sum out of 180 *gr.* the Remainder 50 *gr.* 6 *m.* 3 *sec.* ſhall be the Angle A E F. And then we have two Sides and three Angles, to find the Head-line A F.

As the Sine of	E A F	99 $27\frac{1}{4}$	9.9640502
To the Face	E F	335	2.5250448
			7.4690054
So the Sine of	A E F	50 $6\frac{1}{25}$	9.8848958
To the Head-line	A F	260^{22}	2.4158904

3. If we produce the Face F E until it meet the Cortin in O, we ſhall have the Triangle A E O: wherein, knowing the Side A F, and the three Angles (for knowing two Angles, the third is always known by the Complement unto 180 *gr.*) we may find the other two Sides F O, A O.

As the Sine of	A O F	14 33 *m.* 48 *sec.*	9.4004548
To the Head-line	A F	260^{11}	2.4158904
			6.9845644
So the Sine of	F A O	45 00 00	9.8494850
To the Line	F O	73^{69}	9.8649206
And the Sine of	A F O	30 26 12	9.7046513
To the Line	A O	524^{912}	2.7200869

Take the Gorge N B 140, out of the Side A B 700, there remains 560 for the Line A N. Take this Line A O out of A N, and there remains 35^{088} for O N that part of the Cortin from whence the Face of the Bulwark may be defended.

4. In the Triangle A F N, having two Sides A F, A N, and the Angle between them F A N, we may find the other two Angles at F and N, by the later part of the third Proposition of right-lined Triangles.

As the Sum of the Sides A F, A N		820^{11}	2.9141050
Is to the Difference of those Sides		299^{41}	2.4763245
			4377805
So the Tang. of the half sum of opp. Ang. at F & N		22 30	9.6176153
To the Tang. of half the Diff. between those Ang.		8 36⅓	9.1798348

This half Diff. added to the half sum gives the greater Ang. AFN 31 6⅕
and subtracted the lesser ANF 13 53⅘

As the Sine of	A N F	13 53 48	9.3805157
To the Head-line	A F	260^{110}	2.4158904
			6.9646253
So the Sine of	F A N	45 00 00	9.8494850
To the Line of Defence	F N	767^{111}	2.8848597

5. In the Triangle A B C we have the Side A B, and the three Angles, to find the Side C A or C B from the Center to the Angles of the Fort.

As the Sine of	A C B	90 00 00	10.0000000
To the Side	A B	700	2.8450980
So the Sine of	A B C	45 00 00	9.8494850
To the Line	A C	494^{971}	2.6945830

This Line A C added to the Head-line A F, gives the whole C F, from the Center of the Fort to the uttermost point of the Bulwark to be 755^{121}.

6. In the Triangle C F L (the Side F L being parallel to A B the Side of the Fort) we have the three Angles and the Side C F; by which we may find F L the Distance between the points of the two next Bulwarks.

As

As the Sine of	C L F	45 00 00	9.8494850
To the Line	C F	755 ^{525}	2.8782498
So the Sine of	F C L	90 00 00	10.0000000
To the Line	F L	1068.464	3.0287648

Thus by resolving of six Triangles we have found		*gr. m. sec.*
The Angle at the Gorge	D A E	35 32 15
The Angle of the Bulwark	G F E	60 52 24
The Angle	F E D	104 33 48
The Angle	A N F	13 53 48
		Foot.
The length of the Line	A E	172 047
The Head-line	A F	260 540
The Line on the Cortin	O N	35 088
The Line of Defence	F N	767 113
The Semidiameter	C A	494 975
The Line from the Center to the Bulwark	C F	755 525
The Distance between the Bulwark	F L	1068 464

The principal Lines and Angles belonging to the Bulwark at A.

The rest of the Lines are either parallel unto these, or else they may be found in the same manner.

And all these may be understood by the same in the rest of the Bulwarks belonging to this Fort.

Again, what is said of a Square Fort, the same may be applied to all regular Forts.

And so, resolving the works of other men, it may appear how near they have come to the former grounds.

But that we may not altogether insist upon Examples, I will set down some profitable Suppositions, and from them proceed to find the rest of the Lines and Angles belonging to any Regular Fort.

1. The Angle at the Center A C B, between the Lines C A, C B drawn from the Center to each Bulwark, is found by dividing 360 *gr.* by the number of the Sides. So in a Square Fort, this Angle will be 90 *gr.* In a Pentagonal Fort, where there are five Sides, it will be 72 *gr.* &c.

2. Take this Angle at the Center, out of 180 *gr.* there remains the Angle of the Fort H A D.

3. The Angle A D E between the Flank and the Cortin, may be alway 90 *gr.*

 4. The

4. The uttermoſt Angle of the Bulwark E F G, muſt be leſs than the Angle of the Fort, yet not leſs than 60 *gr.* nor doth it need to be much more than 90 *gr.* If we allow it to be ⅔ of the Angle of the Fort, it may be defended from the Flank and Cortin on either ſide.

5. The Angle at the Gorge D A E, which forms the Flank D E, may be allowed between 35 and 40 *gr.* For in ſmall regular Forts it may be 40 *gr.* But where the Angle of the Fort is great, it may be leſs.

Theſe five Angles being firſt ſetled, the moſt of the other Angles will depend upon them, as in the Table following.

Or howſoever there may be other Angles found to be more convenient, yet theſe are ſufficient to explain the uſe of Triangles.

In a Regular Fort.		Quadr.		Pentag.		Hexag.		Heptag.		Octagon.		Cortin.	
		Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M.
Angle at the Center	*A C B*	90	0	72	0	60	0	51	25	45	0	0	0
Angle of the Fort	*H A D*	90	0	108	0	120	0	128	34	135	0	180	0
Angle of the Flank	*A D E*	90	0	90	0	90	0	90	0	90	0	90	0
Angle of the Bulwark	*G F E*	60	0	72	0	90	0	85	42	90	0	90	6
Angle of the Gorge	*D A E*	40	0	39	0	38	0	37	0	36	0	35	0
The half of H A D is	*C A D*	45	0	54	0	60	0	64	17	67	30	90	0
Half of G F E is	*A F E*	30	0	36	0	40	0	42	51	45	00	45	0
Complement of C A D is	*D A F*	135	0	126	0	120	0	115	43	112	30	90	0
A F E out of C A D leaves	*A O F*	15	0	18	0	20	0	21	25	22	30	45	0
Complement of A O F is	*O E D*	75	0	72	0	70	0	68	35	67	00	45	0
Complement of O E D is	*D E F*	105	0	108	0	110	0	111	26	112	30	135	0
Complement of D A E is	*A E D*	50	0	51	0	52	0	53	0	54	0	55	0
A E D out of D E F leaves	*A E F*	55	0	57	0	58	0	58	26	58	30	80	0
A E F and A F E give	*F A E*	95	0	87	0	82	0	78	43	76	30	55	0

PROP. II.

Having the ordinary Angles, with the Flank and Line of Defence, to find the reſt of the Lines and Angles in a Regular Fort.

SUppoſe the Angles to be ſuch, as in the former Table, the depth of the Flank D E 100 foot, and the Line of Defence F N 720 foot; and that it were required, to find the reſt of the Lines and Angles belonging to a Pentagonal Fort.

1. In the Triangle A D E, having the three Angles and the Flank D E, we may find the length of the Gorge A D, and the Line A E. The Angle of A D E is alway 90 *gr.* but the Fort being Pentagonal, made with five Bulwarks at the five Angles, the Table gives the Angle D A E 39 *gr.* and the Angle A E D 51 *gr.* wherefore,

As

As the Sine of	*D A E*	39 00 00	9 7988718
To the Flank	*D E*	100	2.0000000
			7.7988718
So the Sine of	*A E D*	51 00 00	9.8905076
To the Gorge	*A D*	$123^{\underline{42}}$	2.0916308
And the whole Sine	*A D E*	90 00	10.0000000
To the Line	*A E*	$158^{\underline{20}}$	2.2011282

2. In the Triangle A F E, having the three Angles and the Side A E, we may find the Face of the Bulwark F E, and the Head-line A F.

As the Sine of	*A F E*	36 00 00	9.7692186
To the Line	*A E*	$158^{\underline{20}}$	2.2011282
			7.5680904
So the Sine of	*F A E*	870 00	9.9994044
To the Face	*F E*	$269^{\underline{26}}$	2.4313140
And the Sine of	*A E F*	57 00 00	9.9235914
To the Head-line	*A F*	$226^{\underline{725}}$	2.3555010

3. In the Triangle A F O, having the three Angles and the Side A F, we may find the other two Sides F O and A O.

As the Sine of	*A O F*	18 00 00	9 4899823
To the H ad line	*A F*	$226^{\underline{225}}$	2.3555010
			7 1344813
So the Sine of	*F A O*	126 00 00	9.9074576
To the Line	*F O*	$593^{\underline{51}}$	2.7729763
And the Sine of	*A F O*	26 00 00	9.7692186
To the Line	*A O*	$438^{\underline{126}}$	2.6347373

4. In the Triangle A F N, having the Head-line A F, the Line of Defence F N, and the Angle F A N, we may find the other two Angles at N and F, and the third Side A N.

As the Line of D:fence	*F N*	720	2.8573325
To the Sine of	*F A N*	126 00 00	9.9079576
			7.0506251
So the Head-line	*A F*	$226^{\underline{22}}$	2.3555010
To the Sine of	*A N F*	14 45 33	9 4061261

This Angle A N F added to the Angle F A N, and the Sum of both taken out of 180*gr.* will give the third Angle A F N.

As

As the Sine of	F A N	126 00 00	9.9079576
To the Line of Defence	F N	720	2.8573325
			7.0506251
So the Sine of	A F N	39 14 27	9.8011178
To the Line of	A N	562 ^{98}	2.7504927

Having this Line A N if we add the Gorge N B or A D, the Sum of both ſhall be the Side of the Fort A B.

If we take the Gorge A D, out of this Line A N, the Remainder ſhall be the Cortin D N.

Again if we take the Line A O out of this Line A N, the Remainder ſhall be O N, that part of the Cortin from whence the Face of the Bulwark may be defended. And ſo here,

The length of this Line	A N being	562.98
The Gorge	A D	123.49
The Side of the Fort	A B ſhall be	686.47
The Cortin	D N	439.49
Again taking the Line	A O	431.26
From A N, there remains	O N	131.72

5. In the Triangle A I C, having the three Angles, and the Side A I, the one half of A B the Side of the Fort, we may find both C I, the Semidiameter of the Circle inſcribed, and C A, the Semidiameter of the Circle circumſcribed about the Fort.

As the Sine of	A C I	36 00 00	9.7692186
To the Line	A I	343 ^{23}	2.5355915
			7.2336271
So the Sine of	C A I	54 00 00	9.9079576
To the Line	C I	472.4225	2.6743305
And the whole Sine	C I A	90 00 00	10.0000000
To the Line	C A	583.9466	2.7663729

This Line C A added to the Head-line A F gives the diſtance C F between the Center of the Fort, and the uttermoſt point of the Bulwark.

6. If this Fort ſhall be encompaſſed with a Ditch, whoſe uttermoſt Sides ſhall be parallel to the Face of the Bulwark; ſuppoſing this Ditch to be of a known bredth (and that may be about 100 foot) we have the Triangle F 2 X; wherein knowing the three Angles and the Side F 2, we may find the Line F X.

As

As the Sine of	F X 2	36 00 00	9.7692186
To the Bredth-Line,	F 2	100	2,0000000
So the whole Sine	F 2 X	90 00 00	10,0000000
To the Line	F X	170^{13}	2.2307814

This Line F X added to the Line C F, gives the Diſtance C X between the Center of the Fort, and the uttermoſt Corner of the Ditch: and ſo here:

The Length of the Head-line	A F is	226.72
The Semidiameter	C A	583.95
Both theſe make the Line	C F	810.67
Add unto this the Line	F X	170.13
So C A, A F, F X make	C X	980.80

7. In the Triangle C Y X, having the three Angles and the Side C X, we may find the two other Sides C Y and X Y.

As the Sine of	C Y X	108 00 00	9.9782063
To the Line	C X	980^{80}	2.9915815
			6.9866248
So the Sine of	C X Y	36 00 00	9.7692186
To the Line	C Y	606^{162}	2.7825938
And the Sine	X C Y	36 00 00	9.7692186
To the Line	X Y	606^{162}	2.7825938

Take the Line C I, from this Line C Y, there remains I Y, the bredth of the Ditch from the middle of the Cortin.

8. Then, for the Lines F L, X Z, and ſuch other Parallels to the Side of the Fort A B.

As the Semidiameter	C A	583.95	2.7663729
To the Side of the Fort	A B	686.47	2.8366215
			7.0702486
So the length of	C F	810.67	2.9088444
To the Diſtance	F L	953.00	2.9790930
And the length of	C X	980.80	2.9915815
To the Diſtance	X Z	1152.97	3.0618310

9. The

9. The Perpendiculars C 3, C 4, and such others, let down from the Center upon the former Parallels may be found in the same sort.

As the Semidiameter	C A	583 95	2.7663729
To the Perpendicular	C I	472 42	2.6743305
			920424
So the Length of	C F	810 67	2.9088444
To the Perpendicular	C 3	655 84	2.8168020
And the Length of	C X	980 80	2.9915815
To the Perpendicular	C 4	793 48	2.8995391

10. If we take I R the bredth of the Rampart out of the Perpendicular C I, supposing the bredth of the Rampart to be 100 foot, there remains 372 42 for the Perpendicular C R.

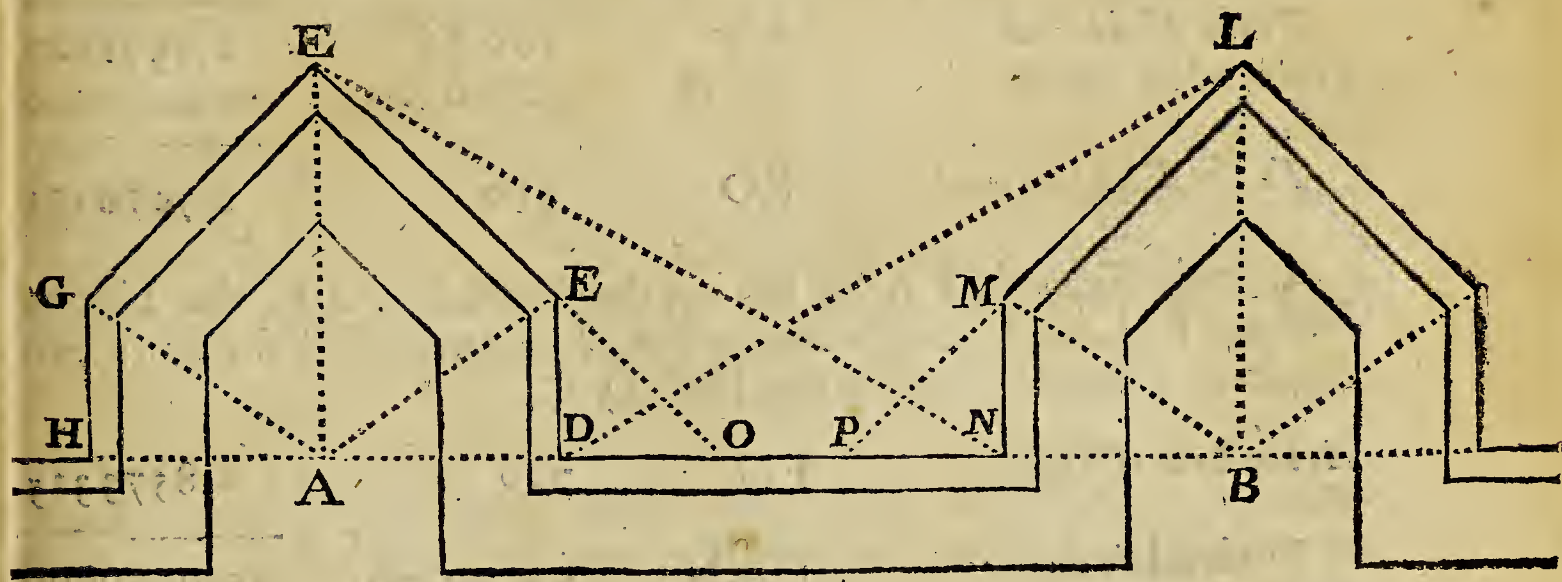
If we take out I T the bredth of the Rampart and Street adjoyning, (the Street being supposed 30 foot broad) there remains 342 42 for the Perpendicular C T.

As the Perpendicular	C I	472 42	2.6743305
To the Side of the Fort	A B	686 47	2.8366215
			1622910
So the Perpendicular	C R	372 42	2.5710358
To the Side of the Rampart	Q S	541 16	2.7333268
And the Perpendicular	C T	342 42	2.5345622
To the inner Side of the Street	V W	497 57	2.6968532
As the Perpendicular	C I	472 42	2.6743305
To the Semidiameter	C A	583 95	2.7663729
			920424
So the Perpendicular	C R	372 42	2.5710378
To the Line	C Q	460 34	2.6630802
And the Perpendicular	C T	342 42	2.5345622
To the Line	C V	423 25	2.6266046

PROP.

PROP. III.

Having the ordinary Angles with the Line of Defence and Face of the Bulwark, to find the rest of the Lines and Angles.



SUppose a long Cortin to be fortified with Bulwarks, the Angle of each Bulwark to be 90 *gr.* the Angle at the Gorge forming the Flank 35 *gr.* the rest as in the former Table, the Line of Defence 720 foot, and the Face of the Bulwark 300 foot.

As the Sine of	F A E	55 00 00	9.9133645
To the Face	F E	300	2.4771212
			7.4362433
So the Sine of	A E F	80 00 00	9.99335[illegible]4
To the Head-line	A F	360.668	2.5571081
And the Sine of	A F E	45 00 00	9.8494850
To the Line	A E	258.965	2.4132417

2. In the Triangle A D E, having the three Angles and the Line A E, we may find both the Flank D E, and the Gorge A D.

As the Sine of	A D E	90 00 00	10.0000000
To the Line	A E	258.96	2.4132417
			7.5867583
So the Sine of	D A E	35 00 00	9.7585913
To the Flank	D E	148.53	2.1718330
And the Sine of	A E D	55 00 00	9.9133645
To the Gorge	A D	212.132	2.3266062

 3. In

3. In the Triangle F A O, having the three Angles, and the two equal Sides A F, A O, we may find the length of F O, the Face produced unto the Cortin.

As the Sine of	A O F	45 00 00	9.8494850
To the Head-line	A F	360.66	2.5571081
So the whole Sine of	F A O	90 00 00	10.0000000
To the Face produced	F O	510	2.7076231

4. In the Triangle F A N, having the Head-line A F, the Line of Defence F N, and the right Angle F A N, we may find the other two Angles at F and N, and the third Side A N.

As the Line of Defence	F N	720	2.8573325
To the whole Sine of	F A N	90 00 00	10.0000000
So the Head-line	A F	360.66	2.5571081
To the Sine of	A N F	30.3 $\frac{2}{3}$	9.6997756
As the Sine of	F A N	90 00 00	10.0000000
To the Line	F N	720	2.8573325
So the Sine of	A F N	59.56 $\frac{1}{3}$	9.9372735
To the Line	A N	623.1697	2.7946060

Having the Line A N, if we add the Gorge N B or A D, the Sum of both ſhall be the Line A B or F L, the Diſtance between both Bulwarks.

If we take the Gorge A D out of this Line A N, the Remainder ſhall be the Cortin D N.

Again, if we take the Line A O out of this Line A N, the Remainder ſhall be O N, that part of the Cortin from whence the Face of the Bulwark may be defended.

Thus the Length of	A N being	623.169
The Gorge N B, or	A D	212.132
The Diſtance F L or	A B ſhall be	835.301
The Cortin	D N	41.037
Again taking the Line	A O	360.668
From A N, there remains	O N	262.501

PROP.

PROP. IV.

Having the Angles of an irregular Fort, with the Side between them, and the Face of the Bulwark, to find the rest of the Lines and Angles.

SUppose the Angles of an old walled Town were to be fortified with new Bulwarks. The Angles of the Bulwark to be either $\frac{2}{3}$ of the Angle at the Wall (or if $\frac{2}{3}$ of the Angle be more than 90 *gr.*) it may suffice that they be 90 *gr.* The Flanks perpendicular to the Cortin, to be formed by an Angle between 35 and 40 *gr.* as shall be found more convenient. And the Face of each Bulwark to be 300 foot.

Let the Angle at A be 126 *gr.* then may E F G, the Angle of the Bulwark be 84 *gr.* and the Angle D A E may be allowed to be 38 *gr.* Let the Angle at B be 140 *gr.* then because $\frac{2}{3}$ of this Angle are above 90 *gr.* the Angle of this Bulwark may well be 90 *gr.* and the Angle at the Gorge N B M, 36 *gr.* And let A B, the Distance between these Angles be 750 foot.

In regular Forts the Bulwarks may be made one like the other, so the Head-lines being produced will all meet in the same Center. In irregular (such as this) there will be some Difference, yet the work though somewhat longer will be still the same.

1. At the Bulwark A in the Triangle A F E, because the Angle of the Fort H A D is 126 *gr.* the half Angle Q A D 63 *gr.* and the Angle at the Gorge D A E supposed to be 38 *gr.* the Angle E A F will be 79 *gr.* Again, the Angle A F E (the half of G F E the Angle of the Bulwark) being 42 *gr.* the Angle A E F will be 59 *gr.* by Complement.

As the Sine of	F A E	79 00 00	9.9919465
To the Face	F E	300	2.4771212
			7.5148253
So the Sine of	A E F	59 00 00	9.9330656
To the Head-line	A F	261 963	2.4182403
And the Sine of	A F E	42 00 00	9.8254109
To the Line	A E	204 496	2.3106856

2. In

In the Rectangle A D E, the Angle at the Gorge D A E being 38 *gr.* the other Angle D E A must be 52 *gr.* by Complement.

As the whole Sine of	A D E	90 00 00	10.0000000
To the Line of	A E	204.496	2.3196856
			7.6803144
So the Sine of	D A E	38 00 00	9.7893419
To the Flank	D E	125.900	2.1000275
And the Sine of	A E D	52 00 00	9.8965321
To the Gorge	A D	161.145	2.2072177

In like manner at the Bulwark B in the Triangle B L M, because the Angle of the Fort is 140 *gr.* the half thereof S B N 70 *gr.* and the Angle at the Gorge N B M supposed to be 36 *gr.* the Angle M B L will be 74 *gr.* And then the Angle B L M (the half of the Angle of the Bulwark) being 45 *gr.* the third Angle B M L, must be 61 *gr.* by Complement.

As the Sine of	M B L	74 00 00	9.9828416
To the Face	M L	300	2.4771212
			7.5057204
So the Sine of	B M L	61 00 00	9.9418192
To the Head-line	B L	272.960	2.4360988
And the Sine of	B L M	45 00 00	9.8494850
To the Line	B M	220.681	2.3437646

And in the Rectangle Triangle B N M, allowing N B M, the Angle at the Gorge to be 36 *gr.* the other Angle B M N must be 54 *gr.* by Complement.

As the whole Sine	B N M	90 00 00	10.0000000
To the Line	B M	220.681	2.3437646
			7.6562354
So the Sine of	N B M	36 00 00	9.7692186
To the Flank	N M	129.713	2.1129832
And the Sine of	B M N	54 00 00	9.9079576
To the Gorge	B N	178.534	2.2517222

3. In

3. In the Triangle A F O, taking the Angle A F O 42 *gr.* out of the Angle Q A O 63 *gr.* there remains 21 *gr.* for the Angle A O F.

As the Sine of	A O F	21 00 00	9.5543291
To the Head line	A F	261.963	2.4182403
			6.1360888
So the Sine of	A F O	42 00 00	9.8255109
To the Line	A O	489 127	2.6894221
And the Sine of	F A O	63 00 00	9 9498808
To the Face produced	F O	651.316	2.8137920

And so in the like Triangle B L P, taking the Angle B L P 45 *gr.* out of the Angle S B P, 70 *gr.* there remains 25 *gr.* for the third Angle B L P.

As the Sine of	B P L	25 00 00	9.6259482
To the Head-line	B L	272.960	2.4360988
			7.1898494
So the Sine of	B L P	45 00 00	9.8494850
To the Line	B P	456.704	2.6596356
And the Sine of	L B P	110 00 00	9.9729858
To the Face produced	L P	606.927	2.7831364

Thus the length of the Side	A B being	750.
The length of the Gorge	B N	178.534
The length of the Line	A N	571.466
Take from this the Line	A O	489.127
There remains for the Line	O N	82.339
Again taking the Gorge	A D	161.145
Out of the Side A B, there remains	B D	588.855
Take from this the Line	B P	456.704
There remains for the Line	D P	132.151
Take A D out of A N, the Cortin	D N is	410.321

4. In

4. In the Triangle A F N, having two Sides A F, A N, and F A N the Angle between them, we may find the other two Angles at N and F, and the Line of Defence F N.

As the Sum of the Sides A F, A N		833.429	2.9208684
Is to the Difference of those Sides		309.503	2.4906636
			430.2048
So the Tang. of the half sum of opp. Ang. at F & N		31 30 0	9.7873193
To the Tang. of half the Diff. between those Ang.		12 49 $\frac{1}{4}$	9.3571145
This half Diff. added to the half sum gives the greater Ang.	AFN		44 19$\frac{1}{4}$
and subtracted gives the lesser	A N F		18 40$\frac{3}{4}$
As the Sine of	A N F	18 40$\frac{3}{4}$	9.5055225
To the Head-line	A F	261.963	2.4182403
			7.0872822
So the Sine of	F A N	63 00 00.	9.9498808
To the Line of Defence	F N	728.783	2.8625986
And the Sine of	A F N	44.19 $\frac{1}{4}$	9.8442725
To the Line	A N	571.465	2.7569903

And in the like Triangle B D L, having two Sides B L, B D, and the Angle between them L B D, we may find the other two Angles at D and L, and the Line of Defence L D.

As the Sum of B L *and* B D		861 815	2.9354138
To the Difference of these Sides		315 895	2.4995421
			4358717
So the Tang. of the half sum of opp. Ang. at L & D		35 00	9.8452267
To the Tangent of		14 23 $\frac{1}{3}$	9.4093550
This half Diff. added to the half sum gives the greater Ang.	BLD		49.23 $\frac{2}{3}$
and subtracted the lesser			20.36 $\frac{1}{3}$
As the Sine of	B L D	20.36 $\frac{2}{3}$	9.5463550
To the Head-line	B L	272.960	2.4360988
			7.1103544
So the Sine of	L B D	70.0.0	9.9729858
To the Line of Defence	L D	728.838	2.86263 14
And the Sine of	B L D	49.23 $\frac{2}{3}$	9.8803627
To the Line	B D	588.855	2.7700083

PROP.

PROP. V.

Having the Lines and Angles of a Regular Fort, to find the Content in Feet and Acres.

THe Content of a Fort may be taken ſeveral ways: either from within the Rampart, or from within the Out-ſide of the Ditch, or elſe we may take in the Out-works: And thoſe may be of ſeveral ſorts, ſuch as are here repreſented or the like.

If we conſider the Content within the Rampart, we have the Triangle Q C S, wherein knowing the Perpendicular C R and the Baſe QS, we may find the Content of the Triangle. And this Content multiplied by the Number of the like Triangles belonging to the Fort, ſhall be the whole Content required.

Thus, in the Pentagonal Fort before deſcribed, where the Perpendicular C R was found to be in feet 372.42, and the Baſe QS 541.16.

As the ſolemn number	2.	0.3010300
Is to the Baſe QS	541.16	2.7333268
		2.4322968
So the Perpendicular C R	372.42	2.5710358
To the Content of the Triangle	100773.25	5.0033326
Add (for five Triangles) the Logarithm of 5		0.6989700
The Content in feet comes to	503866	5.7023026

Then to reduce this Content into Acres, we may either divide the number of feet by 43560 (the number of feet conteined in an Acre) or working by Logarithms, we may ſubtract this ſolemn Logarithm 4.6390878.

Thus from the Logarithm of	503866.25.	5.7023026
Subtract the ſolemn Logarithm	43560	4.6390878
There remains the Logarithm of	11.56	1.0632148

The Content in Acres conteined within the Rampart.

If it be required to find the Content of this Pentagonal Fort within the outward Side of the Ditch, we have ten ſuch Triangles as X C Y, wherein knowing the two Sides C X, C Y, and the Angle between them X C Y, we may let down a Perpendicular from the Angle at Y, upon the Baſe C X; and then with the Perpendicular and the Baſe, we may find the Content of the Triangle as before.

Thus

Thus the Side C X being 980.80. the Side C Y 606.17. and the Angle between them X C Y, 36 00 00.

1. *As the whole Sine of*		90 00 00	10.0000000
To the lesser Side	C Y	607.17	2.7825938
So the Sine of	X C Y	36 00 00	9.7692186
To the Perpendicular			2.5518124
2. *As the solemn number*		2.	0.3010300
To the Base	C X	980.80	2.9915815
			2.6905515
So the Perpendicular			2.5518124
To the Content of the Triangle		174728.60	5.2423639
Add (for ten Triangles) the Logarithm of 10.			1.0000000
The Content in feet comes to		1747286	6.2423639
Again, subtract the Logarithm of		43560	4.6390878
The Content in Acres comes to		40.11	1.6032761

By the same reason, resolving all into Triangles, we may take in the Counterscarp, and the rest of the Out-works; and so find the Content, not only of a Regular Fort, but of any other Piece of Ground.

FINIS.

CANON TRIANGULORUM:

OR, A

TABLE

OF

Artificial Sines and Tangents,

TO A

RADIUS of 10.0000000 Parts

To each MINUTE of the

QUADRANT.

BY

EDMUND GUNTER,

Professor of Astronomy in *Gresham-College*.

LONDON,

Printed by *Andrew Clark*, for *Francis Eglesfield*, and are to be sold at the *Marigold* in S. *Paul* s Church-yard. 1672.

Aaaaa

Honoratissimo Domino

D^{no} JOHANNI

COMITI de *BRIDGEWATER*,

VICECOMITI de *BRACKLEY*,

BARONI de *ELLESMERE*,

Hunc suum

CANONEM TRIANGULORUM

D. D. D.

Edm. Gunter.

The DESCRIPTION of the

CANON.

THis *Canon* hath six Columns. The first is of Degrees and Minutes, from the beginning of the Quadrant unto 45 *gr.* the sixth of Degrees and Minutes from 45 *gr.* unto the end of the Quadrant; the other four contain the *Sines* and *Tangents* belonging to each of those Degrees and Minutes, after the manner of other *Canons.* The difference is in the Numbers: For these *Sines* are not such as half the Chords of the double Ark, nor these *Tangents* Perpendiculars at the end of the Diameter; but other Numbers substituted in their place, for attaining the same end by a more easie way, such as the *Logarithms* of the Lord of *Merchiston*; and thereupon I call them *Artificial Sines and Tangents.* So the second and fourth Columns contain the *Sines* and *Tangents* of the Degrees and Minutes in the first Column; the third and fifth contain the *Sines* and *Tangents* of the sixth Column.

As if it were required to find the *Artificial Sine* belonging to our Latitude, which here at *London* is 51 *gr.* 32 *m.* you may find *Sine* 51 in the lower part of the Page, and *M.* 32 in the sixth Column, the common Angle will give 9.8937452 for the *Sine* required. And in the same Line you have 9.7938317 for the *Sine* of the Complement of this Latitude, which in one word may be called the *Co-sine.* In like manner, the *Tangent* of 51 *gr.* 32 *m.* will be found to be 10.0999134, and the *Co-tangent* 9.9000865.

The Secants (if there were use of them) may easily be supplied, by taking the *Co-sine* out of the double of the *Radius.*

As the double of the *Radius*, being	20.0000000
Take hence the *Co-sine* of 51 *gr.* 32 *m.*	9.7938317
The Secant of 51 *gr.* 32 *m.* will be	10.2061683

The Versed Sine may also be supplied by adding 3010300 unto the double of the *Sine* of half the Ark, and subtracting the *Radius.* As the half of 51 *gr.* 32 *m.* being 25 *gr.* 46 *m.*

Add to the *Sine* of 25 *gr.* 46 *m.*	9.6381968
The same again, and the former Number,	9.6381968
So the *Radius* being subtracted, add	3010300
The *Versed Sine* of 51 *gr.* 32 *m.* will be	9.5774236

Note 3010300 is Log^m of 2.

M	*Sin.* o.		*Tang.* o.		
0	0	10.0000000	0	*Infinitum.*	60
1	6.4637260	9.9999999	6.4637260	13.5362739	59
2	6.7647560	9.9999999	6.7647561	13.2352438	58
3	6.9408473	9.9999998	6.9408474	13.0591525	57
4	7.0657860	9.9999997	7.0657863	12.9342136	56
5	7.1626959	9.9999995	7.1626964	12.9373035	55
6	7.2418771	9.9999993	7.2418778	12.7582221	54
7	7.3088238	9.9999991	7.3088247	12.6911752	53
8	7.3608157	9.9999988	7.3668169	12.6331831	52
9	7.4179681	9.9999985	7.4179696	12.5820303	51
10	7.4637255	9.9999981	7.4637273	12.5362726	50
11	7.5051180	9.9999977	7.5051202	12.4948797	49
12	7.5429064	9.9999973	7.5429091	12.4570908	48
13	7.6776684	9.9999969	7.5776715	12.4223284	47
14	7.6098529	9.9999964	7.6098565	12.3901434	46
15	7.6398160	9.9999958	7.6398201	12.3601798	45
16	7.6678445	9.9999953	7.6678402	12.3321507	44
17	7.6941722	9.9999947	7.6941785	12.3058214	43
18	7.7189966	9.9999940	7.7180026	12.2819974	42
19	7.7424775	9.9999933	7.7424841	12.2575158	41
20	7.7647536	9.9999926	7.7647610	12.2353262	40
21	7.7859427	9.9999919	7.7859508	12.2140491	39
22	7.8061458	9.9999911	7.8061547	12.1938452	38
23	7.8254507	9.9999902	7.8254604	12.1745395	37
24	7.8409238	9.9999894	7.8439444	12.1560556	36
25	7.8616623	9.9999885	7.8616738	12.1382389	35
26	7.8786953	9.9999875	7.8787077	12.1212922	34
27	7.8950854	9.9999866	7.8950988	12.1049012	33
28	7.9108793	9.9999856	7.9108937	12.0891062	32
29	7.9261189	9.9999845	7.9261344	12.0738656	31
30	7.9408418	9.9999834	7.9408584	12.0591416	30
		Sin. 89.		*Tang.* 89.	M

M	Sin. 0.		Tang. 0.		
30	7.7408418	9.9999834	7.9408584	12.0591416	30
31	7.9550819	9.9999823	7.9550996	12.0449004	29
32	7.9688698	9.9999812	7.9688886	12.0311113	28
33	7.9822333	9.9999800	7.9822534	12.0177465	27
34	7.9951979	9.9999787	7.9952192	12.0047808	26
35	8.0077866	9.9999774	8.0078091	11.9921908	25
36	8.0200206	9.9999761	8.0200445	11.9799555	24
37	8.0319194	9.9999748	8.0319446	11.9680553	23
38	8.0435008	9.9999734	8.0435274	11.9564726	22
39	8.0547814	9.9999720	8.0548193	11.9451806	21
40	8.0657763	9.9999706	8.0658057	11.9341942	20
41	8.0764996	9.9999691	8.0765305	11.9234694	19
42	8.0869646	9.9999675	8.0869970	11.9130029	18
43	8.0971832	9.9999660	8.0972172	11.9027827	17
44	8.1071669	9.9999644	8.1072020	11.8927975	16
45	8.1169262	9.9999628	8.1169634	11.8830365	15
46	8.1264709	9.9999611	8.1265098	11.8734901	14
47	8.1358104	9.9999594	8.1358510	11.8641489	13
48	8.1449532	9.9999576	8.1449955	11.8550044	12
49	8.1539075	9.9999558	8.1539516	11.8460483	11
50	8.1626808	9.9999540	8.1627367	11.8372632	10
51	8.1712803	9.9999522	8.1713281	11.8286618	9
52	8.1797129	9.9999503	8.1797626	11.8201374	8
53	8.1879847	9.9999484	8.1880363	11.8119636	7
54	8.1961020	9.9999464	8.1961555	11.8038444	6
55	8.2040702	9.9999444	8.2041258	11.7958741	5
56	8.2118949	9.9999423	8.2119525	11.7880474	4
57	8.2195810	9.9999403	8.2196407	11.7803592	3
58	8.2271335	9.9999382	8.2271953	11.7728046	2
59	8.2345568	9.9999360	8.2345207	11.7653792	1
60	8.2418053	9.9999338	8.2419214	11.7580785	0
		Sin. 89.		Tang. 89.	M

M	Sin. 1.		Tang. 1.		
0	8.2418553	9.9999338	8.2419214	11.7580785	60
1	8.2490331	9.9999316	8.2491015	11.7508984	59
2	8.2560942	9.9999293	8.2561649	11.7438351	58
3	8.2630423	9.9999270	8.2631152	11.7368847	57
4	8.2698810	9.9999247	8.2699562	11.7300437	56
5	8.2766136	9.9999223	8.2766912	11.7233087	55
6	8.2832433	9.9999200	8.2833234	11.7166765	54
7	8.2897734	9.9999175	8.2898559	11.7101440	53
8	8.2962067	9.9999150	8.2962916	11.7037083	52
9	8.3025460	9.9999125	8.3026335	11.6973664	51
10	8.3087941	9.9999099	8.3088842	11.6911158	50
11	8.3149535	9.9999473	8.3150462	11.6849537	49
12	8.3210268	9.9999047	8.3211221	11.6788778	48
13	8.3270163	9.9999020	8.3271142	11.6728857	47
14	8.3329243	9.9998993	8.3330249	11.6669750	46
15	8.3387529	9.9998966	8.3388563	11.6611437	45
16	8.3445043	9.9998938	8.3446104	11.6553895	44
17	8.3501805	9.9998910	8.3502894	11.6497106	43
18	8.3557834	9.9998882	8.3558952	11.6441047	42
19	8.3613149	9.9998853	8.3614296	11.6385703	41
20	8.3667769	9.9998823	8.3668945	11.6331054	40
21	8.3721709	9.9998794	8.3722915	11.6277084	39
22	8.3774988	9.9998764	8.3766223	11.6223776	38
23	8.3827620	9.9998734	8.3828886	11.6171113	37
24	8.3879621	9.9998703	8.3880918	11.6119081	36
25	8.3931007	9.9998672	8.3932335	11.6067664	35
26	8.3981792	9.9998641	8.3983151	11.6016848	34
27	8.4031990	9.9998609	8.4033381	11.5966619	33
28	8.4081613	9.9998576	8.4083036	11.5916963	32
29	8.4130676	9.9998544	8.4132131	11.5867868	31
30	8.4179190	9.9998511	8.4180678	11.5819321	30
		Sin. 88.		Tang. 88.	M

M	Sin. 1.		Tang. 1.		
30	8.4179190	9.9998511	8.4180678	11.5819321	30
31	8.4227168	9.9998788	8.4228689	11.5771310	29
32	8.4274621	9.9998444	8.4276176	11.5723823	28
33	8.4321561	9.9998410	8.4323150	11.5676849	27
34	8.4367998	9.9998376	8.4369622	11.5630377	26
35	8.4413944	9.9998341	8.4415603	11.5584397	25
36	8.4459409	9.9998306	8.4461102	11.5538897	24
37	8.4504402	9.9998271	8.4506131	11.5493868	23
38	8.4548933	9.9998235	8.4550698	11.5449301	22
39	8.4593012	9.9998199	8.4594814	11.5405186	21
40	8.4656648	9.9998162	8.4638486	11.5361513	20
41	8.4679850	9.9998125	8.4681724	11.5318275	19
42	8.4722625	9.9998088	8.4724537	11.5275462	18
43	8.4764983	9.9998050	8.4766933	11.5233066	17
44	8.4806932	9.9998012	8.4808919	11.5191080	16
45	8.4848478	9.9997974	8.4850505	11.5149495	15
46	8.4889631	9.9997935	8.4891696	11.5108303	14
47	8.4930397	9.9997896	8.4932501	11.5067498	13
48	8.4970784	9.9997856	8.4972927	11.5027072	12
49	8.5010798	9.9997816	8.5012981	11.4917018	11
50	8.5050446	9.9997776	8.5052670	11.4947329	10
51	8.5089736	9.9997735	8.5092000	11.4907999	9
52	8.5128673	9.9997694	8.5130978	11.4869021	8
53	8.5167263	9.9997653	8.5169610	11.4830389	7
54	8.5205513	9.9997611	8.5207902	11.4792098	6
55	8.5243429	9.9997569	8.5245860	11.4754140	5
56	8.5281016	9.9997527	8.5283489	11.4716510	4
57	8.5318281	9.9997484	8.5320797	11.4679203	3
58	8.5355228	9.9997441	8.5357787	11.4642212	2
59	8.5391863	9.9997397	8.5394466	11.4605534	1
60	8.5428191	9.9997353	8.5430838	11.4569162	0
		Sin. 88.		Tang. 88.	M

	Sin. 2.		Tang. 2.		
0	8.5428191	9.9997353	8.5430838	11.4569162	60
1	8.5464217	9.9997309	8.5466908	11.4533091	59
2	8.5499947	9.9997264	8.5502683	11.4497317	58
3	8.5535385	9.9997219	8.5538166	11.4461834	57
4	8.5570536	9.9997174	8.5573362	11.4426637	56
5	8.5605404	9.9997128	8.5608276	11.4391724	55
6	8.5639994	9.9997082	8.5642912	11.4357088	54
7	8.5674310	9.9997035	8.5677274	11.4322725	53
8	8.5708357	9.9996988	8.5711368	11.4288631	52
9	8.5742139	9.9996941	8.5745197	11.4254802	51
10	8.5775659	9.9996894	8.5778765	11.4221234	50
11	8.5808923	9.9996846	8.5812076	11.4187923	49
12	8.5841933	9.9996797	8.5845135	11.4154864	48
13	8.5874694	9.9996749	8.5877945	11.4122054	47
14	8.5907209	9.9996700	8.5910509	11.4089490	46
15	8.5939482	9.9996650	8.5943832	11.4057167	45
16	8.5971517	9.9996600	8.5974916	11.4025083	44
17	8.6003317	9.9996550	8.6006766	11.3993233	43
18	8.6034885	9.9996499	8.6038385	11.3961614	42
19	8.6066225	9.9996449	8.6069776	11.3930223	41
20	8.6097341	9.9996397	8.6100943	11.3899056	40
21	8.6128234	9.9996346	8.6131888	11.3868111	39
22	8.6158909	9.9996294	8.6162615	11.3837384	38
23	8.6189369	9.9996241	8.6193127	11.3806872	37
24	8.6219616	9.9996188	8.6223427	11.3776572	36
25	8.6249653	9.9996135	8.6253517	11.3746482	35
26	8.6279484	9.9996082	8.6283402	11.3716598	34
27	8.6399111	9.9996028	8.6313082	11.3686917	33
28	8.6338536	9.9995974	8.6342562	11.3657437	32
29	8,6367763	9.9995919	8.6371844	11.3628155	31
30	8.6396795	9.9995864	8.6400931	11.3599068	30
		Sin. 87.		Tang 87.	M

M	Sin. 2.		Tang. 2.		
30	8.6396795	9.9995864	8.6400931	11.3599068	30
31	8.6425634	9.9995809	8.6429825	11.3570175	29
32	8.6454282	9.9995753	8.6458528	11.3541471	28
33	8.6482741	9.9995697	8.6487044	11.3512955	27
34	8.6511015	9.9995640	8.6515375	11.3484625	26
35	8.6539106	9.9995584	8.6543522	11.3456477	25
36	8.6567016	9.9995527	8.6571489	11.3428510	24
37	8.6594748	9.9995469	8.6599278	11.3400721	23
38	8.6622303	9.9995411	8.6626891	11.3373108	22
39	8.6649684	9.9995353	8.6654330	11.3345669	21
40	8.6676893	9.9995294	8.6681598	11.3318401	20
41	8.6703932	9.9995235	8.6708696	11.3291303	19
42	8.6730803	9.9995176	8.6735627	11.3264372	18
43	8.6757510	9.9995116	8.6762393	11.3237606	17
44	8.6784052	9.9995056	8.6788996	11.3211003	16
45	8.6810433	9.9994995	8.6815437	11.3184562	15
46	8.6836654	9.9994934	8.6841719	11.3158280	14
47	8.6862717	9.9994873	8.6867844	11.3132155	13
48	8.6888625	9.9994812	8.6893813	11.3106186	12
49	8.6914378	9.9994750	8.6919628	11.3080371	11
50	8.6939980	9.9994687	8.6945292	11.3054707	10
51	8.6965431	9.9994625	8.6970806	11.3029193	9
52	8.6990733	9.9994561	8.6996171	11.3003828	8
53	8.7015889	9.9994498	8.7021390	11.2978909	7
54	8.7040899	9.9994434	8.7046464	11.2953535	6
55	8.7065765	9.9994370	8.7071395	11.2928604	5
56	8.7090490	9.9994306	8.7096184	11.2903815	4
57	8.7115074	9.9994241	8.7120833	11.2879166	3
58	8.7139520	9.9994175	8.7145345	11.2854655	2
59	8.7163829	9.9994110	8.7169719	11.2830281	1
60	8.7188001	9.9994044	8.7193957	11.2806042	0
		Sin. 87.		Tang. 87.	M

M	Sin. 3.		Tang. 3.		
0	8.7188001	9.9994044	8.7193957	11.2806042	60
1	8.7212040	9.9993977	8.7218062	11.2781937	59
2	8.7235946	9.9993910	8.7242035	11.2757964	58
3	8.7259720	9.9993843	8.7265877	11.2734123	57
4	8.7283365	9.9993776	8.7289589	11.2710410	56
5	8.7306882	9.9993708	8.7313173	11.2686826	55
6	8.7330271	9.9993640	8.7336631	11.2663368	54
7	8.7353535	9.9993571	8.7359964	11.2640036	53
8	8.7376674	9.9993502	8.7383172	11.2616827	52
9	8.7399691	9.9993433	8.7406258	11.2593742	51
10	8.7422586	9.9993363	8.7429222	11.2570777	50
11	8.7445360	9.9993293	8.7452066	11.2547933	49
12	8.7468015	9.9993223	8.7474792	11.2525207	48
13	8.7490552	9.9993152	8.7497400	11.2502599	47
14	8.7512973	9.9993081	8.7519892	11.2480107	46
15	8.7535278	9.9993009	8.7542268	11.2457731	45
16	8.7557468	9.9992937	8.7564531	11.2435468	44
17	8.7578575	9.9992865	8.7586681	11.2413319	43
18	8.7601511	9.9992792	8.7608719	11.2391280	42
19	8.7623366	9.9992719	8.7630646	11.2369353	41
20	8.7645111	9.9992646	8.7652464	11.2347535	40
21	8.7666747	9.9992572	8.7674174	11.2325825	39
22	8.7688275	9.9992498	8.7695777	11.2304222	38
23	8.7709697	9.9992423	8.7717273	11.2282726	37
24	8.7731013	9.9992349	8.7738664	11.2261335	36
25	8.7752225	9.9992273	8.7759952	11.2240048	35
26	8.7773334	9.9992198	8.7781135	11.2218864	34
27	8.7794340	9.9992122	8.7802217	11.2197782	33
28	8.7815244	9.9992045	8.7823198	11.2176801	32
29	8.7836048	9.9991969	8.7844079	11.2155920	31
30	8.7856752	9.9991892	8.7864860	11.2135139	30
		Sin. 86.		Tang. 86.	M

M	Sin. 3.		Tang. 3.		
30	8.7856752	9.9991892	8.7894860	11.2135139	30
31	8.7877358	9.9991814	8.7885544	11.2114455	29
32	8.7897866	9.9991736	8.7906130	11.2093869	28
33	8.7918278	9.9991658	8.7929619	11.2073380	27
34	8.7938593	9.9991580	8.7947013	11.2052986	26
35	8.7958814	9.9991501	8.7967313	11.2032686	25
36	8.7978940	9.9991421	8.7987519	11.2012480	24
37	8.7998974	9.9991342	8.8007632	11.1992368	23
38	8.8018915	9.9991262	8.8027653	11.1972347	22
39	8.8038794	9.9991181	8.8047582	11.1952417	21
40	8.8058523	9.9991100	8.8067422	11.1932577	20
41	8.8078192	9.9991019	8.8087172	11.1912827	19
42	8.8097772	9.9990938	8.8106834	11.1893166	18
43	8.8117263	9.9990856	8.8126407	11.1873592	17
44	8.8136667	9.9990774	8.8145893	11.1854106	16
45	8.8155985	9.9990691	8.8165293	11.1834706	15
46	8.8175216	9.9990608	8.8184608	11.1815391	14
47	8.8194363	9.9990525	8.8203838	11.1796161	13
48	8.8213425	9.9990441	8.8222984	11.1777016	12
49	8.8232403	9.9990357	8.8242046	11.1757953	11
50	8.8251299	9.9990272	8.8261026	11.1738973	10
51	8.8270112	9.9990188	8.8279924	11.1720075	9
52	8.8288843	9.9990102	8.8298741	11.1701258	8
53	8.8307494	9.9990017	8.8317477	11.1682522	7
54	8.8326065	9.9989931	8.8336134	11.1663865	6
55	8.8344557	9.9989844	8.8354712	11.1645287	5
56	8.8362969	9.9989758	8.8373211	11.1626788	4
57	8.8381304	9.9989671	8.8391632	11.1608367	3
58	8.8399560	9.9989683	8.8409977	11.1590022	2
59	8.8417741	9.9989496	8.8428245	11.1571754	1
60	8.8435845	9.9989407	8.8446437	11.1553562	0
		Sin. 86.		Tang. 86.	M

M	Sin. 4.		Tang. 4.		
0	8.8438545	9.9989408	8.8446437	11.1553562	60
1	8.8453873	9.9989319	8.8464552	11.1535455	59
2	8.8471827	9.9989230	8.8482597	11.1517403	58
3	8.8487906	9.9989141	8.8500565	11.1499934	57
4	8.8507512	9.9989051	8.8518460	11.1481539	56
5	8.8525245	9.9988961	8.8536283	11.1463716	55
6	8.8542905	9.9988871	8.8554034	11.1445966	54
7	8.8560493	9.9988780	8.8571713	11.1428286	53
8	8.8578010	9.9988689	8.8589321	11.1410678	52
9	8.8595456	9.9988597	8.8606858	11.1393141	51
10	8.8612832	9.9988506	8.8624326	11.1375673	50
11	8.8630139	9.9988413	8.8641725	11.1358274	49
12	8.8647376	9.9988321	8.8659055	11.1340944	48
13	8.8664545	9.9988228	8.8676317	11.1323682	47
14	8.8681646	9.9988135	8.8693511	11.1306488	46
15	8.8698679	9.9988041	8.8710638	11.1289361	45
16	8.8716646	9.9987947	8.8727699	11.1272300	44
17	8.8732546	9.9987852	8.8744693	11.1255306	43
18	8.8749380	9.9987758	8.8761622	11.1238377	42
19	8.8766149	9.9987662	8.8778487	11.1221513	41
20	8.8782853	9.9987567	8.8795286	11.1204713	40
21	8.8799493	9.9987471	8.8812022	11.1187978	39
22	8.8816069	9.9987375	8.8828694	11.1171305	38
23	8.8832581	9.9987278	8.8845303	11.1154696	37
24	8.8849031	9.9987181	8.8861849	11.1138150	36
25	8.8865418	9.9987083	8.8878334	11.1121660	35
26	8.8881743	9.9986986	8.8894756	11.1105243	34
27	8.8898006	9.9986888	8.8911118	11.1088881	33
28	8.8914209	9.9986789	8.8927420	11.1072580	32
29	8.8930351	9.9986690	8.8943660	11.1056339	31
30	8.8946433	9.9986591	8.8959841	11.1040158	30
		Sin. 85.		Tang. 85.	M

M	Sin. 4.		Tang. 4.		
30	8.8946433	9.9986591	8.8659841	11.1040158	30
31	8.8962455	9.9986492	8.8975963	11.1024036	29
32	8.8978417	9.9986391	8.8992026	11.1007973	28
33	8.8994322	9.9986291	8.9008030	11.0991969	27
34	8.9010167	9.9986190	8.9023977	11.0976022	26
35	8.9025955	9.9986089	8.9039866	11.0960134	25
36	8.9041685	9.9985988	8.9055697	11.0944302	24
37	8.9057358	9.9985886	8.9071472	11.0928527	23
38	8.9072974	9.9985784	8.9087190	11.0912809	22
39	8.9088534	9.9985681	8.9102853	11.0897146	21
40	8.9104038	9.9985578	8.9118460	11.0881539	20
41	8.9119487	9.9985475	8.9134012	11.0835988	19
42	8.9134880	9.9985371	8.9149508	11.0850491	18
43	8.9150219	9.9985267	8.9164951	11.0835048	17
44	8.9165503	9.9985163	8.9180340	11.0819659	16
45	8.9180733	9.9985058	8.9195675	11.0804324	15
46	8.9195910	9.9984953	8.9210957	11.0789042	14
47	8.9211034	9.9984847	8.9226186	11.0773813	13
48	8.9226104	9.9984742	8.9241362	11.0758637	12
49	8.9241122	9.9984635	8.9256487	11.0743512	11
50	8.9256089	9.9984528	8.9271561	11.0728439	10
51	8.9271003	9.9984422	8.9286581	11.0713418	9
52	8.9285866	9.9984314	8.9301551	11.0698448	8
53	8.9300678	9.9984206	8.9316471	11.0683528	7
54	8.9315439	9.9984098	8.9331340	11.0668659	6
55	8.9330150	9.9983990	8.9346160	11.0653840	5
56	8.9344810	9.9983881	8.9360929	11.0639070	4
57	8.9359421	9.9983772	8.9375649	11.0624350	3
58	8.9373983	9.9983662	8.9390321	11.0609678	2
59	8.9388496	9.9983552	8.9404943	11.0595056	1
60	8.9402960	9.9983442	8.9419517	11.0580482	0
		Sin. 85.		Tang. 85.	M

M	Sin. 5.		Tang. 5.		
0	8.9402960	9.9983442	8.9419517	11.0580482	60
1	8.9417379	9.9983331	8.9434044	11.0565955	59
2	8.9431743	9.9983220	8.9448522	11.0551477	58
3	8.9446063	9.9983109	8.9462954	11.0537046	57
4	8.9460335	9.9982997	8.9477338	11.0522661	56
5	8.9474560	9.9982885	8.9491675	11.0508324	55
6	8.9488739	9.9982772	8.9505966	11.0494033	54
7	8.9502871	9.9982659	8.9520211	11.0479788	53
8	8.9516956	9.9982546	8.9534410	11.0465589	52
9	8.9530996	9.9982432	8.9548564	11.0451436	51
10	8.9544990	9.9982318	8.9562672	11.0537327	50
11	8.9558939	9.9982204	8.9576735	11.0423264	49
12	8.9572843	9.9982089	8.9590754	11.0409245	48
13	8.9586702	9.9981974	8.9604728	11.0395271	47
14	8.9600517	9.9981858	8.9618658	11.0381341	46
15	8.9614287	9.9981742	8.9632544	11.0367455	45
16	8.9628013	9.9981626	8.9646387	11.0353612	44
17	8.9641996	9.9981509	8.9660187	11.0339812	43
18	8.9655337	9.9981392	8.9673944	11.0326055	42
19	8.9668934	9.9981275	8.9687658	11.0312341	41
20	8.9682488	9.9981157	8.9701330	11.0298669	40
21	8.9695998	9.9981039	8.9714949	11.0285050	39
22	8.9709467	9.9980921	8.9728546	11.0271453	38
23	8.9722894	9.9980802	8.9742092	11.0257907	37
24	8.9736280	9.9980682	8.9755597	11.0244402	36
25	8.9749624	9.9980563	8.9769060	11.0230939	35
26	8.9762926	9.9980443	8.9782483	11.0217516	34
27	8.9776187	9.9980323	8.9795864	11.0204135	33
28	8.9789408	9.9980202	8.9809206	11.0190793	32
29	8.9802588	9.9980081	8.9822507	11.0177492	31
30	8.9815728	9.9979959	8.9835769	11.0164230	30
		Sin. 84.		Tang. 84.	M

M	*Sin. 5.*		*Tang. 5.*		
30	8.9815728	9.9979959	8.9835769	11.0164230	30
31	8.9828829	9.9979838	8.9848991	11.0151008	29
32	8.9841889	9.9979715	8.9862173	11.0137826	28
33	8.9854909	9.9979593	8.9875316	11.0124083	27
34	8.9867890	9.9979470	8.9888420	11.0111579	26
35	8.9880833	9.9979347	8.9991486	11.0098513	25
36	8.9893737	9.9979223	8.9914513	11.0085486	24
37	8.9906602	9.9979099	8.9927503	11.0072496	23
38	8.9919429	9.9978974	8.9940454	11.0059545	22
39	8.9932217	9.9978850	8.9953367	11.0046632	21
40	8.9944967	9.9978725	8.9966243	11.0033757	20
41	8.9957680	9.9978599	8.9979081	11.0020918	19
42	8.9970356	9.9978473	8.9991883	11.0008117	18
43	8.9982994	9.9978347	9.0004648	10.9995352	17
44	8.9995595	9.9978220	9.0017375	10.9982624	16
45	9.0008159	9.9978093	9.0030066	10.9969933	15
46	9.0020687	9.9977965	9.0042721	10.9957278	14
47	9.0033178	9.9977838	9.0055340	10.9945659	13
48	9.0045633	9.9977710	9.0067923	10.9932076	12
49	9.0058053	9.9977581	9.0080472	10.9919528	11
50	9.0070436	9.9977452	9.0092984	10.9907016	10
51	9.0082684	9.9977323	9.0105461	10.9894539	9
52	9.0095096	9.9977193	9.0117902	10.9882097	8
53	9.0107373	9.9977063	9.0130310	10.9869690	7
54	9.0119615	9.9976933	9.0142682	10.9857317	6
55	9.0131823	9.9976802	9.0155021	10.9844979	5
56	9.0143996	9.9976671	9.0167325	10.9832675	4
57	9.0156134	9.9976540	9.0179594	10.9820405	3
58	9.0168238	9.9976408	9.0191830	10.9808169	2
59	9.0180309	9.9976276	9.0204033	10.9795967	1
60	9.0192345	9.9976143	9.0216202	10.9783797	0
		Sin. 84.		*Tang. 84.*	M

M	Sin. 6.		Tang. 6.		
0	9.0192345	9.9976143	9.0216202	10.9783797	60
1	9.0204348	9.9976010	9.0228338	10.9771662	59
2	9.0216317	9.9975877	9.0240440	10.9759559	58
3	9.0228254	9.9975743	9.0252510	10.9747489	57
4	9.0240157	9.9975609	9.0264548	10.9735452	56
5	9.0252027	9.9975475	9.0270552	10.9723447	55
6	9.0263864	9.9975340	9.0388524	10.9711475	54
7	9.0275669	9.9975204	9.0300464	10.9699535	53
8	9.0287441	9.9975069	9.0312372	10.9687627	52
9	9.0299182	9.9974933	9.0324249	10.9675751	51
10	9.0310890	9.9974797	9.0336094	10.9663906	50
11	9.0322567	9.9974660	9.0347906	10.9652093	49
12	9.0334211	9.9974523	9.0359688	10.9640311	48
13	9.0345824	9.9974386	9.0371439	10.9628561	47
14	9.0357406	9.9974248	9.0383158	10.9616841	46
15	9.0361957	9.9974110	9.0394848	10.9605152	45
16	9.0380477	9.9973971	9.0406506	10.9593493	44
17	9.0391966	9.9973832	9.0418134	10.9581866	43
18	9.0403424	9.9973693	9.0429731	10.9570268	42
19	9.0414852	9.9973553	9.0441298	10.9558701	41
20	9.0426249	9.9973413	9.0452836	10.9547164	40
21	9.0437616	9.9973273	9.0464353	10.9535656	39
22	9.0448954	9.9973132	9.0475821	10.9524178	38
23	9.0460261	9.9972991	9.0487270	10.9512739	37
24	9.0471538	9.9972849	9.0498689	10.9501311	36
25	9.0482786	9.9972707	9.0510078	10.9489921	35
26	9.0494004	9.9972565	9.0521439	10.9478560	34
27	9.0505194	9.9972423	9.0533771	10.9467228	33
28	9.0516354	9.9972279	9.0544075	10.9455925	32
29	9.0527485	9.9972136	9.0555349	10.9444651	31
30	9.0538587	9.9971992	9.0566595	10.9433405	30
		Sin. 83.		Tang. 83.	M

M	Sin. 6.		Tang. 6.		
30	9.0538588	9.9971993	9.0566595	10.9433405	30
31	9.0549661	9.9971849	9.0577813	10.9422187	29
32	9.0560706	9.9971704	9.0589002	10.9410998	28
33	9.0571723	9.9971559	9.0600164	10.9399836	27
34	9.0582711	9.9971414	9.0611297	10.9388703	26
35	9.0593671	9.9971268	9.0622403	10.9377597	25
36	9.0604604	9.9971122	9.0633482	10.9366518	24
37	9.0615509	9.9970976	9.0644533	10.9355467	23
38	9.0626386	9.9970829	9.0655556	10.9344444	22
39	9.0637235	9.9970682	9.0666553	10.9333447	21
40	9.0648057	9.9970535	9.0677522	10.9322478	20
41	9.0658852	9.9970387	9.0688465	10.9311535	19
42	9.0669619	9.9970239	9.0699381	10.9300619	18
43	9.0680360	9.9970090	9.0710270	10.9289730	17
44	9.0691074	9.9969941	9.0721133	10.9278867	16
45	9.0701761	9.9969792	9.0731969	10.9268031	15
46	9.0712421	9.9969642	9.0742779	10.9257221	14
47	9.0723055	9.9969492	9.0753563	10.9246437	13
48	9.0733663	9.9969342	9.0764321	10.9235679	12
49	9.0744244	9.9969191	9.0775053	10.9224947	11
50	9.0754799	9.9969040	9.0785760	10.9214240	10
51	9.0765329	9.9968888	9.0796441	10.9203559	9
52	9.0775832	9.9968736	9.0807096	109.192904	8
53	9.0786310	9.9968584	9.0817726	10.9182274	7
54	9.0796762	9.9968431	9.0828331	10.9171669	6
55	9.0807189	9.9968278	9.0838911	10.9161089	5
56	9.0817590	9.9968125	9.0849466	10.9150534	4
57	9.0827966	9.9967971	9.0859996	10.9140004	3
58	9.0838317	9.9967817	9.0870501	10.9129499	2
59	9.0848643	9.9967662	9.0880981	10.9119019	1
60	9.0858945	9.9967507	9.0891438	10.9108562	0
		Sin. 83.		Tang. 83.	M

M	Sin. 7.		Tang. 7.		
0	9.0858945	9.9967507	9.0891438	10.9108562	60
1	9.0869221	9.9967352	9.0901869	10.9098131	59
2	9.0879473	9.9967196	9.0912277	10.9087723	58
3	9.0889700	9.9967040	9.0922660	10.9077340	57
4	9.0899903	9.9966884	9.0933020	10.9066980	56
5	9.0910082	9.9966727	9.0943355	10.9056645	55
6	9.0920237	9.9966570	9.0953667	10.9046333	54
7	9.0930367	9.9966412	9.0963955	10.9036045	53
8	9.0940474	9.9966254	9.0974219	10.9025781	52
9	9.0950556	9.9966096	9.0984460	10.9015540	51
10	9.0960615	9.9965937	9.0994678	10.9005322	50
11	9.0970651	9.9965778	9.1004872	10.8995828	49
12	9.0980662	9.9965619	9.1015044	10.8984956	48
13	9.0990651	9.9965419	9.1025192	10.8974808	47
14	9.1000616	9.9965299	9.1035317	10.8964683	46
15	9.1010558	9.9965138	9.1045420	10.8954580	45
16	9.1020477	9.9964977	9.1055500	10.8944500	44
17	9.1930373	9.9964816	9.1065557	10.8934443	43
18	9.1040246	9.9964655	9.1075591	10.8924409	42
19	9.1050096	9.9964493	9.1085604	10.8914396	41
20	9.1059924	9.9964330	9.1095594	10.8904406	40
21	9.1069729	9.9964167	9.1105562	10.8894438	39
22	9.1079512	9.9964004	9.1115508	10.8884492	38
23	9.1089272	9.9963841	9.1125431	10.8874569	37
24	9.1099010	9.9963677	9.1135333	10.8864667	36
25	9.1108726	9.9963513	9.1145212	10.8854787	35
26	9.1118420	9.9963348	9.1155072	10.8844928	34
27	9.1128092	9.9963183	9.1164909	10.8835091	33
28	9.1137742	9.9963018	9.1174724	10.8825276	32
29	9.1147370	9.9962852	9.1184518	10.8815482	31
30	9.1156977	9.9962686	9.1194291	10.8805709	30
		Sin. 82.		Tang. 82.	M

M	Sin. 7.		Tang. 7.		
30	9.1156977	9.9962686	9.1194291	10.8805709	30
31	9.1166562	9.9962519	9.1204043	10.8795957	29
32	9.1176125	9.9962352	9.1213773	10.8786227	28
33	9.1185667	9.9962185	9.1223482	10.8776518	27
34	9.1195188	9.9962017	9.1233171	10.8766829	26
35	9.1204688	9.9961849	9.1242839	10.8757161	25
36	9.1214167	9.9961681	9.1252486	10.8747514	24
37	9.1223624	9.9961512	9.1262112	10.8737888	23
38	9.1233061	9.9961343	9.1271718	10.8728282	22
39	9.1242477	9.9961174	9.1281303	10.8718697	21
40	9.1251872	9.9961004	9.1290868	10.8709132	20
41	9.1261246	9.9960834	9.1300413	10.8699587	19
42	9.1270600	9.9960663	9.1309937	10.8690062	18
43	9.1279934	9.9960492	9.1319442	10.8680558	17
44	9.1289247	9.9960321	9.1328926	10.8671074	16
45	9.1298539	9.9960149	9.1338390	10.8661609	15
46	9.1307812	9.9959977	9.1347835	10.8652165	14
47	9.1317064	9.9959804	9.1357260	10.8642740	13
48	9.1326297	9.9959631	9.1366665	10.8633335	12
49	9.1335509	9.9959458	9.1376051	10.8623949	11
50	9.1344702	9.9959284	9.1385417	10.8614583	10
51	9.1353875	9.9959111	9.1394764	10.8605236	9
52	9.1363028	9.9958936	9.1404092	10.8595908	8
53	9.1372161	9.9958761	9.1413400	10.8586600	7
54	9.1381275	9.9958586	9.1422689	10.8577311	6
55	9.1390370	9.9958411	9.1431959	10.8568041	5
56	9.1399445	9.9958235	9.1441210	10.8558790	4
57	9.1408501	9.9958059	9.1450442	10.8549558	3
58	9.1417537	9.9957882	9.1459655	10.8540345	2
59	9.1426555	9.9957705	9.1468850	10.8531150	1
60	9.1435553	9.9957528	9.1478025	10.8521975	0
		Sin. 82.		Tang. 82.	M

M	Sin. 8.		Tang. 8.		
0	9.1435553	9.9957528	9.1478025	10.8521972	60
1	9.1444532	9.9957350	9.1487182	10.8512818	59
2	9.1453493	9.9957172	9.1496321	10.8503679	58
3	9.1462435	9.9956993	9.1505441	10.8494559	57
4	9.1471358	9.9956815	9.1514543	10.8485457	56
5	9.1480262	9.9956635	9.1523627	10.8476373	55
6	9.1489148	9.9956456	9.1532692	10.8467308	54
7	9.1498015	9.9956276	9.1541739	10.8458261	53
8	9.1506864	9.9956095	9.1550769	10.8449231	52
9	9.1515694	9.9955915	9.1559780	10.8440220	51
10	9.1524507	9.9955734	9.1568773	10.8431227	50
11	9.1533301	9.9955552	9.1577748	10.8422253	49
12	9.1542076	9.9955370	9.1586706	10.8413294	48
13	9.1550834	9.9955188	9.1595646	10.8404354	47
14	9.1559574	9.9955005	9.1604569	10.8395431	46
15	9.1568296	9.9954822	9.1613473	10.8386527	45
16	9.1577000	9.9954639	9.1622361	10.8377639	44
17	9.1585686	9.9954455	9.1631231	10.8368769	43
18	9.1594354	9.9954271	9.1640083	10.8359917	42
19	9.1603005	9.9954087	9.1648919	10.8351081	41
20	9.1611639	9.9953902	9.1657737	10.8342263	40
21	9.1620254	9.9953717	9.1666538	10.8333462	39
22	9.1628853	9.9953531	9.1675322	10.8324678	38
23	9.1637434	9.9953345	9.1684989	10.8315911	37
24	9.1645998	9.9953159	9.1692839	10.8307161	36
25	9.1654544	9.9952972	9.1701572	10.8298428	35
26	9.1663074	9.9952785	9.1710289	10.8289711	34
27	9.1671586	9.9952597	9.1718989	10.8281011	33
28	9.1680082	9.9952409	9.1727672	10.8272328	32
29	9.1688559	9.9952221	9.1736338	10.8263662	31
30	9.1697021	9.9952033	9.1744988	10.8255012	30
		Sin. 81.		Tang. 81.	M

M	Sin. 8.		Tang. 8.		
30	9.1697021	9.9952033	9.1744988	10.8255012	30
31	9.1705465	9.9951844	9.1753622	10.8246378	29
32	9.1713893	9.9951654	9.1762239	10.8237761	28
33	9.1722305	9.9951464	9.1770840	10.8229160	27
34	9.1730699	9.9951274	9.1779425	10.8220575	26
35	9.1739077	9.9951084	9.1787993	10.8212007	25
36	9.1747439	9.9950893	9.1796546	10.8203454	24
37	9.1755784	9.9950702	9.1805082	10.8194918	23
38	9.1764112	9.9950510	9.1813602	10.8186394	22
39	9.1772425	9.9950318	9.1822106	10.8177894	21
40	9.1780721	9.9950126	9.1830595	10.8169405	20
41	9.1789001	9.9949933	9.1839068	10.8160932	19
42	9.1797265	9.9949740	9.1847525	10.8152475	18
43	9.1805512	9.9949546	9.1855966	10.8144034	17
44	9.1813744	9.9949352	9.1864392	10.8135608	16
45	9.1821960	9.9949158	9.1872802	10.8127198	15
46	9.1830160	9.9948964	9.1881196	10.8118804	14
47	9.1838344	9.9948769	9.1889575	10.8110425	13
48	9.1846512	9.9948573	9.1897939	10.8102061	12
49	9.1854665	9.9948377	9.1906287	10.8093713	11
50	9.1862802	9.9948181	9.1914621	10.8085379	10
51	9.1870923	9.9947985	9.1922939	10.8077061	9
52	9.1879029	9.9947788	9.1931241	10.8068759	8
53	9.1887120	9.9947591	9.1939529	10.8060471	7
54	9.1895195	9.9947393	9.1947802	10.8052198	6
55	9.1903254	9.9947195	9.1956059	10.8043941	5
56	9.1911299	9.9946997	9.1964302	10.8035698	4
57	9.1919328	9.9946798	9.1972530	10.8027470	3
58	9.1927342	9.9946599	9.1980743	10.8019257	2
59	9.1935341	9.9946399	9.1988942	10.8011059	1
60	9.1943324	9.9946199	9.1997125	10.8002875	0
		Sin. 81.		Tang. 81.	M

M	Sin. 9.		Tang. 9.		
0	9.1943324	9.9946199	9.1997125	10.8002875	60
1	9.1951293	9.9945999	9.2005294	10.7994706	59
2	9.1959247	9.9945798	9.2013449	10.7986551	58
3	9.1967186	9.9945597	9.2021588	10.7978411	57
4	9.1975110	9.9945396	9.2029714	10.7970286	56
5	9.1983019	9.9945194	9.2037825	10.7962175	55
6	9.1990913	9.9944992	9.2045922	10.7954078	54
7	9.1998793	9.9944789	9.2054004	10.7945996	53
8	9.2006658	9.9944587	9.2062072	10.7937928	52
9	9.2014509	9.9944383	9.2070126	10.7929874	51
10	9.2022345	9.9944180	9.2078165	10.7921835	50
11	9.2030167	9.9943975	9.2086191	10.7913809	49
12	9.2037974	9.9943771	9.2094203	10.7905797	48
13	9.2045767	9.9943566	9.2102200	10.7897800	47
14	9.2053545	9.9943361	9.2110184	10.7889816	46
15	9.2061309	9.9943156	9.2118153	10.7881847	45
16	9.2069059	9.9942950	9.2126109	10.7873891	44
17	9.2076795	9.9942773	9.2134051	10.7865949	43
18	9.2084516	9.9942537	9.2141980	10.7858020	42
19	9.2092224	9.9942330	9.2149894	10.7850106	41
20	9.2099917	9.9942122	9.2157795	10.7842205	40
21	9.2107597	9.9941914	9.2165683	10.7834317	39
22	9.2115263	9.9941706	9.2173556	10.7826444	38
23	9.2122914	9.9941498	9.2181417	10.7818583	37
24	9.2130552	9.9941289	9.2189264	10.7810736	36
25	9.2138176	9.9941079	9.2197097	10.7802902	35
26	9.2145787	9.9940870	9.2204917	10.7785083	34
27	9.2153384	9.9940659	9.2212724	10.7787276	33
28	9.2160967	9.9940449	9.2220518	10.7779482	32
29	9.2168536	9.9940238	9.2228298	10.7771702	31
30	9.2176092	9.9940027	9.2236065	10.7763935	30
		Sin. 80.		Tang. 80.	M

M	Sin. 9.		Tang. 9.		
30	9.2176092	9.9940027	9.2236065	10.7763935	30
31	9.2183635	9.9939815	9.2243819	10.7756181	29
32	9.2191164	9.9939603	9.2251561	10.7748439	28
33	9.2198680	9.9939391	9.2259289	10.7740711	27
34	9.2206182	9.9939178	9.2267004	10.7732996	26
35	9.2213671	9.9938965	9.2274706	10.7725294	25
36	9.2221147	9.9938752	9.2282395	10.7717605	24
37	9.2228609	9.9938538	9.2290071	10.7709929	23
38	9.2236059	9.9938324	9.2297735	10.7702265	22
39	9.2243495	9.9938109	9.2305386	10.7694614	21
40	9.2250918	9.9937894	9.2313024	10.7686976	20
41	9.2258328	9.9937676	9.2320650	10.7679350	19
42	9.2265725	9.9937463	9.2328262	10.7671738	18
43	9.2273110	9.9937247	9.2335863	10.7664137	17
44	9.2280481	9.9937030	9.2343451	10.7656549	16
45	9.2287839	9.9936813	9.2351026	10.7648974	15
46	9.2295185	9.9936596	9.2358589	10.7641411	14
47	9.2302518	9.9936378	9.2366139	10.7633861	13
48	9.2309838	9.9936160	9.2373678	10.7626322	12
49	9.2317145	9.9935942	9.2381203	10.7618797	11
50	9.2324440	9.9935723	9.2388717	10.7611283	10
51	9.2331722	9.9935504	9.2396218	10.7603782	9
52	9.2338992	9.9935285	9.2403708	10.7596292	8
53	9.2346249	9.9935065	9.2411185	10.7588815	7
54	9.2353494	9.9934844	9.2418650	10.7581350	6
55	9.2360726	9.9934624	9.2426103	10.7573897	5
56	9.2367946	9.9934403	9.2433543	10.7566437	4
57	9.2375153	9.9934181	9.2440972	10.7559028	3
58	9.2382349	9.9933959	9.2448389	10.7551611	2
59	9.2389532	9.9933737	9.2455794	10.7544206	1
60	9.2396702	9.9933515	9.2463188	10.7536812	0
		Sin. 80.		Tang. 80.	M

M	Sin. 10.		Tang. 10.		
0	9.2396702	9.9933515	9.2463188	10.7536812	60
1	9.2403861	9.9933292	9.2470569	10.7529430	59
2	9.2411007	9.9933068	9.2477939	10.7522061	58
3	9.2418141	9.9932845	9.2485297	10.7514703	57
4	9.2425264	9.9932621	9.2492643	10.7507357	56
5	9.2432374	9.9932396	9.2499978	10.7500022	55
6	9.2439472	9.9932171	9.2507301	10.7492699	54
7	9.2446558	9.9931946	9.2514612	10.7485388	53
8	9.2453632	9.9931720	9.2521912	10.7478088	52
9	9.2460695	9.9931494	9.2529200	10.7470800	51
10	9.2467746	9.9931268	9.2536477	10.7463522	50
11	9.2474784	9.9931041	9.2543742	10.7456257	49
12	9.2481811	9.9930814	9.2550997	10.7449003	48
13	9.2488827	9.9930587	9.2558240	10.7441760	47
14	9.2495830	9.9930359	9.2565471	10.7434528	46
15	9.2502822	9.9930131	9.2572691	10.7427308	45
16	9.2509803	9.9929902	9.2579901	10.7420099	44
17	9.2516772	9.9929673	9.2587099	10.7412901	43
18	9.2523729	9.9929444	9.2594285	10.7405715	42
19	9.2530675	9.9929214	9.2601461	10.7398539	41
20	9.2537609	9.9928984	9.2608625	10.7391375	40
21	9.2544532	9.9928753	9.2615779	10.7384221	39
22	9.2551444	9.9928522	9.2622921	10.7377079	38
23	9.2558344	9.9928291	9.2630053	10.7369947	37
24	9.2565233	9.9928059	9.2637173	10.7362827	36
25	9.2572110	9.9927827	9.2644283	10.7355717	35
26	9.2578977	9.9927595	9.2651382	10.7348618	34
27	9.2585832	9.9927362	9.2658470	10.7341530	33
28	9.2592676	9.9927129	9.2665547	10.7334453	32
29	9.2599509	9.9926895	9.2672613	10.7327387	31
30	9.2606330	9.9926661	9.2679669	10.7320331	30
		Sin. 79.		Tang. 79.	M

M	Sin. 10.		Tang. 10.		
30	9.2606330	9.9926661	9.2679669	10.7320331	30
31	9.2613141	9.9926427	9.2686714	10.7313286	29
32	9.2619941	9.9926192	9.2693749	10.7306251	28
33	9.2626729	9.9925957	9.2700772	10.7299223	27
34	9.2633507	9.9925722	9.2707786	10.7292214	26
35	9.2640274	9.9925486	9.2714788	10.7285212	25
36	9.2647030	9.9925250	9.2721780	10.7278220	24
37	9.2653775	9.9925013	9.2728763	10.7271238	23
38	9.2660509	9.9924776	9.2735733	10.7264267	22
39	9.2667232	9.9924539	9.2742694	10.7257306	21
40	9.2673945	9.9924301	9.2749644	10.7250356	20
41	9.2680647	9.9924063	9.2756584	10.7243416	19
42	9.2687338	9.9923824	9.2763514	10.7236486	18
43	9.2694019	9.9923585	9.2770434	10.7229566	17
44	9.2700689	9.9923346	9.2777343	10.7222657	16
45	9.2707348	9.9923106	9.2784242	10.7215758	15
46	9.2713997	9.9922866	9.2791130	10.7208869	14
47	9.2720635	9.9922626	9.2798009	10.7201991	13
48	9.2727263	9.9922385	9.2804878	10.7195122	12
49	9.2733880	9.9922144	9.2811736	10.7188264	11
50	9.2740487	9.9921902	9.2818585	10.7181415	10
51	9.2747083	9.9921660	9.2825423	10.7174577	9
52	9.2753669	9.9921418	9.2832251	10.7167749	8
53	9.2760245	9.9921175	9.2839000	10.7160930	7
54	9.2766811	9.9920932	9.2845878	10.7154122	6
55	9.2773366	9.9920689	9.2852677	10.7147323	5
56	9.2779911	9.9920445	9.2859466	10.7140534	4
57	9.2786445	9.9920201	9.2866245	10.7133755	3
58	9.2792970	9.9919956	9.2873014	10.7126986	2
59	9.2799484	9.9919711	9.2879773	10.7120227	1
60	9.2805988	9.9919466	9.2886523	10.7113477	0
		Sin. 79.		Tang. 79.	M

M	*Sin.* 11.		*Tang.* 11.		
0	9.2805988	9.9919466	9.2886523	10.7113477	60
1	9.2812483	9.9919220	9.2893263	10.7106737	59
2	9.2818967	9.9918974	9.2899993	10.7100007	58
3	9.2825441	9.9918727	9.2906713	10.7093287	57
4	9.2831905	9.9918480	9.2913424	10.7086576	56
5	9.2838359	9.9918233	9.2920126	10.7079874	55
6	9.2844803	9.9917989	9.2926817	10.7073183	54
7	9.285 237	9.9917737	9.2933500	10.7066500	53
8	9.2857661	9.9917489	9.2940172	10.7059828	52
9	9.2864076	9.9917240	9.2946836	10.7053164	51
10	9.2870480	9.9916991	9.2953489	10.7046511	50
11	9.2876875	9.9916741	9.2960134	10.7039866	49
12	9.2883260	9.9916492	9.2966769	10.7033231	48
13	9.2889636	9.9916241	9.2973395	10.7026605	47
14	9.2896001	9.9915990	9.2980011	10.7019989	46
15	9.2902357	9.9915739	9.2986618	10.7013382	45
16	9.2908704	9.9915488	9.2993216	10.7006784	44
17	9.2915040	9.9915236	9.2999804	10.7000196	43
18	9.2921367	9.9914984	9.3006383	10.6993617	42
19	9.2927685	9.9914731	9.3012954	10.6987046	41
20	9.2933993	9.9914478	9.3019514	10.6980486	40
21	9.2940291	9.9914225	9.3026066	10.6973934	39
22	9.2946580	9.9913971	9.3032609	10.6967391	38
23	9.2952859	9.9913717	9.3039143	10.6960857	37
24	9.2959129	9.9913462	9.3045667	10.6954333	36
25	9.2965390	9.9913207	9.3052183	10.6947817	35
26	9.2971641	9.9912952	9.3058689	10.6941311	34
27	9.2977883	9.9912696	9.3065187	10.6934813	33
28	9.2984116	9.9912440	9.3071675	10.6928325	32
29	9.2990339	9.9912184	9.3078155	10.6921845	31
30	9.2996553	9.9912927	9.3084626	10.6915374	30
		Sin. 78.		*Tang.* 78.	M

M	Sin. 11.		Tang. 11.		
30	9.2996553	9.9911927	9.3084626	10.6915374	30
31	9.3002758	9.9911670	9.3091088	10.6008912	29
32	9.3008953	9.9911412	9.3097541	10.6902459	28
33	9.3015140	9.9911154	9.3103985	10.6896015	27
34	9.3021317	9.9910896	9.3110421	10.6889579	26
35	9.3027485	9.9910637	9.3116848	10.6883152	25
36	9.3033644	9.9910378	9.3123266	10.6876734	24
37	9.3039794	9.9910119	9.3129675	10.6870325	23
38	9.3045934	9.9909859	9.3136076	10.6863924	22
39	9.3052066	9.9909598	9.3142468	10.6857532	21
40	9.3058189	9.9909338	9.3148851	10.6851149	20
41	9.3064303	9.9909077	9.3155226	10.6844774	19
42	9.3070407	9.9908815	9.3161592	10.6838408	18
43	9.3076503	9.9908553	9.3167950	10.6832050	17
44	9.3082590	9.9908291	9.3174299	10.6825701	16
45	9.3088668	9.9908029	9.3180640	10.6819360	15
46	9.3094737	9.9907766	9.3186972	10.6813028	14
47	9.3100798	9.9907502	9.3193295	10.6806705	13
48	9.3106849	9.9907239	9.3199611	10.6800389	12
49	9.3112892	9.9906974	9.3205918	10.6794082	11
50	9.3118926	9.9906710	9.3212216	10.6787784	10
51	9.3124951	9.9906445	9.3218506	10.6781494	9
52	9.3130968	9.9906180	9.3224788	10.6775212	8
53	9.3136976	9.9905914	9.3231061	10.6768939	7
54	9.3142975	9.9905648	9.3237327	10.6762673	6
55	9.3148965	9.9905382	9.3243584	10.6756416	5
56	9.3154947	9.9905115	9.3249832	10.6750168	4
57	9.3160921	9.9904848	9.3256073	10.6743927	3
58	9.3166885	9.9904580	9.3262305	10.6737695	2
59	9.3172841	9.9904312	9.3268529	10.6731471	1
60	9.3178789	9.9904044	9.3274745	10.6725255	0
		Sin. 78.		Tang. 78.	M

M	Sin. 12.		Tang. 12.		
0	9.3178789	9.9904044	9.3274745	10.6725255	60
1	9.3184728	9.9903775	9.3280953	10.6719047	59
2	9.3190659	9.9903506	9.3287153	10.6712847	58
3	9.3196581	9.9903237	9.3293345	10.6706655	57
4	9.3202495	9.9902967	9.3299528	10.6700472	56
5	9.3208400	9.9902697	9.3305704	10.6699426	55
6	9.3214297	9.9902426	9.3311872	10.6688128	54
7	9.3220186	9.9902155	9.3318031	10.6681969	53
8	9.3226066	9.9901883	9.3324183	10.6675817	52
9	9.3231938	9.9901612	9.3330327	10.6669673	51
10	9.3237802	9.9901339	9.3336463	10.6663537	50
11	9.3243657	9.9901067	9.3342591	10.6657409	49
12	9.3249505	9.9900794	9.3348711	10.6651289	48
13	9.3255344	9.9900521	9.3354823	10.6645177	47
14	9.3261174	9.9900247	9.3360927	10.6639073	46
15	9.3266997	9.9899973	9.3367024	10.6632976	45
16	9.3272811	9.9899698	9.3373113	10.6626887	44
17	9.3278617	9.9899423	9.3379194	10.6620806	43
18	9.3284416	9.9899148	9.3385267	10.6614743	42
19	9.3290206	9.9898873	9.3391333	10.6608667	41
20	9.3295988	9.9898597	9.3397391	10.6602609	40
21	9.3301761	9.9898320	9.3403441	10.6596559	39
22	9.3307527	9.9898043	9.3409484	10.6590516	38
23	9.3313285	9.9897766	9.3415519	10.6584481	37
24	9.3319035	9.9897489	9.3421546	10.6578454	36
25	9.3324777	9.9897211	9.3427566	10.6572434	35
26	9.3330511	9.9896932	9.3433578	10.6566422	34
27	9.3336237	9.9896654	9.3439583	10.6560417	33
28	9.3341955	9.9896374	9.3445580	10.6554420	32
29	9.3347665	9.9896095	9.3451570	10.6548430	31
30	9.3353368	9.9895815	9.3457552	10.6542448	30
		Sin. 77.		Tang. 77.	M

M	Sin. 12.		Tang. 12.		
30	9.3353368	9.9895815	9.3457552	10.6542448	30
31	9.3359062	9.9895535	9.3463527	10.6536473	29
32	9.3364749	9.9895254	9.3469494	10.6530506	28
33	9.3370428	9.9894973	9.3475454	10.6524546	27
34	9.3376099	9.9894692	9.3481407	10.6518593	26
35	9.3381762	9.9894410	9.3487352	10.6512648	25
36	9.3387418	9.9894128	9.3493290	10.6506710	24
37	9.3393065	9.9893845	9.3499220	10.6500780	23
38	9.3398706	9.9893562	9.3505143	10.6494847	22
39	9.3404338	9.9893279	9.3511059	10.6488941	21
40	9.3409963	9.9892995	9.3516968	10.6483032	20
41	9.3415580	9.9892711	9.3522869	10.6477131	19
42	9.3421190	9.9892427	9.3528763	10.6471237	18
43	9.3426792	9.9892142	9.3534650	10.6465350	17
44	9.3432386	9.9891856	9.3540530	10.6459470	16
45	9.3437973	9.9891571	9.3546402	10.6453598	15
46	9.3443552	9.9891285	9.3552267	10.6447733	14
47	9.3449124	9.9890998	9.3558126	10.6441874	13
48	9.3454688	9.9890711	9.3563977	10.6436023	12
49	9.3460245	9.9890424	9.3569821	10.6430179	11
50	9.3465794	9.9890137	9.3575658	10.6424342	10
51	9.3471336	9.9889849	9.3581487	10.6418513	9
52	9.3476870	9:9889560	9.3587310	10.6412690	8
53	9.3482397	9.9889271	9.3593126	10.6406874	7
54	9.3487917	9.9888982	9.3598935	10.6401065	6
55	9.3493429	9.9888693	9.3604736	10.6395264	5
56	9.3498934	9.9888403	9.3610531	10.6389469	4
57	9.3504432	9.9888113	9.3616319	10.6383681	3
58	9.3509922	9.9887822	9.3622100	10.6377900	2
59	9.3515405	9.9887531	9.3627874	10.6372126	1
60	9.3520880	9.9887239	9.3633641	10.6366359	0
		Sin. 77.		Tang. 77.	M

M	Sin. 13.		Tang. 13.		
0	9.3520880	9.9887239	9.3633641	10.6366359	60
1	9.3526349	9.9886947	9.3639401	10.6360599	59
2	9.3531810	9.9886655	9.3645155	10.6354845	58
3	9.3537264	9.9886363	9.3650901	10.6349099	57
4	9.3542710	9.9886070	9.3656641	10.6343359	56
5	9.3548150	9.9885776	9.4662374	10.6337626	55
6	9.3553582	9.9885482	9.3668100	10.6331900	54
7	9.3559008	9.9885188	9.3673819	10.6326181	53
8	9.3564426	9.9884894	9.3679532	10.6320460	52
9	9.3569836	9.9884599	9.3685238	10.6314762	51
10	9.3575240	9.9884303	9.3690937	10.6309063	50
11	9.3580637	9.9884008	9.3696629	10.6303371	49
12	9.3586027	9.9883712	9.3702315	10.6297685	48
13	9.3591409	9.9883415	9.3707994	10.6292006	47
14	9.3596785	9.9883118	9.3713667	10.6286333	46
15	9.3602154	9.9882821	9.3719333	10.6280667	45
16	9.3607515	9.9882523	9.3724992	10.6275008	44
17	9.3612870	9.9882225	9.3730645	10.6269355	43
18	9.3618217	9.9881927	9.3736291	10.6263709	42
19	9.3623558	9.9881628	9.3741930	10.6258070	41
20	9.3628892	9.9881329	9.3747563	10.6252437	40
21	9.3634219	9.9881029	9.3753190	10.6246810	39
22	9.3639539	9.9880729	9.3758810	10.6241190	38
23	9.3644852	9.9880429	9.3764423	10.6235577	37
24	9.3650158	9.9880128	9.3770030	10.6229970	36
25	9.3655458	9.9879827	9.3775631	10.6224369	35
26	9.3660750	9.9879525	9.3781225	10.6218775	34
27	9.3666036	9.9879223	9.1786813	10.6213187	33
28	9.3671315	9.9878921	9.3792394	10.6207606	32
29	9.3676587	9.9878618	9.3797969	10.6202031	31
30	9.3681853	9.9878315	9.3803537	10.6196463	30
		Sin. 76.		Tang. 76.	M

M	Sin. 13.		Tang. 13.		
30	9.3681853	9.9878315	9.3803537	10.6196463	30
31	9.3687111	9.9878012	9.3809100	10.6190900	29
32	9.3692363	9.9877708	9.3814655	10.6185345	28
33	9.3697608	9.9877404	9.3820205	10.6179795	27
34	9.3702847	9.9877099	9.3825748	10.6174252	26
35	9.3708079	9.9876794	9.3831285	10.6168715	25
36	9.3713304	9.9876488	9.3836816	10.6163184	24
37	9.3718523	9.9876183	9.3842340	10.6157660	23
38	9.2723735	9.9875876	9.3847858	10.6152142	22
39	9.3728940	9.9875570	9.3853370	10.6146630	21
40	9.3734139	9.9875263	9.3858876	10.6141124	20
41	9.3739331	9.9874955	9.3864376	10.6135624	19
42	9.3744517	9.9874648	9.3869869	10.6131031	18
43	9.3749696	9.9874339	9.3875356	10.6124644	17
44	9.3754868	9.9874031	9.3880837	10.6119163	16
45	9.3760034	9.9873722	9.3886312	10.6113688	15
46	9.3765194	9.9873413	9.3891781	10.6108219	14
47	9.3770347	9.9873103	9.3897244	10.6102756	13
48	9.3775493	9.9872793	9.3902700	10.6097300	12
49	9.3780633	9.9872482	9.3908151	10.6091849	11
50	9.3785767	9.9872171	9.3913595	10.6086405	10
51	9.3790894	9.9871860	9.3919034	10.6080966	9
52	9.3796015	9.9871549	9.3924466	10.6075534	8
53	9.3801129	9.9871236	9.3929893	10.6070107	7
54	9.3806237	9.9870924	9.3935513	10.6064487	6
55	9.3811339	9.9870611	9.3940727	10.6059273	5
56	9.3816434	9.9870298	9.3946136	10.6053864	4
57	9.3821523	9.9869984	9.3951536	10.6048462	3
58	9.3826605	9.9869670	9.3956935	10.6043065	2
59	9.3831682	9.9869356	9.3962326	10.6037674	1
60	9.3836752	9.9869041	9.3967711	10.6032289	0
		Sin. 76.		Tang. 76.	M

M	Sin. 14.		Tang. 14.		
0	9.3836752	9.9869041	9.3967711	10.6032289	60
1	9.3841815	9.9868726	9.3973089	10.6026911	59
2	9.3846873	9.9868410	9.3978463	10.6021537	58
3	9.3851924	9.9868094	9.3983830	10.6016170	57
4	9.3856969	9.9867778	9.3989191	10.6010809	56
5	9.3862008	9.9867461	9.3994547	10.6005453	55
6	9.3867040	9.9867144	9.3999896	10.6000104	54
7	9.3872067	9.9866827	9.4005240	10.5994760	53
8	9.3877087	9.9866509	9.4010578	10.5989422	52
9	9.3882101	9.9866191	9.4015910	10.5984090	51
10	9.3887109	9.9865872	9.4021237	10.5978763	50
11	9.3892111	9.9865553	9.4026558	10.5973442	49
12	9.3897106	9.9865233	9.4031873	10.5968127	48
13	9.3902096	9.9864913	9.4037182	10.5962818	47
14	9.3907079	9.9864593	9.4042486	10.5957514	46
15	9.3912057	9.9864273	9.4047784	10.5952216	45
16	9.3917028	9.9863952	9.4053076	10.5946924	44
17	9.3921993	9.9863630	9.4058363	10.5941637	43
18	9.3926952	9.9863308	9.4063644	10.5936356	42
19	9.3931905	9.9862986	9.4068919	10.5931081	41
20	9.3936852	9.9862663	9.4074189	10.5925811	40
21	9.3941794	9.9862340	9.4079453	10.5920547	39
22	9.3946729	9.9862017	9.4084712	10.5915288	38
23	9.3951658	9.9861693	9.4089965	10.5910035	37
24	9.3956581	9.9861369	9.4095212	10.5904788	36
25	9.3961499	9.9861045	9.4100454	10.5899546	35
26	9.3966410	9.9860720	9.4105690	10.5894310	34
27	9.3971315	9.9860394	9.4110921	10.5889079	33
28	9.3976215	9.9860069	9.4116146	10.5883854	32
29	9.3981109	9.9859742	9.4121366	10.5878634	31
30	9.3985996	9.9859416	9.4126581	10.5873419	30
		Sin. 75.		Tang. 75.	M

M	Sin. 14.		Tang. 14.		
30	9.3985996	9.9859416	9.4126581	10.5873419	30
31	9.3990878	9.9859089	9.4131789	10.5868211	29
32	9.3995754	9.9858762	9.4136993	10.5863007	28
33	9.4000625	9.9858434	9.4142191	10.5857809	27
34	9.4005489	9.9858106	9.4147383	10.5852617	26
35	9.4010348	9.9857777	9.4152570	10.5847430	25
36	9.4015201	9.9857449	9.4157752	10.5842248	24
37	9.4020048	9.9857119	9.4162928	10.5837072	23
38	9.4024889	9.9856790	9.4168099	10.5831901	22
39	9.4029734	9.9856460	9.4173265	10.5826735	21
40	9.4034554	9.9856129	9.4178425	10.5821575	20
41	9.4039378	9.9855798	9.4183580	10.5816420	19
42	9.4044196	9.9855467	9.4188729	10.5811271	18
43	9.4049009	9.9855135	9.4193874	10.5806126	17
44	9.4053816	9.9854803	9.4199013	10.5800987	16
45	9.4058617	9.9854471	9.4204146	10.5795854	15
46	9.4063413	9.9854138	9.4209275	10.5790725	14
47	9.4068203	9.9853805	9.4214398	10.5785602	13
48	9.4072987	9.9853471	9.4219515	10.5780485	12
49	9.4077766	9.9853138	9.4224628	10.5775372	11
50	9.4082539	9.9852803	9.4229735	10.5770265	10
51	9.4087306	9.9852468	9.4234838	10.5765162	9
52	9.4092068	9.9852133	9.4239935	10.5760065	8
53	9.4096824	9.9851798	9.4245026	10.5754974	7
54	9.4101575	9.9851462	9.4250113	10.5749887	6
55	9.4106320	9.9851125	9.4255194	10.5744806	5
56	9.4111059	9.9850789	9.4260271	10.5739729	4
57	9.4115793	9.9850452	9.4265342	10.5734658	3
58	9.4120522	9.9850114	9.4270408	10.5729592	2
59	9.4125245	9.9859776	9.4275469	10.5724532	1
60	9.4129962	9.9859438	9.4280525	10.5719475	0
		Sin. 75.		Tang. 75.	M

M	Sin. 15.		Tang. 15.		
0	9.4129962	9.9849438	9.4280525	10.5719475	60
1	9.4134674	9.9849099	9.4285575	10.5714425	59
2	9.4139381	9.9848760	9.4290621	10.5709379	58
3	9.4144082	9.9848420	9.4295661	10.5704339	57
4	9.4148778	9.9848081	9.4300697	10.5699303	56
5	9.4153468	9.9847740	9.4305727	10.5694273	55
6	9.4158152	9.9847400	9.4310753	10.5689247	54
7	9.4162832	9.9847059	9.4315773	10.5684227	53
8	9.4167506	9.9846717	9.4320789	10.5679211	52
9	9.4172174	9.9846375	9.4325799	10.5674201	51
10	9.4176837	9.9846033	9.4330804	10.5669196	50
11	9.4181495	9.9845690	9.4335805	10.5664195	49
12	9.4186148	9.9845347	9.4340800	10.5659200	48
13	9.4190795	9.9845004	9.4345791	10.5657209	47
14	9.4195436	9.9844660	9.4350776	10.5649224	46
15	9.4200073	9.9844316	9.4355757	10.5644243	45
16	9.4204704	9.9843971	9.4360733	10.5639267	44
17	9.4209330	9.9843626	9.4365704	10.5634296	43
18	9.4213950	9.9843281	9.4370670	10.5629330	42
19	9.4218566	9.9842935	9.4375631	10.5624369	41
20	9.4223176	9.9842589	9.4380587	10.5619413	40
21	9.4227780	9.9842242	9.4385538	10.5614462	39
22	9.4232380	9.9841895	9.4390485	10.5609515	38
23	9.4236974	9.9841548	9.4395426	10.5604574	37
24	9.4241563	9.9841200	9.4400363	10.5599637	36
25	9.4246147	9.9840852	9.4405295	10.5594705	35
26	9.4250726	9.9840503	9.4410222	10.5589778	34
27	9.4255299	9.9840154	9.4415145	10.5584855	33
28	9.4259867	9.9839805	9.4420062	10.5579938	32
29	9.4264430	9.9839455	9.4429975	10.5575025	31
30	9.4268988	9.9839105	9.4429883	10.5570117	30
		Sin. 74.		Tang. 74.	M

M	Sin. 15.		Tang. 15.		
30	9.4268988	9.9839105	9.4429883	10.5570117	30
31	9.4273541	9.9838755	9.4434786	10.5565214	29
32	9.4278089	9.9838404	9.4439685	10.5560315	28
33	9.4282631	9.9838052	9.4444579	10.5555421	27
34	9.4287169	9.9837701	9.4449468	10.5550532	26
35	9.4291701	9.9837348	9.4454352	10.5545648	25
36	9.4296228	9.9836996	9.4459232	10.5540768	24
37	9.4300750	9.9836643	9.4464107	10.5535893	23
38	9.4305267	9.9836290	9.4468978	10.5531022	22
39	9.4309779	9.9835936	9.4473843	10.5526157	21
40	9.4314286	9.9835582	9.4478704	10.5521262	20
41	9.4318788	9.9835227	9.4483561	10.5516439	19
42	9.4323285	9.9834872	9.4488413	10.5511587	18
43	9.4327777	9.9834517	9.4493260	10.5506740	17
44	9.4332264	9.9834161	9.4498102	10.5501898	16
45	9.4336746	9.9833805	9.4502940	10.5497060	15
46	9.4341223	9.9833449	9.4507774	10.5492226	14
47	9.4345694	9.9833092	9.4512602	10.5487398	13
48	9.4350161	9.9832735	9.4517427	10.5482573	12
49	9.4354623	9.9832377	9.4522246	10.5477754	11
50	9.4359080	9.9832019	9.4527061	10.5472939	10
51	9.4363532	9.9831661	9.4531872	10.5468128	9
52	9.4367980	9.9831302	9.4536678	10.5463322	8
53	9.4372422	9.9830942	9.4541479	10.5458521	7
54	9.4376859	9.9830583	9.4546276	10.5453724	6
55	9.4381292	9.9830223	9.4551069	10.5448931	5
56	9.4385719	9.9829862	9.4555857	10.5444143	4
57	9.4390142	9.9829501	9.4560641	10.5439359	3
58	9.4394560	9.9829140	9.4565420	10.5434580	2
59	9.4398973	9.9828778	9.4570194	10.5429806	1
60	9.4403381	9.9828416	9.4574964	10.5425036	0
		Sin. 74.		Tang. 74.	M

M	*Sin.* 16.		*Tang.* 16.		
0	9.4403381	9.9828416	9.4574964	10.5425036	60
1	9.4407784	9.9828054	9.4579730	10.5420270	59
2	9.4412182	9.9827691	9.4584491	10.5415509	58
3	9.4416576	9.9827328	9.4589248	10.5410752	57
4	9.4420965	9.9826964	9.4594001	10.5405999	56
5	9.4425349	9.9826600	9.4598749	10.5401251	55
6	9.4429728	9.9826236	9.4603492	10.5396508	54
7	9.4434103	9.9825871	9.4608232	10.5391768	53
8	9.4438472	9.9825506	9.4612967	10.5387033	52
9	9.4442837	9.9825140	9.4617697	10.5382303	51
10	9.4447197	9.9824774	9.4622423	10.5377577	50
11	9.4451553	9.9824408	9.4627145	10.5372855	49
12	9.4455904	9.9824041	9.4631863	10.5368137	48
13	9.4460250	9.9823674	9.4636576	10.5363424	47
14	9.4464591	9.9823306	9.4641285	10.5358715	46
15	9.4468927	9.9822938	9.4645990	10.5354010	45
16	9.4473259	9.9822569	9.4650690	10.5349310	44
17	9.4477586	9.9822201	9.4655386	10.5344614	43
18	9.4481909	9.9821831	9.4660078	10.5339922	42
19	9.4486227	9.9821462	9.4664765	10.5335235	41
20	9.4490540	9.9821092	9.4669448	10.5330552	40
21	9.4494849	9.9820721	9.4674127	10.5325873	39
22	9.4499153	9.9820351	9.4678802	10.5321198	38
23	9.4503452	9.9819979	9.4683473	10.5316527	37
24	9.4507747	9.9819608	9.4688139	10.5311861	36
25	9.4512037	9.9819236	9.4692801	10.5307199	35
26	9.4516322	9.9818863	9.4697459	10.5302541	34
27	9.4520603	9.9818490	9.4702112	10.5297888	33
28	9.4524879	9.9818117	9.4706762	10.5293238	32
29	9.4529151	9.9817744	9.4711407	10.5288593	31
30	9.4533418	9.9817370	9.4716048	10.5283952	30
		Sin. 73.		*Tang.* 73.	M

M	Sin. 16.		Tang. 16.		
30	9.4533418	9.9817370	9.4716048	10.5283952	30
31	9.4537681	9.9816995	9.4720685	10.5279315	29
32	9.4541939	9.9816620	9.4725318	10.5274683	28
33	9.4546192	9.9816245	9.4729947	10.5270053	27
34	9.4550441	9.9815870	9.4734571	10.5265428	26
35	9.4554686	9.9815494	9.4739192	10.5260808	25
36	9.4558926	9.9815117	9.4743808	10.5256192	24
37	9.4563161	9.9814740	9.4748421	10.5251579	23
38	9.4567392	9.9814363	9.4753029	10.5246971	22
39	9.4571618	9.9813986	9.4757633	10.5242367	21
40	9.4575840	9.9813608	9.4762233	10.5237767	20
41	9.4580058	9.9813229	9.4766829	10.5233171	19
42	9.4584271	9.9812850	9.4771421	10.5228579	18
43	9.4588480	9.9812471	9.4776009	10.5223991	17
44	9.4592684	9.9812091	9.4780592	10.5219408	16
45	9.4596884	9.9811711	9.4785172	10.5214828	15
46	9.4601079	9.9811331	9.4789748	10.5210251	14
47	9.4605270	9.9810950	9.4794319	10.5205681	13
48	9.4609456	9.9810569	9.4798887	10.5201113	12
49	9.4613638	9.9810187	9.4803451	10.5196549	11
50	9.4617816	9.9809805	9.4808011	10.5191989	10
51	9.4621989	9.9809423	9.4812566	10.5187434	9
52	9.4626158	9.9809040	9.4817118	10.5182882	8
53	9.4630323	9.9808657	9.4821666	10.5178334	7
54	9.4634483	9.9808273	9.4826210	10.5173790	6
55	9.4638639	9.9807889	9.4830750	10.5169250	5
56	9.4642790	9.9807505	9.4835286	10.5164714	4
57	9.4646938	9.9807120	9.4839818	10.5160182	3
58	9.4651081	9.9806735	9.4844346	10.5155654	2
59	9.4655219	9.9806349	9.4848870	10.5151130	1
60	9.4659353	9.9805963	9.4853390	10.5146610	0
		Sin. 73.		Tang. 73.	M

M	Sin. 17.		Tang. 17.		
0	9.4659353	9.9805963	9.4853390	10.5146610	60
1	9.4663483	9.9805577	9.4857907	10.5142093	59
2	9.4667609	9.9805190	9.4862419	10.5137581	58
3	9.4671730	9.9804803	9.4866928	10.5133072	57
4	9.4675848	9.9804415	9.4871433	10.5128567	56
5	9.4679960	9.9804027	9.4875933	10.5124067	55
6	9.4684069	9.9803639	9.4880430	10.5119570	54
7	9.4688173	9.9803250	9.4884924	10.5115076	53
8	9.4692273	9.9802860	9.4889413	10.5110587	52
9	9.4696369	9.9802471	9.4893898	10.5106102	51
10	9.4700461	9.9802081	9.4898380	10.5101620	50
11	9.4704548	9.9801690	9.4902858	10.5097142	49
12	9.4708631	9.9801299	9.4907332	10.5092668	48
13	9.4712710	9.9800908	9.4911802	10.5088198	47
14	9.4716785	9.9800516	9.4916269	10.5083731	46
15	9.4720856	9.9800124	9.4920731	10.5079269	45
16	9.4724922	9.9799732	9.4925190	10.5074810	44
17	9.4728985	9.9799339	9.4929646	10.5070354	43
18	9.4733043	9.9798946	9.4934097	10.5065903	42
19	9.4737097	9.9798552	9.4938545	10.5061455	41
20	9.4741146	9.9798158	9.4942988	10.5057012	40
21	9.4745192	9.9797764	9.4947429	10.5052571	39
22	9.4749234	9.9797369	9.4951865	10.5048135	38
23	9.4753271	9.9796973	9.4956298	10.5043702	37
24	9.4757304	9.9796578	9.4960727	10.5039273	36
25	9.4761334	9.9796182	9.4965152	10.5034848	35
26	9.4765359	9.9795785	9.4969574	10.5030426	34
27	9.4769380	9.9795388	9.4973991	10.5026009	33
28	9.4773396	9.9794991	9.4978406	10.5021594	32
29	9.4777409	9.9794593	9.4982816	10.5017184	31
30	9.4781418	9.9794195	9.4987223	10.5012777	30
		Sin. 72.		Tang. 72.	M

M	Sin. 17.		Tang. 17.		
30	9.4781418	9.9794195	9.4987223	10.5012777	30
31	9.4785423	9.9793796	9.4991626	10.5008374	29
32	9.4789423	9.9793398	9.4996026	10.5003974	28
33	9.4793420	9.9792998	9.5000422	10.4999578	27
34	9.4797412	9.9792599	9.5004814	10.4995186	26
35	9.4801401	9.9792198	9.5009203	10.4990797	25
36	9.4805385	9.9791798	9.5013588	10.4986413	24
37	9.4809366	9.9791397	9.5017969	10.4982031	23
38	9.4813342	9.9790996	9.5022347	10.4977653	22
39	9.4817315	9.9790594	9.5026721	10.4973279	21
40	9.4821283	9.9790192	9.5031092	10.4968908	20
41	9.4825248	9.9789789	9.5035459	10.4964541	19
42	9.4829208	9.9789386	9.5039822	10.4960178	18
43	9.4833165	9.9788983	9.5044182	10.4955818	17
44	9.4837117	9.9788579	9.5048538	10.4951467	16
45	9.4841066	9.9788175	9.5052891	10.4947109	15
46	9.4845010	9.9787770	9.5057240	10.4942760	14
47	9.4848951	9.9787365	9.5061586	10.4938414	13
48	9.4852888	9.9786960	9.5065928	10.4934072	12
49	9.4856820	9.9786554	9.5070267	10.4929733	11
50	9.4860749	9.9786148	9.5074602	10.4925398	10
51	9.4864674	9.9785741	9.5078933	10.4921067	9
52	9.4868595	9.9785334	9.5083261	10.4916739	8
53	9.4872512	9.9784927	9.5087586	10.4912414	7
54	9.4876426	9.9784519	9.5091907	10.4908093	6
55	9.4880335	9.9784111	9.5096224	10.4903776	5
56	9.4884240	9.9783702	9.5100539	10.4899461	4
57	9.4888142	9.9783293	9.5104849	10.4895151	3
58	9.4892040	9.9782883	9.5109156	10.4890844	2
59	9.4895934	9.9782474	9.5113460	10.4886540	1
60	9.4899824	9.9782063	9.5117760	10.4882240	0
		Sin. 72.		Tang. 72.	M

M	Sin. 18.		Tang. 18.		
0	9.4899824	9.9782063	9.5117760	10.4882240	60
1	9.4903710	9.9781653	9.5122057	10.4877943	59
2	9.4907592	9.9781241	9.5126351	10.4873649	58
3	9.4911471	9.9780830	9.5130641	10.4869359	57
4	9.4915345	9.9780418	9.5134927	10.4865073	56
5	9.4919216	9.9780006	9.5139210	10.4860790	55
6	9.4923083	9.9779593	9.5143490	10.4856510	54
7	9.4926646	9.9779180	9.5147766	10.4852234	53
8	9.4930806	9.9778767	9.5152039	10.4847961	52
9	9.4934661	9.9778353	9.5156309	10.4843691	51
10	9.4938513	9.9777938	9.5160575	10.4849425	50
11	9.4942361	9.9777523	9.5164838	10.4835162	49
12	9.4946205	9.9777108	9.5169097	10.4830903	48
13	9.4950046	9.9776693	9.5173353	10.4826647	47
14	9.4953883	9.9776277	9.5177606	10.4822394	46
15	9.4957716	9.9775860	9.5181855	10.4818145	45
16	9.4961545	9.9775444	9.5186101	10.4813899	44
17	9.4965370	9.9775026	9.5190344	10.4809656	43
18	9.4969192	9.9774609	9.5194583	10.4803417	42
19	9.4973010	9.9774191	9.5198819	10.4801181	41
20	9.4976824	9.9773772	9.5103052	10.4796948	40
21	9.4980635	9.9773354	9.5107282	10.4792718	39
22	9.4984442	9.9772934	9.5211508	10.4788492	38
23	9.4988245	9.9772515	9.5215730	10.4784270	37
24	9.4992045	9.9772095	9.5219950	10.4780050	36
25	9.4995840	9.9771674	9.5224166	10.4775834	35
26	9.4999633	9.9771253	9.5228379	10.4771621	34
27	9.5003421	9.9770832	9.5232589	10.4767411	33
28	9.5007206	9.9770410	9.5236795	10.4763205	32
29	9.5010987	9.9769988	9.5240999	10.4759001	31
30	9.5014764	9.9769566	9.5245199	10.4754801	30
		Sin. 71.		Tang. 71.	M

M	Sin. 18.		Tang. 18.		
30	9.5014764	9.9769566	9.5245109	10.4754801	30
31	9.5018538	9.9769143	9.5249395	10.4750005	29
32	9.5022308	9.9768720	9.5253589	10.4746411	28
33	9.5026075	9.9768296	9.5257779	10.4742221	27
34	9.5029838	9.9767872	9.5261966	10.4738034	26
35	9.5033597	9.9767447	9.5266150	10.4733850	25
36	9.5037353	9.9767022	9.5270331	10.4729669	24
37	9.5041105	9.9766597	9.5274508	10.4725492	23
38	9.5044853	9.9766171	9.5278682	10.4721318	22
39	9.5048598	9.9765745	9.5282853	10.4717147	21
40	9.5052339	9.9765318	9.5287021	10.4712979	20
41	9.5056077	9.9764891	9.5291186	10.4708814	19
42	9.5059811	9.9764464	9.5295347	10.4704653	18
43	9.5063542	9.9764036	9.5299505	10.4700495	17
44	9.5067269	9.9763608	9.5303661	10.4696339	16
45	9.5070992	9.9763179	9.5307813	10.4692187	15
46	9.5074712	9.9762750	9.5311961	10.4688039	14
47	9.5078428	9.9762321	9.5316107	10.4683893	13
48	9.5082141	9.9761891	9.5320250	10.4679750	12
49	9.5085850	9.9761461	9.5324389	10.4675611	11
50	9.5089556	9.9761030	9.5328526	10.4671474	10
51	9.5093258	9.9760599	9.5332659	10.4667341	9
52	9.5096956	9.9760167	9.5336789	10.4663211	8
53	9.5100651	9.9759736	9.5340916	10.4659084	7
54	9.5104343	9.9759303	9.5345040	10.4654960	6
55	9.5108031	9.9758870	9.5349161	10.4650839	5
56	9.5111716	9.9758437	9.5353278	10.4646722	4
57	9.5115397	9.9758004	9.5357393	10.4642607	3
58	9.5119074	9.9757570	9.5361505	10.4638495	2
59	9.5122749	9.9757135	9.5365613	10.4634387	1
60	9.5126419	9.9756701	9.5369719	10.4630281	0
		Sin. 71.		Tang. 71.	M

M	Sin. 19.		Tang. 19.		
0	9.5126419	9.9756701	9.5369719	10.4630281	60
1	9.5130086	9.9756265	9.5373821	10.4626179	59
2	9.5133750	9.9755830	9.5377920	10.4622080	58
3	9.5137410	9.9755394	9.5382017	10.4617983	57
4	9.5141067	9.9754957	9.5386110	10.4613890	56
5	9.5144721	9.9754521	9.9390200	10.4609800	55
6	9.5148371	9.9754083	9.5394287	10.4605713	54
7	9.5152017	9.9753646	9.5398371	10.4601629	53
8	9.5155660	9.9753208	9.5402453	10.4597547	52
9	9.5159300	9.9752769	9.5406531	10.4593469	51
10	9.5162036	9.9752330	9.5410605	10.4589394	50
11	9.5166569	9.9751891	9.5414678	10.4585322	49
12	9.5170198	9.9751451	9.5418747	10.4581253	48
13	9.5173824	9.9751011	9.5422813	10.4577187	47
14	9.5177447	9.9750570	9.5426877	10.4573123	46
15	9.5181066	9.9750129	9.5430937	10.4569063	45
16	9.5184682	9.9749688	9.5434994	10.4565005	44
17	9.5188295	9.9749246	9.5439048	10.4560952	43
18	9.5191904	9.9748804	9.5443100	10.4556900	42
19	9.5195510	9.9748361	9.5447148	10.4552852	41
20	9.5199112	9.9747918	9.5451193	10.4548807	40
21	9.5202711	9.9747475	9.5455236	10.4544764	39
22	9.5206307	9.9747031	9.5459276	10.4540724	38
23	9.5209899	9.9746587	9.5463312	10.4536688	37
24	9.5213488	9.9746142	9.5467346	10.4532654	36
25	9.5217074	9.9745697	9.5471377	10.4528623	35
26	9.5220656	9.9745252	9.5475405	10.4524595	34
27	9.5224235	9.9744806	9.5479430	10.4520570	33
28	9.5227811	9.9744359	9.5483452	10.4516548	32
29	9.5231383	9.9743913	9.5487471	10.4512529	31
30	9.5234953	9.9743466	9.5491487	10.4508513	30
		Sin. 70.		Tang. 70.	M

M	Sin. 19.		Tang. 19.		
30	9.5234953	9.9743466	9.5491487	10.4508513	30
31	9.5238518	9.9743018	9.5495500	10.4504500	29
32	9.5242081	9.9742570	9.5499511	10.4500489	28
33	9.5245640	9.9742122	9.5503519	10.4496481	27
34	9.5249196	9.9741673	9.5507523	10.4492477	26
35	9.5252749	9.9741224	9.5511525	10.4488475	25
36	9.5256298	9.9740774	9.5515524	10.4484476	24
37	9.5259844	9.9740324	9.5519521	10.4480479	23
38	9.5263387	9.9739873	9.5523514	10.4476486	22
39	9.5266927	9.9739422	9.5527504	10.4472496	21
40	9.5270463	9.9738971	9.5531492	10.4468508	20
41	9.5273997	9.9738519	9.5535477	10.4464523	19
42	9.5277526	9.9738067	9.5539459	10.4460541	18
43	9.5281053	9.9737615	9.5543438	10.4456562	17
44	9.5284577	9.9737162	9.5547415	10.4452585	16
45	9.5288097	9.9736700	9.5551388	10.4448612	15
46	9.5291614	9.9736255	9.5555359	10.4444641	14
47	9.5295128	9.9735801	9.5559327	10.4440673	13
48	9.5298638	9.9735346	9.5563292	10.4436708	12
49	9.5302146	9.9734891	9.5567255	10.4432745	11
50	9.5305650	9.9734435	9.5571214	10.4428786	10
51	9.5309151	9.9733980	9.5575171	10.4424829	9
52	9.5312649	9.9733523	9.5579125	10.4420875	8
53	9.5316143	9.9733067	9.5583077	10.4416923	7
54	9.5319635	9.9732610	9.5587025	10.4412975	6
55	9.5323123	9.9732152	9.5590971	10.4409029	5
56	9.5326608	9.9731694	9.5594914	10.4405086	4
57	9.5330090	9.9731236	9.5598854	10.4401146	3
58	9.5333569	9.9730777	9.5602792	10.4397207	2
59	9.5337044	9.9730318	9.5606727	10.4393273	1
60	9.5340517	9.9729858	9.5610659	10.4389341	0
		Sin. 70.		Tang. 70.	M

M	*Sin.* 20.		*Tang.* 20.		
0	9.5340517	9.9729858	9.5610656	10.4389341	60
1	9.5343986	9.9729398	9.5614588	10.4385412	59
2	9.5347452	9.9728938	9.5618515	10.4381485	58
3	9.5350915	9.9728477	9.5622439	10.4377561	57
4	9.5354375	9.9728016	9.5626360	10.4373640	56
5	9.5357832	9.9727554	9.9630278	10.4369722	55
6	9.5361286	9.9727092	9.5634194	10.4365806	54
7	9.5364737	9.9726629	9.5638107	10.4361893	53
8	9.5368184	9.9726166	9.5642018	10.4357982	52
9	9.5371628	9.9725703	9.5645925	10.4354075	51
10	9.5375069	9.9725239	9.5649831	10.4350169	50
11	9.5378508	9.9724775	9.5653733	10.4346267	49
12	9.5381943	9.9724310	9.5657633	10.4342367	48
13	9.5385375	9.9723845	9.5661530	10.4338470	47
14	9.5388804	9.9723380	9.5665424	10.4334576	46
15	9.5392230	9.9722914	9.5669316	10.4330684	45
16	9.5395653	9.9722448	9.5673205	10.4326795	44
17	9.5399073	9.9721981	9.5677091	10.4322909	43
18	9.5402489	9.9721514	9.5680975	10.4319025	42
19	9.5405903	9.9721047	9.5684856	10.4315144	41
20	9.5409314	9.9720579	9.5688735	10.4311265	40
21	9.5412721	9.9720110	9.5692611	10.4307389	39
22	9.5416126	9.9719642	9.5696484	10.4303516	38
23	9.5419527	9.9719172	9.5700355	10.4299645	37
24	5422926	9.9718703	9.5704223	10.4295777	36
25	9.5426321	9.9718233	9.5708088	10.4291912	35
26	9.5429713	9.9717762	9.5711951	10.4288049	34
27	9.5433103	9.9717291	9.5715811	10.4284189	33
28	9.5436489	9.9716820	9.5719669	10.4280331	32
29	9.5439873	9.9716348	9.5723524	10.4276476	31
30	9.5443253	9.9715876	9.5727377	10.4272623	30
		Sin. 69.		*Tang.* 69.	M

M	Sin. 20.		Tang. 20.		
30	9.5443253	9.9715876	9.5727377	10.4272623	30
31	9.5446630	9.9715404	9.5731227	10.4268773	29
32	9.5450005	9.9714931	9.5735074	10.4264926	28
33	9.5453376	9.9714457	9.5738919	10.4261081	27
34	9.5456745	9.9713984	9.5742761	10.4257239	26
35	9.5460110	9.9713509	9.5746601	10.4253399	25
36	9.5463472	9.9713035	9.5750438	10.4249562	24
37	9.5466832	9.9712560	9.5754292	10.4245728	23
38	9.5470189	9.9712084	9.5758104	10.4241896	22
39	9.5473542	9.9711608	9.5761934	10.4238066	21
40	9.5476893	9.9711132	9.5765761	10.4234239	20
41	9.5480240	9.9710655	9.5769585	10.4230415	19
42	9.5483585	9.9710178	9.5773407	10.4226593	18
43	9.5486927	9.9709701	9.5777226	10.4222774	17
44	9.5490266	9.9709223	9.5781043	10.4218957	16
45	9.5493602	9.9708744	9.5784858	10.4215142	15
46	9.5496935	9.9708265	9.5788669	10.4211331	14
47	9.5500265	9.9707786	9.5792479	10.4207521	13
48	9.5503592	9.9707306	9.5796286	10.4203714	12
49	9.5506916	9.9706826	9.5800090	10.4199910	11
50	9.5510237	9.9706346	9.5803892	10.4196108	10
51	9.5513556	9.9705865	9.5807691	10.4192309	9
52	9.5516871	9.9705383	9.5811488	10.4188512	8
53	9.5520184	9.9704902	9.5815282	10.4184718	7
54	9.5523494	9.9704419	9.5819074	10.4180926	6
55	9.5526801	9.9703937	9.5822864	10.4177136	5
56	9.5530105	9.9703454	9.5826651	10.4173349	4
57	9.5533406	9.9702970	9.5830435	10.4169565	3
58	9.5536704	9.9702486	9.5834217	10.4165783	2
59	9.5539999	9.9702002	9.5837997	10.4162003	1
60	9.5543292	9.9701517	9.5841774	10.4158226	0
		Sin. 69.		Tang. 69.	M

M	Sin. 21.		Tang. 21.		
0	9.5543292	9.9701517	9.5841774	10.4158226	60
1	9.5546581	9.9701032	9.5845549	10.4154451	59
2	9.5549868	9.9700547	9.5849321	10.4150679	58
3	9.5553152	9.9700061	9.5853091	10.4146909	57
4	9.5556433	9.9699574	9.5856859	10.4143141	56
5	9.5559711	9.9699087	9.5860624	10.4139376	55
6	9.5562987	9.9698600	9.5864386	10.4135614	54
7	9.5566259	9.9698112	9.5868147	10.4131853	53
8	9.5569529	9.9697624	9.5871904	10.4128096	52
9	9.5572796	9.9697136	9.5875660	10.4124340	51
10	9.5576060	9.9696647	9.5879413	10.4120587	50
11	9.5579321	9.9696158	9.5883163	10.4116837	49
12	9.5582579	9.9695668	9.5886912	10.4113088	48
13	9.5585835	9.9695177	9.5890657	10.4109343	47
14	9.5589088	9.9694687	9.5894401	10.4105599	46
15	9.5592338	9.9694196	9.5898142	10.4101858	45
16	9.5595585	9.9693704	9.5901881	10.4098119	44
17	9.5598829	9.9693212	9.5905617	10.4094383	43
18	9.5602071	9.9692720	9.5909351	10.4090649	42
19	9.5605310	9.9692227	9.5913082	10.4086918	41
20	9.5608546	9.9691734	9.5916812	10.4083188	40
21	9.5611779	9.9691240	9.5920539	10.4079461	39
22	9.5615010	9.9690746	9.5924263	10.4075737	38
23	9.5618237	9.9690252	9.5927985	10.4072015	37
24	9.5621462	9.9689757	9.5931705	10.4068295	36
25	9.5624685	9.9689262	9.5935423	10.4064577	35
26	9.5627904	9.9688766	9.5939138	10.4060862	34
27	9.5631121	9.9688270	9.5942851	10.4057149	33
28	9.5634335	9.9687773	9.5946561	10.4053439	32
29	9.5637546	9.9687276	9.5950269	10.4049731	31
30	9.5640754	9.9686779	9.5953975	10.4046025	30
		Sin. 68.		Tang. 68.	M

M	Sin. 21.		Tang. 21.		
30	9.5640754	9.9686779	9.5953975	10.4046025	30
31	9.5643960	9.9686281	9.5957679	10.4042321	29
32	9.5647163	9.9685783	9.5961380	10.4038620	28
33	9.5650363	9.9685284	9.5965079	10.4034921	27
34	9.5653561	9.9684785	9.5968776	10.4031224	26
35	9.5656756	9.9684286	9.5972470	10.4027530	25
36	9.5659948	9.9683786	9.5976162	10.4023838	24
37	9.5663137	9.9683285	9.5979852	10.4020148	23
38	9.5666324	9.9682784	9.5983540	10.4016460	22
39	9.5669508	9.9682283	9.5987225	10.4012775	21
40	9.5673689	9.9681781	9.5990908	10.4009092	20
41	9.5675868	9.9681279	9.5994588	10.4005411	19
42	9.5679044	9.9680777	9.5998267	10.4001733	18
43	9.5682217	9.9680274	9.6001943	10.3998057	17
44	9.5685387	9.9679771	9.6005617	10.3994383	16
45	9.5688555	9.9679267	9.6009289	10.3990711	15
46	9.5691721	9.9678763	9.6012958	10.3987042	14
47	9.5694883	9.9678258	9.6016625	10.3983375	13
48	9.5698043	9.9677753	9.6020290	10.3979710	12
49	9.5701200	9.9677247	9.6023953	10.3976047	11
50	9.5704355	9.9676741	9.6027613	10.3972387	10
51	9.5707506	9.9676235	9.6031271	10.3968729	9
52	9.5710656	9.9675728	9.6034927	10.3965073	8
53	9.5713802	9.9675221	9.6038581	10.3961419	7
54	9.5716946	9.9674713	9.6042233	10.3957766	6
55	9.5720087	9.9674205	9.6045882	10.3954118	5
56	9.5723226	9.9673697	9.6049529	10.3950471	4
57	9.5726362	9.9673188	9.6053174	10.3946826	3
58	9.5729495	9.9672679	9.6056817	10.3943183	2
59	9.5732626	9.9672169	9.6060457	10.3939543	1
60	9.5735754	9.9671659	9.6064096	10.3935904	0
		Sin. 68.		Tang. 68.	M

M	Sin. 22.		Tang. 22.		
0	9.5735754	9.9671659	9.6064096	10.3935904	60
1	9.5738880	9.9671148	9.6067732	10.3932268	59
2	9.5742003	9.9670637	9.6071366	10.3928634	58
3	9.5745123	9.9670125	9.6074997	10.3925003	57
4	9.5748240	9.9669614	9.6078627	10.3921373	56
5	9.5751356	9.9669101	9.6082254	10.3917746	55
6	9.5754468	9.9668588	9.6085880	10.3914120	54
7	9.5757578	9.9668075	9.6089503	10.3910497	53
8	9.5760685	9.9667562	9.6093124	10.3906876	52
9	9.5763790	9.9667048	9.6096742	10.3903258	51
10	9.5766892	9.9666533	9.6100359	10.3899641	50
11	9.5769991	9.9666018	9.6103973	10.3896027	49
12	9.5773088	9.9665503	9.6107586	10.3892414	48
13	9.5776183	9.9664987	9.6111196	10.3888804	47
14	9.5779275	9.9664471	9.6114804	10.3885196	46
15	9.5782364	9.9663954	9.6118409	10.3881591	45
16	9.5785450	9.9663437	9.6122013	10.3877987	44
17	9.5788535	9.9662920	9.6125615	10.3874385	43
18	9.5791616	9.9662402	9.6129214	10.3870786	42
19	9.5794695	9.9661884	9.6132812	10.3867188	41
20	9.5797772	9.9661365	9.6136407	10.3863593	40
21	9.5800845	9.9660846	9.6140000	10.3860000	39
22	9.5803917	9.9660326	9.6143591	10.3856409	38
23	9.5806986	9.9659806	9.6147186	10.3852820	37
24	9.5810052	9.9659285	9.6150766	10.3849234	36
25	9.5813116	9.9658764	9.6154351	10.3845649	35
26	9.5816177	9.9658243	9.6157934	10.3842066	34
27	9.5819236	9.9657721	9.6161514	10.3838486	33
28	9.5822292	9.9657199	9.6165093	10.3834907	32
29	9.5825345	9.9656677	9.6168669	10.3831331	31
30	9.5828397	9.9656153	9.6172243	10.3827757	30
		Sin. 67.		Tang. 67.	M

M	Sin. 22.		Tang. 22.		
30	9.5828397	9.9656153	9.6172243	10.3827757	30
31	9.5831445	9.9655630	9.6175815	10.3824185	29
32	9.5834491	9.9655106	9.6179385	10.3820615	28
33	9.5837535	9.9654582	9.6182953	10.3817047	27
34	9.5840576	9.9654057	9.6186519	10.3813481	26
35	9.5843615	9.9653532	9.6190083	10.3809917	25
36	9.5846651	9.9653006	9.6193645	10.3806355	24
37	9.5849685	9.9652480	9.6197205	10.3802795	23
38	9.5851716	9.9651953	9.6200762	10.3799238	22
39	9.5855745	9.9651426	9.6204318	10.3795682	21
40	9.5858771	9.9650899	9.6207872	10.3792128	20
41	9.5861795	9.9650371	9.6211423	10.3788577	19
42	9.5864816	9.9649843	9.6214974	10.3785026	18
43	9.5867835	9.9649314	9.6218520	10.3781480	17
44	9.5870851	9.9648785	9.6222066	10.3777934	16
45	9.5873865	9.9648256	9.6225609	10.3774391	15
46	9.5876876	9.9647726	9.6229150	10.3770850	14
47	9.5879885	9.9647195	9.6232690	10.3767310	13
48	9.5882892	9.9646665	9.6236227	10.3763773	12
49	9.5885896	9.9646133	9.6239763	10.3760237	11
50	9.5888897	9.9645602	9.6243296	10.3756704	10
51	9.5891897	9.9645069	9.6246827	10.3753173	9
52	9.5894893	9.9644537	9.6250356	10.3749644	8
53	9.5897888	9.9644004	9.6253884	10.3746116	7
54	9.5900880	9.9643470	9.6257409	10.3742591	6
55	9.5903869	9.9642937	9.6260932	10.3739068	5
56	9.5906856	9.9642402	9.6264454	10.3735546	4
57	9.5909841	9.9641868	9.6267973	10.3732027	3
58	9.5912823	9.9641332	9.6271491	10.3728509	2
59	9.5915803	9.9640797	9.6275006	10.3724994	1
60	9.5918780	9.9640261	9.6278519	10.3721481	0
		Sin. 67.		Tang. 67.	M

M	Sin. 23.		Tang. 23.		
0	9.5918780	9.9640261	9.6278519	10.3721481	60
1	9.5921755	9.9639724	9.6282030	10.3717969	59
2	9.5924728	9.9639187	9.6285540	10.3714460	58
3	9.5927698	9.9638650	9.6289048	10.3710952	57
4	9.5930666	9.9638112	9.6292553	10.3707447	56
5	9.5933631	9.9637574	9.6296057	10.3703943	55
6	9.5936594	9.9637036	9.6299558	10.3700442	54
7	9.5939555	9.9636496	9.6303058	10.3696942	53
8	9.5942513	9.9635957	9.6306556	10.3693444	52
9	9.5945469	9.9635417	9.6310052	10.3689948	51
10	9.5948422	9.9634877	9.6313545	10.3686455	50
11	9.5951373	9.9634336	9.6317037	10.3682963	49
12	9.5954322	9.9633795	9.6320527	10.3679473	48
13	9.5957268	9.9633253	9.6324015	10.3675985	47
14	9.5960212	9.9632711	9.6327501	10.3672499	46
15	9.5963154	9.9632168	9.6330985	10.3669015	45
16	9.5966093	9.9631625	9.6334468	10.3665532	44
17	9.5969030	9.9631082	9.6337948	10.3662052	43
18	9.5971965	9.9630538	9.6341426	10.3658574	42
19	9.5974897	9.9629994	9.6344903	10.3655097	41
20	9.5977827	9.9629449	9.6348378	10.3651622	40
21	9.5980754	9.9628904	9.6351850	10.3648150	39
22	9.5983680	9.9628358	9.6355321	10.3644679	38
23	9.5986602	9.9627812	9.6358790	10.3641210	37
24	9.5989523	9.9627266	9.6362257	10.3637743	36
25	9.5992441	9.9626719	9.6365722	10.3634278	35
26	9.5995357	9.9626172	9.6369185	10.3630815	34
27	9.5998270	9.9625624	9.6372646	10.3627354	33
28	9.6001181	9.9625076	9.6376106	10.3623894	32
29	9.6004090	9.9624527	9.6379563	10.3620437	31
30	9.6006997	9.9623978	9.6383019	10.3616981	30
		Sin. 66.		Tang. 66.	M

M	Sin. 23.		Tang. 23.		
30	9.6006997	9.9623978	9.1383019	10.3616981	30
31	9.6009901	9.9623428	9.6386473	10.3613527	29
32	9.6012803	9.9622878	9.6389925	10.3610075	28
33	9.6015703	9.9622328	9.6393375	10.3606625	27
34	9.6018600	9.9621777	9.6396823	10.3603177	26
35	9.6021495	9.9621226	9.6400269	10.3599731	25
36	9.6024388	9.9620674	9.6403714	10.3596286	24
37	9.6027278	9.9620122	9.6407156	10.3592844	23
38	9.6030166	9.9619569	9.6410597	10.3589403	22
39	9.6033052	9.9619966	9.6414036	10.3585964	21
40	9.6035936	9.9618463	9.6417473	10.3582527	20
41	9.6038817	9.9617909	9.6420908	10.3579092	19
42	9.6041696	9.9617355	9.6424342	10.3575658	18
43	9.6044573	9.9616800	9.6427773	10.3572227	17
44	9.6047448	9.9616245	9.6431203	10.3568797	16
45	9.6050320	9.9615689	9.6434631	10.3565370	15
46	9.6053190	9.9615133	9.6438057	10.3561943	14
47	9.6056057	9.9614576	9.6441481	10.3558519	13
48	9.6058923	9.9614020	9.6444903	10.3555097	12
49	9.6061786	9.9613463	9.6448324	10.3551676	11
50	9.6064647	9.9612904	9.6451743	10.3548257	10
51	9.6067506	9.9612346	9.6455160	10.3544840	9
52	9.6070362	9.9611787	9.6458575	10.3541425	8
53	9.6073216	9.9611228	9.6461988	10.3538012	7
54	9.6076068	9.9610668	9.6465400	10.3534600	6
55	9.6078918	9.9610108	9.6468810	10.3531190	5
56	9.6081765	9.9609548	9.6472217	10.3527783	4
57	9.6084611	9.9608967	9.6475624	10.3524376	3
58	9.6087454	9.9608426	9.6479028	10.3520972	2
59	9.6090294	9.9607864	9.6482431	10.3517569	1
60	9.6093133	9.9607302	9.6485831	10.3514169	0
		Sin. 66.		Tang. 66.	M

M	Sin. 24.		Tang. 24.		
0	9.6093133	9.9607302	9.6485831	10.3514169	60
1	9.6095969	9.9606739	9.6489230	10.3510770	59
2	9.6098803	9.9606176	9.6492628	10.3507372	58
3	9.6101635	9.9605612	9.6496023	10.3503977	57
4	9.6104465	9.9605048	9.6499417	10.3500583	56
5	9.6107293	9.9604484	9.6502809	10.3497191	55
6	9.6110118	9.9603919	9.6506199	10.3493801	54
7	9.6112941	9.9603354	9.6509587	10.3490413	53
8	9.6115762	9.9602788	9.6512574	10.3487026	52
9	9.6118580	9.9602222	9.6516359	10.3483641	51
10	9.6121397	9.9601655	9.6519742	10.3480258	50
11	9.6124211	9.9601088	9.6523123	10.3476877	49
12	9.6127023	9.9600520	9.6526502	10.3473497	48
13	9.6129833	9.9599952	9.6529881	10.3470119	47
14	9.6132641	9.9599384	9.6533257	10.3466743	46
15	9.6135446	9.9598815	9.6536631	10.3463369	45
16	9.6138250	9.9598246	9.6540004	10.3459996	44
17	9.6141051	9.9597676	9.6543375	10.3456625	43
18	9.6143850	9.9597106	9.6546744	10.3453256	42
19	9.6146647	9.9596535	9.6550112	10.3449888	41
20	9.6149441	9.9595964	9.6553477	10.3446523	40
21	9.6152234	9.9595393	9.6556841	10.3443159	39
22	9.6155024	9.9594821	9.6560204	10.3439796	38
23	9.6157812	9.9594248	9.6563564	10.3436436	37
24	9.6160598	9.9593675	9.6566923	10.3433077	36
25	9.6163382	9.9593102	9.6570280	10.3429720	35
26	9.6166164	9.9592528	9.6573636	10.3426364	34
27	9.6168944	9.9591954	9.6576989	10.3423011	33
28	9.6171721	9.9591380	9.6580341	10.3419659	32
29	9.6174496	9.9590805	9.6583692	10.3416308	31
30	9.6177270	9.9590229	9.6587041	10.3412960	30
		Sin. 65.		Tang. 65.	M

M	Sin. 24.		Tang. 24.		
30	9.6177270	9.9590299	9.6587041	10.3412960	30
31	9.6180041	9.9589653	9.6590387	10.3409613	29
32	9.6182809	9.9589077	9.6593733	10.3406267	28
33	9.6185576	9.9588500	9.6597076	10.3402924	27
34	9.6188341	9.9587923	9.6600418	10.3399582	26
35	9.6191103	9.9587345	9.6603758	10.3396242	25
36	9.6193864	9.9586767	9.6607097	10.3392903	24
37	9.6196622	9.9586188	9.6610434	10.3389566	23
38	9.6199378	9.9585609	9.6613769	10.3386231	22
39	9.6202132	9.9585030	9.6617103	10.3382897	21
40	9.6204884	9.9584450	9.6620434	10.3379566	20
41	9.6207634	9.9583869	9.6623765	10.3376235	19
42	9.6210382	9.9583288	9.6627093	10.3372907	18
43	9.6213127	9.9582707	9.6630420	10.3369580	17
44	9.6215871	9.9582125	9.6633745	10.3366255	16
45	9.6218612	9.9581543	9.6637069	10.3362931	15
46	9.6221351	9.9580961	9.6640391	10.3359609	14
47	9.6224088	9.9580378	9.6643711	10.3356289	13
48	9.6226824	9.9579794	9.6647030	10.3352970	12
49	9.6229557	9.9579210	9.6650346	10.3349654	11
50	9.6232287	9.9578626	9.6653662	10.3346338	10
51	9.6235016	9.9578041	9.6656975	10.3343025	9
52	9.6237743	9.9577456	9.6660288	10.3339712	8
53	9.6240468	9.9576870	9.6663598	10.3336402	7
54	9.6243190	9.9576284	9.6666907	10.3333093	6
55	9.6245911	9.9575697	9.6670214	10.3329786	5
56	9.6248629	9.9575110	9.6673519	10.3326481	4
57	9.6251346	9.9574522	9.6676823	10.3323177	3
58	9.6254060	9.9573934	9.6680126	10.3319874	2
59	9.6256772	9.9573346	9.6683426	10.3316574	1
60	9.6259483	9.9572757	9.6686725	10.3313275	0
		Sin. 65.		Tang. 65.	M

M	Sin. 25.		Tang. 25.		
0	9.6259483	9.9572757	9.6686725	10.3313275	60
1	9.6262191	9.9572168	9.6690023	10.3309977	59
2	9.6264897	9.9571578	9.6693319	10.3306681	58
3	9.6267601	9.9570988	9.6696613	10.3303387	57
4	9.6270303	9.9570397	9.6699906	10.3300094	56
5	9.6273003	9.9569806	9.6703197	10.3296803	55
6	9.6275701	9.9569215	9.6706486	10.3293514	54
7	9.6278397	9.9568623	9.6709774	10.3290226	53
8	9.6281090	9.9568030	9.6713060	10.3286940	52
9	9.6283782	9.9567437	9.6716345	10.3283655	51
10	9.6286472	9.9566844	9.6719628	10.3280372	50
11	9.6289160	9.9566250	9.6722910	10.3277090	49
12	9.6291845	9.9565656	9.6726190	10.3273810	48
13	9.6294529	9.9565061	9.6729468	10.3270532	47
14	9.6297211	9.9564466	9.6732745	10.3267255	46
15	9.6299890	9.9563870	9.6736020	10.3263980	45
16	9.6302568	9.9563274	9.6739294	10.3260706	44
17	9.6305243	9.9562678	9.6742566	10.3257454	43
18	9.6307917	9.9562081	9.6745836	10.3254164	42
19	9.6310589	9.9561483	9.6749105	10.3250895	41
20	9.6313258	9.9560886	9.6752372	10.3247628	40
21	9.6315926	9.9560287	9.6755638	10.3244362	39
22	9.6318591	9.9559689	9.6758903	10.3241097	38
23	9.6321255	9.9559089	9.6762165	10.3237835	37
24	9.6323916	9.9558490	9.6765426	10.3234574	36
25	9.6326576	9.9557890	9.6768686	10.3231314	35
26	9.6329233	9.9557289	9.6771944	10.3228056	34
27	9.6331889	9.9556688	9.6775201	10.3224799	33
28	9.6334542	9.9556087	9.6778456	10.3221544	32
29	9.6337194	9.9555485	9.6781709	10.3218291	31
30	9.6339844	9.9554882	9.6784961	10.3215039	30
		Sin. 64.		Tang 64.	M

M	Sin. 25.		Tang. 25.		
30	9.6339844	9.9554882	9.6784961	10.3215039	30
31	9.6342491	9.9554280	9.6788211	10.3211788	29
32	9.6345137	9.9553676	9.6791460	10.3208540	28
33	9.6347780	9.9553073	9.6794708	10.3205292	27
34	9.6350422	9.9552469	9.6797953	10.3202047	26
35	9.6353062	9.9551864	9.6801198	10.3198802	25
36	9.6355699	9.9551259	9.6804440	10.3195560	24
37	9.6358335	9.9550653	9.6807682	10.3192318	23
38	9.6360969	9.9550047	9.6810921	10.3189079	22
39	9.6363601	9.9549441	9.6814160	10.3185840	21
40	9.6366231	9.9548834	9.6817396	10.3182604	20
41	9.6368859	9.9548227	9.6820632	10.3179368	19
42	9.6371484	9.9547619	9.6823865	10.3176135	18
43	9.6374108	9.9547011	9.6827098	10.3172902	17
44	9.6376731	9.9546402	9.6830328	10.3169672	16
45	9.6379351	9.9545793	9.6833557	10.3166443	15
46	9.6381969	9.9545184	9.6836785	10.3163215	14
47	9.6384585	9.9544574	9.6840011	10.3159989	13
48	9.6387199	9.9543963	9.6843236	10.3156764	12
49	9.6389812	9.9543352	9.6846459	10.3153541	11
50	9.6392422	9.9542741	9.6849681	10.3150319	10
51	9.6395030	9.9542129	9.6852901	10.3147099	9
52	9.6397637	9.9541517	9.6856120	10.3143880	8
53	9.6400241	9.9540904	9.6859338	10.3140662	7
54	9.6402844	9.9540291	9.6862553	10.3137447	6
55	9.6405445	9.9539677	9.6865768	10.3134232	5
56	9.6408044	9.9539063	9.6868981	10.3131019	4
57	9.6410640	9.9538448	9.6872192	10.3127808	3
58	9.6413235	9.9537833	9.6875402	10.3124598	2
59	9.6415828	9.9537218	9.6878611	10.3121389	1
60	9.6418420	9.9536602	9.6881818	10.3118182	0
		Sin. 64.		Tang. 64.	M

M	Sin. 26.		Tang. 26.		
0	9.6418420	9.9536602	9.6881818	10.3118182	60
1	9.6421009	9.9535985	9.6885023	10.3114977	59
2	9.6423596	9.9535369	9.6888227	10.3111773	58
3	9.6426182	9.9534751	9.6891430	10.3108570	57
4	9.6428765	9.9534134	9.6894631	10.3105369	56
5	9.6431347	9.9533515	9.6897831	10.3102169	55
6	9.6433926	9.9532897	9.6901030	10.3098970	54
7	9.6436504	9.9532278	9.6904226	10.3095774	53
8	9.6439080	9.9531658	9.6907422	10.3092578	52
9	9.6441654	9.9531038	9.6910616	10.3089384	51
10	9.6444226	9.9530418	9.6913809	10.3086191	50
11	9.6446796	9.9529797	9.6917000	10.3083000	49
12	9.6449365	9.9529175	9.6920189	10.3079811	48
13	9.6451931	9.9528553	9.6923378	10.3076622	47
14	9.6454496	9.9527931	9.6926565	10.3073435	46
15	9.6457058	9.9527308	9.6929750	10.3070250	45
16	9.6459619	9.9526685	9.6932934	10.3067066	44
17	9.6462178	9.9526061	9.6936117	10.3063883	43
18	9.6464735	9.9525437	9.6939298	10.3060702	42
19	9.6467290	9.9524813	9.6932478	10.3057522	41
20	9.6469844	9.9524188	9.6945656	10.3054344	40
21	9.6472395	9.9523562	9.6948833	10.3051167	39
22	9.6474945	9.9522936	9.6952009	10.3047991	38
23	9.6477492	9.9522310	9.6955183	10.3044817	37
24	9.6480038	9.9521683	9.6958355	10.3041645	36
25	9.6482582	9.9521055	9.6961527	10.3038473	35
26	9.6485124	9.9520428	9.6964697	10.3035303	34
27	9.6487665	9.9519799	9.6967865	10.3032135	33
28	9.6490203	9.9519171	9.6971032	10.3028968	32
29	9.6492740	9.9518541	9.6974198	10.3025802	31
30	9.6495274	9.9517912	9.6977363	10.3022637	30
		Sin. 63.		Tang. 63.	M

M	Sin. 26.		Tang. 26.		
30	9.6495274	9.9517912	9.6977363	10.3022637	30
31	9.6497807	9.9517282	9.6980526	10.3019474	29
32	9.6500338	9.9516651	9.6983687	10.3016313	28
33	9.6502868	9.9516020	9.6986847	10.3013153	27
34	9.6505395	9.9515389	9.6990006	10.3009994	26
35	9.6507920	9.9514757	9.6993164	10.3006836	25
36	9.6510444	9.9514124	9.6996320	10.3003680	24
37	9.6512966	9.9513492	9.6999474	10.3000526	23
38	9.6515486	9.9512858	9.7002628	10.2997372	22
39	9.6518004	9.9512224	9.7005780	10.2994220	21
40	9.6520521	9.9511590	9.7008930	10.2991070	20
41	9.6523035	9.9510956	9.7012080	10.2987920	19
42	9.6525548	9.9510320	9.7015227	10.2984773	18
43	9.6528059	9.9509685	9.7018374	10.2981626	17
44	9.6530568	9.9509049	9.7021519	10.2978481	16
45	9.6533075	9.9508412	9.7024663	10.2975337	15
46	9.6535581	9.9507775	9.7027805	10.2972195	14
47	9.6538084	9.9507138	9.7030946	10.2969054	13
48	9.6540586	9.9506500	9.7034086	10.2965914	12
49	9.6543086	9.9505861	9.7037225	10.2962775	11
50	9.6545584	9.9505223	9.7040362	10.2959638	10
51	9.6548081	9.9504583	9.7043497	10.2956503	9
52	9.6550575	9.9503944	9.7046632	10.2953368	8
53	9.6553068	9.9503303	9.7049765	10.2950235	7
54	9.6555559	9.9502663	9.7052897	10.2947103	6
55	9.6558048	9.9502022	9.7056027	10.2943973	5
56	9.6560536	9.9501380	9.7059156	10.2940844	4
57	9.6563021	9.9500738	9.7062284	10.2937716	3
58	9.6565505	9.9500095	9.7065410	10.2934590	2
59	9.6567987	9.9499452	9.7068535	10.2931465	1
60	9.6570468	9.9498809	9.7071659	10.2928341	0
		Sin. 63.		Tang. 63.	M

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M	Sin. 27.		Tang. 27.		
0	9.6570468	9.9498809	9.7071659	10.2928341	60
1	9.6572946	9.9498165	9.7074781	10.2925219	59
2	9.6575423	9.9497521	9.7077902	10.2922098	58
3	9.6577898	9.9496876	9.7081022	10.2918978	57
4	9.6580371	9.9496230	9.7084141	10.2915859	56
5	9.6582842	9.9495585	9.7087258	10.2912742	55
6	9.6585312	9.9494938	9.7090374	10.2909626	54
7	9.6587780	9.9494293	9.7093488	10.2906512	53
8	9.6590246	9.9493645	9.7096601	10.2903399	52
9	9.6592710	9.9492997	9.7099713	10.2900287	51
10	9.6595173	9.9492349	9.7102824	10.2897176	50
11	9.6597634	9.9491700	9.7105933	10.2894067	49
12	9.6600093	9.9491051	9.7109041	10.2890959	48
13	9.6602550	9.9490402	9.7112148	10.2887852	47
14	9.6605005	9.9489752	9.7115254	10.2884746	46
15	9.6607459	9.9489101	9.7118358	10.2881642	45
16	9.6609911	9.9488450	9.7121461	10.2878539	44
17	9.6612361	9.9487799	9.7124562	10.2875438	43
18	9.6614810	9.9487147	9.7127662	10.2872338	42
19	9.6617257	9.9486495	9.7130761	10.2869239	41
20	9.6619701	9.9485842	9.7133859	10.2866141	40
21	9.6622145	9.9485189	9.7136956	10.2863044	39
22	9.6624586	9.9484535	9.7140051	10.2859949	38
23	9.6627026	9.9483881	9.7143145	10.2856855	37
24	9.6629464	9.9483227	9.7146237	10.2853763	36
25	9.6631900	9.9482572	9.7149329	10.2850671	35
26	9.6634335	9.9481916	9.7152419	10.2847581	34
27	9.6636768	9.9481260	9.7155508	10.2844492	33
28	9.6639199	9.9480604	9.7158595	10.2841405	32
29	9.6641628	9.9479947	9.7161682	10.2838318	31
30	9.6644056	9.9479289	9.7164767	10.2835233	30
		Sin. 62.		Tang. 62.	M

M	Sin. 27.		Tang. 27.		
30	9.6644056	9.9479289	9.7164767	10.2835233	30
31	9.6646482	9.9478631	9.7167851	10.2832149	29
32	9.6648906	9.9477973	9.7170933	10.2829067	28
33	9.6651329	9.9477314	9.7174014	10.2825986	27
34	9.6653749	9.9476655	9.7177094	10.2822906	26
35	9.6756168	9.9475995	9.7180173	10.2819827	25
36	9.6658586	9.9475335	9.7183251	10.2816749	24
37	9.6661001	9.9474674	9.7186327	10.2813673	23
38	9.6663415	9.9474013	9.7189402	10.2810598	22
39	9.6665828	9.9473352	9.7192476	10.2807524	21
40	9.6668238	9.9472689	9.7195549	10.2804451	20
41	9.6670647	9.9472027	9.7198620	10.2801380	19
42	9.6673054	9.9471364	9.7201690	10.2798309	18
43	9.6675459	9.9470700	9.7204759	10.2795241	17
44	9.6677863	9.9470036	9.7207827	10.2792173	16
45	9.6680265	9.9469372	9.7210893	10.2789107	15
46	9.6682665	9.9468707	9.7213958	10.2786042	14
47	9.6685064	9.9468042	9.7217022	10.2782978	13
48	9.6687461	9.9467376	9.7220085	10.2779915	12
49	9.6689856	9.9466710	9.7223147	10.2776853	11
50	9.6692250	9.9466043	9.7226207	10.2773793	10
51	9.6694642	9.9465376	9.7229266	10.2770734	9
52	9.6697032	9.9464708	9.7232324	10.2767676	8
53	9.6699420	9.9464040	9.7235381	10.2764619	7
54	9.6701807	9.9463371	9.7238436	10.2761564	6
55	9.6704192	9.9462702	9.7241490	10.2758510	5
56	9.6706576	9.9462032	9.7244543	10.2755457	4
57	9.6708958	9.9461362	9.7247595	10.2752405	3
58	9.6711338	9.9460692	9.7250646	10.2749354	2
59	9.6713716	9.9460021	9.7253695	10.2746305	1
60	9.6716093	9.9459349	9.7256744	10.2743256	0
		Sin. 62.		Tang. 62.	M

M	*Sin.* 28.		*Tang.* 28.		
0	9.6716093	9.9459349	9.7256744	10.2743256	60
1	9.6718468	9.9458677	9.7259791	10.2740209	59
2	9.6720841	9.9458005	9.7262837	10.2737163	58
3	9.6723213	9.9457332	9.7265881	10.2734119	57
4	9.6725583	9.9456659	9.7268925	10.2731075	56
5	9.6727952	9.9455985	9.7271967	10.2728033	55
6	9.6730319	9.9455310	9.7275008	10.2724992	54
7	9.6732684	9.9454636	9.7278048	10.2721952	53
8	9.6735047	9.9453960	9.7281087	10.2718913	52
9	9.6737409	9.9453285	9.7284124	10.2715876	51
10	9.6739769	9.9452609	9.7287161	10.2712839	50
11	9.6742128	9.9451932	9.7290196	10.2709804	49
12	9.6744485	9.9451255	9.7293230	10.2706770	48
13	9.6746840	9.9450577	9.7296263	10.2703737	47
14	9.6749194	9.9449899	9.7299295	10.2700705	46
15	9.6751546	9.9449220	9.7302325	10.2697675	45
16	9.6753896	9.9448541	9.7305354	10.2694646	44
17	9.6756245	9.9447862	9.7308383	10.2691617	43
18	9.6758592	9.9447182	9.7311410	10.2688590	42
19	9.6760937	9.9446501	9.7314436	10.2685564	41
20	9.6763281	9.9445821	9.7317460	10.2682540	40
21	9.6765623	9.9445139	9.7320484	10.2679516	39
22	9.6767963	9.9444457	9.7323506	10.2676494	38
23	9.6770302	9.9443775	9.7326527	10.2673473	37
24	9.6772640	9.9443092	9.7329547	10.2670453	36
25	9.6774975	9.9442409	9.7332566	10.2667434	35
26	9.6777309	9.9441725	9.7335584	10.2664416	34
27	9.6779642	9.9441041	9.7338601	10.2661399	33
28	9.6781972	9.9440356	9.7341616	10.2658384	32
29	9.6784301	9.9439671	9.7344631	10.2655369	31
30	9.6786629	9.9438985	9.7347644	10.2652356	30
		Sin. 61.		*Tang.* 61.	M

M	*Sin.* 28.		*Tang.* 28.		
30	9.6786629	9.9438985	9.7347644	10.2652356	30
31	9.6788955	9.9438299	9.7350656	10.2649344	29
32	9.6791279	9.9437612	9.7353667	10.2646333	28
33	9.6793602	9.9436925	9.7356677	10.2643323	27
34	9.6795923	9.9436238	9.7359685	10.2640315	26
35	9.6798243	9.9435549	9.7362693	10.2637307	25
36	9.6800560	9.9434861	9.7365699	10.2634301	24
37	9.6802877	9.9434172	9.7368705	10.2631295	23
38	9.6805191	9.9433482	9.7371709	10.2628291	22
39	9.6807504	9.9432792	9.7374712	10.2625288	21
40	9.6809816	9.9432102	9.7377714	10.2622286	20
41	9.6812126	9.9431411	9.7380715	10.2619285	19
42	9.6814434	9.9430720	9.7383714	10.2616286	18
43	9.6816741	9.9430028	9.7386713	10.2613287	17
44	9.6819046	9.9429335	9.7389710	10.2610290	16
45	9.6821349	9.9428643	9.7392707	10.2607293	15
46	9.6823651	9.9427949	9.7395702	10.2604298	14
47	9.6825952	9.9427255	9.7398696	10.2601304	13
48	9.6828250	9.9426561	9.7401689	10.2598311	12
49	9.6830548	9.9425866	9.7404681	10.2595319	11
50	9.6832843	9.9425171	9.7407672	10.2592328	10
51	9.6835137	9.9424476	9.7410662	10.2589338	9
52	9.6837430	9.9423779	9.7413650	10.2586350	8
53	9.6839720	9.9423083	9.7416638	10.2583362	7
54	9.6842010	9.9422386	9.7419624	10.2580376	6
55	9.6844297	9.9421688	9.7422609	10.2577391	5
56	9.6846583	9.9420990	9.7425594	10.2574406	4
57	9.6848868	9.9420291	9.7428577	10.2571423	3
58	9.6851151	9.9419592	9.7431559	10.2568441	2
59	9.6853432	9.9418893	9.7434540	10.2565460	1
60	9.6855712	9.9418193	9.7437520	10.2562480	0
		Sin. 61.		*Tang.* 61.	M

M	*Sin.* 29.		*Tang.* 29.		
0	9.6855712	9.9418193	9.7437520	10.2562480	60
1	9.6857991	9.9417492	9.7440499	10.2559501	59
2	9.6860267	9.9416791	9.7443476	10.2556524	58
3	9.6862542	9.9416090	9.7446453	10.2553547	57
4	9.6864816	9.9415388	9.7449428	10.2550572	56
5	9.6867088	9.9414685	9.7452403	10.2547597	55
6	9.6869359	9.9413982	9.7455376	10.2544624	54
7	9.6871628	9.9413279	9.7458349	10.2541651	53
8	9.6873895	9.9412575	9.7461320	10.2538680	52
9	9.6876161	9.9411871	9.7464290	10.2535710	51
10	9.6878425	9.9411166	9.7467259	10.2532741	50
11	9.6880688	9.9410461	9.7470227	10.2529773	49
12	9.6882949	9.9409755	9.7473194	10.2526806	48
13	9.6885209	9.9409048	9.7476160	10.2523840	47
14	9.6887467	9.9408342	9.7479125	10.2520875	46
15	9.6889723	9.9407634	9.7482089	10.2517911	45
16	9.6891978	9.9406927	9.7485052	10.2514948	44
17	9.6894232	9.9406219	9.7488013	10.2511987	43
18	9.6896484	9.9405510	9.7490974	10.2509026	42
19	9.6898734	9.9404801	9.7493934	10.2506066	41
20	9.6900983	9.9404091	9.7496892	10.2503108	40
21	9.6903231	9.9403381	9.7499850	10.2500150	39
22	9.6905476	9.9402670	9.7502806	10.2497194	38
23	9.6907721	9.9401959	9.7505762	10.2494238	37
24	9.6909964	9.9401248	9.7508716	10.2491284	36
25	9.6912205	9.9400535	9.7511669	10.2488331	35
26	9.6914445	9.9399823	9.7514622	10.2485378	34
27	9.6916683	9.9399110	9.7517573	10.2482427	33
28	9.6918919	9.9398396	9.7520523	10.2479477	32
29	9.6921155	9.9397682	9.7523472	10.2476528	31
30	9.6923388	9.9396968	9.7526420	10.2473580	30
		Sin. 60.		*Tang* 60.	M

M	*Sin.* 29.		*Tang.* 29.		
30	9.6923388	9.9396968	9.7526420	10.2473580	30
31	9.6925620	9.9396253	9.7529368	10.2470632	29
32	9.6927851	9.9395537	9.7532314	10.2467686	28
33	9.6930080	9.9394821	9.7535259	10.2464741	27
34	9.6932308	9.9394105	9.7538203	10.2461797	26
35	9.6934534	9.9393388	9.7541146	10.2458854	25
36	9.6936758	9.9392671	9.7544088	10.2455912	24
37	9.6938981	9.9391953	9.7547029	10.2452971	23
38	9.6941203	9.9391234	9.7549969	10.2450031	22
39	9.6943423	9.9390515	9.7552908	10.2447092	21
40	9.6945642	9.9389796	9.7555846	10.2444154	20
41	9.6947859	9.9389076	9.7558783	10.2441217	19
42	9.6950074	9.9388356	9.7561718	10.2438282	18
43	9.6952288	9.9387635	9.7564653	10.2435347	17
44	9.6954501	9.9386914	9.7567587	10.2432413	16
45	9.6956712	9.9386192	9.7570520	10.2429480	15
46	9.6958922	9.9385470	9.7573452	10.2426548	14
47	9.6961130	9.9384747	9.7576383	10.2423617	13
48	9.6963336	9.9384024	9.7579313	10.2420687	12
49	9.6965541	9.9383300	9.7582242	10.2417758	11
50	9.6967745	9.9382576	9.7585170	10.2414830	10
51	9.6969947	9.9381851	9.7588096	10.2411904	9
52	9.6972148	9.9381126	9.7591022	10.2408978	8
53	9.6974347	9.9380400	9.7593947	10.2406053	7
54	9.6976545	9.9379674	9.7596871	10.2403129	6
55	9.6978741	9.9378947	9.7599794	10.2400206	5
56	9.6980936	9.9378220	9.7602716	10.2397284	4
57	9.6983129	9.9377492	9.7605637	10.2394363	3
58	9.6985321	9.9376764	9.7608557	10.2391443	2
59	9.6987511	9.9376035	9.7611476	10.2388524	1
60	9.6989700	9.9378506	9.7614394	10.2385606	0
		Sin. 60.		*Tang.* 60.	M

M	Sin. 30.		Tang. 30.		
0	9.6989700	9.9375306	9.7614394	10.2385606	60
1	9.6991887	9.9374577	9.7617311	10.2382689	59
2	9.6994073	9.9373847	9.7620227	10.2379773	58
3	9.6996258	9.9373116	9.7623142	10.2376858	57
4	9.6998441	9.9372385	9.7626056	10.2373944	56
5	9.7000622	9.9371653	9.7628969	10.2371031	55
6	9.7002802	9.9370921	9.7631881	10.2368119	54
7	9.7004981	9.9370189	9.7634792	10.2365208	53
8	9.7007158	9.9369456	9.7637702	10.2362298	52
9	9.7009334	9.9368722	9.7640612	10.2359388	51
10	9.7011508	9.9367988	9.7643520	10.2356480	50
11	9.7013681	9.9367254	9.7646427	10.2353573	49
12	9.7015852	9.9366519	9.7649334	10.2350666	48
13	9.7018022	9.9365783	9.7652239	10.2347761	47
14	9.7020190	9.9365047	9.7655143	10.2344857	46
15	9.7022357	9.9364311	9.7658047	10.2341953	45
16	9.7024523	9.9363574	9.7660949	10.2339051	44
17	9.7026687	9.9362836	9.7663851	10.2336149	43
18	9.7028849	9.9362098	9.7666751	10.2333249	42
19	9.7031011	9.9361360	9.7669651	10.2330349	41
20	9.7033170	9.9360621	9.7672550	10.2327450	40
21	9.7035329	9.9359881	9.7675448	10.2324553	39
22	9.7037488	9.9359141	9.7678344	10.2321656	38
23	9.7039641	9.9358401	9.7681240	10.2318760	37
24	9.7041795	9.9357660	9.7684135	10.2315865	36
25	9.7043947	9.9356918	9.7687029	10.2312971	35
26	9.7046099	9.9356177	9.7689922	10.2310078	34
27	9.7048248	9.9355434	9.7692814	10.2307186	33
28	9.7050397	9.9354691	9.7695705	10.2304295	32
29	9.7052543	9.9353948	9.7698596	10.2301404	31
30	9.7054689	9.9353204	9.7701485	10.2298515	30
		Sin. 59.		Tang 59.	M

M	Sin. 30.		Tang. 30.		
30	9.7054689	9.9353204	9.7701485	10.2298515	30
31	9.7056833	9.9352459	9.7704373	10.2295627	29
32	9.7058975	9.9351715	9.7707261	10.2292739	28
33	9.7061116	9.9350969	9.7710147	10.2289853	27
34	9.7063256	9.9350223	9.7713033	10.2286967	26
35	9.7065394	9.9349477	9.7715917	10.2284082	25
36	9.7067531	9.9348730	9.7718801	10.2281199	24
37	9.7069667	9.9347983	9.7721684	10.2278316	23
38	9.7071801	9.9347235	9.7724566	10.2275434	22
39	9.7093933	9.9346486	9.7727447	10.2272553	21
40	9.7076064	9.9345738	9.7730327	10.2269673	20
41	9.7078194	9.9344988	9.7733206	10.2266794	19
42	9.7080323	9.9344238	9.7736084	10.2263916	18
43	9.7082450	9.9343488	9.7738961	10.2261039	17
44	9.7084575	9.9342737	9.7741838	10.2258162	16
45	9.7086699	9.9341986	9.7744713	10.2255287	15
46	9.7088822	9.9341234	9.7747588	10.2252412	14
47	9.7090943	9.9340482	9.7750462	10.2249538	13
48	9.7093063	9.9339729	9.7753334	10.2246666	12
49	9.7095182	9.9338976	9.7756206	10.2243794	11
50	9.7097299	9.9338222	9.7759077	10.2240923	10
51	9.7099415	9.9337467	9.7761947	10.2238053	9
52	9.7101529	9.9336713	9.7764816	10.2235184	8
53	9.7103642	9.9335957	9.7767685	10.2232315	7
54	9.7105753	9.9335201	9.7770552	10.2229448	6
55	9.7107863	9.9334445	9.7773418	10.2226582	5
56	9.7109972	9.9333688	9.7776284	10.2223716	4
57	9.7112080	9.9332931	9.7779149	10.2220851	3
58	9.7114186	9.9332173	9.7782012	10.2217988	2
59	9.7116290	9.9331415	9.7784875	10.2215125	1
60	9.7118393	9.9330656	9.7787737	10.2212263	0
		Sin. 59.		Tang. 59.	M

M	Sin. 31.		Tang. 31.		
0	9.7118393	9.9330656	9.7787737	10.2212262	60
1	9.7120495	9.9329897	9.7790599	10.2209401	59
2	9.7122596	9.9329137	9.7793459	10.2206541	58
3	9.7124695	9.9328376	9.7796318	10.2203683	57
4	9.7126792	9.9327616	9.7799177	10.2200823	56
5	9.7128889	9.9326854	9.7802034	10.2197966	55
6	9.7130983	9.9326092	9.7804891	10.2195109	54
7	9.7133077	9.9325330	9.7807747	10.2192253	53
8	9.7135169	9.9324567	9.7810602	10.2189398	52
9	9.7137260	9.9323804	9.7813456	10.2186544	51
10	9.7139349	9.9323040	9.7816309	10.2183691	50
11	9.7141437	9.9322276	9.7819162	10.2180838	49
12	9.7143524	9.9321511	9.7822013	10.2177987	48
13	9.7145609	9.9320746	9.7824864	10.2175136	47
14	9.7147693	9.9319980	9.7827713	10.2172287	46
15	9.7149776	9.9319213	9.7830562	10.2169438	45
16	9.7151857	9.9318447	9.7833410	10.2166590	44
17	9.7153937	9.9317679	9.7836258	10.2163742	43
18	9.7156015	9.9316911	9.7839164	10.2160896	42
19	9.7158092	9.9316143	9.7841949	10.2158051	41
20	9.7160168	9.9315374	9.7844794	10.2155206	40
21	9.7162243	9.9314605	9.7847638	10.2152362	39
22	9.7164316	9.9313835	9.7850481	10.2149519	38
23	9.7166387	9.9313065	9.7853323	10.2146677	37
24	9.7168458	9.9312294	9.7856164	10.2143836	36
25	9.7170526	9.9311522	9.7859004	10.2140996	35
26	9.7172594	9.9310750	9.7861844	10.2138156	34
27	9.7174660	9.9309978	9.7864682	10.2135318	33
28	9.7176725	9.9309205	9.7867520	10.2132480	32
29	9.7178789	9.9308432	9.7870357	10.2129643	31
30	9.7180851	9.9307658	9.7873193	10.2126807	30
		Sin. 58.		Tang. 58.	M

M	Sin. 31.		Tang. 31.		
30	9.7180851	9.7873193	9.9307658	10.2123607	30
31	9.7182912	9.9306883	9.7876028	10.2123972	29
32	9.7184971	9.9306109	9.7878863	10.2121137	28
33	9.7187030	9.9305333	9.7881696	10.2118304	27
34	9.7189086	9.9304557	9.7884529	10.2115471	26
35	9.7191142	9.9303781	9.7887361	10.2112639	25
36	9.7193196	9.9303004	9.7890192	10.2109808	24
37	9.7195249	9.9302226	9.7893023	10.2106977	23
38	9.7197300	9.9301448	9.7895852	10.2104148	22
39	9.7199350	9.9300670	9.7898681	10.2101319	21
40	9.7201399	9.9299891	9.7901508	10.2098492	20
41	9.7203447	9.9299112	9.7904335	10.2095665	19
42	9.7205493	9.9298332	9.7907161	10.2092839	18
43	9.7207538	9.9297551	9.7909987	10.2090013	17
44	9.7209581	9.9296770	9.7912811	10.2087189	16
45	9.7211623	9.9295989	9.7915635	10.2084365	15
46	9.7213664	9.9295207	9.7918458	10.2081542	14
47	9.7215704	9.9294424	9.7921280	10.2078720	13
48	9.7217742	9.9293641	9.7924101	10.2075899	12
49	9.7219779	9.9292857	9.7926921	10.2073079	11
50	9.7221814	9.9292073	9.7929741	10.2070259	10
51	9.7223848	9.9291289	9.7932670	10.2067440	9
52	9.7225881	9.9290504	9.7935378	10.2064622	8
53	9.7227913	9.9289718	9.7938195	10.2061805	7
54	9.7229943	9.9288932	9.7941011	10.2058989	6
55	9.7231972	9.9288145	9.7943827	10.2056173	5
56	9.7234000	9.9287358	9.7946641	10.2053359	4
57	9.7236026	9.9286571	9.7949455	10.2050545	3
58	9.7238051	9.9285783	9.7952268	10.2047732	2
59	9.7240075	9.9284994	9.7955081	10.2044919	1
60	9.7242097	9.9284205	9.7957892	10.2042108	0
		Sin. 58.		Tang. 58.	M

M	Sin. 32.		Tang. 32.		
0	9.7242097	9.9284205	9.7957892	10.2042108	60
1	9.7244118	9.9283415	9.7960703	10.2039297	59
2	9.7246138	9.9282625	9.7963513	10.2036487	58
3	9.7248156	9.9281834	9.7966322	10.2033678	57
4	9.7250174	9.9281043	9.7969130	10.2030870	56
5	9.7252189	9.9280251	9.7971938	10.2028062	55
6	9.7254204	9.9279459	9.7974745	10.2025255	54
7	9.7256217	9.9278666	9.7977552	10.2022449	53
8	9.7258229	9.9277873	9.7980356	10.2019644	52
9	9.7260240	9.9277079	9.7983160	10.2016840	51
10	9.7262249	9.9276285	9.7985964	10.2014036	50
11	9.7264257	9.9275490	9.7988767	10.2011233	49
12	9.7266264	9.9274695	9.7991569	10.2008431	48
13	9.7268269	9.9273899	9.7994370	10.2005630	47
14	9.7270273	9.9273103	9.7997170	10.2002830	46
15	9.7272276	9.9272306	9.7999970	10.2000030	45
16	9.7274278	9.9271509	9.8002769	10.1997231	44
17	9.7276278	9.9270711	9.8005567	10.1994433	43
18	9.7278277	9.9269913	9.8008365	10.1991635	42
19	9.7280275	9.9269114	9.8011161	10.1988839	41
20	9.7282271	9.9268314	9.8013957	10.1986043	40
21	9.7284267	9.9267514	9.8016752	10.1983248	39
22	9.7286260	9.9266714	9.8019546	10.1980454	38
23	9.7288253	9.9265913	9.8022340	10.1977660	37
24	9.7290244	9.9265112	9.8025133	10.1974867	36
25	9.7292234	9.9264310	9.8027925	10.1972075	35
26	9.7294223	9.9263507	9.8030716	10.1969284	34
27	9.7296211	9.9262704	9.8033506	10.1966494	33
28	9.7298197	9.9261901	9.8036296	10.1963704	32
29	9.7300 82	9.9261096	9.8039085	10.1960915	31
30	9.7302165	9.9260292	9.8041873	10.1958127	30
		Sin. 57.		Tang. 57.	M

M	Sin. 32.		Tang. 32.		
30	9.7302165	9.9260292	9.8041873	10.1958127	30
31	9.7304148	9.9259487	9.8044661	10.1955339	29
32	9.7306129	9.9258681	9.8047447	10.1952553	28
33	9.7308109	9.9257875	9.8050233	10.1949767	27
34	9.7310087	9.9257069	9.8053019	10.1946981	26
35	9.7312064	9.9256261	9.8055803	10.1944197	25
36	9.7314040	9.9255454	9.8058587	10.1941413	24
37	9.7316015	9.9254646	9.8061370	10.1938630	23
38	9.7317989	9.9253837	9.8064151	10.1935848	22
39	9.7319961	9.9253028	9.8066933	10.1933067	21
40	9.7321932	9.9252218	9.8069714	10.1930286	20
41	9.7323902	9.9251408	9.8072494	10.1927506	19
42	9.7325870	9.9250597	9.8075273	10.1924727	18
43	9.7327837	9.9249786	9.8078052	10.1921948	17
44	9.7329803	9.9248974	9.8080829	10.1919171	16
45	9.7331768	9.9248161	9.8083606	10.1916394	15
46	9.7333731	9.9247349	9.8086383	10.1913617	14
47	9.7335693	9.9246535	9.8089158	10.1910842	13
48	9.7337654	9.9245721	9.8091933	10.1908067	12
49	9.7339614	9.9244907	9.8094707	10.1905292	11
50	9.7341572	9.9244092	9.8097480	10.1902520	10
51	9.7343529	9.9243277	9.8100253	10.1899747	9
52	9.7345485	9.9242461	9.8103025	10.1896975	8
53	9.7347440	9.9241644	9.8105796	10.1894204	7
54	9.7349393	9.9240827	9.8108566	10.1891434	6
55	9.7351345	9.9240010	9.8111336	10.1888664	5
56	9.7353296	9.9239191	9.8114105	10.1885895	4
57	9.7355246	9.9238373	9.8116873	10.1883127	3
58	9.7357195	9.9237554	9.8119641	10.1880359	2
59	9.7359141	9.9236734	9.8122408	10.1877592	1
60	9.7361088	9.9235914	9.8125174	10.1874826	0
		Sin. 57.		Tang. 57.	M

M	Sin. 33.		Tang. 33.		
0	9.7361088	9.9235914	9.8125174	10.1874826	60
1	9.7363032	9.9235093	9.8127939	10.1872061	59
2	9.7364976	9.9234272	9.8130704	10.1869296	58
3	9.7366918	9.9233450	9.8133468	10.1866532	57
4	9.7368859	9.9232628	9.8136231	10.1863769	56
5	9.7370799	9.9231865	9.8138993	10.1861007	55
6	9.7372737	9.9230982	9.8141755	10.1858243	54
7	9.7374675	9.9230158	9.8144516	10.1855484	53
8	9.7376611	9.9229334	9.8147277	10.1852723	52
9	9.7378546	9.9228509	9.8150036	10.1849964	51
10	9.7380479	9.9227684	9.8152795	10.1847205	50
11	9.7382412	9.9226858	9.8155554	10.1844446	49
12	9.7384343	9.9226032	9.8158311	10.1841689	48
13	9.7386273	9.9225205	9.8161068	10.1838932	47
14	9.7388201	9.9224377	9.8163824	10.1836176	46
15	9.7390129	9.9223549	9.8166580	10.1833420	45
16	9.7392055	9.9222721	9.8169335	10.1830665	44
17	9.7393980	9.9221891	9.8172089	10.1827991	43
18	9.7395904	9.9221061	9.8174842	10.1825158	42
19	9.7397827	9.9220232	9.8177595	10.1822405	41
20	9.7399748	9.9219401	9.8180347	10.1819653	40
21	9.7401668	9.9218570	9.8183098	10.1816902	39
22	9.7403587	9.9217738	9.8185849	10.1814151	38
23	9.7405505	9.9216906	9.8188599	10.1811401	37
24	9.7407421	9.9216073	9.8191348	10.1808652	36
25	9.7409337	9.9215240	9.8194096	10.1805904	35
26	9.7411251	9.9214406	9.8196844	10.1803156	34
27	9.7413164	9.9213572	9.8199592	10.1800408	33
28	9.7415075	9.9212737	9.8202338	10.1797662	32
29	9.7416986	9.9211902	9.8205084	10.1794916	31
30	9.7418895	9.9211066	9.8207829	10.1792171	30
		Sin. 56.		Tang. 56.	M

M	Sin. 33.		Tang. 33.		
30	9.7418895	9.9211066	9.8207829	10.1792171	30
31	9.7420803	9.9210229	9.8210574	10.1789426	29
32	9.7422710	9.9209393	9.8213317	10.1786683	28
33	9.7424616	9.9208555	9.8216060	10.1783940	27
34	9.7426520	9.9207717	9.8218803	10.1781197	26
35	9.7428423	9.9206876	9.8221545	10.1778455	25
36	9.7430325	9.9206039	9.8224286	10.1775714	24
37	9.7432226	9.9205200	9.8227026	10.1772974	23
38	9.7434126	9.9204360	9.8229766	10.1770234	22
39	9.7436024	9.9203519	9.8232505	10.1767495	21
40	9.7437921	9.9202678	9.8235244	10.1764756	20
41	9.7439817	9.9201836	9.8237981	10.1762019	19
42	9.7441712	9.9200994	9.8240719	10.1759281	18
43	9.7443606	9.9200151	9.8243455	10.1756545	17
44	9.7445498	9.9199308	9.8246191	10.1753809	16
45	9.7447390	9.9198464	9.8248926	10.1751074	15
46	9.7449280	9.9197619	9.8251660	10.1748340	14
47	9.7451169	9.9196775	9.8254394	10.1745606	13
48	9.7453056	9.9195929	9.8257127	10.1742873	12
49	9.7454943	9.9195083	9.8259860	10.1740140	11
50	9.7456828	9.9194237	9.8262592	10.1737408	10
51	9.7458712	9.9193390	9.8265323	10.1734677	9
52	9.7460595	9.9192542	9.8268053	10.1731947	8
53	9.7462477	9.9191694	9.8270783	10.1729217	7
54	9.7464358	9.9190845	9.8273513	10.1726487	6
55	9.7466237	9.9189996	9.8276241	10.1723759	5
56	9.7468115	9.9189146	9.8278969	10.1721031	4
57	9.7469992	9.9188296	9.8281696	10.1718304	3
58	9.7471868	9.9187445	9.8284423	10.1715577	2
59	9.7473742	9.9186594	9.8287149	10.1712851	1
60	9.7475617	9.9185742	9.8289874	10.1720126	0
		Sin. 56.		Tang. 56.	M

M	Sin. 34.		Tang. 34.		
0	9.7475617	9.9185742	9.8239874	10.1710126	60
1	9.7477489	9.9184890	9.8292599	10.1707401	59
2	9.7479360	9.9184037	9.8295323	10.1704677	58
3	9.7481230	9.9183183	9.8298047	10.1701953	57
4	9.7483099	9.9182329	9.8300769	10.1699231	56
5	9.7484967	9.9181475	9.8303492	10.1696508	55
6	9.7486833	9.9180620	9.8306213	10.1693787	54
7	9.7488699	9.9179764	9.8308934	10.1691066	53
8	9.7490562	9.9178908	9.8311654	10.1688346	52
9	9.7492425	9.9178051	9.8314374	10.1685626	51
10	9.7494287	9.9177194	9.8317093	10.1682907	50
11	9.7496148	9.9176336	9.8319811	10.1680189	49
12	9.7498007	9.9175478	9.8322529	10.1677471	48
13	9.7499866	9.9174619	9.8325246	10.1674754	47
14	9.7501723	9.9173760	9.8327963	10.1672037	46
15	9.7503579	9.9172900	9.8330679	10.1669321	45
16	9.7505434	9.9172040	9.8333394	10.1666606	44
17	9.7507287	9.9171179	9.8336109	10.1663891	43
18	9.7509140	9.9170317	9.8338823	10.1661177	42
19	9.7510991	9.9169453	9.8341536	10.1658404	41
20	9.7512842	9.9168593	9.8344249	10.1655751	40
21	9.7514691	9.9167730	9.8346961	10.1653039	39
22	9.7516538	9.9166866	9.8349673	10.1650327	38
23	9.7518385	9.9166002	9.8352384	10.1647616	37
24	9.7520231	9.9165137	9.8355094	10.1644906	36
25	9.7522075	9.9164272	9.8357804	10.1642196	35
26	9.7523919	9.9163406	9.8360513	10.1639487	34
27	9.7525761	9.9162539	9.8363221	10.1636779	33
28	9.7527603	9.9161673	9.8365929	10.1634071	32
29	9.7529442	9.9160805	9.8368636	10.1631364	31
30	9.7531280	9.9159937	9.8371342	10.1628657	30
		Sin. 55.		Tang. 55.	M

M	*Sin.* 34.		*Tang.* 34.		
30	9.7531280	9.9159937	9.8371343	10.1626857	30
31	9.7533118	9.9159069	9.8374049	10.1625951	29
32	9.7534954	9.9158200	9.8376755	10.1623245	28
33	9.7536790	9.9157330	9.8379460	10.1620540	27
34	9.7538624	9.9156460	9.8382164	10.1617836	26
35	9.7540457	9.9155589	9.8384867	10.1615133	25
36	9.7542288	9.9154718	9.8387571	10.1612429	24
37	9.7544119	9.9153846	9.8390273	10.1609727	23
38	9.7545949	9.9152974	9.8392975	10.1607025	22
39	9.7547777	9.9152101	9.8395676	10.1604324	21
40	9.7549604	9.9151228	9.8398377	10.1601623	20
41	9.7551431	9.9150354	9.8401077	10.1598923	19
42	9.7553256	9.9149479	9.8403776	10.1596224	18
43	9.7555080	9.9148604	9.8406475	10.1593525	17
44	9.7556902	9.9147729	9.8409174	10.1590826	16
45	9.7558724	9.9146852	9.8411871	10.1588129	15
46	9.7569544	9.9145971	9.8414569	10.1585431	14
47	9.7562364	9.9145099	9.8417265	10.1582735	13
48	9.7564182	9.9144221	9.8419961	10.1580039	12
49	9.7565999	9.9143342	9.8422657	10.1577343	11
50	9.7567815	9.9142464	9.8425351	10.1584649	10
51	9.7569630	9.9141584	9.8428046	10.1571954	9
52	9.7571444	9.9140704	9.8430739	10.1569261	8
53	9.7573256	9.9139824	9.8433432	10.1566568	7
54	9.7575068	9.9138943	9.8436125	10.1563875	6
55	9.7576878	9.9138061	9.8438817	10.1561183	5
56	9.7578687	9.9137179	9.8441508	10.1558492	4
57	9.7580495	9.9136296	9.8444199	10.1555801	3
58	9.7582302	9.9135413	9.8446889	10.1553111	2
59	9.7584108	9.9134530	9.8449569	10.1550421	1
60	9.7585913	9.9133645	9.8452268	10.1547732	0
		Sin. 55.		*Tang.* 55.	M

M	Sin. 35.		Tang. 35.		
0	9.7585913	9.9133645	9.8452268	10.1547732	60
1	9.7587717	9.9132760	9.8454956	10.1545044	59
2	9.7589519	9.9131875	9.8457644	10.1542356	58
3	9.7591321	9.9130989	9.8460332	10.1539668	57
4	9.7593121	9.9130102	9.8463018	10.1536982	56
5	9.7594920	9.9129215	9.8465705	10.1534295	55
6	9.7596718	9.9128328	9.8468390	10.1531610	54
7	9.7598515	9.9127440	9.8471075	10.1528925	53
8	9.7600311	9.9126551	9.8473760	10.1526240	52
9	9.7602106	9.9125662	9.8476444	10.1523556	51
10	9.7603899	9.9124772	9.8479127	10.1520873	50
11	9.7605692	9.9123882	9.8481810	10.1518190	49
12	9.7607483	9.9122991	9.8484492	10.1515508	48
13	9.7609274	9.9122099	9.8487174	10.1512826	47
14	9.7611063	9.9121207	9.8489855	10.1510145	46
15	9.7612851	9.9120315	9.8492536	10.1507464	45
16	9.7614638	9.9119422	9.8495216	10.1504784	44
17	9.7616424	9.9118528	9.8497896	10.1502104	43
18	9.7618208	9.9117634	9.8500575	10.1499425	42
19	9.7619992	9.9116739	9.8503253	10.1496747	41
20	9.7621775	9.9115844	9.8505931	10.1494069	40
21	9.7623556	9.9114948	9.8508608	10.1491392	39
22	9.7625337	9.9114051	9.8511285	10.1488715	38
23	9.7627116	9.9113155	9.8513961	10.1486039	37
24	9.7628894	9.9112257	9.8516637	10.1483363	36
25	9.7630671	9.9111359	9.8519312	10.1480688	35
26	9.7632447	9.9110460	9.8521987	10.1478013	34
27	9.7634222	9.9109561	9.8524661	10.1475339	33
28	9.7635996	9.9108661	9.8527335	10.1472665	32
29	9.7637769	9.9107761	9.8530008	10.1469992	31
30	9.7639540	9.9106860	9.8532680	10.1467320	30
		Sin. 54.		Tang. 54.	M

M	Sin. 35.		Tang. 35.		
30	9.7639540	9.9106860	9.8532680	10.1467320	30
31	9.7641311	9.9105959	9.8535352	10.1464648	29
32	9.7643080	9.9105057	9.8538023	10.1461977	28
33	9.7644849	9.9104155	9.8540694	10.1459306	27
34	9.7646616	9.9103251	9.8543365	10.1456635	26
35	9.7648382	9.9102348	9.8546034	10.1453966	25
36	9.7650147	9.9101444	9.8548704	10.1451296	24
37	9.7651911	9.9100538	9.8551372	10.1448628	23
38	9.7653674	9.9099633	9.8554041	10.1445959	22
39	9.7655436	9.9098728	9.8556708	10.1443292	21
40	9.7657197	9.9097821	9.8559376	10.1440624	20
41	9.7658957	9.9096915	9.8562042	10.1437958	19
42	9.7660715	9.9096007	9.8564708	10.1435292	18
43	9.7662473	9.9095099	9.8567374	10.1432626	17
44	9.7664229	9.9094190	9.8570039	10.1429961	16
45	9.7665985	9.9093281	9.8572704	10.1427296	15
46	9.7667739	9.9092371	9.8575368	10.1424632	14
47	9.7669492	9.9091461	9.8578030	10.1421969	13
48	9.7671244	9.9090550	9.8580694	10.1419306	12
49	9.7672996	9.9089639	9.8583357	10.1416643	11
50	9.7674746	9.9088727	9.8586019	10.1413981	10
51	9.7676494	9.9087814	9.8588680	10.1411320	9
52	9.7678242	9.9086901	9.8591341	10.1408659	8
53	9.7679989	9.9085988	9.8594002	10.1405998	7
54	9.7681735	9.9085073	9.8596661	10.1403339	6
55	9.7683480	9.9084159	9.8599321	10.1400679	5
56	9.7685223	9.9083243	9.8601980	10.1398020	4
57	9.7686966	9.9082327	9.8604638	10.1395362	3
58	9.7688707	9.9081411	9.8607296	10.1392704	2
59	9.7690448	9.9080494	9.8609954	10.1390046	1
60	9.7692187	9.9079576	9.8612610	10.1387390	0
		Sin. 54.		Tang. 54.	M

M	Sin. 36.		Tang. 36.		
0	9.7692187	9.9079576	9.8612610	10.1387390	60
1	9.7693925	9.9078650	9.8615267	10.1384733	59
2	9.7695662	9.9077740	9.8617923	10.1382077	58
3	9.7697398	9.9076820	9.8620578	10.1379422	57
4	9.7699134	9.9075901	9.8623233	10.1376767	56
5	9.7700868	9.9074980	9.8625887	10.1374113	55
6	9.7702601	9.9074059	9.8628541	10.1371459	54
7	9.7704332	9.9073138	9.8631195	10.1368805	53
8	9.7706063	9.9072216	9.8633848	10.1366152	52
9	9.7707793	9.9071293	9.8636500	10.1363500	51
10	9.7709522	9.9070370	9.8639152	10.1360848	50
11	9.7711249	9.9069446	9.8641803	10.1358197	49
12	9.7712976	9.9068522	9.8644454	10.1355546	48
13	9.7714702	9.9067597	9.8647105	10.1352895	47
14	9.7716426	9.9066671	9.8649755	10.1350245	46
15	9.7718150	9.9065745	9.8652404	10.1347596	45
16	9.7719872	9.9064819	9.8655053	10.1344948	44
17	9.7721593	9.9063892	9.8657702	10.1342298	43
18	9.7723314	9.9062964	9.8660350	10.1339650	42
19	9.7725033	9.9062036	9.8662997	10.1337003	41
20	9.7726751	9.9061107	9.8665644	10.1334356	40
21	9.7728468	9.9060177	9.8668291	10.1331709	39
22	9.7730185	9.9059247	9.8670937	10.1329063	38
23	9.7731900	9.9058317	9.8673583	10.1326417	37
24	9.7733614	9.9057386	9.8676228	10.1323772	36
25	9.7735327	9.9056454	9.8678873	10.1321127	35
26	9.7737039	9.9055522	9.8681517	10.1318483	34
27	9.7738749	9.9054589	9.8684160	10.1315840	33
28	9.7740459	9.9053656	9.8686804	10.1313196	32
29	9.7742168	9.9052722	9.8689446	10.1310554	31
30	9.7743876	9.9051787	9.8692089	10.1307911	30
		Sin. 53.		Tang. 53.	M

M	Sin. 36.		Tang. 36.		
30	9.7743876	9.9051787	9.8692089	10.1307911	30
31	9.7745583	9.9050852	9.8694831	10.1305269	29
32	9.7747288	9.9049916	9.8697372	10.1302628	28
33	9.7748993	9.9048980	9.8700013	10.1299987	27
34	9.7750697	9.9048043	9.8702653	10.1297347	26
35	9.7752390	9.9047106	9.8705293	10.1294707	25
36	9.7754101	9.9046168	9.8707933	10.1292067	24
37	9.7755801	9.9045230	9.8710572	10.1289428	23
38	9.7757501	9.9044291	9.8713210	10.1286790	22
39	9.7759199	9.9043351	9.8715848	10.1284152	21
40	9.7760897	9.9042411	9.8718486	10.1281514	20
41	9.7762593	9.9041470	9.8721123	10.1278877	19
42	9.7764289	9.9040529	9.8723760	10.1276240	18
43	9.7765983	9.9039587	9.8726396	10.1273604	17
44	9.7767676	9.9038644	9.8729032	10.1270968	16
45	9.7769369	9.9037701	9.8731668	10.1268332	15
46	9.7771060	9.9036758	9.8734302	10.1265698	14
47	9.7772750	9.9035813	9.8736937	10.1263063	13
48	9.7774439	9.9034868	9.8739571	10.1260429	12
49	9.7776128	9.9033923	9.8742204	10.1257796	11
50	9.7777815	9.9032977	9.8744838	10.1255162	10
51	9.7779501	9.9032031	9.8747470	10.1252530	9
52	9.7781186	9.9031084	9.8750102	10.1249898	8
53	9.7782870	9.9030136	9.8752734	10.1247266	7
54	9.7784553	9.9029188	9.8755365	10.1244635	6
55	9.7786235	9.9028239	9.8757996	10.1242004	5
56	9.7787916	9.9027289	9.8760627	10.1239373	4
57	9.7789596	9.9026339	9.8763257	10.1236743	3
58	9.7791275	9.9025389	9.8765886	10.1234114	2
59	9.7792953	9.9024438	9.8768515	10.1231485	1
60	9.7794630	9.9023486	9.8771144	10.1228856	0
		Sin. 53.		Tang. 53.	M

M	*Sin.* 37.		*Tang.* 37.		
0	9.7794630	9.9023486	9.8771144	10.1228856	60
1	9.7796306	9.9022534	9.8773772	10.1226228	59
2	9.7797981	9.9021581	9.8776400	10.1223600	58
3	9.7799655	9.9020628	9.8779027	10.1220973	57
4	9.7801328	9.9019674	9.8781654	10.1218345	56
5	9.7803000	9.9018719	9.8784281	10.1215719	55
6	9.7804671	9.9017764	9.8786907	10.1213093	54
7	9.7806341	9.9016808	9.8789533	10.1210467	53
8	9.7808010	9.9015852	9.8792158	10.1207842	52
9	9.7809677	9.9014895	9.8794782	10.1205218	51
10	9.7811344	9.9013938	9.8797407	10.1202592	50
11	9.7813010	9.9012980	9.8800031	10.1199969	49
12	9.7814675	9.9012021	9.8802654	10.1197346	48
13	9.7816339	9.9011062	9.8805277	10.1194723	47
14	9.7818002	9.9010102	9.8807900	10.1192100	46
15	9.7819664	9.9009142	9.8810522	10.1189478	45
16	9.7821324	9.9008181	9.8813144	10.1186856	44
17	9.7822984	9.9007219	9.8815765	10.1184235	43
18	9.7824643	9.9006257	9.8818386	10.1181614	42
19	9.7826301	9.9005294	9.8821007	10.1178993	41
20	9.7827958	9.9004331	9.8823627	10.1176374	40
21	9.7829614	9.9003367	9.8826246	10.1173754	39
22	9.7831268	9.9002403	9.8828866	10.1171134	38
23	9.7832922	9.9001438	9.8831484	10.1168516	37
24	9.7834575	9.9000472	9.8834103	10.1165897	36
25	9.7836227	9.8999506	9.8836721	10.1163279	35
26	9.7837878	9.8998539	9.8839338	10.1160662	34
27	9.7839528	9.8997572	9.8841956	10.1158044	33
28	9.7841877	9.8996604	9.8844572	10.1155428	32
29	9.7842824	9.8995636	9.8847189	10.1152811	31
30	9.7844471	9.8994667	9.8849805	10.1150195	30
		Sin. 52.		*Tang.* 52.	M

M	Sin. 37.		Tang. 36.		
30	9.7844471	9.8994667	9.8849805	10.1150195	30
31	9.7846117	9.8993697	9.8852420	10.1147508	29
32	9.7847762	9.8992727	9.8855035	10.1144965	28
33	9.7849406	9.8991756	9.8857650	10.1142350	27
34	9.7851049	9.8990784	9.8860264	10.1139736	26
35	9.7852691	9.8989812	9.8862878	10.1137122	25
36	9.7854332	9.8988840	9.8865492	10.1134508	24
37	9.7855972	9.8987867	9.8868105	10.1131895	23
38	9.7857611	9.8986893	9.8870718	10.1129282	22
39	9.7859249	9.8985919	9.8873330	10.1126670	21
40	9.7860886	9.8984944	9.8875942	10.1124058	20
41	9.7862522	9.8983968	9.8878554	10.1121446	19
42	9.7864157	9.8982992	9.8881165	10.1118835	18
43	9.7865791	9.8982015	9.8883775	10.1116225	17
44	9.7867424	9.8981038	9.8886386	10.1113614	16
45	9.7869056	9.8980060	9.8888996	10.1111004	15
46	9.7870687	9.8979082	9.8891605	10.1108395	14
47	9.7872317	9.8978103	9.8894214	10.1105786	13
48	9.7873946	9.8977123	9.8896823	10.1103177	12
49	9.7875574	9.8976143	9.8899432	10.1100568	11
50	9.7877202	9.8975162	9.8902040	10.1097960	10
51	9.7878828	9.8974181	9.8904647	10.1095353	9
52	9.7880453	9.8973199	9.8907254	10.1092746	8
53	9.7882077	9.8972216	9.8909861	10.1090139	7
54	9.7883701	9.8971233	9.8912468	10.1087532	6
55	9.7885323	9.8970249	9.8915074	10.1084926	5
56	9.7886944	9.8969265	9.8917679	10.1082321	4
57	9.7888565	9.8968280	9.8920285	10.1079715	3
58	9.7890184	9.8967294	9.8922890	10.1077110	2
59	9.7891802	9.8966308	9.8925494	10.1074506	1
60	9.7893420	9.8965321	9.8928098	10.1071902	0
		Sin. 52.		Tang. 52.	M

M	Sin. 38.		Tang. 38.		
0	9.7893420	9.8965321	9.8928098	10.1071902	60
1	9.7895036	9.8964334	9.8930702	10.1069298	59
2	9.7896652	9.8963346	9.8933306	10.1066694	58
3	9.7898266	9.8962358	9.8935909	10.1064091	57
4	9.7899880	9.8961369	9.8938511	10.1061489	56
5	9.7901493	9.8960379	9.8941114	10.1058886	55
6	9.7903104	9.8959389	9.8943715	10.1056285	54
7	9.7904715	9.8958398	9.8946317	10.1053683	53
8	9.7906325	9.8957406	9.8948918	10.1051082	52
9	9.7907933	9.8956414	9.8951519	10.1048481	51
10	9.7909541	9.8955422	9.8954119	10.1045881	50
11	9.7911148	9.8954429	9.8956719	10.1043281	49
12	9.7912754	9.8953435	9.8959319	10.1040081	48
13	9.7914359	9.8952440	9.8961918	10.1038083	47
14	9.7915963	9.8951445	9.8964517	10.1035483	46
15	9.7917566	9.8950450	9.8967116	10.1032884	45
16	9.7919168	9.8949453	9.8969714	10.1030286	44
17	9.7920769	9.8948457	9.8972312	10.1027688	43
18	9.7922369	9.8947459	9.8974910	10.1025090	42
19	9.7923968	9.8946461	9.8977507	10.1022493	41
20	9.7925566	9.8945463	9.8980104	10.1019896	40
21	9.7927163	9.8944463	9.8982700	10.1017300	39
22	9.7928760	9.8943464	9.8985296	10.1014704	38
23	9.7930355	9.8942463	9.8987892	10.1012108	37
24	9.7931949	9.8941462	9.8990487	10.1009513	36
25	9.7933543	9.8940461	9.8993082	10.1006918	35
26	9.7935135	9.8939458	9.8995677	10.1004323	34
27	9.7936727	9.8938456	9.8998271	10.1001729	33
28	9.7938317	9.8937452	9.9000865	10.0999135	32
29	9.7939907	9.8936448	9.9003459	10.0996541	31
30	9.7941496	9.8935444	9.9006052	10.0993948	30
		Sin. 51.		Tang. 51.	M

M	Sin. 38.		Tang. 38.		
30	9.7941496	9.8935444	9.9006052	10.0993948	30
31	9.7943083	9.8934439	9.9008645	10.0991355	29
32	9.7944670	9.8933433	9.9011237	10.0988763	28
33	9.7946256	9.8932426	9.9013830	10.0986170	27
34	9.7947841	9.8931419	9.9016422	10.0983578	26
35	9.7949425	9.8930412	9.9019013	10.0980987	25
36	9.7951008	9.8929404	9.9021604	10.0978396	24
37	9.7952590	9.8928395	9.9024195	10.0975805	23
38	9.7954171	9.8927385	9.9026786	10.0973214	22
39	9.7955751	9.8926375	9.9029376	10.0970624	21
40	9.7957330	9.8925365	9.9031966	10.0968034	20
41	9.7958909	9.8924354	9.9034555	10.0965445	19
42	9.7960486	9.8923342	9.9037144	10.0962856	18
43	9.7962062	9.8922329	9.9039733	10.0960267	17
44	9.7963638	9.8921316	9.9042321	10.0957679	16
45	9.7965212	9.8920303	9.9044910	10.0955090	15
46	9.7966786	9.8919289	9.9047497	10.0952503	14
47	9.7968359	9.8918274	9.9050085	10.0949915	13
48	9.7969930	9.8917258	9.9052672	10.0947328	12
49	9.7971501	9.8916242	9.9055259	10.0944741	11
50	9.7973071	9.8915226	9.9057845	10.0942155	10
51	9.7974640	9.8914208	9.9060431	10.0939569	9
52	9.7976208	9.8913191	9.9063017	10.0936983	8
53	9.7977775	9.8912172	9.9065603	10.0934397	7
54	9.7979341	9.8911153	9.9068188	10.0931812	6
55	9.7980906	9.8910133	9.9070773	10.0929227	5
56	9.7982470	9.8909113	9.9073357	10.0926643	4
57	9.7984034	9.8908092	9.9075942	10.0924059	3
58	9.7985596	9.8907071	9.9078525	10.0921475	2
59	9.7987158	9.8906049	9.9081109	10.0918891	1
60	9.7988718	9.8905026	9.9083692	10.0916308	0
		Sin. 51.		Tang. 51.	M

M	Sin. 39.		Tang. 39.		
0	9.7988718	9.8905026	9.9083692	10.0916308	60
1	9.7990278	9.8904003	9.9086275	10.0913725	59
2	9.7991836	9.8902979	9.9088858	10.0911141	58
3	9.7993394	9.8901954	9.9091440	10.0908560	57
4	9.7994951	9.8900929	9.9094022	10.0905978	56
5	9.7996507	9.8899903	9.9096603	10.0903397	55
6	9.7998062	9.8898877	9.9099185	10.0900815	54
7	9.7999616	9.8897850	9.9101766	10.0898234	53
8	9.8001169	9.8896822	9.9104347	10.0895653	52
9	9.8002721	9.8895794	9.9106927	10.0893073	51
10	9.8004272	9.8894765	9.9109507	10.0890493	50
11	9.8005823	9.8893736	9.9112087	10.0887913	49
12	9.8007372	9.8892706	9.9114666	10.0885334	48
13	9.8008921	9.8891675	9.9117245	10.0882755	47
14	9.8010468	9.8890644	9.9119824	10.0880176	46
15	9.8012015	9.8889612	9.9122403	10.0877597	45
16	9.8013561	9.8888580	9.9124981	10.0875019	44
17	9.8015106	9.8887547	9.9127559	10.0872441	43
18	9.8016649	9.8886513	9.9130127	10.0869863	42
19	9.8018190	9.8885479	9.9132714	10.0867286	41
20	9.8019735	9.8884444	9.9135291	10.0864709	40
21	9.8021276	9.8883408	9.9137868	10.0862132	39
22	9.8022816	9.8882372	9.9140444	10.0856556	38
23	9.8024355	9.8881335	9.9143020	10.0856980	37
24	9.8025894	9.8880298	9.9145596	10.0854404	36
25	9.8027431	9.8879260	9.9148171	10.0851829	35
26	9.8028968	9.8878221	9.9150747	10.0849253	34
27	9.8030504	9.8877182	9.9153322	10.0846678	33
28	9.8032038	9.8876142	9.9155896	10.0844104	32
29	9.8033572	9.8875102	9.9158471	10.0841529	31
30	9.8035105	9.8874061	9.9161045	10.0838955	30
		Sin. 50.		Tang. 50.	M

M	Sin. 39.		Tang. 39.		
30	9.8035105	9.8874061	9.9161045	10.0838955	30
31	9.8036637	9.8873019	9.9163618	10.0836382	29
32	9.8038168	9.8871977	9.9166192	10.0833808	28
33	9.8039699	9.8870934	9.9168765	10.0831235	27
34	9.8041228	9.8869890	9.9171338	10.0828662	26
35	9.8042757	9.8868846	9.9173911	10.0826089	25
36	9.8044284	9.8867801	9.9176483	10.0823517	24
37	9.8045811	9.8866756	9.9179055	10.0820945	23
38	9.8047336	9.8865710	9.9181627	10.0818373	22
39	9.8048861	9.8864663	9.9184198	10.0815802	21
40	9.8050385	9.8863616	9.9186769	10.0813231	20
41	9.8051908	9.8862568	9.9189340	10.0810660	19
42	9.8053430	9.8861519	9.9191911	10.0808089	18
43	9.8054951	9.8860470	9.9194481	10.0805519	17
44	9.8056472	9.8859420	9.9197051	10.0802949	16
45	9.8057991	9.8858370	9.9199621	10.0800379	15
46	9.8059510	9.8857319	9.9202191	10.0797806	14
47	9.8061027	9.8856267	9.9204760	10.0795240	13
48	9.8062544	9.8855215	9.9207329	10.0792671	12
49	9.8064060	9.8854162	9.9209898	10.0790102	11
50	9.8065575	9.8853109	9.9212466	10.0787534	10
51	9.8067089	9.8852055	9.9215034	10.0784966	9
52	9.8068601	9.8851000	9.9217602	10.0782398	8
53	9.8070114	9.8849945	9.9220170	10.0779830	7
54	9.8071626	9.8848889	9.9222737	10.0777263	6
55	9.8073136	9.8847832	9.9225304	10.0774696	5
56	9.8074646	9.8846775	9.9227871	10.0772129	4
57	9.8076154	9.8845717	9.9230437	10.0769563	3
58	9.8077662	9.8844659	9.9233004	10.0766996	2
59	9.8079169	9.8843599	9.9235570	10.0764430	1
60	9.8080675	9.8842540	9.9238135	10.0761865	0
		Sin. 50.		Tang. 50.	M

M	*Sin.* 40.		*Tang.* 40.		
0	9.8080675	9.8842540	9.9238135	10.0761865	60
1	9.8082180	9.8841479	9.9240701	10.0759299	59
2	9.8083684	9.8840418	9.9243266	10.0756734	58
3	9.8085188	9.8839357	9.9245831	10.0754169	57
4	9.8086690	9.8838294	9.9248396	10.0751604	56
5	9.8088192	9.8837232	9.9250960	10.0749040	55
6	9.8089692	9.8836168	9.9253524	10.0746476	54
7	9.8091192	9.8835104	9.9256088	10.0743912	53
8	9.8092691	9.8834039	9.9258652	10.0741348	52
9	9.8094189	9.8832974	9.9261215	10.0738785	51
10	9.8095686	9.8831908	9.9263778	10.0736222	50
11	9.8097182	9.8830841	9.9266341	10.0733659	49
12	9.8098678	9.8829774	9.9268904	10.0731096	48
13	9.8100172	9.8828706	9.9271466	10.0728534	47
14	9.8101666	9.8827638	9.9274028	10.0725972	46
15	6.8103159	9.8826568	9.9276590	10.0723410	45
16	9.8104650	9.8825499	9.9179152	10.0720848	44
17	9.8106141	9.8824428	9.9281713	10.0718287	43
18	9.8107631	9.8823357	9.9284274	10.0715726	42
19	9.8109121	9.8822285	9.9286835	10.0713165	41
20	9.8110609	9.8821213	9.9289396	10.0710604	40
21	9.8112096	9.8820140	9.9291956	10.0708044	39
22	9.8113583	9.8819067	9.9294516	10.0705484	38
23	9.8115069	9.8817992	9.9297076	10.0702924	37
24	9.8116554	9.8816918	9.9299636	10.0700364	36
25	9.8118038	9.8815842	9.9302195	10.0697805	35
26	9.8119521	9.8814766	9.9304755	10.0695245	34
27	9.8121003	9.8813689	9.9307314	10.0692686	33
28	9.8122484	9.8812612	9.9309872	10.0690128	32
29	9.8123965	9.8811534	9.9312431	10.0687569	31
30	9.8125444	9.8810455	9.9314989	10.0685011	30
		Sin. 49.		*Tang.* 49.	M

M	Sin. 40.		Tang. 40.		
30	9.8125444	9.8810455	9.9314989	10.0685011	30
31	9.8126923	9.8809376	9.9317547	10.0682453	29
32	9.8128401	9.8808296	9.9320105	10.0679895	28
33	9.8129878	9.8807215	9.9322662	10.0677338	27
34	9.8131354	9.8806134	9.9325220	10.0674780	26
35	9.8132829	9.8805052	9.9327777	10.0672223	25
36	9.8134303	9.8803970	9.9330334	10.0669666	24
37	9.8135777	9.8802887	9.9332890	10.0667110	23
38	9.8137250	9.8801803	9.9335446	10.0664554	22
39	9.8138721	9.8800719	9.9338003	10.0661997	21
40	9.8140192	9.8799634	9.9340559	10.0659441	20
41	9.8141662	9.8798548	9.9343114	10.0656886	19
42	9.8143131	9.8797462	9.9345670	10.0654330	18
43	9.8144600	9.8796375	9.9348225	10.0651775	17
44	9.8146067	9.8795287	9.9350780	10.0649220	16
45	9.8147534	9.8794199	9.9353335	10.0646665	15
46	9.8148999	9.8793110	9.9355889	10.0644111	14
47	9.8150464	9.8792021	9.9358444	10.0641556	13
48	9.8151928	9.8790930	9.9360998	10.0639002	12
49	9.8153391	9.8789840	9.9363552	10.0636448	11
50	9.8154854	9.8788748	9.9366105	10.0633895	10
51	9.8156315	9.8787656	9.9368659	10.0631341	9
52	9.8157776	9.8786563	9.9371212	10.0628788	8
53	9.8159235	9.8785470	9.9373765	10.0626235	7
54	9.8160694	9.8784376	9.9376318	10.0623682	6
55	9.8162152	9.8783281	9.9378871	10.0621129	5
56	9.8163609	9.8782186	9.9381423	10.0618577	4
57	9.8165066	9.8781090	9.9383975	10.0616025	3
58	9.8166521	9.8779994	9.9386527	10.0613473	2
59	9.8167975	9.8778896	9.9389079	10.0610921	1
60	9.8169425	9.8777799	9.9391631	10.0608369	0
		Sin. 49.		Tang. 49.	M

M	Sin. 41.		Tang. 41.		
0	9.8169429	9.8777799	9.9391630	10.0608369	60
1	9.8170882	9.8776700	9.9394182	10.0605818	59
2	9.8172334	9.8775601	9.9396733	10.0603267	58
3	9.8173785	9.8774501	9.9399284	10.0600716	57
4	9.8175235	9.8773401	9.9401835	10.0598165	56
5	9.8176685	9.8772300	9.9404385	10.0595615	55
6	9.8178133	9.8771198	9.9406936	10.0593064	54
7	9.8179581	9.8770096	9.9409486	10.0590514	53
8	9.8181028	9.8768993	9.9412036	10.0587964	52
9	9.8182474	9.8767889	9.9414585	10.0585415	51
10	9.8183919	9.8766785	9.9417135	10.0582865	50
11	9.8185364	9.8765680	9.9419684	10.0580316	49
12	9.8186807	9.8764574	9.9422233	10.0577767	48
13	9.8188250	9.8763468	9.9424782	10.0575218	47
14	9.8189692	9.8762361	9.9427331	10.0572669	46
15	9.8191133	9.8761253	9.9429879	10.0570121	45
16	9.8192573	9.8760145	9.9432428	10.0567572	44
17	9.8194012	9.8759036	9.9434976	10.0565024	43
18	9.8195450	9.8757927	9.9437524	10.0562476	42
19	9.8196887	9.8756816	9.9440072	10.0559928	41
20	9.8198325	9.8755706	9.9442619	10.0557381	40
21	9.8199761	9.8754594	9.9445166	10.0554834	39
22	9.8201196	9.8753482	9.9447714	10.0552286	38
23	9.8202630	9.8752369	9.9450261	10.0549739	37
24	9.8204063	9.8751256	9.9442807	10.0547193	36
25	9.8205496	9.8750142	9.9455354	10.0544646	35
26	9.8206927	9.8749027	9.9457900	10.0542100	34
27	9.8208358	9.8747912	9.9460447	10.0539553	33
28	9.8209788	9.8746795	9.9462993	10.0537007	32
29	9.8211217	9.8745679	9.9465539	10.0534461	31
30	9.8212646	9.8744561	9.9468084	10.0531916	30
		Sin. 48.		Tang. 48.	M

M	*Sin.* 41.		*Tang.* 41.		
30	9.8212646	9.8744561	9.9468084	10.0531916	30
31	9.8214073	9.8743443	9.9470630	10.0529370	29
32	9.8215500	9.8742325	9.9473175	10.0526825	28
33	9.8216926	9.8741205	9.9475720	10.0524280	27
34	9.8218351	9.8740085	9.9478265	10.0521735	26
35	9.8219775	9.8738965	9.9480810	10.0519190	25
36	9.8221198	9.8737844	9.9483355	10.0516645	24
37	9.8222621	9.8736722	9.9485899	10.0514101	23
38	9.8224042	9.8735599	9.9488443	10.0511557	22
39	9.8225463	9.8734476	9.9490987	10.0509013	21
40	9.8226883	9.8733352	9.9493531	10.0506469	20
41	9.8228302	9.8732227	9.9496075	10.0503925	19
42	9.8229721	9.8731102	9.9498619	10.0501381	18
43	9.8231138	9.8729976	9.9501162	10.0498838	17
44	9.8232555	9.8728849	9.9503705	10.0496295	16
45	9.8233971	9.8727722	9.9506248	10.0493752	15
46	9.8235386	9.8726594	9.9508791	10.0491209	14
47	9.8236800	9.8725466	9.9511334	10.0488666	13
48	9.8238213	9.8724337	9.9513876	10.0486124	12
49	9.8239626	9.8723207	9.9516419	10.0483581	11
50	9.8241037	9.8722076	9.9518961	10.0481039	10
51	9.8242448	9.8720945	9.9521503	10.0478497	9
52	9.8243858	9.8719813	9.9524045	10.0475955	8
53	9.8245267	9.8718682	9.9526587	10.0473413	7
54	9.8246676	9.8717548	9.9529128	10.0470872	6
55	9.8248083	9.8716414	9.9531670	10.0468330	5
56	9.8249490	9.8715279	9.9534211	10.0465789	4
57	9.8250896	9.8714144	9.9536752	10.0463248	3
58	9.8252301	9.8713008	9.9539293	10.0460707	2
59	9.8253705	9.8711872	9.9541834	10.0458166	1
60	9.8255109	9.8710735	9.9544374	10.0455626	0
		Sin. 48.		*Tang.* 48.	M

M	*Sin.* 42.		*Tang.* 42.		
0	9.8255109	9.8710735	9.9544374	10.0455626	60
1	9.8256512	9.8709597	9.9546915	10.0453085	59
2	9.8257913	9.8708458	9.9549455	10.0450545	58
3	9.8259314	9.8707319	9.9551995	10.0448005	57
4	9.8260715	9.8706179	9.9554535	10.0445465	56
5	9.8262114	9.8705039	9.9557075	10.0442925	55
6	9.8263512	9.8703898	9.9559615	10.0440285	54
7	9.8264910	9.8702756	9.9562154	10.0437846	53
8	9.8266307	9.8701613	9.9564693	10.0435307	52
9	9.8267703	9.8700470	9.9567233	10.0432767	51
10	9.8269098	9.8699326	9.9569772	10.0430228	50
11	9.8270493	9.8698182	9.9572311	10.0427689	49
12	9.8271887	9.8697037	9.9574850	10.0425150	48
13	9.8273279	9.8695891	9.9577389	10.0422611	47
14	9.8274671	9.8694744	9.9579927	10.0420073	46
15	9.8276063	9.8693597	9.9582465	10.0417535	45
16	9.8277453	9.8692449	9.9589004	10.0414996	44
17	9.8278843	9.8691301	9.9587542	10.0412458	43
18	9.8280231	9.8690152	9.9590080	10.0409920	42
19	9.8281619	9.8689002	9.9592618	10.0407382	41
20	9.8283006	9.8687851	9.9595155	10.0404845	40
21	9.8284393	9.8686700	9.9597693	10.0402307	39
22	9.8285778	9.8685548	9.9600230	10.0399770	38
23	9.8287163	9.8684396	9.9602767	10.0397233	37
24	9.8288547	9.8683242	9.9605305	10.0394695	36
25	9.8289930	9.8682088	9.9607842	10.0392158	35
26	9.8291312	9.8680934	9.9610378	10.0389622	34
27	9.8292694	9.8679779	9.9612915	10.0387085	33
28	9.8294075	9.8678623	9.9615452	10.0384548	32
29	9.8295454	9.8677466	9.9617988	10.0382012	31
30	9.8296833	9.8676309	9.9620525	10.0379475	30
		Sin. 47.		*Tang.* 47.	M

M	Sin. 42.		Tang. 42.		
30	9.8296833	9.8675309	9.9620525	10.0379475	30
31	9.8298212	9.8675151	9.9623061	10.0376939	29
32	9.8299589	9.8673992	9.9625597	10.0374403	28
33	9.8300966	9.8672833	9.9628133	10.0371867	27
34	9.8302342	9.8671673	9.9630669	10.0369331	26
35	9.8303717	9.8670512	9.9633204	10.0366796	25
36	9.8305091	9.8669351	9.9635740	10.0364260	24
37	9.8306464	9.8668189	9.9638275	10.0361725	23
38	9.8307837	9.8667026	9.9640811	10.0359189	22
39	9.8309209	9.8665863	9.9643346	10.0356654	21
40	9.8310580	9.8664699	9.9645881	10.0354119	20
41	9.8311950	9.8663534	9.9648416	10.0351584	19
42	9.8313320	9.8662369	9.9650951	10.0349049	18
43	9.8314688	9.8661202	9.9653486	10.0346514	17
44	9.8316056	9.8660036	9.9656020	10.0343980	16
45	9.8317422	9.8658868	9.9658555	10.0341445	15
46	9.8318789	9.8657700	9.9661089	10.0338911	14
47	9.8320155	9.8656531	9.9663623	10.0336377	13
48	9.8321519	9.8655362	9.9666157	10.0333843	12
49	9.8322883	9.8654192	9.9668692	10.0331308	11
50	9.8324246	9.8653021	9.9671225	10.0328775	10
51	9.8325609	9.8651849	9.9673759	10.0326241	9
52	9.8326970	9.8650677	9.9676293	10.0323707	8
53	9.8328331	9.8649504	9.9678827	10.0321173	7
54	9.8329691	9.8648331	9.9681360	10.0318640	6
55	9.8331050	9.8647156	9.9683893	10.0316197	5
56	9.8332408	9.8645981	9.9686427	10.0313573	4
57	9.8333766	9.8644806	9.9688960	10.0311040	3
58	9.8335122	9.8643629	9.9691493	10.0308507	2
59	9.8336478	9.8642452	9.9694025	10.0305974	1
60	9.8337833	9.8641275	9.9696559	10.0303441	0
		Sin. 47.		Tang. 47.	M

M	*Sin.* 43.		*Tang.* 43.		
0	9.8337833	9.8641275	9.9696559	10.0303441	60
1	9.8339188	9.8640096	9.9699091	10.0300909	59
2	9.8340541	9.8638917	9.9701624	10.0298376	58
3	9.8341894	9.8637737	9.9704157	10.0295843	57
4	9.8343246	9.8636557	9.9706689	10.0293311	56
5	9.8344597	9.8635376	9.9709221	10.0290779	55
6	9.8345948	9.8634194	9.9711754	10.0288246	54
7	9.8347297	9.8633011	9.9714287	10.0285713	53
8	9.8348646	9.8631828	9.9716818	10.0283182	52
9	9.8349994	9.8630644	9.9719351	10.0280659	51
10	9.8351341	9.8629460	9.9721882	10.0278118	50
11	9.8352688	9.8628274	9.9724413	10.0275587	49
12	9.8354032	9.8627088	9.9726945	10.0273055	48
13	9.8355378	9.8625902	9.9729477	10.0270523	47
14	9.8356722	9.8624714	9.9732008	10.0267992	46
15	9.8358066	9.8623526	9.9734539	10.0265461	45
16	9.8359408	9.8622338	9.9737071	10.0262929	44
17	9.8360750	9.8621148	9.9739602	10.0260398	43
18	9.8362091	9.8619958	9.9742133	10.0257867	42
19	9.8363431	9.8618767	9.9744664	10.0255336	41
20	9.8364771	9.8617576	9.9747195	10.0252805	40
21	9.8366109	9.8616383	9.9749726	10.0250274	39
22	9.8367447	9.8615190	9.9752257	10.0247743	38
23	9.8368784	9.8613997	9.9754787	10.0245213	37
24	9.8370121	9.8612803	9.9757318	10.0242682	36
25	9.8371456	9.8611608	9.9759849	10.0240151	35
26	9.8372791	9.8610412	9.9762379	10.0237621	34
27	9.8374125	9.8609215	9.9764909	10.0235091	33
28	9.8375458	9.8608018	9.9767440	10.0232560	32
29	9.8376790	9.8606821	9.9769970	10.0230030	31
30	9.8378122	9.8605622	9.9772500	10.0227500	30
		Sin. 46.		*Tang.* 46.	M

M	*Sin.* 43.		*Tang.* 43.		
30	9.8378122	9.8605622	9.9772500	10.0227500	30
31	9.8379453	9.8604423	9.9775030	10.0224970	29
32	9.8380783	9.8603223	9.9777560	10.0222440	28
33	9.8382112	9.8602022	9.9780090	10.0219910	27
34	9.8383441	9.8600821	9.9782620	10.0217380	26
35	9.8384769	9.8599619	9.9785149	10.0214851	25
36	9.8386096	9.8598416	9.9787679	10.0212321	24
37	9.8387422	9.8597213	9.9790209	10.0209791	23
38	9.8388747	9.8596009	9.9792738	10.0207262	22
39	9.8390072	9.8594804	9.9795268	10.0204732	21
40	9.8391396	9.8593599	9.9797797	10.0202203	20
41	9.8392719	9.8592393	9.9800326	10.0199674	19
42	9.8394041	9.8591186	9.9802856	10.0197144	18
43	9.8395363	9.8589978	9.9805385	10.0194615	17
44	9.8396684	9.8588770	9.9807914	10.0192086	16
45	9.8398004	9.8587561	9.9810443	10.0189557	15
46	9.8399323	9.8586351	9.9812972	10.0187028	14
47	9.8400642	9.8585141	9.9815501	10.0184499	13
48	9.8401959	9.8583929	9.9818030	10.0181970	12
49	9.8403276	9.8582718	9.9820559	10.0179441	11
50	9.8404593	9.8581505	9.9823087	10.0176913	10
51	9.8405908	9.8580292	9.9825616	10.0174384	9
52	9.8407223	9.8579078	9.9828145	10.0171855	8
53	9.8408537	9.8577863	9.9830673	10.0169327	7
54	9.8409850	9.8576648	9.9833202	10.0166798	6
55	9.8411162	9.8575432	9.9835730	10.0164270	5
56	9.8412474	9.8574215	9.9838259	10.0161741	4
57	9.8413785	9.8572998	9.9840787	10.0159213	3
58	9.8415095	9.8571779	9.9843315	10.0156685	2
59	9.8416404	9.8570561	9.9845844	10.0154156	1
60	9.8417713	9.8569341	9.9848372	10.0151628	0
		Sin. 46.		*Tang.* 46.	M

M	Sin. 44.		Tang. 44.		
0	9.8417713	9.8569341	9.9848372	10.0151628	60
1	9.8419021	9.8568121	9.9850900	10.0149100	59
2	9.8420328	9.8566900	9.9853428	10.0146572	58
3	9.8421634	9.8565678	9.9855956	10.0144044	57
4	9.8422939	9.8564455	9.9858484	10.0141516	56
5	9.8424244	9.8563232	9.9861012	10.0138988	55
6	9.8425548	9.8562008	9.9863540	10.0136460	54
7	9.8426851	9.8560784	9.9866068	10.0133932	53
8	9.8428154	9.8559558	9.9868596	10.0131404	52
9	9.8429456	9.8558332	9.9871123	10.0128877	51
10	9.8430757	9.8557106	9.9873651	10.0126349	50
11	9.8432057	9.8555878	9.9876179	10.0123821	49
12	9.8433356	9.8554650	9.9878706	10.0121294	48
13	9.8434655	9.8553421	9.9881234	10.0118766	47
14	9.8435953	9.8552192	9.9883761	10.0116239	46
15	9.8437250	9.8550961	9.9886289	10.0113711	45
16	9.8438547	9.8549730	9.9888816	10.0111184	44
17	9.8439842	9.8548499	9.9891344	10.0108656	43
18	9.8441137	9.8547266	9.9893871	10.0106129	42
19	9.8442432	9.8546033	9.9896399	10.0103601	41
20	9.8443725	9.8544799	9.9898926	10.0101074	40
21	9.8445018	9.8543564	9.9901453	10.0098547	39
22	9.8446310	9.8542329	9.9903981	10.0096019	38
23	9.8447601	9.8541093	9.9906508	10.0093492	37
24	9.8448891	9.8539856	9.9909035	10.0090965	36
25	9.8450181	9.8538619	9.9911562	10.0088438	35
26	9.8451470	9.8537381	9.9914089	10.0085911	34
27	9.8452758	9.8536142	9.9916616	10.0083384	33
28	9.8454045	9.8534902	9.9919143	10.0080857	32
29	9.8455332	9.8533662	9.9921670	10.0078330	31
30	9.8456618	9.8532421	9.9924197	10.0075863	30
		Sin. 45.		Tang. 45.	M

M	Sin. 44.		Tang. 44.		
30	9.8456618	9.8532420	9.9924197	10.0075802	30
31	9.8457903	9.8531178	9.9926724	10.0073275	29
32	9.8459187	9.8529936	9.9929251	10.0070748	28
33	9.8460471	9.8528693	9.9931778	10.0068221	27
34	9.8461754	9.8527449	9.9934305	10.0065694	26
35	9.8463036	9.8526204	9.9936832	10.0063167	25
36	9.8464317	9.8524958	9.9939359	10.0060641	24
37	9.8465598	9.8523712	9.9941886	10.0058114	23
38	9.8466878	9.8522465	9.9944412	10.0055587	22
39	9.8468157	9.8521218	9.9946939	10.0053060	21
40	9.8469436	9.8519969	9.9949466	10.0050533	20
41	9.8470713	9.8518720	9.9951993	10.0048006	19
42	9.8471991	9.8517471	9.9954520	10.0045480	18
43	9.8473267	9.8516220	9.9957047	10.0042953	17
44	9.8474542	9.8514969	9.9959573	10.0040426	16
45	9.8475817	9.8513717	9.9962100	10.0037899	15
46	9.8477091	9.8512464	9.9964627	10.0035373	14
47	9.8478364	9.8511211	9.9967153	10.0032846	13
48	9.8479637	9.8509957	9.9969680	10.0030319	12
49	9.8480909	9.8508701	9.9972207	10.0027793	11
50	9.8482180	9.8507446	9.9974733	10.0025266	10
51	9.8483450	9.8506190	9.9977260	10.0022739	9
52	9.8484720	9.8504933	9.9979787	10.0020213	8
53	9.8485988	9.8503675	9.9982312	10.0017686	7
54	9.8487257	9.8502416	9.9984840	10.0015159	6
55	9.8488524	9.8501157	9.9987367	10.0012633	5
56	9.8489790	9.8499897	9.9989893	10.0010106	4
57	9.8491056	9.8498636	9.9992420	10.0007579	3
58	9.8492221	9.8497375	9.9994946	10.0005053	2
59	9.8493586	9.8496113	9.9997473	10.0002526	1
60	9.8494850	9.8494850	10.0000000	10.0000090	0
		Sin. 45.		Tang. 45.	M

Lectori practicæ Matheseos studioso, S.P.

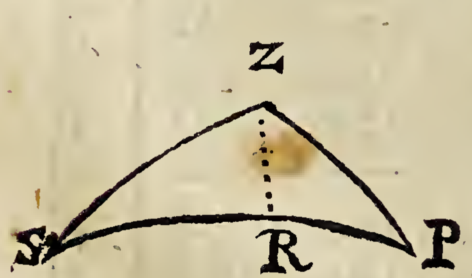
CANON noster usum habet in Triangulorum sphæricorum solutione, eundem quem tabulæ Sinuum rectorum & Tangentium ab aliis editæ, sed praxin paulo faciliorem; Nam eorum multiplicationem per additionem, & divisionem per subtractionem & extractionem radicis quadratæ per bipartitionem evitamus.

Ut si datis tribus lateribus quæratur angulus, erit

Ut Rectangulum sub sinibus crurum,
ad quadratum Radii;
Ita Rectangulum sub sinibus semisummæ trium laterum, & differentiæ inter hanc semisummam & Basin,
Ad quadratum Co-sinus semianguli quæsiti.

Ut in Triangulo P Z S, (referente Polum, Zenith & Solem) datis lateribus, P S gr. 70, *& Z P* gr. 38, *m.* 30, *& Z S* gr. 40, *si quæratur angulus P Z S, cujus Basis est P S: summa laterum erit* gr. 148, *m.* 30, *semisumma* gr. 74 *m.* 15, *differentia inter semisummam & basin,* gr. 4, m. 15.

Hic nos pro quadrato Radii ponimus 2.00000000 *Radii duplum, cui addimus* 9.9833805 *Sinum* gr. 74, m. 15, 8.8698679 *Sinum* gr. 4, m. 14, *fient* 38.8532484. *Deinde pro rectangulo divisore addentes* 9.7941495 *Sinum* gr. 38, m. 30, *&* 9.8080675 *Sinum* gr. 40 *facimus* 19.6022170, *& auferimus è* 38.8532484, *ita restant* 19.2510314. *Horum semissis est* 9.6255157 *Sinus semianguli externi* gr. 24, m.58, s.24: *& Co-sinus semianguli interni* gr. 65, m. 1, s. 36, *& proinde totus angulus quæsitus est* gr. 130, m. 3, s. 12.



Quod

Quod si quis pro Sinibus auferendis addat eorum complementa ad Radium, non alia indigebit subtractione ; ut patere potest ex collatione utriusq; praxeos.

Gr. M.			
70 00	20.0000000		
38 30	9.7941495		.2058505
40 00	9.8080675		.1919325
148 30	19.6022170		
74 15	9.9833805		9.9833805
04 15	8.8698679		8.8698679
	20.0000000		
	38.8532484		
Gr. M. S.	19.2510314	Gr. M. S.	19.2510314
24 58 24	9.6255157	65 1 36	9.6255157
49 56 48		130 3 12	

Eadem ratione, sed majori compendio solvuntur cætera quæ quæri solent in Triangulis sphericis, sine ope Secantium aut Sinuum versorum, ut pluribus non sit opus aut præceptis aut exemplis.

Idem si desideres in Triangulis rectilineis, adjunge nostris Amici & Collegæ Henrici Brigii *Logarithmos. Nam eo nitimur fundamento, eodem utimur operandi modo.*

Vale, & si hæc tibi grata fuerint, plura à nobis in hoc genere expecta.

FINIS.

TEN CHILIADES
OF
LOGARITHMS
of Abſolute Numbers, from an Unite to Ten thouſand.

The Uſe of the CANON.

THis *Canon* hath like uſe as the Tables of Right Sines and Tangents ſet forth by others, but the Practice ſomewhat more eaſie. For keeping to their Rules, and working by theſe Tables, you may uſe Addition inſtead of their Multiplication, and Subtraction inſtead of their Diviſion: And ſo reſolve all Spherical Triangles without the help of Secants or Verſed Sines.

The like may be done for the Solution of right-lined Triangles, by help of the Logarithms of my old Collegue and worthy Friend Mr. *Henry Briggs* (10000 whereof follow.) For both proceed from the ſame ground, and ſo require the ſame manner of work, as I often ſhew in my publick Lectures in *Greſham-College*: Where I reſt a Friend to all that are ſtudious of Mathematical Practice,

E. G.

Num.	Logarithm.	Num	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
1	0.0000000	36	1.5563025	71	1.8512583	106	2.0253058
2	0.3010300	37	1.5682017	72	1.8573325	107	2.0293838
3	0.4771212	38	1.5797836	73	1.8633228	108	2.0334237
4	0.6020600	39	1.5910646	74	1.8692317	109	2.0374265
5	0.6989700	40	1.6020600	75	1.8750612	110	2.0413927
6	0.7781512	41	1.6127838	76	1.8808136	111	2.0453230
7	0.8450980	42	1.6232493	77	1.8864907	112	2.0492180
8	0.9030900	43	1.6334684	78	1.8920946	113	2.0530784
9	0.9542425	44	1.6434527	79	1.8976271	114	2.0569048
10	1.0000000	45	1.6532125	80	1.9030900	115	2.0606978
11	1.0413927	46	1.6627578	81	1.9084850	116	2.0644580
12	1.0791812	47	1.6720978	82	1.9138138	117	2.0681858
13	1.1139433	48	1.6812412	83	1.9190781	118	2.0718820
14	1.1461280	49	1.6901961	84	1.9242793	119	2.0755469
15	1.1760912	50	1.6989700	85	1.9294189	120	2.0791812
16	1.2041200	51	1.7075702	86	1.9344984	121	2.0827854
17	1.2304489	52	1.7160033	87	1.9395192	122	2.0863598
18	1.2552725	53	1.7242759	88	1.9444827	123	2.0899051
19	1.2787536	54	1.7323937	89	1.9493900	124	2.0934217
20	1.2010300	55	1.7403627	90	1.9542425	125	2.0969100
21	1.3222193	56	1.7481880	91	1.9590414	126	2.1003705
22	1.3424227	57	1.7558748	92	1.9637878	127	2.1038037
23	1.3617278	58	1.7634280	93	1.9684829	128	2.1072099
24	1.3802112	59	1.7708520	94	1.9731278	129	2.1105897
25	1.3979400	60	1.7781512	95	1.9777236	130	2.1139433
26	1.4149733	61	1.7853298	96	1.9822712	131	2.1172713
27	1.4313637	62	1.7923917	97	1.9867717	132	2.1205739
28	1.4471580	63	1.7993405	98	1.9912261	133	2.1238516
29	1.4623980	64	1.8061800	99	1.9956352	134	2.1271048
30	1.4771212	65	1.8129133	100	2.0000000	135	2.1303337
31	1.4913627	66	1.8195439	101	3.0043213	136	2.1335389
32	1.5051500	67	1.8260748	102	2.0086002	137	2.1367205
33	1.5185139	68	1.8325089	103	2.0128372	138	2.1398791
34	1.5314789	69	1.8388491	104	2.0170333	139	2.1430148
35	1.5440680	70	1.8450980	105	2.0211893	140	2.1461280

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
141	2.1492191	176	2.2455126	211	2.3242824	246	2.3909351
142	2.1522883	177	2.2479732	212	2.3263358	247	2.3926969
143	2.1553360	178	2.2504200	213	2.3283796	248	2.3944517
144	2.1583625	179	2.2528530	214	2.3304138	249	2.3961993
145	2.1613680	180	2.2552725	215	2.3324384	250	2.3979400
146	2.1643528	181	2.2576786	216	2.3344537	251	2.3996737
147	2.1673173	182	2.2600714	217	2.3364597	252	2.4014005
148	2.1702617	183	2.2624511	218	2.3384565	253	2.4031205
149	2.1731862	184	2.2648178	219	2.3404441	254	2.4048337
150	2.1760912	185	2.2671717	220	2.3424226	255	2.4065402
151	2.1789769	186	2.2695129	221	2.3443923	256	2.4082399
152	2.1818436	187	2.2718416	222	2.3463530	257	2.4099331
153	2.1846914	188	2.2741578	223	2.3483048	258	2.4116197
154	2.1875207	189	2.2764618	224	2.3502480	259	2.4132997
155	2.1903317	190	2.2787536	225	2.3521825	260	2.4149733
156	2.1931246	191	2.2810333	226	2.3541084	261	2.4166405
157	2.1958996	192	2.2833012	227	2.3560258	262	2.4183013
158	2.1986571	193	2.2855573	228	2.3579348	263	2.4199557
159	2.2013971	194	2.2878017	229	2.3598355	264	2.4216039
160	2.2041200	195	2.2900346	230	2.3617278	265	2.4232459
161	2.2068258	196	2.2922561	231	2.3636120	266	2.4248816
162	2.2095150	197	2.2944662	232	2.3654880	267	2.4265112
163	2.2121876	198	2.2966652	233	2.3673559	268	2.4281348
164	2.2148438	199	2.2988531	234	2.3692158	269	2.4297523
165	2.2174839	200	2.3010300	235	2.3710678	270	2.4313637
166	2.2201081	201	2.3031960	236	2.3729120	271	2.4329693
167	2.2227164	202	2.3053513	237	2.3747483	272	2.4345689
168	2.2253093	203	2.3074960	238	2.3765770	273	2.4361626
169	2.2278867	204	2.3096301	239	2.3783979	274	2.4377505
170	2.2304489	205	2.3117538	240	2.3802112	275	2.4393327
171	2.2329961	206	2.3138672	241	2.3820170	276	2.4409091
172	2.2355284	207	2.3159703	242	2.3838153	277	2.4424797
173	2.2380461	208	2.3180633	243	2.3856063	278	2.4440448
174	2.2405492	209	2.3201463	244	2.3873898	279	2.4456042
175	2.2430380	210	2.3222193	245	2.3891660	280	2.4471580

Num.	Logarithm.	Num	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
281	2.4487063	316	2.4996871	351	2.5453071	386	2.5865873
282	2.4502491	317	2.5010593	352	2.5465426	387	2.5877110
283	2.4517864	318	2.5024271	353	2.5477747	388	2.5888317
284	2.4533183	319	2.5037907	354	2.5490033	389	2.5899496
285	2.4548448	320	2.5051500	355	2.5502283	390	2.5910646
286	2.4563660	321	2.5065050	356	2.5514500	391	2.5921767
287	2.4578819	322	2.5078559	357	2.5526682	392	2.5932861
288	2.4593925	323	2.5092025	358	2.5538830	393	2.5943925
289	2.4608978	324	2.5105450	359	2.5550944	394	2.5954962
290	2.4623980	325	2.5118833	360	2.5563025	395	2.5965971
291	2.4638930	326	2.5132176	361	2.5575072	396	2.5976952
292	2.4653828	327	2.5145477	362	2.5587086	397	2.5987905
293	2.4668676	328	2.5158738	363	2.5599066	398	2.5998831
294	2.4683473	329	2.5171959	364	2.5611014	399	2.6009729
295	2.4698220	330	2.5185139	365	2.5622929	400	2.6020600
296	2.4712917	331	2.5198280	366	2.5634810	401	2.6031444
297	2.4727564	332	2.5211381	367	2.5646661	402	2.6042260
298	2.4742162	333	2.5224442	368	2.5658478	403	2.6053050
299	2.4756712	334	2.5237464	369	2.5670263	404	2.6063813
300	2.4771212	335	2.5250448	370	2.5682017	405	2.6074550
301	2.4785665	336	2.5263393	371	2.5693739	406	2.6085260
302	2.4800069	337	2.5276299	372	2.5705429	407	2.6095944
303	2.4814426	338	2.5289167	373	2.5717088	408	2.6106602
304	2.4828736	339	2.5301997	374	2.5728716	409	2.6117233
305	2.4842998	340	2.5314789	375	2.5740313	410	2.6127838
306	2.4857214	341	2.5327544	376	2.5751878	411	2.6138418
307	2.4871384	342	2.5340261	377	2.5763413	412	2.6148972
308	2.4885507	343	2.5352941	378	2.5774918	413	2.6159500
309	2.4899585	344	2.5365584	379	2.5786392	414	2.6170003
310	2.4913617	355	2.5378191	380	2.5797836	415	2.6180481
311	2.4927604	346	2.5390761	381	2.5809250	416	2.6190933
312	2.4941546	347	2.5403295	382	2.5820634	417	2.6201360
313	2.4955443	348	2.5415792	383	2.5831988	418	2.6211763
314	2.4969296	349	2.5428254	384	2.5843312	419	2.6222140
315	2.4983105	350	2.5440680	385	2.5854607	420	2.6232493

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
421	2.6242821	456	2.6589648	491	2.6910815	526	2.7209857
422	2.6253124	457	2.6599162	492	2.6919651	527	2.7218106
423	2.6263403	458	2.6608655	493	2.6928469	528	2.7226339
424	2.6273657	459	2.6618127	494	2.6937269	529	2.7234557
425	2.6283889	460	2.6627578	495	2.6946052	530	2.7242759
426	2.6294096	461	2.6637009	496	2.6954817	531	2.7250945
427	2.6304279	462	2.6646420	497	2.6963564	532	2.7259116
428	2.6314438	463	2.6655810	498	2.6972293	533	2.7267272
429	2.6324573	464	2.6665180	499	2.6981005	534	2.7275412
420	2.6334684	465	2.6674529	500	2.6989700	535	2.7283538
431	2.6344773	466	2.6683859	501	2.6998377	536	2.7291648
432	2.6354837	467	2.6693169	502	2.7007037	537	2.7299743
433	2.6364879	468	2.6702458	503	2.7015680	538	2.7307823
434	2.6374897	469	2.6711728	504	2.7024305	539	2.7315888
435	2.6384892	470	2.6720978	505	2.7032914	540	2.7323937
436	2.6394865	471	2.6730209	506	2.7041505	541	2.7331973
437	2.6407814	472	2.6739420	507	2.7050079	542	2.7339993
438	2.6414741	473	2.6748611	508	2.7058637	543	2.7347998
439	2.6424645	474	2.6757783	509	2.7067178	544	2.7355989
440	2.6434527	475	2.6766936	510	2.7075702	455	2.7363965
441	2.6444386	476	2.6776069	511	2.7084209	546	2.7371926
442	2.6454223	477	2.6785184	512	2.7092699	547	2.7379873
443	2.6464037	478	2.6794279	513	2.7101174	548	2.7387805
444	2.6473830	479	2.6803355	514	2.7109631	549	2.7395723
445	2.6483600	480	2.6812412	515	2.7118072	550	2.7403627
446	2.6493348	481	2.6821451	516	2.7126497	551	2.7411516
447	2.6503075	482	2.6830470	517	2.7134905	552	2.7419391
448	2.6512780	483	2.6839471	518	2.7143297	553	2.7427251
449	2.6522463	484	2.6848453	519	2.7151673	554	2.7435098
450	2.6532125	485	2.6857417	520	2.7160033	555	2.7442930
451	2.6541765	486	2.6866363	521	2.7168377	556	2.7450748
452	2.6551384	487	2.6875289	522	2.7176705	557	2.7458552
453	2.6560982	488	2.6884198	523	2.7185017	558	2.7466342
454	2.6570558	489	2.6893088	524	2.7193313	559	2.7474118
455	2.6580114	490	2.6901961	525	2.7201593	560	2.7481880

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
561	2.7489628	596	2.7752462	631	2.8000293	666	2.8234742
562	2.7497363	597	2.7759743	632	2.8007171	667	2.8241258
563	2.7505084	598	2.7767012	633	2.8014037	668	2.8247765
564	2.7512791	599	2.7774268	634	2.8020892	669	2.8254261
565	2.7520484	600	2.7781512	635	2.8027737	670	2.8260748
566	2.7528164	601	2.7788745	636	2.8034571	671	2.8267225
567	2.7535830	602	2.7795965	637	2.8041394	672	2.8273693
568	2.7543483	603	2.7803173	638	2.8048206	673	2.8280151
569	2.7551123	604	2.7810369	639	2.8055008	674	2.8286599
570	2.7558748	605	2.7817554	640	2.8061800	675	2.8293038
571	2.7566361	606	2.7824726	641	2.8068580	676	2.8299467
572	2.7573960	607	2.7831887	642	2.8075350	677	2.8305887
573	2.7581546	608	2.7839036	643	2.8082110	678	2.8312297
574	2.7589119	609	2.7846173	644	2.8088859	679	2.8318698
575	2.7596678	610	2.7853298	645	2.8095597	680	2.8325089
576	2.7604225	611	2.7860412	646	2.8102325	681	2.8331471
577	2.7611758	612	2.7867514	647	2.8109043	682	2.8337844
578	2.7619278	613	2.7874605	648	2.8115750	683	2.8344207
579	2.7626785	614	2.7881684	649	2.8122447	684	2.8350561
580	2.7634280	615	2.7888751	650	2.8129133	685	2.8356906
581	2.7641761	616	2.7895807	651	2.8135810	686	2.8363241
582	2.7649230	617	2.7902851	652	2.8142476	687	2.8369567
583	2.7656685	618	2.7909885	653	2.8149132	688	2.8375884
584	2.7664128	619	2.7916906	654	2.8155777	689	2.8382192
585	2.7671558	620	2.7923917	655	2.8162413	690	2.8388491
586	2.7678976	621	2.7930916	656	2.8169038	691	2.8394780
587	2.7686381	622	2.7937904	657	2.8172654	692	2.8401061
588	2.7693773	623	2.7944880	658	2.8182259	693	2.8407332
589	2.7701153	624	2.7951846	659	2.8188854	694	2.8413595
590	2.7708520	625	2.7958800	660	2.8195439	695	2.8410848
591	2.7715875	626	2.7965743	661	2.8202014	696	2.8426092
592	2.7723217	627	2.7972675	662	2.8208580	697	2.8432328
593	2.7730547	628	2.7979596	663	2.8215135	698	2.8438554
594	2.7737864	629	2.7986506	664	2.8221681	699	2.8444772
595	2.7745169	630	2.7993405	665	2.8228216	700	2.8450980

Num.	Logarithm.	Num	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
701	2.8457180	736	2.8668778	771	2.8870544	806	2.9063350
702	2.8463371	737	2.8674675	772	2.8876170	807	2.9068735
703	2.8469553	738	2.8680564	773	2.8881795	808	2.9074114
704	2.8475726	739	2.8686444	774	2.8887410	809	2.9079485
705	2.8481891	740	2.8692317	775	2.8893017	810	2.9084850
706	2.8488047	741	2.8698182	776	2.8898617	811	2.9090208
707	2.8494194	742	2.8704039	777	2.8904210	812	2.9095560
708	2.8500332	743	2.8709888	778	2.8909796	813	2.9100905
709	2.8506462	744	2.8715729	779	2.8915374	814	2.9106244
710	2.8512583	745	2.8721563	780	2.8920946	815	2.9111576
711	2.8518696	746	2.8727388	781	2.8926510	816	2.9116901
712	2.8524800	747	2.8733206	782	2.8932067	817	2.9122220
713	2.8530895	748	2.8739016	783	2.8937618	818	2.9127533
714	2.8536982	749	2.8744818	784	2.8943161	819	2.9132839
715	2.8543060	750	2.8750613	785	2.8948696	820	2.9138138
716	2.8549130	751	2.8756399	786	2.8954225	821	2.9143431
717	2.8555191	752	2.8762178	787	2.8959747	822	2.9148718
718	2.8561244	753	2.8767950	788	2.8965262	823	2.9153998
719	2.8567289	754	2.8773713	789	2.8970770	824	2.9159272
720	2.8573325	755	2.8779469	790	2.8976271	825	2.9164539
721	2.8579353	756	2.8785218	791	2.8981765	826	2.9169800
722	2.8585372	757	2.8790959	792	2.8987252	827	2.9175055
723	2.8591383	758	2.8796692	793	2.8992732	828	2.9180303
724	2.8597387	759	2.8802418	794	2.8998205	829	2.9185545
725	2.8603380	760	2.8808136	795	2.9003671	830	2.9190781
726	2.8609366	761	2.8813846	796	2.9009131	831	2.9196010
727	2.8615344	762	2.8819550	797	2.9014583	832	2.9201233
728	2.8621314	763	2.8825245	798	2.9020029	833	2.9206450
729	2.8627275	764	2.8830933	799	2.9025468	834	2.9211660
730	2.8633228	765	2.8836614	800	2.9030900	835	2.9216865
731	2.8639174	766	2.8842288	801	2.9036325	836	2.9222063
732	2.8645111	767	2.8847953	802	2.9041744	837	2.9227254
733	2.8651041	768	2.8853612	803	2.9047155	838	2.9232440
734	2.8656960	769	2.8859263	804	2.9052560	839	2.9237620
735	2.8662873	770	2.8864907	805	2.9057959	840	2.9242793

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
841	2.9247960	876	2.9425041	911	2.9595184	946	2.9758911
842	2.9253121	877	2.9429996	912	2.9599948	947	2.9763500
843	2.9258276	878	2.9434945	913	2.9604708	948	2.9768083
844	2.9263424	879	2.9439889	914	2.9609462	949	2.9772662
845	2.9268567	880	2.9444827	915	2.9614211	950	2.9777236
846	2.9273704	881	2.9449759	916	2.9618955	951	2.9781805
847	2.9278834	882	2.9454686	917	2.9623693	952	2.9786369
848	2.9283958	883	2.9459607	918	2.9628427	953	2.9790929
849	2.9289077	884	2.9464523	919	2.9633155	954	2.9795484
850	2.9294189	885	2.9469433	920	2.9637878	955	2.9800034
851	2.9299296	886	2.9474337	921	2.9642596	956	2.9804579
852	2.9304396	887	2.9479236	922	2.9647308	957	2.9809119
853	2.9309490	888	2.9484130	923	2.9652017	958	2.9813655
854	2.9314579	889	2.9489018	924	2.9656720	959	2.9818186
855	2.9319661	890	2.9493900	925	2.9661417	960	2.9822712
856	2.9324738	891	2.9498777	926	2.9666110	961	2.9827234
857	2.9329808	892	2.9503648	927	2.9670797	962	2.9831750
858	2.9334873	893	2.9508514	928	2.9675480	963	2.9836263
859	2.9339932	894	2.9513375	929	2.9680157	964	2.9840770
860	2.9344984	895	2.9518230	930	2.9684829	965	2.9845273
861	2.9350031	896	2.9523080	931	2.9689497	966	2.9849771
862	2.9355073	897	2.9527924	932	2.9694159	967	2.9854265
863	2.9360108	898	2.9532763	933	2.9698816	968	2.9858753
864	2.9365137	899	2.9537597	934	2.9703470	969	2.9863238
865	2.9370162	900	2.9542425	935	2.9708116	970	2.9867717
866	2.9375179	901	2.9547248	936	2.9712758	971	2.9872192
867	2.9380191	902	2.9552065	937	2.9717396	972	2.9876663
868	2.9385197	903	2.9556877	938	2.9722028	973	2.9881128
869	2.9390198	904	2.9561684	939	2.9726656	974	2.9885589
870	2.9395192	905	2.9566486	940	2.9731278	975	2.9890046
871	2.9400181	906	2.9571282	941	2.9735896	976	2.9894498
872	2.9405165	907	2.9576073	942	2.9740509	977	2.9898946
873	2.9410142	908	2.9580858	943	2.9745117	978	2.9903388
874	2.9415114	909	2.9585639	944	2.9749720	979	2.9907827
875	2.9420080	910	2.9590414	945	2.9754318	980	2.9912261

Num.	Logarithm.	Num	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
981	2.9916690	1016	3.0068937	1051	3.0216027	1086	3.0358298
982	2.9921115	1017	3.0073209	1052	3.0220157	1087	3.0362295
983	2.9925535	1018	3.0077478	1053	3.0224284	1088	3.0366289
984	2.9929951	1019	3.0081742	1054	3.0228406	1089	3.0370279
985	2.9934362	1020	3.0086002	1055	3.0232524	1090	3.0374265
986	2.9938769	1021	3.0090257	1056	3.0236639	1091	3.0378247
987	2.9943171	1022	3.0094509	1057	3.0240750	1092	3.0382226
988	2.9947569	1023	3.0098756	1058	3.0244857	1093	3.0386201
989	2.9951963	1024	3.0102999	1059	3.0248960	1094	3.0390173
990	2.9956352	1025	3.0107239	1060	3.0253059	1095	3.0394141
991	2.9960736	1026	3.0111473	1061	3.0257154	1096	3.0398105
992	2.9965117	1027	3.0115704	1062	3.0261245	1097	3.0402066
993	2.9969492	1028	3.0119931	1063	3.0265333	1098	3.0406023
994	2.9973864	1029	3.0124154	1064	3.0269416	1099	3.0409977
995	2.9978231	1030	3.0128372	1065	3.0273496	1100	3.0413927
996	2.9982593	1031	3.0132587	1066	3.0277572	1101	3.0417873
997	2.9986951	1032	3.0136797	1067	3.0281644	1102	3.0421816
998	2.9991305	1033	3.0141003	1068	3.0285712	1103	3.0425755
999	2.9995655	1034	3.0145205	1069	3.0289777	1104	3.0429691
1000	3.0000000	1035	3.0149403	1070	3.0293838	1105	3.0433623
1001	3.0004341	1036	3.0153597	1071	3.0297895	1106	3.0437551
1002	3.0008677	1037	3.0157787	1072	3.0301948	1107	3.0441476
1003	3.0013009	1038	3.0161973	1073	3.0305997	1108	3.0445398
1004	3.0017337	1039	3.0166155	1074	3.0310043	1109	3.0449315
1005	3.0021661	1040	3.0170333	1075	3.0314085	1110	3.0453230
1006	3.0025980	1041	3.0174507	1076	3.0318124	1111	3.0457140
1007	3.0030295	1042	3.0178677	1077	3.0322157	1112	3.0461048
1008	3.0034605	1043	3.0182843	1078	3.0326188	1113	3.0464952
1009	3.0038912	1044	3.0187005	1079	3.0330214	1114	3.0468852
1010	3.0043214	1045	3.0191163	1080	3.0334237	1115	3.0472749
1011	3.0047511	1046	3.0195317	1081	3.0338257	1116	3.0476642
1012	3.0051805	1047	3.0199467	1082	3.0342273	1117	3.0480532
1013	3.0056094	1048	3.0203613	1083	3.0346284	1118	3.0484418
1014	3.0060379	1049	3.0207755	1084	3.0350293	1119	3.0488301
1015	3.0064660	1050	3.0211893	1085	3.0354297	1120	3.0492180

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num	Logarithm
1121	3.0490056	1156	3.0629578	1191	3.0759118	1220	3.0884905
1122	3.0499928	1157	3.0633333	1192	3.0762762	1227	3.0888446
1123	3.0503797	1158	3.0637085	1193	3.0766404	1228	3.0891984
1124	3.0507663	1159	3.0640834	1194	3.0770043	1229	3.0895519
1125	3.0511525	1160	3.0644580	1195	3.0773679	1230	3.0899051
1126	3.0515384	1161	3.0648322	1196	3.0777312	1231	3.0902580
1127	3.0519239	1162	3.0652061	1197	3.0780941	1232	3.0906107
1128	3.0523091	1163	3.0655797	1198	3.0784568	1233	3.0909631
1129	3.0526939	1164	3.0659530	1199	3.0788192	1234	3.0913151
1130	3.0530784	1165	3.0663259	1200	3.0791812	1235	3.0916669
1131	3.0534626	1166	3.0666985	1201	3.0795430	1236	3.0920185
1132	3.0538464	1167	3.0670708	1202	3.0799045	1237	3.0923696
1133	3.0542299	1168	3.0674428	1203	3.0802656	1238	3.0927206
1134	3.0546130	1169	3.0678145	1204	3.0806265	1239	3.0930712
1135	3.0549958	1170	3.0681859	1205	3.0809870	1240	3.0934217
1136	3.0553783	1171	3.0685569	1206	3.0813473	1241	3.0937718
1137	3.0557604	1172	3.0689276	1207	3.0817073	1242	3.0941216
1138	3.0561423	1173	3.0692980	1208	3.0820669	1243	3.0944711
1139	3.0565237	1174	3.0696681	1209	3.0824263	1244	3.0948204
1140	3.0569048	1175	3.0700379	1210	3.0827854	1245	3.0951693
1141	3.0572856	1176	3.0704073	1211	3.0831441	1246	3.0955180
1142	3.0576661	1177	3.0707765	1212	3.0835026	1247	3.0958664
1143	3.0580462	1178	3.0711453	1213	3.0838608	1248	3.0962146
1144	3.0584260	1179	3.0715138	1214	3.0842187	1249	3.0965624
1145	3.0588055	1180	3.0718820	1215	3.0845763	1250	3.0969100
1146	3.0591846	1181	3.0722499	1216	3.0849336	1251	3.0972573
1147	3.0595634	1182	3.0726175	1217	3.0852906	1252	3.0976043
1148	3.0599419	1183	3.0729847	1218	3.0856473	1253	3.0979511
1149	3.0603200	1184	3.0733517	1219	3.0860037	1254	3.0982975
1150	3.0606978	1185	3.0737183	1220	3.0863598	1255	3.0986437
1151	3.0610753	1182	3.0740847	1221	3.0867156	1256	3.0989896
1152	3.0614525	1187	3.0744507	1222	3.0870712	1257	3.0993353
1153	3.0618293	1188	3.0748164	1223	3.0874264	1258	3.0996806
1154	3.0622058	1189	3.0751818	1224	3.0877814	1259	3.1000257
1155	3.0625820	1190	3.0755470	1225	3.0881361	1260	3.1003705

Num	Logarithm.	Num	Logarithm.	Num.	Logarithm	Num.	Logarithm.
1261	3.1007151	1296	3.1126050	1331	3.1241780	1366	3.1354507
1262	3.1010593	1297	3.1129400	1332	3.1245042	1367	3.1357685
1263	3.1014033	1298	3.1132747	1333	3.1248301	1368	3.1360861
1264	3.1017471	1299	3.1136091	1334	3.1251558	1369	3.1364034
1265	3.1020905	1300	3.1139433	1335	3.1254813	1370	3.1367206
1266	3.1024337	1301	3.1142773	1336	3.1258064	1371	3.1370374
1267	3.1027766	1302	3.1146110	1337	3.1261314	1372	3.1373541
1268	3.1031192	1303	3.1149444	1338	3.1264561	1373	3.1376705
1269	3.1034616	1304	3.1152776	1339	3.1267806	1374	3.1379867
1270	3.1038037	1305	3.1156105	1340	3.1271048	1375	3.1383027
1271	3.1041455	1306	3.1159432	1341	3.1274288	1376	3.1386184
1272	3.1044871	1307	3.1162756	1342	3.1277525	1377	3.1389334
1273	3.1048284	1308	3.1166077	1343	3.1280760	1378	3.1392492
1274	3.1051694	1309	3.1169396	1344	3.1283993	1379	3.1395643
1275	3.1055102	1310	3.1172713	1345	3.1287223	1380	3.1398791
1276	3.1058507	1311	3.1176027	1346	3.1290450	1381	3.1401937
1277	3.1061909	1312	3.1179338	1347	3.1293678	1382	3.1405080
1278	3.1065308	1313	3.1182647	1348	3.1296899	1383	3.1408222
1279	3.1068705	1314	3.1185954	1349	3.1300119	1384	3.1411361
1280	3.1072100	1315	3.1189257	1350	3.1303338	1385	3.1414498
1281	3.1075491	1316	3.1192559	1351	3.1306553	1386	3.1417632
1282	3.1078880	1317	3.1195858	1352	3.1309767	1387	3.1420765
1283	3.1082266	1318	3.1199154	1353	3.1312978	1388	3.1423895
1284	3.1085650	1319	3.1202448	1354	3.1316187	1389	3.1427022
1285	3.1089031	1320	3.1205739	1355	3.1319393	1390	3.1430148
1286	3.1092410	1321	3.1209028	1356	3.1322597	1391	3.1433271
1287	3.1095785	1322	3.1212314	1357	3.1325798	1392	3.1436392
1288	3.1099159	1323	3.1215598	1358	3.1328998	1393	3.1439511
1289	3.1102529	1324	3.1218880	1359	3.1332195	1394	3.1442628
1290	3.1105897	1325	3.1222159	1360	3.1335389	1395	3.1445742
1291	3.1109262	1326	3.1225435	1361	3.1338581	1396	3.1448854
1292	3.1112625	1327	3.1228709	1362	3.1341771	1397	3.1451964
1293	3.1115985	1328	3.1231980	1363	3.1344958	1398	3.1455071
1294	3.1119343	1329	3.1235250	1364	3.1348144	1399	3.1458177
1295	3.1122698	1330	3.1238516	1365	3.1351326	1400	3.1461280

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm
1401	3.1464381	1436	3.1571544	1471	3.1676127	1506	3.1778250
1402	3.1467480	1437	3.1574568	1472	3.1679078	1507	3.1781132
1403	3.1470577	1438	3.1577589	1473	3.1682027	1508	3.1784013
1404	3.1473671	1439	3.1580608	1474	3.1684975	1509	3.1786892
1405	3.1476763	1440	3.1583625	1475	3.1687920	1510	3.1789769
1406	3.1479853	1441	3.1586640	1476	3.1690863	1511	3.1792645
1407	3.1482941	1442	3.1589653	1477	3. 693805	1512	3.1795518
1408	3.1486026	1443	3.1592663	1478	3.1696744	1513	3.1798389
1409	3.1489110	1444	3.1595672	1479	3.1699682	1514	3.1801259
1410	3.1492191	1445	3.1598678	1480	3.1702617	1515	3.1804126
1411	3.1495270	1446	3.1601683	1481	3.1705550	1516	3.1806992
1412	3.1498347	1447	3.1604685	1482	3.1708482	1517	3.1809852
1413	3.1501422	1448	3.1607686	1483	3.1711411	1518	3.1812718
1414	3.1504494	1449	3.1610684	1484	3.1714339	1519	3.1815578
1415	3.1507564	1450	3.1613680	1485	3.1717264	1520	3.1818436
1416	3.1510632	1451	3.1616674	1486	3.1720188	1521	3.1821292
1417	3.1513698	1452	3.1619666	1487	3.1723110	1522	3.1824146
1418	3.1516762	1453	3.1622656	1488	3.1726029	1523	3.1826999
1419	3.1519824	1454	3.1625644	1489	3.1728947	1524	3.1829850
1420	3.1522883	1455	3.1628630	1490	3.1731863	1525	3.1832698
1421	3.1525941	1456	3.1631614	1491	3.1734776	1526	3.1835545
1422	3.1528996	1457	3.1634595	1492	3.1737688	1527	3.1838390
1423	3.1532049	1458	3.1637575	1493	3.1740598	1528	3.1841233
1424	3.1535100	1459	3.1640553	1494	3.1743506	1529	3.1844075
1425	3.1538149	1460	3.1643528	1495	3.1746412	1530	3.1846914
1426	3.1541195	1461	3.1646502	1496	3.1749316	1531	3.1849752
1427	3.1544240	1462	3.1649474	1497	3.1752218	1532	3.1852588
1428	3.1547282	1463	3.1652443	1498	3.1755118	1533	3.1855421
1429	3.1550322	1464	3.1655411	1499	3.1758016	1534	3.1858253
1430	3.1553360	1465	3.1658376	1500	3.1760913	1535	3.1861084
1431	3.1556396	1466	3.1661340	1501	3.1763807	1536	3.1863912
1432	3.1559430	1467	3.1664301	1502	3.1766699	1537	3.1866739
1433	3.1562462	1468	3.1667260	1503	3.1769590	1538	3.1869563
1434	3.1565491	1469	3.1670218	1504	3.1772478	1539	3.1872366
1435	3.1568519	1470	3.1673173	1505	3.1775365	1540	3.1875207

Num	Logarithm.	Num	Logarithm.	Num.	Logarithm	Num.	Logarithm.
1541	3.1878026	1576	3.1975562	1611	3.2070955	1646	3.2164298
1542	3.1880844	1577	3.19783 7	1612	3.2073650	1647	3.2166936
1543	3.1883659	1578	3.1981070	1613	3.2076344	1648	3.2169572
1544	3.1886473	1579	3.1983821	1614	3.2079035	1649	3.2172206
1545	3.1889285	1580	3.1986571	1615	3.2081725	1650	3.2174839
1546	3.1892095	1581	3.1989319	1616	3.2084414	1651	3.2177471
1547	3.1894903	1582	3.1992065	1617	3.2087100	1652	3.2180100
1548	3.1897709	1583	3.1994809	1618	3.2089785	1653	3.2182728
1549	3.1900514	1584	3.1997552	1619	3.2092468	1654	3.2185355
1550	3.1903317	1585	3.2000293	1620	3.2095150	1655	3.2187980
1551	3.1906118	1586	3.2003032	1621	3.2097830	1656	3.2190603
1552	3.1908917	1587	3.2005769	1622	3.2100508	1657	3.2193225
1553	3.1911714	1588	3.2008505	1623	3.2103185	1658	3.2195845
1554	3.1914510	1589	3.2011239	1624	3.2105860	1659	3.2198464
1555	3.1917304	1590	3.2013971	1625	3.2108534	1660	3.2201081
1556	3.1920096	1591	3.2016702	1626	3.2111205	1661	3.2203696
1557	3.1922886	1592	3.2019431	1627	3.2113876	1662	3.2206310
1558	3.1925674	1593	3.2022158	1628	3.2116544	1663	3.2208921
1559	3.1928461	1594	3.2024883	1629	3.2119211	1664	3.2211533
1560	3.1931246	1595	3.2027607	1630	3.2121876	1665	3.2214142
1561	3.1934029	1596	3.2030329	1631	3.2124540	1666	3.2216750
1562	3.1936810	1597	3.2033049	1632	3.2127201	1667	3.2219356
1563	3.1939590	1598	3.2035768	1633	3.2129862	1668	3.2221960
1564	3.1942367	1599	3.2038485	1634	3.2132521	1669	3.2224563
1565	3.1945143	1600	3.2041200	1635	3.2135178	1670	3.2227165
1566	3.1947917	1601	3.2043913	1636	3.2137833	1671	3.2229764
1567	3.1950690	1602	3.2046625	1637	3.2140487	1672	3.2232363
1568	3.1953460	1603	3.2049335	1638	3.2143139	1673	3.2234959
1569	3.1956229	1604	3.2052044	1639	3.2145789	1674	3.2237555
1570	3.1958996	1605	3.2054750	1640	3.2148438	1675	3.2240148
1571	3.1961762	1606	3.2057455	1641	3.2151086	1676	3.2242740
1572	3.1964525	1607	3.2060159	1642	3.2153732	1677	3.2245331
1573	3.1967287	1608	3.2062869	1643	3.2156376	1678	3.2247920
1574	3.1970047	1609	3.2065560	1644	3.2159018	1679	3.2250507
1575	3.1972806	1610	3.2068259	1645	3.2161659	1680	3.2253093

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
1681	3.2255677	1716	3.2345173	1751	3.2432861	1786	3.2518815
1682	3.2258260	1717	3.2347703	1752	3.2435341	1787	3.2521246
1683	3.2260841	1718	3.2350232	1753	3.2437819	1788	3.2523675
1684	3.2263421	1719	3.2352759	1754	3.2440296	1789	3.2526103
1685	3.2265999	1720	3.2355284	1755	3.2442771	1790	3.2528530
1686	3.2268576	1721	3.2357809	1756	3.2445245	1791	3.2530956
1687	3.2271151	1722	3.2360331	1757	3.2447718	1792	3.2533380
1688	3.2273724	1723	3.2362853	1758	3.2450189	1793	3.2535803
1689	3.2276296	1724	3.2365373	1759	3.2452658	1794	3.2538224
1690	3.2278867	1725	3.2367891	1760	3.2455127	1795	3.2540645
1691	3.2281436	1626	3.2370408	1761	3.2457593	1796	3.2543063
1692	3.2284004	1627	3.2372923	1762	3.2460059	1797	3.2545481
1693	3.2286570	1628	3.2375437	1763	3.2462523	1798	3.2547897
1694	3.2289134	1629	3.2377950	1764	3.2464986	1799	3.2550312
1695	3.2291697	1630	3.2380461	1765	3.2467447	1800	3.2552725
1696	3.2294258	1631	3.2382971	1766	3.2469907	1801	3.2555137
1697	3.2296818	1632	3.2385479	1767	3.2472365	1802	3.2557548
1698	3.2299377	1633	3.2387986	1768	3.2474823	1803	3.2559957
1699	3.2301934	1634	3.2390491	1769	3.2477278	1804	3.2562365
1700	3.2304489	1635	3.2392995	1770	3.2479733	1805	3.2564772
1701	3.2307043	1636	3.2395497	1771	3.2482186	1806	3.2567177
1702	3.2309596	1637	3.2397998	1772	3.2484636	1807	3.2569582
1703	3.2312146	1638	3.2400498	1773	3.2487087	1808	3.2571984
1704	3.2314696	1639	3.2402996	1774	3.2489536	1809	3.2574386
1705	3.2317244	1640	3.2405492	1775	3.2491984	1810	3.2576786
1706	3.2319790	1641	3.2407988	1776	3.2494430	1811	3.2579184
1707	3.2322335	1642	3.2410481	1777	3.2496874	1812	3.2581582
1708	3.2324879	1643	3.2412974	1778	3.2499318	1813	3.2583978
1709	3.2327421	1644	3.2415465	1779	3.2501759	1814	3.2586373
1710	3.2329961	1645	3.24[illegible]7954	1780	3.2504200	1815	3.2588766
1711	3.2332500	1646	3.2420442	1781	3.2506636	1816	3.2591158
1712	3.2335038	1647	3.2422929	1782	3.2509077	1817	3.2593549
1713	3.2337574	1648	3.2425413	1783	3.2511513	1818	3.2595939
1714	3.2340108	1649	3.2427898	1784	3.2513948	1819	3.2598327
1715	3.2342641	1650	3.2430380	1785	3.2516382	1820	3.2600714

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
1821	3.2603099	1856	3.2685780	1891	3.2766915	1926	3.2846563
1822	3.2605484	1857	3.2688119	1892	3.2769211	1927	3.2848817
1823	3.2607867	1858	3.2690457	1893	3.2771506	1928	3.2851070
1824	3.2610248	1859	3.2692794	1894	3.2773800	1929	3.2853322
1825	3.2612629	1860	3.2695 29	1895	3.2776092	1930	3.2855573
1826	3.2615008	1861	3.2697464	1896	3.2778383	1931	3.2857823
1827	3.2617385	1862	3.2699797	1897	3.2780673	1932	3.2860071
1828	3.2619762	1863	3.2702128	1898	3.2782962	1933	3.2862318
1829	3.2622137	1864	3.2704459	1899	3.2785250	1934	3.2864565
1830	3.2624511	1865	3.2706788	1900	3.2783536	1935	3.2866810
1831	3.2626883	1866	3.2709116	1901	3.2789821	1936	3.2869054
1832	3.2629255	1867	3.2711443	1902	3.2792105	1937	3.2871296
1833	3.2631625	1868	3.2713769	1903	3.2794388	1938	3.2873538
1834	3.2633993	1869	3.2716093	1904	3.2796669	1939	3.2875778
1835	3.2636361	1870	3.2718416	1905	3.2798950	1940	3.2878017
1836	3.2638727	1871	3.2720738	1906	3.2801229	1941	3.2880255
1837	3.2641092	1872	3.2723058	1907	3.2803507	1942	3.2882492
1838	3.2643455	1873	3.2725378	1908	.2805784	1943	3.2884728
1839	3.2645817	1874	3.2727696	1909	3.2808059	1944	3.2886963
1840	3.2648178	1875	3.2730013	1910	3.2810034	1945	3.2889196
1841	3.2650538	1876	3.2732328	1911	3.2812607	1946	3.2891428
1842	3.2652896	1877	3.2734643	1912	3.2814879	1947	3.2893659
1843	3.2655253	1878	3.2736956	1913	3.2817150	1948	3.2895889
1844	3.2657609	1879	3.2739268	1914	3.2819419	1949	3.2898118
1845	3.2659964	1880	3.2741578	1915	3.2821688	1950	3.2900346
1846	3.2662317	1881	3.2743888	1916	3.2823955	1951	3.2902573
1847	3.2664669	1882	3.2746196	1917	3.2826221	1952	3.2904798
1848	3.2667020	1883	3.2748503	1918	3.2828486	1953	3.2907022
1849	3.2669369	1884	3.2750809	1919	3.2830750	1954	3.2909246
1850	3.2671[illegible]7	1885	3.2753113	1920	3.2833012	1955	3.2911468
1851	3.2674064	1886	3.2755417	1921	3.2835274	1956	3.2913688
1852	3.2676410	1887	3.2757719	1922	3.2837534	1957	3.2915908
1853	3.2678754	1888	3.2760020	1923	3.2839793	1958	3.2918127
1854	3.2681097	1889	3.2762320	1924	3.2842051	1959	3.2920344
1855	3.2683439	1890	3.2764618	1925	3.2844307	1960	3.2922561

Num.	Logarithm.	Num	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
1961	3.2924776	1996	3.3001605	2031	3.3077099	2066	3.3151303
1962	3.2926990	1997	3.3003781	2032	3.3079237	2067	3.3153405
1963	3.2929203	1998	3.3005955	2033	3.3081374	2068	3.3155505
1964	3.2931415	1999	3.3008128	2034	3.3083509	2069	3.3157605
1965	3.2933626	2000	3.3010300	2035	3.3085644	2070	3.3159703
1966	3.2935835	2001	3.3012471	2036	3.3087778	2071	3.3161801
1967	3.2938041	2002	3.3014641	2037	3.3089910	2072	3.3163897
1968	3.2940251	2003	3.3016809	2038	3.3092042	2073	3.3165993
1969	3.2942457	2004	3.3018977	2039	3.3094172	2074	3.3168087
1970	3.2944662	2005	3.3021144	2040	3.3096302	2075	3.3170181
971	3.2946866	2006	3.3023309	2041	3.3098430	2076	3.3172273
1972	3.2949069	2007	3.3025474	2042	3.3100557	2077	3.3174365
1973	3.2951271	2008	3.3027637	2043	3.3102684	2078	3.3176455
1974	3.2953471	2009	3.3029799	2044	3.3104809	2079	3.3178545
1975	3.2955671	2010	3.3031961	2045	3.3106933	2080	3.3180633
1976	3.2957869	2011	3.3034121	2046	3.3109056	2981	3.3182721
1977	3.2960067	2012	3.3036280	2047	3.3111178	2082	3.3184807
1978	3.2962263	2013	3.3038438	2048	3.3113299	2083	3.3186893
1979	3.2964458	2014	3.3040595	2049	3.3115420	2084	3.3188977
1980	3.2966652	2015	3.3042751	2050	3.3117539	2085	3.3191061
1981	3.2968845	2016	3.3044905	2051	3.3119657	2086	3.3193143
1982	3.2971036	2017	3.3047059	2052	3.3121774	2087	3.3195224
1983	3.2973227	2018	3.3049212	2053	3.3123889	2088	3.3197305
1984	3.2975417	2019	3.3051363	2054	3.3126004	2089	3.3199384
1985	3.2977605	2020	3.3053514	2055	3.3128118	2090	3.3201463
1986	3.2979792	2021	3.3055663	2056	3.3130231	2091	3.3203540
1987	3.2981979	2022	3.3057812	2057	3.3132343	2092	3.3205617
1988	3.2984164	2023	3.3059959	2058	3.3134454	2093	3.3207692
1989	3.2986348	2024	3.3062105	2059	3.3136563	2094	3.3209767
1990	3.2988531	2025	3.3064250	2060	3.3138672	2095	3.3211840
1991	3.2990713	2026	3.3066394	2061	3.3140780	2096	3.3213913
1992	3.2992893	2027	3.3068537	2062	3.3142887	2097	3.3215984
1993	3.2995073	2028	3.3070679	2063	3.3144992	2098	3.3218055
1994	3.2997251	2029	3.3072820	2064	3.3147097	2099	3.3220124
1995	3.2999429	2030	3.3074960	2065	3.3149200	2100	3.3222193

Num.	Logarithm.	Num	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
2101	3.3224260	2136	3.3296012	2171	3.3366598	2206	3.3430055
2102	3.3226327	2137	3.3298045	2172	3.3368598	2207	3.3438023
2103	3.3228393	2138	3.3300077	2173	3.3370597	2208	3.3439991
2104	3.3230457	2139	3.3302108	2174	3.3372595	2209	3.3441957
2105	3.3232521	2140	3.3304138	2175	3.3374593	2210	3.3443923
2106	3.3234584	2141	3.3306167	2176	3.3376589	2211	3.3445887
2107	3.3236645	2142	3.3308195	2177	3.3378584	2212	3.3447851
2108	3.3238706	2143	3.3310222	2178	3.3380579	2213	3.3449814
2109	3.3240766	2144	3.3312248	2179	3.3382572	2214	3.3451776
2110	3.3242825	2145	3.3314273	2180	3.3384565	2215	3.3453737
2111	3.3244882	2146	3.3316297	2181	3.3386557	2216	3.3455698
2112	3.3246939	2147	3.3318320	2182	3.3388547	2217	3.3457657
2113	3.3248995	2148	3.3320343	2183	3.3390537	2218	3.3459615
2114	3.3251050	2149	3.3322364	2184	3.3392526	2219	3.3461573
2115	3.3253104	2150	3.3324385	2185	3.3394514	2220	3.3463530
2116	3.3255157	2151	3.3326404	2186	3.3396501	2221	3.3465486
2117	3.3257209	2152	3.3328423	2187	3.3398488	2222	3.3467441
2118	3.3259260	2153	3.3330440	2188	3.3400473	2223	3.3469395
2119	3.3261310	2154	3.3332457	2189	3.3402458	2224	3.3471348
2120	3.3263359	2155	3.3334473	2190	3.3404441	2225	3.3473300
2121	3.3265407	2156	3.3336488	2191	3.3406424	2226	3.3475252
2122	3.3267454	2157	3.3338501	2192	3.3408405	2227	3.3477202
2123	3.3269500	2158	3.3340514	2193	3.3410386	2228	3.3479152
2124	3.3271545	2159	3.3342526	2194	3.3412366	2229	3.3481101
2125	3.3273589	2160	3.3344537	2195	3.3414345	2230	3.3483049
2126	3.3275633	2161	3.3346548	2196	3.3416323	2231	3.3484996
2127	3.3277675	2162	3.3348557	2197	3.3418301	2232	3.3486942
2128	3.3279716	2163	3.3350565	2198	3.3420277	2233	3.3488887
2129	3.3281757	2164	3.3352572	2199	3.3422252	2234	3.3490832
2130	3.3283796	2165	3.3354579	2200	3.3424227	2235	3.3492775
2131	3.3285834	2166	3.3356585	2201	3.3426200	2236	3.3494718
2132	3.3287872	2167	3.3358589	2202	3.3428173	2237	3.3496660
2133	3.3289909	2168	3.3360593	2203	3.3430145	2238	3.3498601
2134	3.3291944	2169	3.3362596	2204	3.3432116	2239	3.3500541
2135	3.3293979	2170	3.3364597	2205	3.3434086	2240	3.3502480

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
2241	3.3504419	2276	3.3571722	2311	3.3638000	2346	3.3703280
2242	3.3506356	2277	3.3573630	2312	3.3639878	2347	3.3705131
2243	3.3508293	2278	3.3575537	2313	3.3641756	2348	3.3706981
2244	3.3510228	2279	3.3577443	2314	3.3643633	2349	3.3708830
2245	3.3512163	2280	3.3579348	2315	3.3645510	2350	3.3710678
2246	3.3514098	2281	3.3581253	2316	3.3647386	2351	3.3712526
2247	3.3516031	2282	3.3583156	2317	3.3649260	2352	3.3714373
2248	3.3517963	2283	3.3585059	2318	3.3651134	2353	3.3716219
2249	3.3519895	2284	3.3586961	2319	3.3653007	2354	3.3718065
2250	3.3521825	2285	3.3588862	2320	3.3654880	2355	3.3719909
2251	3.3523755	2286	3.3590762	2321	3.3656751	2356	3.3721753
2252	3.3525684	2287	3.3592662	2322	3.3658622	2357	3.3723596
2253	3.3527613	2288	3.3594560	2323	3.3660492	2358	3.3725438
2254	3.3529539	2289	3.3596458	2324	3.3662361	2359	3.3727279
2255	3.3531465	2290	3.3598355	2325	3.3664230	2360	3.3729120
2256	3.3533391	2291	3.3600251	2326	3.3666097	2361	3.3730960
2257	3.3535316	2292	3.3602146	2327	3.3667964	2362	3.3732799
2258	3.3537239	2293	3.3604041	2328	3.3669830	2363	3.3734637
2259	3.3539162	2294	3.3605934	2329	3.3671695	2364	3.3736475
2260	3.3541084	2295	3.3607827	2330	3.3673559	2365	3.3738311
2261	3.3543006	2296	3.3609719	2331	3.3675423	2366	3.3740147
2262	3.3544926	2297	3.3611610	2332	3.3677285	2367	3.3741983
2263	3.3546846	2298	3.3613500	2333	3.3679147	2368	3.3743817
2264	3.3548764	2299	3.3615390	2334	3.3681008	2369	3.3745651
2265	3.3550682	2300	3.3617278	2335	3.3682869	2370	3.3747483
2266	3.3552599	2301	3.3619166	2336	3.3684728	2371	3.3749316
2267	3.3554515	2302	3.3621053	2337	3.3686587	2372	3.3751147
2268	3.3556430	2303	3.3622939	2338	3.3688445	2373	3.3752977
2269	3.3558345	2304	3.3624825	2339	3.3690302	2374	3.3754807
2270	3.3560259	2305	3.3626709	2340	3.3692159	2375	3.3756636
2271	3.3562171	2306	3.3628593	2341	3.3694014	2376	3.3758464
2272	3.3564083	2307	3.3630476	2342	3.3695869	2377	3.3760292
2273	3.3565994	2308	3.3632358	2343	3.3697723	2378	3.3762118
2274	3.3567905	2309	3.3634239	2344	3.3699576	2379	3.3763944
2275	3.3569814	2310	3.3636120	2345	3.3701428	2380	3.3765769

Num.	Logarithm.	Num	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
2381	3.3767594	2416	3.3830969	2451	3.3893433	2486	3.3955011
2382	3.3769418	2417	3.3832766	2452	3.3895205	2487	3.3956758
2383	3.3771240	2418	3.3834563	2453	3.3896975	2488	3.3958504
2384	3.3773062	2419	3.3836359	2454	3.3898746	2489	3.3960249
2385	3.3774884	2420	3.3838154	2455	3.3900515	2490	3.3961993
2386	3.3776704	2421	3.3839948	2456	3.3902284	2491	3.3963737
2387	3.3778524	2422	3.3841741	2457	3.3904052	2492	3.3965480
2388	3.3780343	2423	3.3843534	2458	3.3905819	2493	3.3967223
2389	3.3782161	2424	3.3845326	2459	3.3907585	2494	3.3968964
2390	3.3783979	2425	3.3847117	2460	3.3909351	2495	3.3970705
2391	3.3785796	2426	3.3848908	2461	3.3911116	2496	3.3972446
2392	3.3787612	2427	3.3850698	2462	3.3912880	2497	3.3974185
2393	3.3789427	2428	3.3852487	2463	3.3914644	2498	3.3975924
2394	3.3791241	2429	3.3854275	2464	3.3916407	2499	3.3977662
2395	3.3793055	2430	3.3856063	2465	3.3918169	2500	3.3979400
2396	3.3794868	2431	3.3857850	2466	3.3919931	2501	3.3981137
2397	3.3796680	2432	3.3859636	2467	3.3921691	2502	3.3982873
2398	3.3798492	2433	3.3861421	2468	3.3923452	2503	3.3984608
2399	3.3800302	2434	3.3863206	2469	3.3925211	2504	3.3986343
2400	3.3802112	2435	3.3864990	2470	3.3926969	2505	3.3988077
2401	3.3803922	2436	3.3866773	2471	3.3928727	2506	3.3989811
2402	3.3805730	2437	3.3868555	2472	3.3930485	2507	3.3991543
2403	3.3807538	2438	3.3870337	2473	3.3932241	2508	3.3993275
2404	3.3809345	2439	3.3872118	2474	3.3933997	2509	3.3995005
2405	3.3811151	2440	3.3873898	2475	3.3935752	2510	3.3996737
2406	3.3812956	2441	3.3875678	2476	3.3937506	2511	3.3998467
2407	3.3814761	2442	3.3877457	2477	3.3939260	2512	3.4000196
2408	3.3816565	2443	3.3879235	2478	3.3941013	2513	3.4001925
2409	3.3818368	2444	3.3881012	2479	3.3942765	2514	3.4003653
2410	3.3820170	2445	3.3882789	2480	3.3944517	2515	3.4005380
2411	3.3821972	2446	3.3884565	2481	3.3946268	2516	3.4007106
2412	3.3823773	2447	3.3886340	2482	3.3948018	2517	3.4008832
2413	3.3825573	2448	3.3888114	2483	3.3949767	2518	3.4010557
2414	3.3827373	2449	3.3889888	2484	3.3951516	2519	3.4012282
2415	3.3829171	2450	3.3891661	2485	3.3953264	2520	3.4014005

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
2521	3.4015728	2556	3.4075608	2591	3.4134674	2626	3.4192947
2522	3.4017451	2557	3.4077307	2592	3.4136350	2627	3.4194601
2523	3.4019173	2558	3.4079005	2593	3.4138025	2628	3.4196254
2524	3.4020893	2559	3.4080703	2594	3.4139700	2629	3.4197906
2525	3.4022614	2560	3.4082400	2595	3.4141374	2630	3.4199557
2526	3.4024333	2561	3.4084096	2596	3.4143047	2631	3.4201208
2527	3.4026052	2562	3.4085791	2597	3.4144719	2632	3.4202859
2528	3.4027771	2563	3.4087486	2598	3.4146391	2633	3.4204509
2529	3.4029488	2564	3.4089180	2599	3.4148063	2634	3.4206158
2530	3.4031205	2565	3.4090874	2600	3.4149733	2635	3.4207806
2531	3.4032921	2566	3.4092567	2601	3.4151404	2636	3.4209454
2532	3.4034637	2567	3.4094259	2602	3.4153073	2637	3.4211101
2533	3.4036352	2568	3.4095950	2603	3.4154742	2638	3.4212748
2534	3.4038066	2569	3.4097641	2604	3.4156410	2639	3.4214394
2535	3.4039780	2570	3.4099331	2605	3.4158078	2640	3.4216039
2536	3.4041492	2571	3.4101021	2606	3.4159744	2641	3.4217684
2537	3.4043205	2572	3.4102710	2607	3.4161410	2642	3.4219328
2538	3.4044916	2573	3.4104398	2608	3.4163076	2643	3.4220972
2539	3.4046627	2574	3.4106085	2609	3.4164741	2644	3.4222614
2540	3.4048337	2575	3.4107771	2610	3.4166405	2645	3.4224257
2541	3.4050047	2576	3.4109459	2611	3.4168069	2646	3.4225898
2542	3.4051755	2577	3.4111144	2612	3.4169732	2647	3.4227539
2543	3.4053463	2578	3.4112829	2613	3.4171394	2648	3.4229180
2544	3.4055171	2579	3.4114513	2614	3.4173056	2649	3.4230820
2545	3 4056878	2580	3.4116197	2615	3.4174717	2650	3.4232459
2546	3.4058584	2581	3.4117880	2616	3.4176377	2651	3.4234097
2547	3.4060289	2582	3.4119562	2617	3.4178037	2652	3.4235735
2548	3.4061994	2583	3.4121244	2618	3.4179696	2653	3.4237372
2549	3.4063698	2584	3.4122925	2619	3.4181355	2654	3.4239009
2550	3.4065402	2585	3.4124605	2620	3.4183013	2655	3.4240645
2551	3.4067105	2586	3.4126285	2621	3.4184670	2656	3.4242281
2552	3.4068807	2587	3.4127964	2622	3.4186327	2657	3.4243916
2553	3.4070508	2588	3.4129642	2623	3.4187983	2658	3.4245550
2554	3.4072209	2589	3.4131320	2624	3.4189638	2659	3.4247183
2555	3.4073909	2590	3.4132998	2625	3.4191293	2660	3.4248816

Num.	Logarithm.	Num	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
2661	3.4250449	2696	3.4307199	2731	3.4363217	2766	3.4418522
2662	3.4252080	2697	3.4308809	2732	3.4364807	2767	3.4420092
2663	3.4253712	2698	3.4310419	2733	3.4366396	2768	3.4421661
2664	3.4255342	2699	3.4312029	2734	3.4367985	2769	3.4423229
2665	3.4256972	2700	3.4313638	2735	3.4369573	2770	3.4424798
2666	3.4258601	2701	3.4315246	2736	3.4371161	2771	3.4426365
2667	3.4260230	2702	3.4316853	2737	3.4372748	2772	3.4427932
2668	3.4261858	2703	3.4318460	2738	3.4374334	2773	3.4429499
2669	3.4263486	2704	3.4320066	2739	3.4375920	2774	3.4431065
2670	3.4265113	2705	3.4321673	2740	3.4377506	2775	3.4432630
2671	3.4266739	2706	3.4323278	2741	3.4379090	2776	3.4434195
2672	3.4268365	2707	3.4324883	2742	3.4380674	2777	3.4435759
2673	3.4269990	2708	3.4326487	2743	3.4382258	2778	3.4437322
2674	3.4271614	2709	3.4328090	2744	3.4383841	2779	3.4438885
2675	3.4273238	2710	3.4329693	2745	3.4385423	2780	3.4440448
2676	3.4274861	2711	3.4331295	2746	3.4387005	2781	3.4442010
2677	3.4276484	2712	3.4332897	2747	3.4388587	2782	3.4443571
2678	3.4278106	2713	3.4334498	2748	3.4390167	2783	3.4445132
2679	3.4279727	2714	3.4336098	2749	3.4391747	2784	3.4446692
2680	3.4281348	2715	3.4337698	2750	3.4393327	2785	3.4448252
2681	3.4282968	2716	3.4339298	2751	3.4394906	2786	3.4449811
2682	3.4284588	2717	3.4340896	2752	3.4396484	2787	3.4451370
2683	3.4286207	2718	3.4342494	2753	3.4398062	2788	3.4452928
2684	3.4287825	2719	3.4344092	2754	3.4399639	2789	3.4454485
2685	3.4289442	2720	3.4345689	2755	3.4401216	2790	3.4456042
2686	3.4291060	2721	3.4347285	2756	3.4402792	2791	3.4457598
2687	3.4292677	2722	3.4348881	2757	3.4404368	2792	3.4459154
2688	3.4294293	2723	3.4350476	2758	3.4405943	2793	3.4460709
2689	3.4295908	2724	3.4352071	2759	3.4407517	2794	3.4462264
2690	3.4297522	2725	3.4353665	2760	3.4409091	2795	3.4463818
2691	3.4299137	2726	3.4355258	2761	3.4410664	2796	3.4465371
2692	3.4300751	2727	3.4356851	2762	3.4412237	2797	3.4466925
2693	3.4302364	2728	3.4358444	2763	3.4413809	2798	3.4468477
2694	3.4303976	2729	3.4360035	2764	3.4415380	2799	3.4470029
2695	3.4305588	2730	3.4361626	2765	3.4416951	2800	3.4471580

Num.	Logarithm.	Num	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
2801	3.4473131	2836	3.4527062	2871	3.4580332	2906	3.4632956
2802	3.4474681	2837	3.4528593	2872	3.4581844	2907	3.4634450
2803	3.4476231	2838	3.4530124	2873	3.4583356	2908	3.4635944
2804	3.4477780	2839	3.4531654	2874	3.4584868	2909	3.4637437
2805	3.4479329	2840	3.4533183	2875	3.4586378	2910	3.4638930
2806	3.4480877	2841	3.4534712	2876	3.4587889	2911	3.4640422
2807	3.4482424	2842	3.4536241	2877	3.4589399	2912	3.4641914
2808	3.4483971	2843	3.4537769	2878	3.4590908	2913	3.4643405
2809	3.4485517	2844	3.4539296	2879	3.4592417	2914	3.4644895
2810	3.4487063	2845	3.4540823	2880	3.4593925	2915	3.4646386
2811	3.4488608	2846	3.4542349	2881	3.4595433	2916	3.4647875
2812	3.4490153	2847	3.4543875	2882	3.4596940	2917	3.4649364
2813	3.4491697	2848	3.4545400	2883	3.4598446	2918	3.4650853
2814	3.4493241	2849	3.4546924	2884	3.4599953	2919	3.4652341
2815	3.4494784	2850	3.4548449	2885	3.4601458	2920	3.4653828
2816	3.4496326	2851	3.4549972	2886	3.4602963	2921	3.4655316
2817	3.4497868	2852	3.4551495	2887	3.4604468	2922	3.4656802
2818	3.4499410	2853	3.4553018	2888	3.4605972	2923	3.4658288
2819	3.4500951	2854	3.4554540	2889	3.4607475	2924	3.4659775
2820	3.4502491	2855	3.4556061	2890	3.4608978	2925	3.4661259
2821	3.4504031	2856	3.4557582	2891	3.4610481	2926	3.4662743
2822	3.4505570	2857	3.4559102	2892	3.4611983	2927	3.4664227
2823	3.4507109	2858	3.4560622	2893	3.4613484	2928	3.4665711
2824	3.4508647	2859	3.4562141	2894	3.4614985	2929	3.4667194
2825	3.4510184	2860	3.4563660	2895	3.4616486	2930	3.4668676
2826	3.4511721	2861	3.4565179	2896	3.4617986	2931	3.4670158
2827	3.4513258	2862	3.4566696	2897	3.4619485	2932	3.4671640
2828	3.4514794	2863	3.4568213	2898	3.4620984	2933	3.4673120
2829	3.4516329	2864	3.4569730	2899	3.4622482	2934	3.4674601
2830	3.4517864	2865	3.4571246	2900	3.4623980	2935	3.4676081
2831	3.4519399	2866	3.4572762	2901	3.4625477	2936	3.4677560
2832	3.4520932	2867	3.4574276	2902	3.4626974	2937	3.4679039
2833	3.4522466	2868	3.4575791	2903	3.4628470	2938	3.4680518
2834	3.4523998	2869	3.4577305	2904	3.4629966	2939	3.4681996
2835	3.4525531	2870	3.4578819	2905	3.4631461	2940	3.4683473

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
2941	3.4684950	2976	3.4736329	3011	3.4787108	3046	3.4837299
2942	3.4686427	2977	3.4737788	3012	3.4788550	3047	3.4838725
2943	3.4687903	2978	3.4739247	3013	3.4789991	3048	3.4840150
2944	3.4689378	2979	3.4740705	3014	3.4791432	3049	3.4841574
2945	3.4690853	2980	3.4742163	3015	3.4792873	3050	3.4842998
2946	3.4692327	2981	3.4743620	3016	3.4794313	3051	3.4844422
2947	3.4693801	2982	3.4745076	3017	3.4795753	3052	3.4845845
2948	3.4695275	2983	3.4746533	3018	3.4797192	3053	3.4847268
2949	3.4696748	2984	3.4747988	3019	3.4798631	3054	3.4848690
2950	3.4698220	2985	3.4749443	3020	3.4800069	3055	3.4850112
2951	3.4699692	2986	3.4750898	3021	3.4801507	3056	3.4851533
2952	3.4701163	2987	3.4752352	3022	3.4802945	3057	3.4852954
2953	3.4702634	2988	3.4753806	3023	3.4804381	3058	3.4854375
2954	3.4704105	2989	3.4755259	3024	3.4805818	3059	3.4855795
2955	3.4705575	2990	3.4756712	3025	3.4807254	3060	3.4857214
2956	3.4707044	2991	3.4758164	3026	3.4808689	3061	3.4858633
2957	3.4708513	2992	3.4759616	3027	3.4810124	3062	3.4860052
2958	3.4709982	2993	3.4761067	3028	3.4811559	3063	3.4861470
2959	3.4711450	2994	3.4762518	3029	3.4812993	3064	3.4862888
2960	3.4712917	2995	3.4763968	3030	3.4814426	3065	3.4864305
2961	3.4714384	2996	3.4765418	3031	3.4815859	3066	3.4865721
2962	3.4715852	2997	3.4766867	3032	3.4817292	3067	3.4867138
2963	3.4717317	2998	3.4768316	3033	3.4818724	3068	3.4868554
2964	3.4718782	2999	3.4769765	3034	3.4820156	3069	3.4869969
2965	3.4720247	3000	3.4771212	3035	3.4821587	3070	3.4871384
2966	3.4721711	3001	3.4772660	3036	3.4823018	3071	3.4872798
2967	3.4723175	3002	3.4774107	3037	3.4824448	3072	3.4874212
2968	3.4724639	3003	3.4775553	3038	3.4825878	3073	3.4875626
2969	3.4726102	3004	3.4776999	3039	3.4827307	3074	3.4877039
2970	3.4727564	3005	3.4778445	3040	3.4828736	3075	3.4878451
2971	3.4729027	3006	3.4779890	3041	3.4830164	3076	3.4879863
2972	3.4730488	3007	3.4781334	3042	3.4831592	3077	3.4881275
2973	3.4731949	3008	3.4782778	3043	3.4833019	3078	3.4882686
2974	3.4733410	3009	3.4784222	3044	3.4834446	3079	3.4884097
2975	3.4734870	3010	3.4785665	3045	3.4835873	3080	3.4885507

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
3081	3.4886917	3116	3.4935974	3151	3.4984484	3186	3.5032458
3082	3.4888326	3117	3.4937368	3152	3.4985862	3187	3.5033821
3083	3.4889735	3118	3.4938761	3153	3.4987240	3188	3.5035183
3084	3.4891144	3119	3.4940154	3154	3.4988617	3189	3.5036545
3085	3.4892552	3120	3.4941546	3155	3.4989994	3190	3.5037907
3086	3.4893959	3121	3.4942938	3156	3.4991370	3191	3.5039268
3087	3.4895366	3122	3.4944329	3157	3.4992746	3192	3.5040629
3088	3.4896773	3123	3.4945720	3158	3.4994121	3193	3.5041989
3089	3.4898179	3124	3.4947110	3159	3.4995496	3194	3.5043349
3090	3.4899585	3125	3.4948500	3160	3.4996871	3195	3.5044709
3091	3.4900990	3126	3.4949890	3161	3.4998245	3196	3.5046068
3092	3.4902395	3127	3.4951279	3162	3.4999619	3197	3.5047426
3093	3.4903799	3128	3.4952667	3163	3.5000992	3198	3.5048785
3094	3.4905203	3129	3.4954056	3164	3.5002365	3199	3.5050142
3095	3.4906607	3130	3.4955443	3165	3.5003737	3200	3.5051500
3096	3.4908009	3131	3.4956831	3166	3.5005109	3201	3.5052857
3097	3.4909412	3132	3.4958218	3167	3.5006481	3202	3.5054213
3098	3.4910814	3133	3.4959604	3168	3.5007852	3203	3.5055569
3099	3.4912216	3134	3.4960990	3169	3.5009222	3204	3.5056925
3100	3.4913617	3135	3.4962375	3170	3.5010593	3205	3.5058280
3101	3.4915018	3136	3.4963761	3171	3.5011962	3206	3.5059635
3102	3.4916418	3137	3.4965145	3172	3.5013332	3207	3.5060990
3103	3.4917818	3138	3.4966529	3173	3.5014701	3208	3.5062344
3104	3.4919217	3139	3.4967913	3174	3.5016069	3209	3.5063697
3105	3.4920616	3140	3.4969296	3175	3.5017437	3210	3.5065090
3106	3.4922014	3141	3.4970679	3176	3.5018805	3211	3.5066403
3107	3.4923413	3142	3.4972062	3177	3.5020172	3212	3.5067755
3108	3.4924810	3143	3.4973444	3178	3.5021539	3213	3.5069107
3109	3.4926207	3144	3.4974825	3179	3.5022905	3214	3.5070459
3110	3.4927604	3145	3.4976206	3180	3.5024271	3215	3.5071810
3111	3.4929000	3146	3.4977587	3181	3.5025637	3216	3.5073160
3112	3.4930396	3147	3.4978967	3182	3.5027001	3217	3.5074511
3113	3.4931791	3148	3.4980347	3183	3.5028366	3218	3.5075860
3114	3.4933186	3149	3.4981727	3184	3.5029731	3219	3.5077210
3115	3.4934580	3150	3.4983106	3185	3.5030094	3220	3.5078559

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
3221	3.5079907	3256	3.5126844	3291	3.5173279	3326	3.5219222
3222	3.5081255	3257	3.5128178	3292	3.5174598	3327	3.5220528
3223	3.5082603	3258	3.5129511	3293	3.5175917	3328	3.5221833
3224	3.5083950	3259	3.5130844	3294	3.5177236	3329	3.5223138
3225	3.5085297	3260	3.5132176	3295	3.5178554	3330	3.5224442
3226	3.5086644	3261	3.5133508	3296	3.5179872	3331	3.5225746
3227	3.5087990	3262	3.5134840	3297	3.5181189	3332	3.5227050
3228	3.5089335	3263	3.5136171	3298	3.5182506	3333	3.5228353
3229	3.5090680	3264	3.5137501	3299	3.5183823	3334	3.5229656
3230	3.5092025	3265	3.5138832	3300	3.5185139	3335	3.5230958
3231	3.5093370	3266	3.5140162	3301	3.5186455	3336	3.5232260
3232	3.5094713	3267	3.5141491	3302	3.5187771	3337	3.5233562
3233	3.5096057	3268	3.5142820	3303	3.5189086	3338	3.5234863
3234	3.5097400	3269	3.5144149	3304	3.5190400	3339	3.5236164
3235	3.5098743	3270	3.5145478	3305	3.5191715	3340	3.5237465
3236	3.5100085	3271	3.5146805	3306	3.5193028	3341	3.5238765
3237	3.5101427	3272	3.5148133	3307	3.5194342	3342	3.5240064
3238	3.5102768	3273	3.5149460	3308	3.5195655	3343	3.5241364
3239	3.5104109	3274	3.5150787	3309	3.5196968	3344	3.5242663
3240	3.5105450	3275	3.5152113	3310	3.5198280	3345	3.5243961
3241	3.5106790	3276	3.5153439	3311	3.5199592	3346	3.5245259
3242	3.5108130	3277	3.5154764	3312	3.5200903	3347	3.5246551
3243	3.5109469	3278	3.5156089	3313	3.5202214	3348	3.5247854
3244	3.5110808	3279	3.5157414	3314	3.5203525	3349	3.5249151
3245	3.5112147	3280	3.5158738	3315	3.5204835	3350	3.5250448
3246	3.5113485	3281	3.5160062	3316	3.5206145	3351	3.5251744
3247	3.5114823	3282	3.5161386	3317	3.5207455	3352	3.5253040
3248	3.5116160	3283	3.5162709	3318	3.5208764	3353	3.5254335
3249	3.5117497	3284	3.5164031	3319	3.5210073	3354	3.5255631
3250	3.5118834	3285	3.5165354	3320	3.5211381	3355	3.5256925
3251	3.5120170	3286	3.5166676	3321	3.5212689	3356	3.5258219
3252	3.5121505	3287	3.5167997	3322	3.5213996	3357	3.5259513
3253	3.5122841	3288	3.5169318	3323	3.5215303	3358	3.5260806
3254	3.5124175	3289	3.5170639	3324	3.5216610	3359	3.5262100
3255	3.5125510	3290	3.5171959	3325	3.5217916	3360	3.5263393

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
3361	3.5264685	3396	3.5309677	3431	3.5354207	3466	3.5398286
3362	3.5265977	3397	3.5310955	3432	3.5355473	3467	3.5399538
3363	3.5267269	3398	3.5312234	3433	3.5356738	3468	3.5400791
3364	3.5268560	3399	3.5313512	3434	3.5358003	3469	3.5402043
3365	3.5269851	3400	3.5314789	3435	3.5359267	3470	3.5403295
3366	3.5271141	3401	3.5316066	3436	3.5360532	3471	3.5404546
3367	3.5272431	3402	3.5317343	3437	3.5361795	3472	3.5405797
3368	3.5273721	3403	3.5318619	3438	3.5363059	3473	3.5407048
3369	3.5275010	3404	3.5319895	3439	3.5364322	3474	3.5408298
3370	3.5276299	3405	3.3521171	3440	3.5365584	3475	3.5409548
3371	3.5277588	3406	3.5322446	3441	3.5366847	3476	3.5410798
3372	3.5278876	3407	3.5323721	3442	3.5368109	3477	3.5422047
3373	3.5280163	3408	3.5324996	3443	3.5369370	3478	3.5413296
3374	3.5281451	3409	3.5326270	3444	3.5370631	3479	3.5414544
3375	3.5282738	3410	3.5327544	3445	3.5321892	3580	3.5415792
3376	3.5284024	3411	3.5328817	3446	3.5373153	3481	3.5417040
3377	3.5285311	3412	3.5330090	3447	3.5374413	3482	2.5418288
3378	3.5286596	3413	3.5331363	3448	3.5375672	3483	3.5419535
3379	3.5287882	3414	3.5332635	3449	3.5376932	3484	3.5420781
3380	3.5289167	3415	3.5333907	3450	3.5378191	3485	3.5422028
3381	3.5290452	3416	3.5335179	3451	3.5379450	3486	3.5423274
3382	3.5291736	3417	3.5336450	3452	3.5380708	3487	3.5424519
3383	3.5293020	3418	3.5337721	3453	3.5381966	3488	3.5425765
3384	3.5294303	3419	3.5338991	3454	3.5383223	3489	3.5427010
3385	3.5295587	3420	3.5340261	3455	3.5384481	3490	3.5428254
3386	3.5296869	3421	3.5341531	3456	3.5385737	3491	3.5429498
3387	3.5298152	3422	3.5342800	3457	3.5386994	3492	3.5430742
3388	3.5299434	3423	3.5344069	3458	3.5388250	3493	3.5431986
3389	3.5300716	3424	3.5345338	3459	3.5389506	3494	3.5433229
3390	3.5301997	3425	3.5346606	3460	3.5390761	3495	3.5434472
3391	3.5303278	3426	3.5347874	3461	3.5392016	3496	3.5435714
3392	3.5304558	3427	3.5349141	3462	3.5393271	3497	3.5436956
3393	3.5305839	3428	3.5350408	3463	3.5394525	3498	3.5438198
3394	3.5307118	3429	3.5351675	3464	3.5395779	3499	3.5439439
3395	3.5308398	3430	3.5352941	3465	3.5397032	3500	3.5440630

Num.	*Logarithm.*	Num.	*Logarithm.*	Num.	*Logarithm.*	Num.	*Logarithm.*
3501	3.5441921	3536	3.5485123	3571	3.5527898	3606	3.5570257
3502	3.5443161	3537	3.5486351	3562	3.5529114	3607	3.5571461
3503	3.5444401	3538	3.5487578	3573	3.5530330	3608	3.5572665
3504	3.5445641	3539	3.5488806	3574	3.5531545	3609	3.5573869
3505	3.5446880	3540	3.5490033	3575	3.5532760	3610	3.5575072
3506	3.5448119	3541	3.5491259	3576	3.5533975	3611	3.5576275
3507	3.5449358	3542	.5492486	3577	3.5535189	3612	3.5577477
3508	3.5450596	3543	3.5493712	3578	3.5536403	3613	3.5578680
3509	3.5451834	3544	3.5494937	3579	3.5537617	3614	3.5579881
3510	3.5453071	3545	3.5496162	3580	3.5538830	3615	3.5581083
3511	3.5454308	3546	3.5497387	3581	3.5540043	3616	3.5582284
3512	3.5455545	3547	3.5498612	3582	3.5541256	3617	3.5583485
3513	3.5456781	3548	3.5499836	3583	3.5542468	3618	3.5584686
3514	3.5458017	3549	3.5501060	3584	3.5543680	3619	3.5585886
3515	3.5459253	3550	3.5502283	3585	3.5544892	3620	3.5587086
3516	3.5460489	3551	3.5503507	3586	3.5546103	3621	3.5588285
3517	3.5461724	3552	3.5504730	3587	3.5547314	3622	3.5589484
3518	3.5462958	3553	3.5505952	3588	3.5548524	3623	3.5590683
3519	3.5464193	3554	3.5507174	3589	3.5549735	3624	3.5591882
3520	3.5465427	3555	3.5508396	3590	3.5550944	3625	3.5593080
3521	3.5466660	3556	3.5509618	3591	3.5552154	3626	3.5594278
3522	3.5467894	3557	3.5510839	3592	3.5553363	3627	3.5595476
3523	3.5469126	3558	3.5512059	3593	3.5554572	3628	3.5596673
3524	3.5470359	3559	3.5513280	3594	3.5555781	3629	3.5597870
3525	3.5471591	3560	3.5514500	3595	3.5556989	3630	3.5599066
3526	3.5472823	3561	3.5515720	3596	3.5558197	3631	3.5600262
3527	3.5474055	3562	3.5516939	3597	3.5559404	3632	3.5601458
3528	3.5475286	3563	3.5518158	3598	3.5560612	3633	3.5602654
3529	3.5476517	3564	3.5519377	3599	3.5561818	3634	3.5603849
3530	3.5477748	3565	3.5520595	3600	3.5563025	3635	3.5605044
3531	3.5478977	3566	3.5421813	3601	3.5564231	3636	3.5606339
3532	3.5480207	3567	3.5523031	3602	3.5565437	3637	3.5607433
3533	3.5481436	3568	3.5524248	3603	3.5566643	3638	3.5608627
3534	3.5482665	3569	3.5525465	3604	3.5567848	3639	3,5609820
3535	3.5483896	3570	3.5526682	3605	3.5569053	3640	3,5611014

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
3041	3.5612207	3676	3.5653755	3711	3.5694910	3746	3.5735678
3642	3.5613399	3677	3.5654936	3712	3.5696080	3747	3.5736837
3643	3.5614592	3678	3.5656117	3713	3.5697249	3748	3.5737996
3644	3.5615784	3679	3.5657298	3714	3.5698419	3749	3.5739154
3645	3.5616975	3680	3.5658478	3715	3.5699588	3750	3.5740313
3646	3.5618167	3681	3.5659658	3716	3.5700757	3751	3.5741471
3647	3.5619358	3682	3.5660838	3717	3.5701926	3752	3.5742628
3648	3.5620548	3683	3.5662017	3718	3.5703094	3753	3.5743786
3649	3.5621739	3684	3.5663196	3719	3.5704262	3754	3.5744943
3650	3.5622929	3685	3.5664375	3720	3.5705429	3755	3.5746099
3651	3.5624118	3686	3.5665553	3721	3.5706597	3756	3.5747256
3652	3.5625308	3687	3.5666731	3722	3.5707764	3757	3.5748412
3653	3.5626497	3688	3.5667909	3723	3.5708930	3758	3.5749568
3654	3.5627685	3689	3.5669087	3724	3.5710097	3759	3.5750723
3655	3.5628874	3690	3.5670264	3725	3.5711263	3760	3.5751878
3656	3.5630062	3691	3.5671440	3726	3.5712428	3761	3.5753033
3657	3.5631250	3692	3.5672617	3727	3.5713594	3762	3.5754188
3658	3.5632437	3693	3.5673793	3728	3.5714759	3763	3.5755342
3659	3.5633624	3694	3.5674969	3729	3.5715924	3764	3.5756496
3660	3.5634811	3695	3.5676144	3730	3.5717087	3765	3.5757650
3661	3.5635997	3696	3.5677320	3731	3.5718252	3766	3.5758803
3662	3.5637183	3697	3.5678494	3732	3.5719416	3767	3.5759956
3663	3.5638369	3698	3.5679669	3733	3.5720580	3768	3.5761109
3664	3.5639555	3699	3.5680843	3734	3.5721743	3769	3.5762261
3665	3.5640740	3700	3.5682017	3735	3.5722906	3770	3.5763413
3666	3.5641925	3701	3.5683192	3736	3.5724069	3771	3.5764565
3667	3.5643109	3702	3.5684364	3737	3.5725231	3772	3.5765717
3668	3.5644293	3703	3.5685537	3738	3.5726393	3773	3.5766868
3669	3.5645477	3704	3.5686710	3739	3.5727555	3774	3.5768019
3670	3.5646661	3705	3.5687882	3740	3.5728716	3775	3.5769169
3671	3.5647844	3706	3.5689054	3741	3.5739877	3776	3.5770321
3672	3.5649027	3707	3.5690226	3742	3.5731038	3777	3.5771470
3673	3.5650209	3708	3.5691397	3743	3.5732198	3778	3.5772620
3674	3.5651392	3709	3.5692568	3744	3.5733358	3779	3.5773769
3675	3.5652573	3710	3.5693739	3745	3.5734518	3780	3.5774917

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm
3781	3.5776067	3816	3.5816084	3851	3.5855735	3886	3.5895028
3782	3.5777215	3817	3.5817222	3852	3.5856863	3887	3.5896145
3783	3.5778363	3818	3.5818359	3853	3.5857990	3888	3.5897262
3784	3.5779511	3819	3.5819497	3854	3.5859117	3889	3.5898379
3785	3.5780659	3820	3.5820634	3855	3.5860244	3890	3.5899496
3786	3.5781806	3821	3.5821770	3856	3.5861370	3891	3.5900612
3787	3.5782953	3822	3.5822907	3857	3.5862496	3892	3.5901728
3788	3.5784100	3823	3.5824043	3858	3.5863622	3893	3.5902844
3789	3.5785246	3824	3.5825179	3859	3.5864748	3894	3.5903959
3790	3.5786392	3825	3.5826314	3860	3.5865873	3895	3.5905075
3791	3.5787538	3826	3.5827450	3861	3.5866998	3896	3.5906189
3792	3.5788683	3827	3.5828585	3862	3.5868123	3897	3.5907304
3793	3.5789828	3828	3.5821799	3863	3.5869247	3898	3.5908418
3794	3.5790973	3829	3.5830854	3864	3.5870371	3899	3.5909532
3795	3.5792118	3830	3.5831988	3865	3.5871495	3900	3.5910646
3796	3.5793262	3831	3.5833122	3866	3.5872618	3901	3.5911759
3797	3.5794406	3832	3.5834255	3867	3.5873742	3902	3.5912873
3798	3.5795550	3833	3.5835388	3868	3.5874865	3903	3.5913985
3799	3.5796693	3834	3.5836521	3869	3.5875987	3904	3.5915098
3800	3.5797836	3835	3.5837654	3870	3.5877110	3905	3.5916210
3801	3.5798979	3846	3.5838786	3871	3.5878232	3906	3.5917322
3802	3.5800121	3847	3.5839918	3872	3.5879353	3907	3.5918434
3803	3.5801263	3848	3.5841050	3873	3.5880475	3908	3.5919546
3804	3.5802405	3849	3.5842181	3874	3.5881596	3909	3.5920657
3805	3.5803547	3840	3.5843312	3875	3.5882717	3910	3.5921768
3806	3.5804688	3841	3.5844443	3876	3.5883838	3911	3.5922878
3807	3.5805829	3842	3.5845574	3877	3.5884958	3912	3.5923988
3808	3.5806969	3843	3.5846704	3878	3.5886078	3913	3.5925098
3809	3.5808110	3844	3.5847834	3879	3.5887198	3914	3.5926208
3810	3.5809250	3845	3.5848963	3880	3.5888317	3915	3.5927318
3811	3.5810389	3846	3.5850093	3881	3.5889436	3916	3.5928427
3812	3.5811529	3847	3.5851222	3882	3.5890555	3917	3.5929536
3813	3.5812668	3848	3.5852351	3883	3.5891674	3918	3.5930644
3814	3.5813807	3849	3.5853479	3884	3.5892792	3919	3.5931753
3815	3.5814945	3850	3.5854607	3885	3.5893910	3920	3.5932861

Num.	Logarithm.	Num	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
3921	3.5933968	3956	3.5972563	3991	3.6010817	4026	3.6048738
3922	3.5935076	3957	3.5973660	3992	3.6011905	4027	3.6049816
3923	3.5936183	3958	3.5974758	3993	3.6012993	4028	3.6050895
3924	3.5937290	3959	3.5975855	3994	3.6014080	4029	3.6051973
3925	3.5938397	3960	3.5976952	3995	3.6015168	4030	3.6053050
3926	3.5939503	3961	3.5978048	3996	3.6016255	4031	3.6054128
3927	3.5940609	3962	3.5979145	3997	3.6017341	4032	3.6055205
3928	3.5941715	3963	3.5980241	3998	3.6018428	4033	3.6056282
3929	3.5942820	3964	3.5981336	3999	3.6019514	4034	3.6057359
3930	3.5943925	3965	3.5982423	4000	3.6020600	4035	3.6058435
3931	3.5945030	3966	3.5983527	4001	3.6021685	4036	3.6059512
3932	3.5946135	3967	3.5984622	4002	3.6022771	4037	3.6060587
3933	3.5947239	3968	3.5985717	4003	3.6023856	4038	3.6061663
3934	3.5948344	3969	3.5986811	4004	3.6024941	4039	3.6062738
3935	3.5949447	3970	3.5987905	4005	3.6026025	4040	3.6063814
3936	3.5950551	3971	3.5988999	4006	3.6027109	4041	3.6064888
3937	3.5951654	3972	3.5990092	4007	3.6028193	4042	3.6065963
3938	3.5952757	3973	3.5991186	4008	3.6029277	4043	3.6067037
3939	3.5953860	3974	3.5992279	4009	3.6030361	4044	3.6068111
3940	3.5954962	3975	3.5993371	4010	3.6031444	4045	3.6069185
3941	3.5956064	3976	3.5994464	4011	3.6032527	4046	3.6070259
3942	3.5957166	3977	3.5995556	4012	3.6033609	4047	3.6071331
3943	3.5958268	3978	3.5996648	4913	3.6034692	4048	3.6072405
3944	3.5959369	3979	3.5997739	4014	3.6035774	4049	3.6073478
3945	3.5960470	3980	3.5998831	4015	3.6036855	4050	3.6074550
3946	3.5961571	3981	3.5999922	4016	3.6037937	4051	3.6075622
3947	3.5962671	3982	3.6001013	4017	3.6039018	4052	3.6076694
3948	3.5963771	3983	3.6002103	4018	3.6040099	4053	3.6077766
3949	3.5964871	3984	3.6003193	4019	3.6041180	4054	3.6078837
3950	3.5965971	3985	3.6004283	4020	3.6042261	4055	3.6079909
3951	3.5967070	3986	3.6005373	4021	3.6043341	4056	3.6080979
3952	3.5968169	3987	3.6006462	4022	3.6044421	4057	3.6082050
3953	3.5969268	3988	3.6007551	4023	3.6045500	4058	3.6083120
3954	3.5970367	3989	3.6008640	4024	3.6046580	4059	3.6084190
3655	3.5971465	3990	3.6009729	4025	3.6047659	4060	3.6085260

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
4061	3.6086330	4096	3.6123599	4131	3.6160552	4166	3.6197193
4062	3.6087399	4097	3.6124660	4132	3.6161603	4167	3.6198235
4063	3.6088468	4098	3.6125720	4133	3.6162654	4168	3.6199277
4064	3.6089537	4099	3.6126779	4134	3.6163705	4169	3.6200319
4065	3.6090605	4100	3.6127839	4135	3.6164755	4170	3.6201360
4066	3.6091674	4101	3.6128898	4136	3.6165805	4171	3.6202402
4067	3.6092742	4102	3.6129957	4137	3.6166855	4172	3.6203443
4068	3.6093809	4103	3.6131015	4138	3.6167905	4173	3.6204484
4069	3.6094877	4104	3.6132073	4139	3.6168954	4174	3.6205524
4070	3.6095944	4105	3.6133132	4140	3.6170003	4175	3.6206565
4071	3.6097011	4106	3.6134189	4141	3.6171052	4176	3.6207605
4072	3.6098078	4107	3.6135247	4142	3.6172101	4177	3.6208645
4073	3.6099144	4108	3.6136304	4143	3.6173149	4178	3.6209684
4074	3.6100210	4109	3.6137361	4144	3.6174197	4179	3.6210724
4075	3.6101276	4110	3.6138418	4145	3.6175245	4180	3.6211763
4076	3.6102342	4111	3.6139475	4146	3.6176293	4181	3.6212802
4077	3.6103407	4112	3.6140531	4147	3.6177340	4182	3.6213840
4078	3.6104472	4113	3.6141587	4148	3.6178387	4183	3.6214879
4079	3.6105537	4114	3.6142642	4149	3.6179434	4184	3.6215917
4080	3.6106602	4115	3.6143698	4150	3.6180481	4185	3.6216955
4081	3.6107666	4116	3.6144754	4151	3.6181527	4186	3.6217992
4082	3.6108730	4117	3.6145809	4152	3.6182573	4187	3.6219030
4083	3.6109794	4118	3.6146863	4153	3.6183619	4188	3.6220067
4084	3.6110857	4119	3.6147918	4154	3.6184665	4189	3.6221104
4085	3 6111921	4120	3.6148972	4155	3.6185710	4190	3.6222140
4086	3.6112984	4121	3.6150026	4156	3.6186755	4191	3.6223177
4087	3.6114046	4122	3.6151080	4157	3.6187800	4192	3.6224213
4088	3.6115109	4123	3.6152133	4158	3.6188845	4193	3.6225249
4089	3.6116171	4124	3.6153187	4159	3.6189889	4194	3.6226284
4090	3.6117233	4125	3.6154240	4160	3.6190933	4195	3.6227320
4091	3.6118295	4126	3.6155292	4161	3.6191977	4196	3.6228355
4092	3.6119356	4127	3.6156345	4162	3.6193021	4197	3.6229390
4093	3.6120417	4128	3.6157397	4163	3.6194064	4198	3.6230424
4094	3.6121478	4129	3.6158449	4164	3.6195107	4199	3.6231459
4095	3.6122539	4130	3.6159501	4165	3.6196150	4200	3.6232493

Num.	Logarithm.	Num	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
4201	3.6233527	4236	3.6269559	4271	3.6305296	4306	3.6340740
4202	3.6234560	4237	3.6270585	4272	3.6306312	4307	3.6341749
4203	3.6235594	4238	3.6271610	4273	3.6307329	4308	3.6342757
4204	3.6236627	4239	3.6272634	4274	3.6308345	4309	3.6343765
4205	3.6237660	4240	3.6273659	4275	3.6309362	4310	3.6344773
4206	3.6238693	4241	3.6274683	4276	3.6310377	4311	3.6345780
4207	3.6239725	4242	3.6275706	4277	3.6311392	4312	3.6346788
4208	3.6240757	4243	3.6276730	4278	3.6312408	4313	3.6347795
4209	3.6241789	4244	3.6277754	4279	3.6313423	4314	3.6348801
4210	3.6242821	4245	3.6278777	4280	3.6314438	4315	3.6349808
4211	3.6243852	4246	3.6279800	4281	3.6315452	4316	3.6350814
4212	3.6244883	4247	3.6280823	4282	3.6316467	4317	3.6351820
4213	3.6245915	4248	3.6281845	4283	3.6317481	4318	3.6352822
4214	3.6246945	4249	3.6282867	4284	3.6318495	4319	3.6353836
4215	3.6247976	4250	3.6283889	4285	3.6319508	4320	3.6354837
4216	3.6249006	4241	3.6284911	4286	3.6320522	4321	3.6355843
4217	3.6250036	4252	3.6285933	4287	3.6321535	4322	3.6356848
4218	3.6251066	4253	3.6286954	4288	3.6322548	4323	3.6357852
4219	3.6252095	4254	3.6287975	4289	3.6323560	4324	3.6358857
4220	3.6253124	4255	3.6288996	4290	3.6324573	4325	3.6359861
4221	3.6254153	4256	3.9290016	4291	3.6325585	4326	3.6360865
4222	3.6255182	4257	3.6291036	4292	3.6326597	4327	3.6361869
4223	3.6256211	4258	3.6292057	4293	3.6327609	4328	3.6362872
4224	3.6257239	4259	3.6293076	4294	3.6328620	4329	3.6363876
4225	3.6258267	4260	3.6294096	4295	3.6329632	4330	3.6364879
4226	3.6259295	4261	3.6295115	4296	3.6330643	4331	3.6365882
4227	3.6260322	4262	3.6296134	4297	3.6331653	4332	3.6366884
4228	3.6261350	4263	3.6297153	4298	3.6332664	4333	3.6367887
4229	3.6262377	4264	3.9298172	4299	3.6333674	4334	3.6368889
4230	3.6263404	4265	3.6299190	4300	3.6334685	4335	3.6369891
4231	3.6264430	4266	3.6300208	4301	3.6335694	4336	3.6370893
4232	3.6265457	4267	3.6301226	4302	3.6336704	4337	3.6371894
4233	3.6266483	4268	3.6302244	4303	3.6337713	4338	3.6372895
4234	3.6267509	4269	3.6303262	4304	3.6338723	4339	3.6373896
4235	3.6268534	4270	3.6304279	4305	3.6339732	4340	3.6374897

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
4341	3.6375898	4376	3.6410773	4411	3.6445371	4446	3.6479695
4342	3.6376898	4377	3.6411765	4412	3.6446355	4447	3.6480671
4343	3.6377898	4378	3.6412758	4413	3.6447339	4448	3.6481648
4344	3.6378898	4379	3.6413749	4414	3.6448323	4449	3.6482624
4345	3.6379898	4380	3.6414741	4415	3.6449307	4450	3.6483600
4346	3.6380897	4381	3.6415733	4416	3.6450291	4451	3.6484576
4347	3.6381896	4382	3.6416724	4417	3.6451274	4452	3.6485552
4348	3.6382895	4383	3.6417715	4418	3.6452257	4453	3.6486527
4349	3.6383894	4384	3.6418705	4419	3.6453240	4454	3.6487502
4350	3.6384893	4385	3.6419696	4420	3.6454223	4455	3.6488477
4351	3.6385891	4386	3.6420686	4421	3.6455205	4456	3.6489452
4352	3.6386886	4387	3.6421676	4422	3.6456187	4457	3.6490426
4353	3.6387887	4388	3.6422666	4423	3.6457169	4458	3.6491401
4354	3.6388884	4389	3.6423656	4424	3.6458151	4459	3.6492375
4355	3.6389882	4390	3.6424645	4425	3.6459133	4460	3.6493349
4356	3.6390879	4391	3.6425634	4426	3.6460114	4461	3.6494322
4357	3.6391878	4392	3.6426623	4427	3.6461095	4462	3.6495296
4358	3.6392872	4393	3.6427612	4428	3.6462076	4463	3.6496269
4359	3.6393869	4394	3.6428601	4429	3.6463057	4464	3.6497242
4360	3.6394865	4395	3.6429589	4430	3.6464037	4465	3.6498215
4361	3.6395861	4396	3.6430577	4431	3.6465017	4466	3.6499187
4362	3.6396857	4397	3.6431565	4432	3.6465997	4467	3.6500160
4363	3.6397852	4398	3.6432552	4433	3.6466977	4468	3.6501132
4364	3.6398847	4399	3.6433540	4434	3.6467957	4469	3.6502104
4365	3.6399842	4400	3.6434527	4435	3.6468936	4470	3.6503075
4366	3.6400837	4401	3.6435514	4436	3.6469915	4471	3.6504047
4367	3.6401832	4402	3.6436500	4437	3.6470894	4472	3.6505018
4368	3.6402826	4403	3.6437487	4438	3.6471873	4473	3.6505989
4369	3.6403820	4404	3.6438473	4439	3.6472851	4474	3.6506960
4370	3.6404814	4405	3.6439459	4440	3.6473830	4475	3.6507930
4371	3.6405808	4406	3.6440445	4441	3.6474808	4476	3.6508901
4372	3.6406802	4407	3.6441430	4442	3.6475785	4477	3.6509871
4373	3.6407795	4408	3.6442416	4443	3.6476763	4478	3.6510841
4374	3.6408788	4409	3.6443401	4444	3.6477740	4479	3.6511811
4375	3.6409781	4410	3.6444386	4445	3.6478718	4480	3.6512780

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
4481	3.6513750	4516	3.6547539	4551	3.6581068	4586	3.6614340
4482	3.6514719	4517	3.6548501	4552	3.6582023	4587	3.6615287
4483	3.6515687	4518	3.6549462	4553	3.6582976	4588	3.6616234
4484	3.6516656	4519	3.6550423	4554	3.6583930	4589	3.6617181
4485	3.6517624	4520	3.6551384	4555	3.6584884	4590	3.6618127
4486	3.6518593	4521	3.6552345	4556	3.6585837	4591	3.6619073
4487	3.6519561	4522	3.6553306	4557	3.6586790	4592	3.6620019
4488	3.6520528	4523	3.6554266	4558	3.6587743	4593	3.6620964
4489	3.6521496	4524	3.6555226	4559	3.6588696	4594	3.6621910
4490	3.6522463	4525	3.6556186	4560	3.6589648	4595	3.6622855
4491	3.6523430	4526	3.6557145	4561	3.6590601	4596	3.6623800
4492	3.6524397	4527	3.6558105	4562	3.6591553	4597	3.6624745
4493	3.6525364	4528	3.6559064	4563	3.6592505	4598	3.6625690
4494	3.6526331	4529	3.6560023	4564	3.6593456	4599	3.6626634
4495	3.6527297	4530	3.6560982	4565	3.6594408	4600	3.6627578
4496	3.6528263	4531	3.6561941	4566	3.6595359	4601	3.6628523
4497	3.6529229	4532	3.6562899	4567	3.6596310	4602	3.6629466
4498	3.6530195	4533	3.6563857	4568	3.6597261	4603	3.6630410
4499	3.6531160	4534	3.6564815	4569	3.6598212	4604	3.6631353
4500	3.6532125	4535	3.6565773	4570	3.6599162	4605	3.6632296
4501	3.6533090	4536	3.6566730	4571	3.6600112	4606	3.6633239
4502	3.6534055	4537	3.6567688	4572	3.6601062	4607	3.6634182
4503	3.6535019	4538	3.6568645	4573	3.6602012	4608	3.6635125
4504	3.6535984	4539	3.6569690	4574	3.6602962	4609	3.6636067
4505	3.6536948	4540	3.6570558	4575	3.6603911	4610	3.6637009
4506	3.6537912	4541	3.6571515	4576	3.6604860	4611	3.6637951
4507	3.6538876	4542	3.6572471	4577	3.6605809	4612	3.6638893
4508	3.6539839	4543	3.6573427	4578	3.6606758	4613	3.6639835
4509	3.6540802	4544	3.6574383	4579	3.6607706	4614	3.6640776
4510	3.6541765	4545	3.6575339	4580	3.6608655	4615	3.6641717
4511	3.6542728	4546	3.6576294	4581	3.6609603	4616	3.6642658
4512	3.6543691	4547	3.6577250	4582	3.6610551	4617	3.6643599
4513	3.6544653	4548	3.6578205	4583	3.6611499	4618	3.6644539
4514	3.6545616	4549	3.6579159	4584	3.6612445	4619	3.6645480
4515	3.6546578	4550	3.6580114	4585	3.6613393	4620	3.6646420

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
4621	3.6647360	4656	3.6680130	4691	3.6712654	4726	3.6744937
4622	3.6648299	4657	3.6681062	4692	3.6713580	4727	3.6745856
4623	3.6649239	4658	3.6681995	4693	3.6714506	4728	3.6746775
4624	3.6650175	4659	3.6682927	4694	3.6715431	4729	3.6747693
4625	3.6651117	4660	3.6683859	4695	3.6716356	4730	3.6748611
4626	3.6652056	4661	3.6684791	4696	3.6717281	4731	3.6749529
4627	3.6652995	4662	3.6685723	4697	3.6718206	4732	3.6750447
4628	3.6653934	4663	3.6686654	4698	3.6719130	4733	3.6751365
4629	3.6654873	4664	3.6687585	4699	3.6720054	4734	3.6752283
4630	3.6655810	4665	3.6688516	4700	3.6720979	4735	3.6753200
4631	3.6656748	4666	3.6689447	4701	3.6721903	4736	3.6754117
4632	3.6657685	4667	3.6690378	4702	3.6722826	4737	3.6755034
4633	3.6658623	4668	3.6691308	4703	3.6723750	4738	3.6755951
4634	3.6659560	4669	3.6692239	4704	3.6724673	4739	3.6756867
4635	3.6660497	4670	3.6693169	4705	3.6725596	4740	3.6757783
4636	3.6661434	4671	3.6694099	4706	3.6726519	4741	3.6758700
4637	3.6662371	4672	3.6695028	4707	3.6727442	4742	3.6759615
4638	3.6963307	4673	3.6695958	4708	3.6728365	4743	3.6760531
4639	3.6664244	4674	3.6696887	4709	3.6729287	4744	3.6761447
4640	3.6665180	4675	3.6697816	4710	3.6730209	4745	3.6762362
4641	3.6666116	4676	3.6698745	4711	3.6731131	4746	3.6763277
4642	3.6667051	4677	3.6699674	4712	3.6732053	4747	3.6764192
4643	3.6667987	4678	3.6700602	4713	3.6732974	4748	3.6765106
4644	3.6668922	4679	3.6701530	4714	3.6733896	4749	3.6766022
4645	3.6669857	4680	3.6702459	4715	3.6734817	4750	3.6766936
4646	3.6670792	4681	3.6703386	4716	3.6735738	4751	3.6767850
4647	3.6671727	4682	3.6704314	4717	3.6736659	4752	3.6768764
4648	3.6672661	4683	3.6705242	4718	3.6737579	4753	3.6769678
4649	3.6673595	4684	3.6706169	4719	3.6738500	4754	3.6770592
4650	3.6674530	4685	3.6707096	4720	3.6739420	4755	3.6771505
4851	3.6675463	4686	3.6708023	4721	3.6740340	4756	3.6772418
4652	3.6676397	4687	3.6708950	4722	3.6741260	4757	3.6773332
4653	3.6677331	4688	3.6709876	4723	3.6742179	4758	3.6774244
4654	3.6678264	4689	3.6710802	4724	3.6743099	4759	3.6775157
4655	3.6679197	4690	3.6711728	4725	3.6744018	4760	3.6776069

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
4761	3.6776982	4796	3.6808792	4831	3.6840370	4866	3.6871721
4762	3.6777894	4797	3.6809697	4832	3.6841269	4867	3.6872613
4763	3.6778806	4798	3.6810602	4833	3.6842168	4868	3.6873506
4764	3.6779718	4799	3.6811507	4834	3.6843066	4869	3.6874398
4765	3.6780629	4800	3.6812412	4835	3.6843965	4870	3.6875290
4766	3.6781540	4801	3.6813317	4836	3.6844863	4871	3.6876181
4767	3.6782452	4802	3.6814222	4837	3.6845761	4872	3.6877073
4768	3.6783362	4803	3.6815126	4838	3.6846659	4873	3.6877964
4769	3.6784273	4804	3.6816030	4839	3.6847556	4874	3.6878855
4770	3.6785184	4805	3.6816934	4840	3.6848455	4875	3.6879746
4771	3.6786094	4806	3.6817838	4841	3.6849351	4876	3.6880637
4772	3.6787004	4807	3.6818742	4842	3.6850248	4877	3.6881528
4773	3.6787914	4808	3.6819645	4843	3.6851145	4878	3.6882418
4774	3.6788824	4809	3.6820548	4844	3.6852041	4879	3.6883308
4775	3.6789734	4810	3.6821451	4845	3.6852938	4880	3.6884198
4776	3.6790643	4811	3.6822354	4846	3.6853834	4881	3.6885088
4777	3.6791552	4812	3.6823256	4847	3.6854730	4882	3.6885978
4778	3.6792461	4813	3.6824159	4848	3.6855626	4883	3.6886867
4779	3.6793370	4814	3.6825061	4849	3.6856522	4884	3.6887756
4780	3.6794279	4815	3.6825963	4850	3.6857417	4885	3.6888646
4781	3.6795187	4816	3.6826865	4851	3.6858313	4886	3.6889535
4782	3.6796096	4817	3.6827766	4852	3.6859208	4887	3.6890423
4783	3.6797004	4818	3.6828668	4853	3.6860103	4888	3.6891312
4784	3.6797912	4819	3.6829569	4854	3.6860998	4889	3.6892200
4785	3.6798819	4820	3.6830470	4855	3.6861892	4890	3.6893089
4786	3.6799727	4821	3.6831371	4856	3.6862787	4891	3.6893977
4787	3.6800634	4822	3.6832272	4857	3.6863681	4892	3.6894864
4788	3.6801541	4823	3.6833173	4858	3.6864575	4893	3.6895752
4789	3.6802448	4824	3.6834073	4859	3.6865469	4894	3.6896640
4790	3.6803355	4825	3.6834973	4860	3.6866363	4895	3.6897527
4791	3.6804262	4826	3.6835873	4861	3.6867256	4896	3.6898414
4792	3.6805168	4827	3.6836773	4862	3.6868149	4897	3.6899301
4793	3.6806074	4828	3.6837673	4863	3.6869043	4898	3.6900188
4794	3.6806980	4829	3.6838572	4864	3.6869936	4899	3.6901074
4795	3.6807886	4830	3.6839471	4865	3.6870828	4900	3.6901961

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
4901	3.6902847	4936	3.6933752	4971	3.6964438	5006	3.6994908
4902	3.6903733	4937	3.6934631	4972	3.6965311	5007	3.6995776
4903	3.6904619	4938	3.6935511	4973	3.6966185	5008	3.6996643
4904	3.6905505	4939	3.6936390	4974	3.6967058	5009	3.6997510
4905	3.6906390	4940	3.6937269	4975	3.6967931	5010	3.6998377
4906	3.6907275	4941	3.6938148	4976	3.6968804	5011	3.6999244
4907	3.6908161	4942	3.6939027	4977	3.6969676	5012	3.7000111
4908	3.6909046	4943	3.6939906	4978	3.6970549	5013	3.7000977
4909	3.6909930	4944	3.6940785	4979	3.6971421	5014	3.7001843
4910	3.6910815	4945	3.6941663	4980	3.6972293	5015	3.7002709
4911	3.6911699	4946	3.6942541	4981	3.6973165	5016	3.7003575
4912	3.6912584	4947	3.6943419	4982	3.6974037	5017	3.7004441
4913	3.6913467	4948	3.6944297	4983	3.6974909	5018	3.7005307
4914	3.6914352	4949	3.6945174	4984	3.6975780	5019	3.7006172
4915	3.6915235	4950	3.6946052	4985	3.6976652	5020	3.7007037
4916	3.6916119	4951	3.6946929	4986	3.6977523	5021	3.7007902
4917	3.6917002	4952	3.6947806	4987	3.6978394	5022	3.7008767
4918	3.6917885	4953	3.6948683	4988	3.6979264	5023	3.7009632
4919	3.6918768	4954	3.6949560	4989	3.6980135	5024	3.7010496
4920	3.6919651	4955	3.6950437	4990	3.6981005	5025	3.7011361
4921	3.6920534	4956	3.6951313	4991	3.6981876	5026	3.7012225
4922	3.6921416	4957	3.6952189	4992	3.6982746	5027	3.7013089
4923	3.6922298	4958	3.6953065	4993	3.6983616	5028	3.7013952
4924	3.6923180	4959	3.6953941	4994	3.6984485	5029	3.7014816
4925	3.6924062	4960	3.6954817	4995	3.6985355	5030	3.7015680
4926	3.6924944	4961	3.6955692	4996	3.6986224	5031	3.7016543
4927	3.6925825	4962	3.6956568	4997	3.6987093	5032	3.7017406
4928	3.6926707	4963	3.6957443	4998	3.6987963	5033	3.7018269
4929	3.6927588	4964	3.6958318	4999	3.6988831	5034	3.7019132
4930	3.6928469	4965	3.6959193	5000	3.6989700	5035	3.7019995
4931	3.6929350	4966	3.6960067	5001	3.6990569	5036	3.7020857
4932	3.6930231	4967	3.6960942	5002	3.6991437	5037	3.7021719
4933	3.6931111	4968	3.6961816	5003	3.6992305	5038	3.7022582
4934	3.6931991	4969	3.6962690	5004	3.6993173	5039	3.7023444
4935	3.6932872	4970	3.6963564	5005	3.6994041	5040	3.7024305

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
5041	3.7025167	5076	3.7055216	5111	3.7085059	5146	3.7114698
5042	3.7026028	5077	3.7056072	5112	3.7085908	5147	3.7115542
5043	3.7026890	5078	3.7056927	5113	3.7086758	5148	3.7116385
5044	3.7027751	5079	3.7057782	5114	3.7087607	5149	3.7117229
5045	3.7028612	5080	3.7058637	5115	3.7088456	5150	3.7118072
5046	3.7029472	5081	3.7059492	5116	3.7089305	5151	3.7118915
5047	3.7030333	5082	3.7060347	5117	3.7090154	5152	3.7119759
5048	3.7031193	5083	3.7061201	5118	3.7091003	5153	3.7120601
5049	3.7032054	5084	3.7062055	5119	3.7091851	5154	3.7121444
5050	3.7032914	5085	3.7062910	5120	3.7092700	5155	3.7122287
5051	3.7033774	5086	3.7063764	5121	3.7093548	5156	3.7123129
5052	3.7034633	5087	3.7064617	5122	3.7094396	5157	3.7123971
5053	3.7035493	5088	3.7065471	5123	3.7095244	5158	3.7124813
5054	3.7036352	5089	3.7066324	5124	3.7096091	5159	3.7125655
5055	3.7037212	5090	3.7067178	5125	3.7096939	5160	3.7126497
5056	3.7038071	5091	3.7068030	5126	3.7097786	5161	3.7127339
5057	3.7038929	5092	3.7068884	5127	3.7098633	5162	3.7128180
5058	3.7039788	5093	3.7069737	5128	3.7099480	5163	3.7129021
5059	3.7040647	5094	3.7070589	5129	3.7100327	5164	3.7129862
5060	3.7041505	5095	3.7071442	5130	3.7101174	5165	3.7130703
5061	3.7042363	5096	3.7072294	5131	3.7102020	5166	3.7131544
5062	3.7043221	5097	3.7073146	5132	3.7102866	5167	3.7132385
5063	3.7044079	5098	3.7073998	5133	3.7103713	5168	3.7133225
5064	3.7044937	5099	3.7074850	5134	3.7104559	5169	3.7134065
5065	3.7045794	5100	3.7075702	5145	3.7105404	5170	3.7134905
5066	3.7046652	5101	3.7076553	5136	3.7106250	5171	3.7135745
5067	3.7047509	5102	3.7077405	5137	3.7107096	5172	3.7136585
5068	3.7048366	5103	3.7078256	5138	3.7107941	5173	3.7137425
5069	3.7049223	5104	3.7079107	5139	3.7108786	5174	3.7138264
5070	3.7050080	5105	3.7079957	5140	3.7109631	5175	3.7139104
5071	3.7050936	5106	3.7080808	5141	3.7110476	5176	3.7139943
5072	3.7051792	5107	3.7081659	5142	3.7111321	5177	3.7140782
5073	3.7052649	5108	3.7082509	5143	3.7112165	5178	3.7141620
5074	3.7053505	5109	3.7083359	5144	3.7113010	5179	3.7142459
5075	3.7054360	5110	3.7084209	5145	3.7113854	5180	3.7143298

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
5321	3.7259933	5356	3.7288406	5391	3.7316693	5426	3.7344798
5322	3.7260749	5357	3.7289216	5392	3.7317499	5427	3.7345598
5323	3.7261565	5358	3.7290027	5393	3.7318304	5428	3.6346398
5324	3.7262380	5359	3.7290838	5394	3.7319109	5429	3.7347198
5325	3.7263196	5360	3.7291648	5395	3.7319914	5430	3.7347998
5326	3.7264012	5361	3.7292458	5396	3.7320719	5431	3.7348798
5327	3.7264827	5362	3.7293268	5397	3.7321524	5432	3.7349598
5328	3.7265642	5363	3.7294078	5398	3.7322329	5433	3.7350397
5329	3.7266457	5364	3.7294888	5399	3.7323133	5434	3.7351196
5330	3.7267272	5365	3.7295697	5400	3.7323938	5435	3.7351995
5331	3.7268087	5366	3.7296507	5401	3.7324742	5436	3.7352794
5332	3.7268901	5367	3.7297316	5402	3.7325546	5437	3.7353593
5333	3.7269716	5368	3.7298125	5403	3.7326350	5438	3.7354392
5334	3.7270531	5369	3.7298934	5404	3.7327153	5439	3.7355191
5335	3.7271344	5370	3.7299743	5405	3.7327957	5440	3.7355989
5336	3.7272158	5371	3.7300551	5406	3.7328760	5441	3.7356787
5337	3.7272972	5372	3.7301360	5407	3.7329564	5442	3.7357585
5338	3.7273786	5373	3.7302168	5408	3.7330367	5443	3.7358383
5339	3.7274599	5374	3.7302977	5409	3.7331170	5444	3.7359181
5340	3.7275413	5375	3.7303785	5410	3.7331973	5445	3.7359979
5341	3.7276226	5376	3.7304593	5411	3.7332775	5446	3.7360776
5342	3.7277039	5377	3.7305400	5412	3.7333578	5447	3.7361574
5343	3.7277852	5378	3.7306208	5413	3.7334380	5448	3.7362371
5344	3.7278664	5379	3.7307015	5414	3.7335181	5449	3.7363168
5345	3.7279477	5380	3.7307823	5415	3.7335985	5450	3.7363965
5346	3.7280290	5381	3.7308630	5416	3.7336787	5451	3.7364762
5347	3.7281101	5382	3.7309437	5417	3.7337588	5452	3.7365558
5348	3.7281914	5383	3.7310244	5418	3.7338990	5453	3.7366355
5349	3.7282726	5384	3.7311551	5419	3.7339191	5454	3.7367151
5350	3.7283538	5385	3.7311857	5420	3.7339993	5455	3.7367948
5351	3.7284349	5386	3.7312663	5421	3.7340794	5456	3.7368744
5352	3.7285161	5387	3.7313470	5422	3.7341595	5457	3.7369540
5353	3.7285972	5388	3.7314276	5423	3.7342396	5458	3.7370335
5354	3.7286784	5389	3.7315082	5424	3.7343197	5459	3.7371131
5355	3.7287595	5390	3.7315888	5425	3.7343997	5460	3.7371926

Num.	*Logarithm.*	Num.	*Logarithm.*	Num.	*Logarithm.*	Num.	*Logarithm.*
5181	3.7144136	5216	3.7173376	5251	3.7202420	5286	3.7231272
5182	3.7144974	5217	3.7174208	5252	3.7203247	5287	3.7232093
5183	3.7145812	5218	3.7175041	5153	3.7204074	5288	3.7232914
5184	3.7146650	5219	3.7175873	5254	3.7204901	5289	3.6233736
5185	3.7147488	5220	3.7276705	5255	3.7205727	5290	3.7234557
5186	3.7148325	5221	3.7177537	5256	3.7206554	5291	3.7235378
5187	3.7149162	5222	3.7178369	5257	3.7207380	5292	3.7236198
5188	3.7150000	5223	3.7179200	5258	3.7208206	5293	3.7237019
5189	3.7150837	5224	3.7180032	5259	3.7209032	5294	3.7237839
5190	3.7151674	5225	3.7180863	5260	3.7209857	5295	3.7238660
5191	3.7152510	5226	3.7181694	5261	3.7210683	5296	3.7239480
5192	3.7153347	5227	3.7182525	5262	3.7211508	5297	3.7240300
5193	3.7154183	5228	3.7183356	5263	3.7212334	5298	3.7241120
5194	3.7155019	5229	3.7184186	5264	3.7213159	5299	3.7241939
5195	3.7155856	5230	3.7185017	5265	3.7213984	5300	3.7242759
5196	3.7156691	5231	3.7185847	5266	3.7214809	5301	3.7243578
5197	3.7157527	5232	3.7186677	5267	3.7215633	5302	3.7244397
5198	3.7158363	5233	3.7187507	5268	3.7216458	5303	3.7245216
5199	3.7159198	5234	3.7188337	5269	3.7217282	5304	3.7246035
5200	3.7160033	5235	3.7189167	5270	3.7218106	5305	3.7246854
5201	3.7160869	5236	3.7189996	5271	3.7218930	5306	3.7247672
5202	3.7161703	5237	3.7190826	5272	3.7219754	5307	3.7248491
5203	3.7162538	5238	3.7191655	5273	3.7220578	5308	3.7249309
5204	3.7163373	5239	3.7192484	5274	3.7221401	5309	3.7250127
5205	3.7164207	5240	3.7193313	5275	3.7222225	5310	3.7250945
5206	3.7165042	5241	3.7194142	5276	3.7223048	5311	3.7251763
5207	3.7165876	5242	3.7194970	5277	3.7223871	5312	3.7252581
5208	3.7166710	5243	3.7195799	5278	3.7224694	5313	3.7253398
5209	3.7167544	5244	3.7196627	5279	3.7225517	5314	3.7254215
5210	3.7168377	5245	3.7197455	5280	3.7226339	5315	3.7255033
5211	3.7169211	5246	3.7198283	5281	3.7227162	5316	3.7255850
5212	3.7170044	5247	3.7199111	5282	3.7227984	5317	3.7256667
5213	3.7170877	5248	3.7199938	5283	3.7228806	5318	3.7257483
5214	3.7171710	5249	3.7200766	5284	3.7229628	5319	3.7258300
5215	3.7172543	5250	3.7201593	5285	3.7230450	5320	3.7259116

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm
5461	3.7372722	5496	3.7400467	5531	3.7428037	5566	3.7455432
5462	3.7373517	5497	3.7401257	5532	3.7428822	5567	3.7456212
5463	3.7374312	5498	3.7402047	5533	3.7429607	5568	3.7456992
5464	3.7375107	5499	3.7402837	5534	3.74303[illegible]2	5569	3.7457772
5465	3.7375902	5500	3.7403627	5535	3.7431176	5570	3.7458552
5466	3.7376696	5501	3.7404416	5536	3.7431961	5571	3.7459332
5467	3.7377491	5502	3.7405206	5537	3.7432745	5572	3.7460111
5468	3.7378285	5503	3.7405995	5538	3.7433530	5573	3.7460890
5469	3.7379079	5504	3.7406784	5539	3.7434314	5574	3.7461670
5470	3.7379873	5505	3.7407573	5540	3.7435098	5575	3.7462449
5471	3.7380667	5506	3.7408362	5541	3.7435881	5576	3.7463228
5472	3.7381461	5507	3.7409151	5542	3.7436665	5577	3.7464006
5473	3.7382254	5508	3.7409939	5543	3.7437449	5578	3.7464785
5474	3.7383048	5509	3.7410728	5544	3.7438232	5579	3.7465564
5475	3.7383841	5510	3.7411516	5545	3.7439015	5580	3.7466342
5476	3.7384634	5511	3.7412304	5546	3.7439799	5581	3.7467120
5477	3.7385427	5512	3.7413092	5547	3.7440582	5582	3.7467898
5478	3.7386220	5513	3.7413880	5548	3.7441365	5533	3.7468676
5479	3.7387013	5514	3.7414668	5549	3.7442147	5584	3.7469454
5480	3.7387806	5515	3.7415455	5550	3.7442930	5585	3.7470232
5481	3.7388598	5516	3.7416243	5551	3.7443712	5586	3.7471009
5482	3.7389390	5517	3.7417030	5552	3.7444495	5587	3.7471787
5483	3.7390182	5518	3.7417817	5553	3.7445277	5588	3.7472564
5484	3.7390974	5519	3.7418604	5554	3.7446059	5589	3.7473341
5485	3.7391766	5520	3.7419391	5555	3.7446841	5590	3.7474118
5486	3.7392558	5521	3.7420177	5556	3.7447622	5591	3.7474895
5487	3.7393350	5522	3.7420964	5557	3.7448404	5592	3.7475672
5488	3.7394141	5523	3.7421750	5558	3.7449185	5593	3.7476448
5489	3.7394932	5524	3.7422537	5559	3.7449967	5594	3.7477225
5490	3.7395723	5525	3.7423323	5560	3.7450748	5595	3.7478001
5491	2.7396514	5526	3.7424109	5561	3.7451529	5596	3.7478778
5492	3.7397305	5527	3.7424895	5562	3.7452310	5597	3.7479553
5493	3.7398096	5528	3.7425680	5563	3.7453091	5598	3.7480329
5494	3.7398886	5529	3.7426466	5564	3.7453871	5599	3.7481105
5495	3.7399677	5530	3.7427251	5565	3.7454652	5600	3.7481880

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
5601	3.7482656	5636	3.7509709	5671	3.7536596	5706	3.7563318
5602	3.7483431	5637	3.7510480	5672	3.7537362	5707	3.7564079
5603	3.7484206	5638	3.7511251	5673	3.7538128	5708	3.7564840
5604	3.7484981	5639	3.7512021	5674	3.7538893	5709	3.7565600
5605	3.7485756	5640	3.7512791	5675	3.7539659	5710	3.7566361
5606	3.7486531	5641	3.7513561	5676	3.7540424	5711	3.7567122
5607	3.7487306	5642	3.7514331	5677	3.7541189	5712	3.7567882
5608	3.7488080	5643	3.7515100	5678	3.7541954	5713	3.7568642 2
5609	3.7488854	5644	3.7515870	5679	3.7542719	5714	3.7569402
5610	3.7489629	5645	3.7516639	5680	3.7543483	5715	3.7570162
5611	3.7490403	5646	3.7517409	5681	3.7544248	5716	3.7570922
5612	3.7491177	5647	3.7518178	5682	3.7545012	5717	3.7571682
5613	3.7491950	5648	3.7518947	5683	3.7545777	5718	3.7572441
5614	3.7492724	5649	3.7519716	5684	3.7546541	5719	3.7573201
5615	3.7493498	5650	3.7520484	5685	3.7547305	5720	3.7573960
5616	3.7494271	5651	3.7521253	5686	3.7548069	5721	3.7574719
5617	3.7495044	5652	3.7522022	5687	3.7548832	5722	3.7575479
5618	3.7495817	5653	3.7522790	5688	3.7549596	5723	3.7576237
5619	3.7496590	5654	3.7523558	5689	3.7550359	5724	3.7576996
5620	3.7497363	5655	3.7524326	5690	3.7551123	5725	3.7577755
5621	3.7498136	5656	3.7525094	5691	3.7551886	5726	3.7578513
5622	3.7498908	5657	3.7525862	5692	3.7552649	5727	3.7579272
5623	3.7499681	5658	3.7526629	5693	3.7553412	5728	3.7580030
5624	3.7500453	5659	3.7527397	5694	3.7554175	5729	3.7580788
5625	3.7501225	5660	3.7528164	5695	3.7554937	5730	3.7581546
5626	3.7501997	5661	3.7528932	5696	3.7555700	5731	3.7582304
5627	3.7502769	5662	3.7529699	5697	3.7556462	5732	3.7583062
5628	3.7503541	5663	3.7530466	5698	3.7557224	5733	3.7583819
5629	3.7504312	5664	3.7531232	5699	3.7557987	5734	3.7584577
5630	3.7505084	5665	3.7531999	5700	3.7558749	5735	3.7585334
5631	3.7505855	5666	3.7532766	5701	3.7559510	5736	3.7586091
5632	3.7506626	5667	3.7533532	5702	3.7560273	5737	3.7586848
5633	3.7507398	5668	3.7534298	5703	3.7561034	5738	3.7587605
5634	3.7508168	5669	3.7535065	5704	3.7561795	5739	3.7588362
5635	3.7508939	5670	3.7535831	5705	3.7562556	5740	3.7589119

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm
5741	3.7589875	5776	3.7616272	5811	3.7642509	5846	3.7668588
5742	3.7590632	5777	3.7617024	5812	3.7643256	5847	3.7669331
5743	3.7591388	5778	3.7617775	5813	3.7644003	5848	3.7670074
5744	3.7592144	5779	3.7618527	5814	3.7644750	5849	3.7670815
5745	3.7592900	5780	3.7619278	5815	3.7645497	5850	3.7671559
5746	3.7593656	5781	3.7620030	5816	3.7646244	5851	3.7672301
5747	3.7594412	5782	3.7620781	5817	3.7646991	5852	3.7673043
5748	3.7595168	5783	3.7621532	5818	3.7647737	5853	3.7673785
5749	3.7595923	5784	3.7622283	5819	3.7648484	5854	3.7674527
5750	3.7596678	5785	3.7623034	5820	3.7649230	5855	3.7675269
5751	3.7597434	5786	3.7623784	5821	3.7649976	5856	3.7676011
5752	3.7598189	5787	3.7624535	5822	3.7650722	5857	3.7676752
5753	3.7598944	5788	3.7625285	5823	3.7651468	5858	3.7677494
5754	3.7599699	5789	3.7626035	5824	3.7652214	5859	3.7678235
5755	3.7600453	5790	3.7626786	5825	3.7652959	5860	3.7678976
5756	3.7601208	5791	3.7627536	5826	3.7653705	5861	3.7679717
5757	3.7601962	5792	3.7628286	5827	3.7654450	5862	3.7680458
5758	3.7602717	5793	3.7629035	5828	3.7655195	5863	3.7681199
5759	3.7603471	5794	3.7629785	5829	3.7655941	5864	3.7681940
5760	3.7604225	5795	3.7630534	5830	3.7656686	5865	3.7682680
5761	3.7604979	5796	3.7631284	5831	3.7657430	5866	3.7683421
5762	3.7605733	5797	3.7632033	5832	3.7658175	5867	3.7684161
5763	3.7606486	5798	3.7632782	5833	3.7658920	5868	3.7684901
5764	3.7607240	5799	3.7633531	5834	3.7659664	5869	3.7685641
5765	3.7607993	5700	3.7634280	5835	3.7660409	5870	3.7686381
5766	3.7608746	5801	3.7635029	5836	3.7661153	5871	3.7687121
5767	3.7609500	5802	3.7635777	5837	3.7661897	5872	3.7687860
5768	3.7610253	5803	3.7636526	5838	3.7662642	5873	3.7688600
5769	3.7611005	5804	3.7637274	5839	3.7663385	5874	3.7689339
5770	3.7611758	5805	3.7638022	5840	3.7664128	5875	3.7690079
5771	3.7612511	5806	3.7638770	5841	3.7664872	5876	3.7690818
5772	3.7613263	5807	3.7639518	5842	3.7665616	5877	3.7691557
5773	3.7614016	5808	3.7640266	5843	3.7666359	5878	3.7692296
5774	3.7614768	5809	3.7641014	5844	3.7667102	5879	3.7693035
5775	3.7615520	5810	3.7641761	5845	3.7667845	5880	3.7693773

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
5881	3.7694512	5916	3.7720282	5951	3.7745899	5986	3.7771367
5882	3.7695250	5917	3.7721016	5952	3.7746629	5987	3.7772093
5883	3.7695988	5918	3.7721750	5953	3.7747359	5988	3.7772818
5884	3.7696727	5919	3.7722483	5954	3.7748088	5989	3.7773543
5885	3.7697465	5920	3.7723217	5955	3.7748818	5990	3.7774268
5886	3.7698203	5921	3.7723951	5956	3.7749547	5991	3.7774993
5887	3.7698940	5922	3.7724684	5957	3.7750276	5992	3.7775718
5888	3.7699678	5923	3.7725417	5958	3.7751005	5993	3.7776443
5889	3.7700416	5924	3.7726150	5959	3.7751734	5994	3.7777167
5890	3.7701153	5925	3.7726884	5960	3.7752463	5995	3.7777892
5891	3.7701890	5926	3.7727616	5961	3.7753191	5996	3.7778616
5892	3.7702627	5927	3.7728349	5962	3.7753920	5997	3.7779340
5893	3.7703364	5928	3.7729082	5963	3.7754648	5998	3.7780065
5894	3.7704101	5929	3.7729814	5964	3.7755376	5999	3.7780789
5895	3.7704838	5930	3.7730547	5965	3.7756104	6000	3.7781513
5896	3.7705575	5931	3.7731279	5966	3.7756832	6001	3.7782236
5897	3.7706311	5932	3.7732011	5967	3.7757560	6002	3.7782960
5898	3.7707048	5933	3.7732743	5968	3.7758288	6003	3.7783683
5899	3.7707784	5934	3.7733475	5969	3.7759016	6004	3.7784407
5900	3.7708520	5935	3.7734207	5970	3.7759743	6005	3.7785130
5901	3.7709256	5936	3.7734939	5971	3.7760471	6006	3.7785853
5902	3.7709992	5937	3.7735670	5972	3.7761198	6007	3.7786576
5903	3.7710728	5938	3.7736402	5973	3.7761925	6008	3.7787299
5904	3.7711463	5939	3.7737133	5974	3.7762652	6009	3.7788022
5905	3.7712199	5940	3.7737864	5975	3.7763379	6010	3.7788745
5906	3.7712934	5941	3.7738596	5976	3.7764106	6011	3.7789467
5907	3.7713670	5942	3.7739326	5977	3.7764833	6012	3.7790190
5908	3.7714405	5943	3.7740057	5978	3.7765559	6013	3.7790912
5909	3.7715140	5944	3.7740788	5979	3.7766286	6014	3.7791634
5910	3.7715875	5945	3.7741519	5980	3.7767012	6015	3.7792356
5911	3.7716610	5946	3.7742249	5981	3.7766738	6016	3.7793078
5912	3.7717344	5947	3.7742979	5982	3.7768464	6017	3.7793800
5913	3.7718079	5948	3.7743710	5983	3.7769190	6018	3.7794522
5914	3.7718813	5949	3.7744440	5984	3.7769916	6019	3.7795243
5915	3.7719547	5950	3.7745170	5985	3.7770642	6020	3.7795965

Num.	*Logarithm.*	Num.	*Logarithm.*	Num.	*Logarithm.*	Num	*Logarithm*
6021	3.7796686	6056	3.7821859	6091	3.7846886	6126	3.7871770
6022	3.7797408	6057	3.7822576	6092	3.7847599	6127	3.7872479
6023	3.7798129	6058	3.7823293	6093	3.7848312	6128	3.7873188
6024	3.7798850	6059	3.7824010	6094	3.7849024	6129	3.7873896
6025	3.7799571	6060	3.7824726	6095	3.7849737	6130	3.7874605
6026	3.7800291	6061	3.7825443	6096	3.7850450	6131	3.7875313
6027	3.7801012	6062	3.7826159	6097	3.7851162	6132	3.7876021
6028	3.7801732	6063	3.7826876	6098	3.7851874	6133	3.7876730
6029	3.7802453	6064	3.7827592	6099	3.7852586	6134	3.7877438
6030	3.7803173	6065	3.7828308	6100	3.7853298	6135	3.7878146
6031	3.7803893	6066	3.7829024	6101	3.7854010	6136	3.7878853
6032	3.7804613	6067	3.7829740	6102	3.7854722	6137	3.7879561
6033	3.7805333	6068	3.7830456	6103	3.7855434	6138	3.7880269
6034	3.7806053	6069	3.7831171	6104	3.7856145	6139	3.7880976
6035	3.7806773	6070	3.7831887	6105	3.7856857	6140	3.7881684
6036	3.7807492	6071	3.7832602	6106	3.7857568	6141	3.7882391
6037	3.7808212	6072	3.7833318	6107	3.7858279	6142	3.7883098
6038	3.7808931	6073	3.7834033	6108	3.7858990	6143	3.7883805
6039	3.7809650	6074	3.7834748	6109	3.7859701	6144	3.7884512
6040	3.7810369	6075	3.7835463	6110	3.7860412	6145	3.7885219
6041	3.7811088	6076	3.7836178	6111	3.7861124	6146	3.7885926
6042	3.7811807	6077	3.7836892	6112	3.7861833	6147	3.7886632
6043	3.7812526	6078	3.7837607	6113	3.7862544	6148	3.7887339
6044	3.7813245	6079	3.7838321	6114	3.7863254	6149	3.7888045
6045	3.7813963	6080	3.7839036	6115	3.7863965	6150	3.7888751
6046	3.7814681	6081	3.7839750	6116	3.7864675	6151	3.7889457
6047	3.7815400	6082	3.7840464	6117	3.7865385	6152	3.7890163
6048	3.7816118	6083	3.7841178	6118	3.7866095	6153	3.7890869
6049	3.7816836	6084	3.7841892	6119	3.7866805	6154	3.7891575
6050	3.7817554	6085	3.7842606	6120	3.7867514	6155	3.7892281
6051	3.7818272	6086	3.7843319	6121	3.7868224	6156	3.7892986
6052	3.7818989	6087	3.7844033	6122	3.7868933	6157	3.7893691
6053	3.7819707	6088	3.7844746	6123	3.7869643	6158	3.7894397
6054	3.7820424	6089	3.7845460	6124	3.7870352	6159	3.7895102
6055	3.7821141	6090	3.7846173	6125	3.7871061	6160	3.7895807

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
6161	3.7896513	6196	3.7921114	6231	3.7945578	6266	3.7969904
6162	3.7897217	6197	3.7921815	6232	3.7946274	6267	3.7970597
6163	3.7897922	6198	3.7922516	6233	3.7946971	6268	3.7971290
6164	3.7898626	6199	3.7923216	6234	3.7947668	6269	3.7971983
6165	3.7899331	6200	3.7923917	6235	3.7948365	6270	3.7972675
6166	3.7900035	6201	3.7924617	6236	3.7949061	6271	3.7973368
6167	3.7900739	6202	3.7925318	6237	3.7949757	6272	3.7974060
6168	3.7901444	6203	3.7926018	6238	3.7950454	6273	3.7974753
6169	3.7902148	6204	3.7926718	6239	3.7951150	6274	3.7975445
6170	3.7902852	6205	3.7927418	6240	3.7951846	6275	3.7976137
6171	3.7903555	6206	3.7928118	6241	3.7952542	6276	3.7976829
6172	3.7904259	6207	3.7928817	6242	3.7953238	6277	3.7977521
6173	3.7904963	6208	3.7929517	6243	3.7953933	6278	3.7978213
6174	3.7905666	6209	3.7930217	6244	3.7954629	6279	3.7978905
6175	3.7906370	6210	3.7930916	6245	3.7955324	6280	3.7979596
6176	3.7907073	6211	3.7931615	6246	3.7956020	6281	3.7980288
6177	3.7907776	6212	3.7932314	6247	3.7956715	6282	3.7980979
6178	3.7908479	6213	3.7933014	6248	3.7957410	6283	3.7981671
6179	3.7909182	6214	3.7933712	6249	3.7958105	6284	3.7982362
6180	3.7909885	6215	3.7934411	6250	3.7958800	6285	3.7983053
6181	3.7910587	6216	3.7935110	6251	3.7959495	6286	3.7983744
6182	3.7911290	6217	3.7935809	6252	3.7960190	6287	3.7984435
6183	3.7911992	6218	3.7936507	6253	3.7960884	6288	3.7985125
6184	3.7912695	6219	3.7937206	6254	3.7961579	6289	3.7985816
6185	3.7913397	6220	3.7937904	6255	3.7962273	6290	3.7986506
6186	3.7914099	6221	3.7938602	6256	3.7962967	6291	3.7987197
6187	3.7914801	6222	3.7939300	6257	3.7963662	6292	3.7987887
6188	3.7915503	6223	3.7939998	6258	3.7964356	6293	3.7988577
6189	3.7916205	6224	3.7940696	6259	3.7965050	6294	3.7989267
6190	3.7916906	6225	3.7941394	6260	3.7965743	6295	3.7989957
6191	3.7917608	6226	3.7942091	6261	3.7966437	6296	3.7990647
6192	3.7918309	6227	3.7942789	6262	3.7967131	6297	3.7991337
6193	3.7919011	6228	3.7943486	6263	3.7967824	6298	3.7992027
6194	3.7919712	6229	3.7944183	6264	3.7968517	6299	3.7992716
6195	3.7920413	6230	3.7944880	6265	3.7969211	6300	3.7993405

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
6301	3.7994095	6336	3.8018152	6371	3.8042076	6406	3.8065869
6302	3.7994784	6337	3.8018837	6372	3.8042758	6407	3.8066547
6303	3.7995473	6338	3.8019522	6373	3.8043439	6408	3.8067225
6304	3.7996162	6339	3.8020208	6374	3.8044121	6409	3.8067903
6305	3.7996851	6340	3.8020893	6375	3.8044802	6410	3.8068580
6306	3.7997540	6341	3.8021578	6376	3.8045483	6011	3.8069258
6307	3.7998228	6342	3.8022262	6377	3.8046164	6012	3.8069935
6308	3.7998917	6343	3.8022947	6378	3.8046845	6013	3.8070612
6309	3.7999605	6344	3.8023632	6379	3.8047526	6014	3.8071290
6310	3.8000294	6345	3.8024316	6380	3.8048207	6015	3.8071967
6311	3.8000982	6346	3.8025001	6381	3.8048887	6416	3.8072643
6312	3.8001670	6347	3.8025685	6382	3.8049568	6417	3.8073320
6313	3.8002358	6348	3.8026369	6383	3.8050248	6418	3.8073997
6314	3.8003046	6349	3.8027053	6384	3.8050929	6419	3.8074674
6315	3.8003734	6350	3.8027737	6385	3.8051609	6420	3.8075350
6316	3.8004421	6351	3.8028421	6386	3.8052289	6421	3.8076027
6317	3.8005109	6352	3.8029105	6387	3.8052969	6422	3.8076703
6318	3.8005796	6353	3.8029789	6388	3.8053649	6423	3.8077379
6319	3.8006484	6354	3.8030472	6389	3.8054329	6424	3.8078055
6320	3.8007171	6355	3.8031156	6390	3.8055009	6425	3.8078731
6321	3.8007858	6356	3.8031839	6391	3.8055688	6426	3.8079407
6322	3.8008545	6357	3.8032522	6392	3.8056368	6427	3.8080083
6323	3.8009232	6358	3.8033205	6393	3.8057047	6428	3.8080759
6324	3.8009919	6359	3.8033888	6394	3.8057726	6429	3.8081434
6325	3 8010605	6360	3.8034571	6395	3.8058405	6430	3.8082110
6326	3.8011292	6361	3.8035254	6396	3.8059085	6431	3.8082785
6327	3.8011978	6362	3.8035937	6397	3.8059763	6432	3.8083460
6328	3.8012665	6363	3.8036619	6398	3.8060442	6433	3.8084136
6329	3.8013351	6364	3.8037302	6399	3.8061121	6434	3.8084811
6330	3.8014037	6365	3.8037984	6400	3.8061800	6435	3.8085485
6331	3.8014723	6366	3.8038666	6401	3.8062478	6436	3.8086160
6332	3.8015409	6367	3.8039348	6402	3.8063157	6437	3.8086835
6333	3.8016095	6368	3.8040031	6403	3.8063835	6438	3.8087510
6334	3.8016781	6369	3.8040712	6404	3.8064513	6439	3.8088184
6335	3.8017466	6370	3.8041394	6405	3.8065191	6440	3.8088859

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
6441	3.8089533	6476	3.8113068	6511	3.8136477	6546	3.8159760
6442	3.8090207	6477	3.8113739	6512	3.8137144	6547	3.8160423
6443	3.8090881	6478	3.8114409	6513	3.8137811	6548	3.8161087
6444	3.8091555	6479	3.8115080	6514	3.8138478	6549	3.8161750
6445	3.8092229	6480	3.8115750	6515	3.8139144	6550	3.8162413
6446	3.8092903	6481	3.8116420	6516	3.8139811	6551	3.8163076
6447	3.8093577	6482	3.8117090	6517	3.8140477	6552	3.8163739
6448	3.8094250	6483	3.8117760	6518	3.8141144	6553	3.8164402
6449	3.8094924	6484	3.8118430	6519	3.8141810	6554	3.8165064
6450	3.8095597	6485	3.8119100	6520	3.8142476	6555	3.8165726
6451	3.8096270	6486	3.8119769	6521	3.8143142	6556	3.8166389
6452	3.8096944	6487	3.8120439	6522	3.8143808	6557	3.8167052
6453	3.8097617	6488	3.8121108	6523	3.8144474	6558	3.8167714
6454	3.8098290	6489	3.8121778	6524	3.8145140	6559	3.8168376
6455	3.8098962	6490	3.8122447	6525	3.8145805	6560	3.8169038
6456	3.8099635	6491	3.8123116	6526	3.8146471	6561	3.8169700
6457	3.8100308	6492	3.8123785	6527	3.8147136	6562	3.8170362
6458	3.8100980	6493	3.8124454	6528	3.8147801	6563	3.8171024
6459	3.8101653	6494	3.8125123	6529	3.8148467	6564	3.8171686
6460	3.8102325	6495	3.8125792	6530	3.8149132	6565	3.8172343
6461	3.8102997	6496	3.8126460	6531	3.8149797	6566	3.8173009
6462	3.8103670	6497	3.8127129	6532	3.8150462	6567	3.8173670
6463	3.8104342	6498	3.8127797	6533	3.8151127	6568	3.8174331
6464	3.8105013	6499	3.8128465	6534	3.8151791	6569	3.8174993
6465	3.8105685	6500	3.8129134	6535	3.8152456	6570	3.8175654
6466	3.8106357	6501	3.8129802	6536	3.8153120	6571	3.8176315
6467	3.8107029	6502	3.8130470	6537	3.8153785	6572	3.8176976
6468	3.8107700	6503	3.8131138	6538	3.8154449	6573	3.8177636
6469	3.8108371	6504	3.8131805	6539	3.8155113	6574	3.8178297
6470	3.8109043	6505	3.8132473	6540	3.8155777	6575	3.8178958
6471	3.8109714	6506	3.8133141	6541	3.8156441	6576	3.8179618
6472	3.8110385	6507	3.8133808	6542	3.8157105	6577	3.8180278
6473	3.8111056	6508	3.8134475	6543	3.8157769	6578	3.8180939
6474	3.8111727	6509	3.8135143	6544	3.8158433	6579	3.8181599
6475	3.8112398	6510	3.8135810	6545	3.8159096	6580	3.8182259

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
6581	3.8182919	6616	3.8205955	6651	3.8228869	6686	3.8251664
6582	3.8183579	6617	3.8206611	6652	3.8229522	6687	3.8252313
6583	3.8184239	6618	3.8207268	6653	3.8230175	6688	3.8252963
6584	3.8184898	6619	3.8207924	6654	3.8230828	6689	3.8253612
6585	3.8185558	6620	3.8208580	6655	3.8231481	6690	3.8254261
6586	3.8186217	6621	3.8209236	6656	3.8232133	6691	3.8254910
6587	3.8186877	6622	3.8209892	6657	3.8232786	6692	3.8255559
6588	3.8187536	6623	3.8210548	6658	3.8233438	6693	3.8256208
6589	3.8188195	6624	3.8211203	6659	3.8234090	6694	3.8256857
6590	3.8188854	6625	3.8211859	6660	3.8234742	6695	3.8257506
6591	3.8189513	6626	3.8212514	6661	3.8235394	6699	3.8258154
6592	3.8190172	6627	3.8213170	6662	3.8236046	6697	3.8258803
6593	3.8190831	6628	3.8213825	6663	3.8236698	6698	3.8259451
6594	3.8191489	6629	3.8214480	6664	3.8237350	6699	3.8260100
6595	3.8192148	6630	3.8215135	6665	3.8238002	6700	3.8260748
6596	3.8192806	6631	3.8215790	6666	3.8238653	6701	3.8261396
6597	3.8193465	6632	3.8216445	6667	3.8239305	6702	3.8262044
6598	3.8194123	6633	3.8217100	6668	3.8239956	6703	3.8262692
6599	3.8194781	6634	3.8217755	6669	3.8240607	6704	3.8263340
6600	3.8195439	6635	3.8218409	6670	3.8241258	6705	3.8263988
6601	3.8196097	6636	3.8219064	6671	3.8241909	6706	3.8264635
6602	3.8196755	6637	3.8219718	6672	3.8242560	6707	3.8265283
6603	3.8197413	6638	3.8220372	6673	3.8243211	6708	3.8265932
6604	3.8198071	6639	3.8221027	6674	3.8243862	6709	3.8266578
6605	3.8198728	6640	3.8221681	6675	3.8244513	6710	3.8267225
6606	3.8199386	6641	3.8222335	6676	3.8245163	6711	3.8267872
6607	3.8200043	6642	3.8222989	6677	3.8245814	6712	3.8268519
6608	3.8200700	6643	3.8223643	6678	3.8246464	6713	3.8269166
6609	3.8201358	6644	3.8224296	6679	3.8247114	6714	3.8269813
6610	3.8202015	6645	3.8224950	6680	3.8247765	6715	3.8270460
6611	3.8202672	6646	3.8225603	6681	3.8248415	6716	3.8271107
6612	3.8203328	6647	3.8226257	6682	3.8249065	6717	3.8271753
6613	3.8203985	6648	3.8226910	6683	3.8249715	6718	3.8272400
6614	3.8204642	6649	3.8227563	6684	3.8250364	6719	3.8273046
6615	3.8205898	6650	3.8228216	6685	3.8251014	6720	3.8273693

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num	Logarithm
6721	3.8274339	6756	3.8296896	6791	3.8319337	0826	3.8341663
6722	3.8274985	6757	3.8297539	6792	3.8319977	6827	3.8342299
6723	3.8275631	6758	3.8298182	6793	3.8320616	6828	3.8342935
6724	3.8276277	6759	3.8298824	6794	3.8321255	6829	3.8343571
6725	3.8276923	6760	3.8299467	6795	3.8321895	6830	3.8344207
6726	3.8277569	6761	3.8300109	6796	3.8322534	6831	3.8344843
6727	3.8278214	6762	3.8300752	6797	3.8323173	6832	3.8345479
6728	3.8278860	6763	3.8301394	6798	3.8323812	6833	3.8346114
6729	3.8279505	6764	3.8302036	6799	3.8324450	6834	3.8346750
6730	3.8280151	6765	3.8302678	6800	3.8325089	6835	3.8347385
6731	3.8280796	6766	3.8303320	6801	3.8325728	6836	3.8348021
6732	3.8281441	6767	3.8303962	6802	3.8326366	6837	3.8348656
6733	3.8282086	6768	3.8304603	6803	3.8327005	6838	3.8349291
6734	3.8282731	6769	3.8305245	6804	3.8327643	6839	3.8349926
6735	3.8283376	6770	3.8305887	6805	3.8328281	6840	3.8350561
6736	3.8284022	6771	3.8306528	6806	3.8328919	6841	3.8351196
6737	3.8284665	6772	3.8307169	6807	3.8329558	6842	3.8351831
6738	3.8285310	6773	3.8307811	6808	3.8330195	6843	3.8352465
6739	3.8285955	6774	3.8308452	6809	3.8330833	6844	3.8353100
6740	3.8286599	6775	3.8309093	6810	3.8331471	6845	3.8353735
6741	3.8287243	6776	3.8309734	6811	3.8332109	6846	3.8354369
6742	3.8287887	6777	3.8310375	6812	3.8332746	6847	3.8355003
6743	3.8288532	6778	3.8311016	6813	3.8333384	6848	3.8355638
6744	3.8289176	6779	3.8311656	6814	3.8334021	6849	3.8356272
6745	3.8289820	6780	3.8312297	6815	3.8334659	6850	3.8356906
6746	3.8290463	6781	3.8312937	6816	3.8335296	6851	3.8357540
6747	3.8291107	6782	3.8313578	6817	3.8335933	6852	3.8358174
6748	3.8291751	6783	3.8314218	6818	3.8336570	6853	3.8358807
6749	3.8292394	6784	3.8314858	6819	3.8337207	6854	3.8359441
6750	3.8293038	6785	3.8315499	6820	3.8337844	6855	3.8360075
6751	3.8293681	6786	3.8316139	6821	3.8338480	6856	3.8360708
6752	3.8294324	6787	3.8316778	6822	3.8339117	6857	3.8361341
6753	3.8294967	6788	3.8317418	6823	3.8339754	6858	3.8361975
6754	3.8295611	6789	3.8318058	6824	3.8340390	6859	3.8362608
6755	3.8296254	6790	3.8318698	6825	3.8341027	6860	3.8363241

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
6861	3.8363874	6896	3.8385973	6931	3.8407959	6966	3.8429835
6862	3.8364507	6897	3.8386602	6932	3.8408586	6967	3.8430458
6863	3.8365140	6898	3.8387232	6933	3.8409212	6968	3.8431081
6864	3.8365773	6899	3.8387861	6934	3.8409838	6969	3.8431705
6865	3.8366405	6900	3.8388491	6935	3.8410465	6970	3.8432328
6866	3.8367038	6901	3.8389120	6936	3.8411091	6971	3.8432951
6867	3.8367670	6902	3.8389750	6937	3.8411717	6972	3.8433574
6868	3.8368303	6903	3.8390379	6938	3.8412343	6973	3.8434197
6869	3.8368935	6904	3.8391008	6939	3.8412969	6974	3.8434819
6870	3.8369567	6905	3.8391637	6940	3.8413595	6975	3.8435442
6871	3.8370199	6906	3.8392266	6941	3.8414220	6976	3.8436065
6872	3.8370832	6907	3.8392895	6942	3.8414846	6977	3.8436687
6873	3.8371463	6908	3.8393523	6943	3.8415472	6978	3.8437310
6874	3.8372095	6909	3.8394152	6944	3.8416097	6979	3.8437932
6875	3.8372727	6910	3.8394780	6945	3.8416722	6980	3.8438554
6876	3.8373359	6911	3.8395409	6946	3.8417348	6981	3.8439176
6877	3.8373990	6912	3.8396037	6947	3.8417973	6982	3.8439798
6878	3.8374622	6913	3.8396666	6948	3.8418598	6983	3.8440420
6879	3.8375253	6914	3.8397294	6949	3.8419223	6984	3.8441042
6880	3.8375884	6915	3.8397922	6950	3.8419848	6985	3.8441664
6881	3.8376516	6916	3.8398550	6951	3.8420473	6986	3.8442286
6882	3.8377147	6917	3.8399178	6952	3.8421098	6987	3.8442907
6883	3.8377778	6918	3.8399806	6953	3.8421722	6988	3.8443529
6884	3.8378409	6919	3.8400433	6954	3.8422347	6989	3.8444150
6885	3.8379039	6920	3.8401061	6955	3.8422971	6990	3.8444772
6886	3.8379670	6921	3.8401684	6956	3.8423596	6991	3.8445393
6887	3.8380301	6922	3.8402316	6957	3.8424220	6992	3.8446014
6888	3.8380931	6923	3.8402943	6958	3.8424844	6993	3.8446635
6889	3.8381562	6924	3.8403571	6959	3.8425468	6994	3.8447256
6890	3.8382192	6925	3.8404198	6960	3.8426092	6995	3.8447877
6891	3.8382822	6926	3.8404825	6961	3.8426716	6996	3.8448498
6892	3.8383453	6927	3.8405452	6962	3.8427340	6997	3.8449119
6893	3.8384083	6928	3.8406079	6963	3.8427964	6998	3.8449739
6894	3.8384713	6929	3.8406706	6964	3.8428588	6999	3.8450360
6895	3.8385343	6930	3.8407332	6965	3.8429211	7000	3.8450980

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm
7001	3.8451601	7036	3.8473258	7071	3.8494808	7106	3.8516252
7002	3.8452221	7037	3.8473876	7072	3.8495423	7107	3.8516863
7003	3.8452841	7038	3.8474493	7073	3.8496037	7108	3.8517474
7004	3.8453461	7039	3.8475110	7074	3.8496651	7109	3.8518085
7005	3.8454081	7040	3.8475727	7075	3.8497264	7110	3.8518696
7006	3.8454701	7041	3.8476343	7076	3.8497878	7111	3.8519307
7007	3.8455321	7042	3.8476960	7077	3.8498492	7112	3.8519917
7008	3.8455941	7043	3.8477577	7078	3.8499106	7113	3.8520528
7009	3.8456561	7044	3.8478193	7079	3.8499719	7114	3.8521139
7010	3.8457180	7045	3.8478810	7080	3.8500333	7115	3.8521749
7011	3.8457800	7046	3.8479426	7081	3.8500946	7116	3.8522359
7012	3.8458419	7047	3.8480043	7082	3.8501559	7117	3.8522970
7013	3.8459038	7048	3.8480659	7083	3.8502172	7118	3.8523580
7014	3.8459658	7049	3.8481275	7084	3.8502786	7119	3.8524190
7015	3.8460277	7050	3.8481891	7085	3.8503399	7120	3.8524800
7016	3.8460896	7051	3.8482507	7086	3.8504011	7121	3.8525410
7017	3.8461515	7052	3.8483123	7087	3.8504624	7122	3.8526020
7018	3.8462134	7053	3.8483739	7088	3.8505237	7123	3.8526629
7019	3.8462752	7054	3.8484355	7089	3.8505850	7124	3.8527239
7020	3.8463371	7055	3.8484970	7090	3.8506462	7125	3.8527849
7021	3.8463990	7056	3.8485586	7091	3.8507075	7126	3.8528458
7022	3.8464608	7057	3.8486201	7092	3.8507687	7127	3.8529068
7023	3.8465227	7058	3.8486817	7093	3.8508300	7128	3.8529677
7024	3.8465845	7059	3.8487472	7094	3.8508912	7129	3.8530286
7025	3.8466463	7060	3.8488087	7095	3.8509524	7130	3.8530895
7026	3.8467081	7061	3.8488662	7096	3.8510136	7131	3.8531504
7027	3.8467700	7062	3.8489277	7097	3.8510748	7132	3.8532113
7028	3.8468318	7063	3.8489892	7098	3.8511360	7133	3.8532722
7029	3.8468935	7064	3.8490507	7099	3.8511972	7134	3.8533331
7030	3.8469553	7065	3.8491122	7100	3.8512583	7135	3.8533940
7031	3.8470171	7066	3.8491736	7101	3.8513195	7136	3.8534548
7032	3.8470789	7067	3.8492351	7102	3.8513807	7137	3.8535157
7033	3.8471406	7068	3.8492965	7103	3.8514418	7138	3.8535765
7034	3.8472024	7069	3.8493580	7104	3.8515030	7139	3.8536374
7035	3.8472641	7070	3.8494194	7105	3.8515641	7140	3.8536982

Num.	Logarithm.	Num	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
7141	3.8537590	7176	3.8558824	7211	3.8579955	7246	3.8600983
7142	3.8538198	7177	3.8559429	7212	3.8580557	7247	3.8601583
7143	3.8538806	7178	3.8560035	7213	3.8581159	7248	3.8602183
7144	3.8539414	7179	3.8560640	7214	3.8581761	7249	3.8602781
7145	3.8540022	7180	3.8561244	7215	3.8582363	7250	3.8603380
7146	3.8540630	7181	3.8561849	7216	3.8582965	7251	3.8603979
7147	3.8541238	7182	3.8562454	7217	3.8583567	7252	3.8604578
7148	3.8541845	7183	3.8563059	7218	3.8584169	7253	3.8605177
7149	3.8542453	7184	3.8563663	7219	3.8584770	7254	3.8605776
7150	3.8543060	7185	3.8564268	7220	3.8585372	7255	3.8606374
7151	3.8543668	7186	3.8564872	7221	3.8585973	7256	3.8606973
7152	3.8544275	7187	3.8565476	7222	3.8586575	7257	3.8607571
7153	3.8544882	7188	3.8566081	7223	3.8587176	7258	3.8608170
7154	3.8545489	7189	3.8566685	7224	3.8587777	7259	3.8608768
7155	3.8546096	7190	3.8567289	7225	3.8588379	7260	3.8609366
7156	3.8546703	7191	3.8567893	7226	3.8588980	7261	3.8609964
7157	3.8547310	7192	3.8568497	7227	3.8589581	7262	3.8610562
7158	3.8547917	7193	3.8569101	7228	3.8590181	7263	3.8611160
7159	3.8548524	7194	3.8569704	7229	3.8590782	7264	3.8611758
7160	3.8549130	7195	3.8570308	7230	3.8591383	7265	3.8612356
7161	3.8549737	7196	3.8570912	7231	3.8591984	7266	3.8612954
7162	3.8550343	7197	3.8571515	7232	3.8592584	7267	3.8613552
7163	3.8550949	7198	3.8572118	7233	3.8593185	7268	3.8614149
7164	3.8551556	7199	3.8572722	7234	3.8593785	7269	3.8614747
7165	3.8552162	7200	3.8573325	7235	3.8594385	7270	3.8615344
7166	3.8552768	7201	3.8573928	7236	3.8594986	7271	3.8615941
7167	3.8553374	7202	3.8574531	7237	3.8595586	7272	3.8616539
7168	3.8553980	7203	3.8575134	7238	3.8596186	7273	3.8617136
7169	3.8554586	7204	3.8575737	7239	3.8596786	7274	3.8617733
7170	3.8555192	7205	3.8576340	7240	3.8597386	7275	3.8618330
7171	3.8555797	7206	3.8576943	7241	3.8597985	7276	3.8618927
7172	3.8556403	7207	3.8577545	7242	3.8598585	7277	3.8619524
7173	3.8557008	7208	3.8578148	7243	3.8599185	7278	3.8620120
7174	3.8557614	7209	3.8578750	7244	3.8599784	7279	3.8620717
7175	3.8558219	7210	3.8579353	7245	3.8600384	7280	3.8621314

Num.	*Logarithm.*	Num.	*Logarithm.*	Num.	*Logarithm.*	Num.	*Logarithm.*
7281	3.8621910	7316	3.8642737	7351	3.8663464	7386	3.8684093
7282	3.8622507	7317	3.8643331	7352	3.8664055	7387	3.8684681
7283	3.8623103	7318	3.8643924	7353	3.8664646	7388	3.8685269
7284	3.8623699	7319	3.8644517	7354	3.8665236	7389	3.8685857
7285	3.8624296	7320	3.8645111	7355	3.8665827	7390	3.8686444
7286	3.8624892	7321	3.8645704	7356	3.8666417	7391	3.8687032
7287	3.8625488	7322	3.8646297	7357	3.8667008	7392	3.8687620
7288	3.8626084	7323	3.8646890	7358	3.8667598	7393	3.8688207
7289	3.8626679	7324	3.8647483	7359	3.8668188	7394	3.8688794
7290	3.8627275	7325	3.8648076	7360	3.8668778	7395	3.8689382
7291	3.8627871	7326	3.8648669	7361	3.8669368	7396	3.8689969
7292	3.8628467	7327	3.8649262	7362	3.8669958	7397	3.8690556
7293	3.8629062	7328	3.8649855	7363	3.8670547	7398	3.8691143
7294	3.8629658	7329	3.8650447	7364	3.8671138	7399	3.8691730
7295	3.8630253	7330	3.8651040	7365	3.8671728	7400	3.8692317
7296	3.8630848	7331	3.8651632	7366	3.8672317	7401	3.8692904
7297	3.8631443	7332	3.8652225	7367	3.8672907	7402	3.8693491
7298	3.8632039	7333	3.8652817	7368	3.8673496	7403	3.8694077
7299	3.8632634	7334	3.8653409	7369	3.8674086	7404	3.8694664
7300	3.8633229	7335	3.8654001	7370	3.8674675	7405	3.8695251
7301	3.8633823	7336	3.8654593	7371	3.8675264	7406	3.8695837
7302	3.8634418	7337	3.8655185	7372	3.8675853	7407	3.8696423
7303	3.8635013	7338	3.8655777	7373	3.8676442	7408	3.8697010
7304	3.8635608	7339	3.8656369	7374	3.8677031	7409	3.8697596
7305	3.8636202	7340	3.8656961	7375	3.8677620	7410	3.8698182
7306	3.8636797	7341	3.8657552	7376	3.8678209	7411	3.8698768
7307	3.8637391	7342	3.8658144	7377	3.8678798	7412	3.8699354
7308	3.8637985	7343	3.8658735	7378	3.8679387	7413	3.8699940
7309	3.8638580	7344	3.8659327	7379	3.8679975	7414	3.8700526
7310	3.8639174	7345	3.8659918	7380	3.8680564	7415	3.8701112
7311	3.8639768	7346	3.8660509	7381	3.8681152	7416	3.8701697
7312	3.8640362	7347	3.8661100	7382	3.8681740	7417	3.8702283
7313	3.8640956	7348	3.8661691	7383	3.8682329	7418	3.8702868
7314	3.8641550	7349	3.8662282	7384	3.8682917	7419	3.8703454
7315	3.8642143	7350	3.8662873	7385	3.8683505	7420	3.8704039

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
7421	3.8704624	7456	3.8725059	7491	3.8745398	7526	3.8765642
7422	3.8705209	7457	3.8725641	7492	3.8745978	7527	3.8766219
7423	3.8705795	7458	3.8726224	7493	3.8746557	7528	3.8766796
7424	3.8706380	7459	3.8726806	7494	3.8747137	7529	3.8767373
7425	3.8706965	7460	3.8727388	7495	3.8747716	7530	3.8767950
7426	3.8707549	7461	3.8727970	7496	3.8748296	7531	3.8768526
7427	3.8708134	7462	3.8728552	7497	3.8748875	7532	3.8769103
7428	3.8708719	7463	3.8729134	7498	3.8749454	7533	3.8769680
7429	3.8709304	7464	3.8729716	7499	3.8750034	7534	3.8770256
7430	3.8709888	7465	3.8730298	7500	3.8750613	7535	3.8770833
7431	3.8710473	7466	3.8730880	7501	3.8751192	7536	3.8771409
7432	3.8711057	7467	3.8731461	7502	3.8751771	7537	3.8771985
7433	3.8711641	7468	3.8732043	7503	3.8752349	7538	3.8772561
7434	3.8712226	7469	3.8732625	7504	3.8752928	7539	3.8773137
7435	3.8712810	7470	3.8733206	7505	3.8753507	7540	3.8773713
7436	3.8713394	7471	3.8733788	7506	3.8754086	7541	3.8774289
7437	3.8713978	7472	3.8734369	7507	3.8754664	7542	3.8774865
7438	3.8714562	7473	3.8734950	7508	3.8755243	7543	3.8775441
7439	3.8715146	7474	3.8735531	7509	3.8755821	7544	3.8776017
7440	3.8715729	7475	3.8736112	7510	3.8756399	7545	3.8776592
7441	3.8716313	7476	3.8736693	7511	3.8756978	7546	3.8777168
7442	3.8716897	7477	3.8737274	7512	3.8757556	7547	3.8777743
7443	3.8717480	7478	3.8737855	7513	3.8758134	7548	3.8778319
7444	3.8718064	7479	3.8738435	7514	3.8758712	7549	3.8778894
7445	3.8718647	7480	3.8739016	7515	3.8759290	7550	3.8779469
7446	3.8719230	7481	3.8739597	7516	3.8759868	7551	3.8780045
7447	3.8719814	7482	3.8740177	7517	3.8760445	7552	3.8780620
7448	3.8720397	7483	3.8740757	7518	3.8761023	7553	3.8781195
7449	3.8720980	7484	3.8741338	7519	3.8761601	7554	3.8781770
7450	3.8721563	7485	3.8741918	7520	3.8762178	7555	3.8782345
7451	3.8722146	7486	3.8742498	7521	3.8762756	7556	3.8782919
7452	3.8722728	7487	3.8743078	7522	3.8763333	7557	3.8783493
7453	3.8723311	7488	3.8743658	7523	3.8763911	7558	3.8784069
7454	3.8723894	7489	3.8744238	7524	3.8764488	7559	3.8784643
7455	3.8724476	7490	3.8744818	7525	3.8765065	7560	3.8785218

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
7561	3.8785792	7596	3.8805850	7631	3.8825815	7666	3.8845688
7562	3.8786367	7597	3.8806421	7632	3.8826384	7667	3.8846255
7563	3.8786941	7598	3.8806993	7633	3.8826953	7668	3.8846821
7564	3.8787515	7599	3.8807564	7634	3.8827522	7669	3.8847387
7565	3.8788089	7600	3.8808136	7635	3.8828090	7670	3.8847954
7566	3.8788663	7601	3.8808707	7636	3.8828659	7671	3.8848520
7567	3.8789237	7602	3.8809279	7637	3.8829228	7672	3.8849086
7568	3.8789811	7603	3.8809850	7638	3.8829797	7673	3.8849652
7569	3.8790385	7604	3.8810421	7639	3.8830365	7674	3.8850218
7570	3.8790959	7605	3.8810992	7640	3.8830934	7675	3.8850784
7571	3.8791532	7606	3.8811563	7641	3.8831502	7676	3.8851350
7572	3.8792106	7607	3.8812134	7642	3.8832070	7677	3.8851915
7573	3.8792686	7608	3.8812705	7643	3.8832639	7678	3.8852481
7574	3.8793253	7609	3.8813276	7644	3.8833207	7679	3.8853047
7575	3.8793826	7610	3.8813847	7645	3.8833775	7680	3.8853612
7576	3.8794400	7611	3.8814417	7646	3.8834343	7681	3.8854178
7577	3.8794973	7612	3.8814988	7647	3.8834911	7682	3.8854743
7578	3.8795546	7613	3.8815558	7648	3.8835479	7683	3.8855308
7579	3.8796119	7614	3.8816129	7649	3.8836047	7684	3.8855874
7580	3.8796692	7615	3.8816699	7650	3.8836614	7685	3.8856439
7581	3.8797265	7616	3.8817269	7651	3.8837182	7686	3.8857004
7582	3.8797838	7617	3.8817840	7652	3.8837750	7687	3.8857569
7583	3.8798411	7618	3.8818410	7653	3.8838317	7688	3.8858134
7584	3.8798983	7619	3.8818980	7654	3.8838885	7689	3.8858699
7585	3.8799556	7620	3.8819550	7655	3.8839452	7690	3.8859263
7586	3.8800128	7621	3.8820120	7656	3.8840019	7691	3.8859828
7587	3.8800701	7622	3.8820689	7657	3.8840586	7692	3.8860393
7588	3.8801273	7623	3.8821259	7658	3.8841154	7693	3.8860957
7589	3.8801846	7624	3.8821829	7659	3.8841721	7694	3.8861522
7590	3.8802418	7625	3.8822398	7660	3.8842288	7695	3.8862086
7591	3.8802990	7626	3.8822968	7661	3.8842855	7696	3.8862651
7592	3.8803562	7627	3.8823537	7662	3.8843421	7697	3.8863215
7593	3.8804134	7628	3.8824107	7663	3.8843988	7698	3.8863779
7594	3.8804706	7629	3.8824676	7664	3.8844555	7699	3.8864343
7595	3.8805278	7630	3.8825245	7665	3.8845122	7700	3.8864907

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
7701	3.8865471	7736	3.8885165	7771	3.8904769	7806	3.8924285
7702	3.8866035	7737	3.8885726	7772	3.8905328	7807	3.8924842
7703	3.8866599	7738	3.8886287	7773	3.8905887	7808	3.8925398
7704	3.8867163	7739	3.8886848	7774	3.8906445	7809	3.8925954
7705	3.8867726	7740	3.8887410	7775	3.8907004	7810	3.8926510
7706	3.8868290	7741	3.8887971	7776	3.8907562	7811	3.8927066
7707	3.8868854	7742	3.8888531	7777	3.8908120	7812	3.8927622
7708	3.8869417	7743	3.8889092	7778	3.8908679	7813	3.8928178
7709	3.8869980	7744	3.8889653	7779	3.8909238	7814	3.8928734
7710	3.8870544	7745	3.8890214	7780	3.8909796	7815	3.8929290
7711	3.8871107	7746	3.8890775	7781	3.8910354	7816	3.8929846
7712	3.8871670	7747	3.8891336	7782	3.8910912	7817	3.8930401
7713	3.8872233	7748	3.8891896	7783	3.8911470	7818	3.8930957
7714	3.8872796	7749	3.8892457	7784	3.8912028	7819	3.8931512
7715	3.8873359	7750	3.8893017	7785	3.8912586	7820	3.8932068
7716	3.8873922	7751	3.8893577	7786	3.8913144	7821	3.8932623
7717	3.8874485	7752	3.8894138	7787	3.8913702	7822	3.8933178
7718	3.8875048	7753	3.8894698	7788	3.8914259	7823	3.8933733
7719	3.8875610	7754	3.8895258	7789	3.8914817	7824	3.8934288
7720	3.8876173	7755	3.8895818	7790	3.8915375	7825	3.8934843
7721	3.8876736	7756	3.8896378	7791	3.8915932	7826	3.8935398
7722	3.8877298	7757	3.8896938	7792	3.8916489	7827	3.8935953
7723	3.8877860	7758	3.8897498	7793	3.8917047	7828	3.8936508
7724	3.8878423	7759	3.8898058	7794	3.8917604	7829	3.8937063
7725	3.8878985	7760	3.8898617	7795	3.8918161	7830	3.8937618
7726	3.8879547	7761	3.8899177	7796	3.8918718	7831	3.8938172
7727	3.8880109	7762	3.8899736	7797	3.8919275	7832	3.8938727
7728	3.8880671	7763	3.8900296	7798	3.8919832	7833	3.8939281
7729	3.8881233	7764	3.8900855	7799	3.8920389	7834	3.8939836
7730	3.8881795	7765	3.8901415	7800	3.8920946	7835	3.8940390
7731	3.8882357	7766	3.8901974	7801	3.8921503	7836	3.8940944
7732	3.8882918	7767	3.8902533	7802	3.8922059	7837	3.8941498
7733	3.8883480	7768	3.8903092	7803	3.8922616	7838	3.8942053
7734	3.8884042	7769	3.8903651	7804	3.8923173	7839	3.8942607
7735	3.8884603	7770	3.8904210	7805	3.8923729	7840	3.8943161

Num.	*Logarithm.*	Num	*Logarithm.*	Num.	*Logarithm.*	Num.	*Logarithm.*
7841	3.8943715	7876	3.8963058	7911	3.8982314	7946	3.9001486
7842	3.8944268	7877	3.8963608	7912	3.8982863	7947	3.9002032
7843	3.8944822	7878	3.8964160	7913	3.8983412	7948	3.9002579
7844	3.8945376	7879	3.8964711	7914	3.8983960	7949	3.9003125
7845	3.8945929	7880	3.8965262	7915	3.8984509	7950	3.9003671
7846	3.8946483	7881	3.8965813	7916	3.8985058	7951	3.9004218
7847	3.8947037	7882	3.8966364	7917	3.8985606	7952	3.9004764
7848	3.8947590	7883	3.8966915	7918	3.8986155	7953	3.9005310
7849	3.8948143	7884	3.8967466	7919	3.8986703	7954	3.9005856
7850	3.8948697	7885	3.8968017	7920	3.8987252	7955	3.9006402
7851	3.8949250	7886	3.8968568	7921	3.8987800	7956	3.9006948
7852	3.8949803	7887	3.8969118	7922	3.8988348	7957	3.9007494
7853	3.8950356	7888	3.8969669	7923	3.8988897	7958	3.9008039
7854	3.8950909	7889	3.8970220	7924	3.8989445	7959	3.9008585
7855	3.8951462	7890	3.8970770	7925	3.8989993	7960	3.9009131
7856	3.8952015	7891	3.8971320	7926	3.8990541	7961	3.9009676
7857	3.8952568	7892	3.8971871	7927	3.8991089	7962	3.9010222
7858	3.8953120	7893	3.8972421	7928	3.8991636	7963	3.9010767
7859	3.8953673	7894	3.8972971	7929	3.8992184	7964	3.9011313
7860	3.8954225	7895	3.8973521	7930	3.8992732	7965	3.9011858
7861	3.8954778	7896	3.8974071	7931	3.8993279	7966	3.9012403
7862	3.8955330	7897	3.8974621	7932	3.8993827	7967	3.9012948
7863	3.8955883	7898	3.8975171	7933	3.8994375	7968	3.9013493
7864	3.8956435	7899	3.8975721	7934	3.8994922	7969	3.9014038
7865	3.8956987	7900	3.8976271	7935	3.8995469	7970	3.9014583
7866	3.8957539	7901	3.8976821	7936	3.8996017	7971	3.9015128
7867	3.8958091	7902	3.8977370	7937	3.8996564	7972	3.9015673
7868	3.8958643	7903	3.8977920	7938	3.8997111	7973	3.9016218
7869	3.8959195	7904	3.8978469	7939	3.8997658	7974	3.9016762
7870	3.8959747	7905	3.8979019	7940	3.8998205	7975	3.9017307
7871	3.8960299	7906	3.8979568	7941	3.8998752	7976	3.9017851
7872	3.8960851	7907	3.8980117	7942	3.8999299	7977	3.9018396
7873	3.8961403	7908	3.8980667	7943	3.8999846	7978	3.9018940
7874	3.8961954	7909	3.8981216	7944	3.9000392	7979	3.9019485
7875	3.8962506	7910	3.8981765	7945	3.9000939	7980	3.9020029

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm
7981	3.9020573	8016	3.9039577	8051	3.9058498	8086	3.9077337
7982	3.9021117	8017	3.9040119	8052	3.9059038	8087	3.9077874
7983	3.9021661	8018	3.9040661	8053	3.9059577	8088	3.9078411
7984	3.9022205	8019	3.9041202	8054	3.9060116	8089	3.9078948
7985	3.9022749	8020	3.9041744	8055	3.9060656	8090	3.9079485
7986	3.9023293	8021	3.9042285	8056	3.9061195	8091	3.9080022
7987	3.9023837	8022	3.9042827	8057	3.9061734	8092	3.9080559
7988	3.9024381	8023	3.9043368	8058	3.9062273	8093	3.9081095
7989	3.9024924	8024	3.9043909	8059	3.9062812	8094	3.9081632
7990	3.9025468	8025	3.9044450	8060	3.9063351	8095	3.9082169
7991	3.9026011	8026	3.9044992	8061	3.9063889	8096	3.9082705
7992	3.9026555	8027	3.9045533	8062	3.9064428	8097	3.9083241
7993	3.9027098	8028	3.9046074	8063	3.9064967	8098	3.9083778
7994	3.9027641	8029	3.9046615	8064	3.9065505	8099	3.9084314
7995	3.9028185	8030	3.9047155	8065	3.9066044	8100	3.9084850
7996	3.9028728	8031	3.9047696	8066	3.9066582	8101	3.9085386
7997	3.9029271	8032	3.9048237	8067	3.9067121	8102	3.9085922
7998	3.9029814	8033	3.9048778	8068	3.9067659	8103	3.9086458
7999	3.9030357	8034	3.9049318	8069	3.9068197	8104	3.9086994
8000	3.9030900	8035	3.9049859	8070	3.9068735	8105	3.9087530
8001	3.9031443	8036	3.9050399	8071	3.9069273	8106	3.9088066
8002	3.9031985	8037	3.9050940	8072	3.9069812	8107	3.9088602
8003	3.9032528	8038	3.9051480	8073	3.9070350	8108	3.9089137
8004	3.9033071	8039	3.9052020	8074	3.9070888	8109	3.9089673
8005	3.9043613	8040	3.9052560	8075	3.9071425	8110	3.9090209
8006	3.9034156	8041	3.9053101	8076	3.9071963	8111	3.9090744
8007	3.9034698	8042	3.9053641	8077	3.9072501	8112	3.9091279
8008	3.9035241	8043	3.9054181	8078	3.9073088	8113	3.9091815
8009	3.9035783	8044	3.9054721	8079	3.9073576	8114	3.9092350
8010	3.9036325	8045	3.9055261	8080	3.9074114	8115	3.9092885
8011	3.9036867	8046	3.9055801	8081	3.9074651	8116	3.9093420
8012	3.9037409	8047	3.9056341	8082	3.9075188	8117	3.9093955
8013	3.9037951	8048	3.9056880	8083	3.9075726	8118	3.9094490
8014	3.9038493	8049	3.9057420	8084	3.9076263	8119	3.9095025
8015	3.9039035	8050	3.9057960	8085	3.9076800	8120	3.9095560

Num.	Logarithm.	Num	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
8121	3.9096095	8156	3.9114772	8191	3.9133369	8226	3.9151887
8122	3.9096630	8157	3.9115305	8192	3.9133899	8227	3.9152415
8123	3.9097165	8158	3.9115837	8193	3.9134430	8228	3.9152943
8124	3.9097699	8159	3.9116369	8194	3.9134960	8229	3.9153471
8125	3.9098234	8160	3.9116902	8195	3.9135490	8230	3.9153998
8126	3.9098768	8161	3.9117434	8196	3.9136019	8231	3.9154526
8127	3.9099303	8162	3.9117966	8197	3.9136549	8232	3.9155054
8128	3.9099837	8163	3.9118498	8198	3.9137079	8233	3.9155581
8129	3.9100371	8164	3.9119030	8199	3.9137609	8234	3.9156109
8130	3.9100905	8165	3.9119562	8200	3.9138139	8235	3.9156636
8131	3.9101440	8166	3.9120094	8201	3.9138668	8236	3.9157163
8132	3.9101974	8167	3.9120626	8202	3.9139198	8237	3.9157691
8133	3.9102508	8168	3.9121157	8203	3.9139727	8238	3.9158218
8134	3.9103042	8169	3.9121689	8204	3.9140257	8239	3.9158745
8135	3.9103576	8170	3.9122220	8205	3.9140786	8240	3.9159272
8136	3.9104109	8171	3.9122752	8206	3.9141315	8241	3.9159799
8137	3.9104643	8172	3.9123234	8207	3.9141844	8242	3.9160326
8138	3.9105177	8173	3.9123815	8208	3.9142373	8243	3.9160853
8139	3.9105710	8174	3.9124346	8209	3.9142903	8244	3.9161380
8140	3.9106244	8175	3.9124878	8210	3.9143432	8245	3.9161907
8141	3.9106778	8176	3.9125409	8211	3.9143961	8246	3.9162433
8142	3.9107311	8177	3.9125940	8212	3.9144489	8247	3.9162960
8143	3.9107844	8178	3.9126471	8213	3.9145018	8248	3.9163487
8144	3.9108378	8179	3.9127002	8214	3.9145547	8249	3.9164013
8145	3.9108911	8180	3.9127533	8215	3.9146076	8250	3.9164539
8146	3.9109444	8181	3.9128064	8216	3.9146604	8251	3.9165066
8147	3.9109977	8182	3.9128595	8217	3.9147133	8252	3.9165592
8148	3.9110510	8183	3.9129126	8218	3.9147661	8253	3.9166118
8149	3.9111043	8184	3.9129656	8219	3.9148190	8254	3.9166645
8150	3.9111576	8185	3.9130187	8220	3.9148718	8255	3.9167171
8151	3.9112109	8186	3.9130717	8221	3.9149246	8256	3.9167697
8152	3.9112642	8187	3.9131248	8222	3.9149775	8257	3.9168223
8153	3.9113174	8188	3.9131778	8223	3.9150303	8258	3.9168749
8154	3.9113707	8189	3.9132309	8224	3.9150831	8259	3.9169275
8155	3.9114240	8190	3.9132839	8225	3.9151359	8260	3.9169800

Num.	*Logarithm.*	Num.	*Logarithm.*	Num.	*Logarithm.*	Num.	*Logarithm*
8261	3.9170326	8296	3.9188687	8331	3.9206971	8366	3.9225179
8262	3.9170852	8297	3.9189211	8332	3.9207493	8367	3.9225698
8263	3.9171378	8298	3.9189734	8333	3.9208014	8368	3.9226217
8264	3.9171903	8299	3.9190258	8334	3.9208535	8369	3.9226736
8265	3.9172429	8300	3.9190781	8335	3.9209056	8370	3.9227255
8266	3.9172954	8301	3.9191304	8336	3.9209577	8371	3.9227773
8267	3.9173479	8302	3.9191827	8337	3.9210098	8372	3.9228292
8268	3.9174005	8303	3.9192350	8338	3.9210619	8373	3.9228811
8269	3.9174530	8304	3.9192873	8339	3.9211140	8374	3.9229330
8270	3.9175055	8305	3.9193396	8340	3.9211661	8375	3.9229848
8271	3.9175580	8306	3.9193919	8341	3.9212181	8376	3.9230367
8272	3.9176108	8307	3.9194442	8342	3.9212701	8377	3.9230885
8273	3.9176630	8308	3.9194965	8343	3.9213222	8378	3.9231404
8274	3.9177155	8309	3.9195488	8344	3.9213743	8379	3.9231922
8275	3.9177680	8310	3.9196010	8345	3.9214263	8380	3.9232440
8276	3.9178205	8311	3.9196533	8346	3.9214784	8381	3.9232958
8277	3.9178730	8312	3.9197055	8347	3.9215304	8382	3.9233477
8278	3.9179254	8313	3.9197578	8348	3.9215824	8383	3.9233995
8279	3.9179779	8314	3.9198100	8349	3.9216345	8384	3.9234513
8280	3.9180303	8315	3.9198623	8350	3.9216865	8385	3.9235031
8281	3.9180828	8316	3.9199145	8351	3.9217385	8386	3.9235549
8282	3.9181352	8317	3.9199667	8352	3.9217905	8387	3.9236066
8283	3.9181877	8318	3.9200189	8353	3.9218425	8388	3.9236584
8284	3.9182401	8319	3.9200711	8354	3.9218945	8389	3.9237102
8285	3.9182925	8320	3.9201233	8355	3.9219465	8390	3.9237620
8286	3.9183449	8321	3.9201755	8356	3.9219984	8391	3.9238137
8287	3.9183973	8322	3.9202277	8357	3.9220504	8392	3.9238655
8288	3.9184497	8323	3.9202799	8358	3.9221024	8393	3.9239172
8289	3.9185021	8324	3.9203321	8359	3.9221543	8394	3.9239690
8290	3.9185545	8325	3.9203842	8360	3.9222063	8395	3.9240207
8291	3.9186069	8326	3.9204364	8361	3.9222582	8396	3.9240724
8292	3.9186593	8327	3.9204886	8362	3.9223 02	8397	3.9241242
8293	3.9187117	8328	3.9205407	8363	3.9223621	8398	3.9241759
8294	3.9187640	8329	3.9205929	8364	3.9224140	8399	3.9242276
8295	3.9188164	8330	3.9206450	8365	3.9224659	8400	3.9242793

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
8401	3.9243310	8436	3.9261366	8471	3.9279347	8506	3.9297254
8402	3.9243827	8437	3.9261880	8472	3.9279859	8507	3.9297764
8403	3.9244344	8438	3.9262395	8473	3.9280372	8508	3.9298275
8404	3.9244860	8439	3.9262910	8474	3.9280885	8509	3.9298785
8405	3.9245377	8440	3.9263424	8475	3.9281397	8510	3.9299296
8406	3.9245894	8441	3.9263939	8476	3.9281909	8511	3.9299806
8407	3.9246410	8442	3.9264453	8477	3.9282422	8512	3.9300316
8408	3.9246927	8443	3.9264968	8478	3.9282934	8513	3.9300826
8409	3.9247444	8444	3.9265482	8479	3.9283446	8514	3.9301336
8410	3.9247960	8445	3.9265997	8480	3.9283959	8515	3.9301847
8411	3.9248476	8446	3.9266511	8481	3.9284471	8516	3.9302357
8412	3.9248993	8447	3.9267025	8482	3.9284983	8517	3.9302866
8413	3.9249509	8448	3.9267539	8483	3.9285495	8518	3.9303376
8414	3.9250025	8449	3.9268053	8484	3.9286007	8519	3.9303886
8415	3.9250541	8450	3.9268567	8485	3.9286518	8520	3.9304396
8416	3.9251057	8451	3.9269081	8486	3.9287030	8521	3.9304906
8417	3.9251573	8452	3.9269595	8487	3.9287542	8522	3.9305415
8418	3.9252089	8453	3.9270109	8488	3.9288054	8523	3.9305925
8419	3.9252605	8454	3.9270622	8489	3.9288565	8524	3.9306434
8420	3.9253121	8455	3.9271136	8490	3.9289077	8525	3.9306944
8421	3.9253637	8456	3.9271650	8491	3.9289588	8526	3.9307453
8422	3.9254152	8457	3.9272163	8492	3.9290100	8527	3.9307963
8423	3.9254668	8458	3.9272677	8493	3.9290611	8528	3.9308472
8424	3.9255184	8459	3.9273190	8494	3.9291123	8529	3.9308981
8425	3.9255699	8460	3.9273704	8495	3.9291634	8530	3.9309490
8426	3.9256215	8461	3.9274217	8496	3.9292145	8531	3.9309999
8427	3.9256730	8462	3.9274730	8497	3.9292656	8532	3.9310508
8428	3.9257245	8463	3.9275243	8498	3.9293167	8533	3.9311017
8429	3.9257761	8464	3.9275757	8499	3.9293678	8534	3.9311526
8430	3.9258276	8465	3.9276270	8500	3.9294189	8535	3.9312035
8431	3.9258791	8466	3.9276783	8501	3.9294700	8536	3.9312544
8432	3.9259306	8467	3.9277296	8502	3.9295211	8537	3.9313053
8433	3.9259821	8468	3.9277808	8503	3.9295722	8538	3.9313561
8434	3.9260336	8469	3.9278321	8504	3.9296233	8539	3.9314070
8435	3.9260851	8470	3.9278834	8505	3.9296743	8540	3.9314579

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm	Num.	Logarithm.
8541	3.9315087	8576	3.9332848	8611	3.9350536	8646	3.9368182
8542	3.9315596	8577	3.9333354	8612	3.9351040	8647	3.9368655
8543	3.9316154	8578	3.9333860	8613	3.9351544	8648	3.9369157
8544	3.9316612	8579	3.9334367	8614	3.9352049	8649	3.9369659
8545	3.9317121	8580	3.9334873	8615	3.9352553	8650	3.9370161
8546	3.9317629	8581	3.9335379	8616	3.9353057	8651	3.9370663
8547	3.9318137	8582	3.9335885	8617	3.9353561	8652	3.9371165
8548	3.9318645	8583	3.9336392	8618	3.9354065	8653	3.9371667
8549	3.9319153	8584	3.9336897	8619	3.9354569	8654	3.9372169
8550	3.9319661	8585	3.9337403	8620	3.9355073	8655	3.9372671
8551	3.9320169	8586	3.9337909	8621	3.9355576	8656	3.8373172
8552	3.9320677	8587	3.9338415	8622	3.9356080	8657	3.9373674
8553	3.9321185	8588	3.9338920	8623	3.9356584	8658	3.9374176
8554	3.9321692	8589	3.9339426	8624	3.9357087	8659	3.9374677
8555	3.9322200	8590	3.9339932	8625	3.9357591	8660	3.9375179
8556	3.9322708	8591	3.9340437	8626	3.9358095	8661	3.9375680
8557	3.9323215	8592	3.9340943	8627	3.9358598	8662	3.9376182
8558	3.9323723	8593	3.9341448	8628	3.9359101	8663	3.9376683
8559	3.9324230	8594	3.9341953	8629	3.9359605	8664	3.9377184
8560	3.9324738	8595	3.9342459	8630	3.9360108	8665	3.9377686
8561	3.9325245	8596	3.9342964	8631	3.9360611	8666	3.9378187
8562	3.9325752	8597	3.9343469	8632	3.9361114	8667	3.9378688
8563	3.9326259	8598	3.9343974	8633	3.9361617	8668	3.9379189
8564	3.9326767	8599	3.9344479	8634	3.9362120	8669	3.9379690
8565	3.9327274	8600	3.9344984	8635	3.9362623	8670	3.9380191
8566	3.9327781	8601	3.9345489	8636	3.9363126	8671	3.9380692
8567	3.9328288	8602	3.9345994	8637	3.9363629	8672	3.9381193
8568	3.9328795	8603	3.9346499	8638	3.9364132	8673	3.9381693
8569	3.9329301	8604	3.9347004	8639	3.9364635	8674	3.9382194
8570	3.9329808	8605	3.9347509	8640	3.9365137	8675	3.9382695
8571	3.9330315	8606	3.9348013	8641	3.9365640	8676	3.9383195
8572	3.9330822	8607	3.9348518	8642	3.9366143	8677	3.9383696
8573	3.9331328	8608	3.9349022	8643	3.9366645	8678	3.9384196
8574	3.9331835	8609	3.9349527	8644	3.9367148	8679	3.9384697
8575	3.9332341	8610	3.9350032	8645	3.9367650	8680	3.9385197

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
8681	3.9385698	8716	3.9403172	8751	3.9420577	8786	3.9437912
8682	3.9386198	8717	3.9403670	8752	3.9421073	8787	3.9438406
8683	3.9386698	8718	3.9404169	8753	3.9421569	8788	3.9438900
8684	3.9387198	8719	3.9404667	8754	3.9422065	8789	3.9439395
8685	3.9387698	8720	3.9405165	8755	3.9422561	8790	3.9439880
8686	3.9388198	8721	3.9405663	8756	3.9423058	8791	3.9440383
8687	3.9388698	8722	3.9406161	8757	3.9423553	8792	3.9440877
8688	3.9389198	8723	3.9406659	8758	3.9424049	8793	3.9441371
8689	3.9389698	8724	3.9407157	8759	3.9424545	8794	3.9441865
8690	3.9390198	8725	3.9407654	8760	3.9425041	8795	3.9442358
8691	3.9390697	8726	3.9408152	8761	3.9425537	8796	3.9442852
8692	3.9391197	8727	3.9408650	8762	3.9426032	8797	3.9443346
8693	3.9391697	8728	3.9409147	8763	3.9426528	8798	3.9443840
8694	3.9392196	8729	3.9409645	8764	3.9427024	8799	3.9444333
8695	3.9392696	8730	3.9410142	8765	3.9427519	8800	3.9444827
8696	3.9393195	8731	3.9410640	8766	3.9428015	8801	3.9445320
8697	3.9393695	8732	3.9411137	8767	3.9428510	8802	3.9445814
8698	3.9394194	8733	3.9411635	8768	3.9429005	8803	3.9446307
8699	3.9394693	8734	3.9412132	8769	3.9429501	8804	3.9446800
8700	3.9395193	8735	3.9412629	8770	3.9429996	8805	3.9447294
8701	3.9395692	8736	3.9413126	8771	3.9430491	8806	3.9447787
8702	3.9396191	8737	3.9413623	8772	3.9430986	8807	3.9448280
8703	3.9396690	8738	3.9414120	8773	3.9431481	8808	3.9448773
8704	3.9397189	8739	3.9414617	8774	3.9431976	8809	3.9449266
8705	3.9397688	8740	3.9415114	8775	3.9432471	8810	3.9449759
8706	3.9398187	8741	3.9415611	8776	3.9432966	8811	3.9450252
8707	3.9398685	8742	3.9416108	8777	3.9433461	8812	3.9450745
8708	3.9399184	8743	3.9416605	8778	3.9433956	8813	3.9451238
8709	3.9399683	8744	3.9417101	8779	3.9434450	8814	3.9451730
8710	3.9400182	8745	3.9417598	8780	3.9434945	8815	3.9452223
8711	3.9400680	8746	3.9418095	8781	3.9435440	8816	3.9452716
8712	3.9401179	8747	3.9418591	8782	3.9435934	8817	3.9453208
8713	3.9401677	8748	3.9419088	8783	3.9436429	8818	3.9453701
8714	3.9402176	8749	3.9419584	8784	3.9436923	8819	3.9454192
8715	3.9402674	8750	3.9420081	8785	3.9437418	8820	3 9454686

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
8821	3.9455178	8856	3.9472376	8891	3.9489506	8926	3.9506569
8822	3.9455671	8857	3.9472866	8892	3.9489994	8927	3.9507055
8823	3.9456163	8858	3.9473357	8893	3.9490483	8928	3.9507542
8824	3.9456655	8859	3.9473847	8894	3.9490971	8929	3.9508028
8825	3.9457147	8860	3.9474337	8895	3.9491460	8930	3.9508515
8826	3.9457639	8861	3.9474827	8896	3.9491948	8931	3.9509001
8827	3.9458131	8862	3.9475317	8897	3.9492436	8932	3.9509487
8828	3.9458623	8863	3.9475807	8898	3.9492924	8933	3.9509973
8829	3.9459115	8864	3.9476297	8899	3.9493412	8934	3.9510459
8830	3.9459607	8865	3.9476787	8900	3.9493900	8935	3.9510946
8831	3.9460099	8866	3.9477277	8901	3.9494388	8936	3.9511432
8832	3.9460591	8867	3.9477767	8902	3.9494876	8937	3.9511918
8833	3.9461082	8868	3.9478257	8903	3.9495364	8938	3.9512404
8834	3.9461574	8869	3.9478747	8904	3.9495851	8939	3.9512889
8835	3.9462066	8870	3.9479236	8905	3.9496339	8940	3.9513375
8836	3.9462557	8871	3.9479726	8906	3.9496827	8941	3.9513861
8837	3.9463048	8872	3.9480215	8907	3.9497314	8942	3.9514347
8838	3.9463540	8873	3.9480705	8908	3.9497802	8943	3.9514832
8839	3.9464031	8874	3.9481194	8909	3.9498290	8944	3.9515318
8840	3.9464523	8875	3.9481684	8910	3.9498777	8945	3.9515803
8841	3.9465014	8876	3.9482173	8911	3.9499264	8946	3.9516289
8842	3.9465505	8877	3.9482662	8912	3.9499752	8947	3.9516774
8843	3.9465996	8878	3.9483151	8913	3.9500239	8948	3.9517260
8844	3.9466487	8879	3.9483641	8914	3.9500726	8949	3.9517745
8845	3 9466978	8880	3.9484130	8915	3.9501213	8950	3.9518230
8846	3.9467469	8881	3.9484619	8916	3.9501701	8951	3.9518716
8847	3.9467960	8882	3.9485108	8917	3.9502188	8952	3.9519201
8848	3.9468451	8883	3.9485597	8918	3.9502675	8953	3.9519686
8849	3.9468942	8884	3.9486085	8919	3.9503162	8954	3.9520171
8850	3.9469433	8885	3.9486574	8920	3.9503649	8955	3.9520656
8851	3.9469923	8886	3.9487063	8921	3.9504135	8956	3.9521141
8852	3.9470414	8887	3.9487552	8922	3.9504622	8957	3.9521626
8853	3.9470905	8888	3.9488040	8923	3.9505109	8958	3.9522111
8854	3.9471395	8889	3.9488529	8924	3.9505596	8959	3.9522595
8855	3.9471886	8890	3.9489018	8925	3.9506082	8960	3.9523080

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm
8961	3.9523565	8996	3.9540494	9031	3.9557358	9066	3.9574157
8962	3.9524049	8997	3.9540977	9032	3.9557839	9067	3.9574636
8963	3.9524534	8998	3.9541460	9033	3.9558320	9068	3.9575115
8964	3.9525018	8999	3.9541943	9034	3.9558800	9069	3.9575594
8965	3.9525503	9000	3.9542425	9035	3.9559282	9070	3.9576073
8966	3.9525987	9001	3.9542908	9036	3.9559762	9071	3.9576552
8967	3.9526472	9002	3.9543390	9037	3.9560243	9072	3.9577030
8968	3.9526956	9003	3.9543872	9038	3.9560723	9073	3.9577509
8969	3.9527440	9004	3.9544355	9039	3.9561204	9074	3.9577988
8970	3.9527924	9005	3.9544837	9040	3.9561684	9075	3.9578466
8971	3.9528409	9006	3.9545319	9041	3.9562165	9076	3.9578945
8972	3.9528893	9007	3.9545802	9042	3.9562645	9077	3.9579423
8973	3.9529377	9008	3.9546284	9043	3.9563125	9078	3.9579902
8974	3.9529861	9009	3.9546766	9044	3.9563605	9079	3.9580380
8975	3.9530345	9010	3.9547248	9045	3.9564686	9080	3.9580858
8976	3.9530828	9011	3.9547730	9046	3.9564566	9081	3.9581337
8977	3.9531312	9012	3.9548212	9047	3.9565046	9082	3.9581815
8978	3.9531796	9013	3.9548694	9048	3.9565526	9083	3.9582294
8979	3.9532280	9014	3.9549176	9049	3.9566006	9084	3.9582771
8980	3.9532763	9015	3.9549657	9050	3.9566486	9085	3.9583249
8981	3.9533247	9016	3.9550139	9051	3.9566966	9086	3.9583727
8982	3.9533730	9017	3.9550621	9052	3.9567445	9087	3.9584205
8983	3.9534214	9018	3.9551102	9053	3.9567925	9088	3.9584683
8984	3.9534697	9019	3.9551584	9054	3.9568405	9089	3.9585161
8985	3.9535181	9020	3.9552065	9055	3.9568885	9090	3.9585639
8986	3.9535664	9021	3.9552547	9056	3.9569364	9091	3.9586117
8987	3.9536147	9022	3.9553028	9057	3.9569844	9092	3.9586594
8988	3.9536631	9023	3.9553510	9058	3.9570323	9093	3.9587072
8989	3.9537114	9024	3.9553991	9059	3.9570803	9094	3.9587549
8990	3.9537597	9025	3.9554472	9060	3.9571282	9095	3.9588027
8991	3.9538080	9026	3.9554953	9061	3.9571761	9096	3.9588505
8992	3.9538563	9027	3.9555434	9062	3.9572241	9097	3.9588982
8993	3.9539046	9028	3.9555915	9063	3.9572720	9098	3.9589459
8994	3.9539529	9029	3.9556397	9064	3.9573199	9099	3.9589937
8995	3.9540012	9030	3.9556877	9065	3.9573678	9100	3.9590414

Num.	Logarithm.	Num	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
9101	3.9590891	9136	3.9607561	9171	3.9624167	9206	3.9640710
9102	3.9591368	9137	3.9608036	9172	3.9624640	9207	3.9641181
9103	3.9591845	9138	3.9608512	9173	3.9625114	9208	3.9641653
9104	3.9592322	9139	3.9608987	9174	3.9625587	9209	3.9642125
9105	3.9592799	9140	3.9609462	9175	3.9626061	9210	3.9642596
9106	3.9593276	9141	3.9609937	9176	3.9626534	9211	3.9643068
9107	3.9593753	9142	3.9610412	9177	3.9627007	9212	3.9643539
9108	3.9594230	9143	3.9610887	9178	3.9627481	9213	3.9644011
9109	3.9594707	9144	3.9611362	9179	3.9627954	9214	3.9644482
9110	3.9595184	9145	3.9611837	9180	3.9628427	9215	3.9644953
9111	3.9595660	9146	3.9612312	9181	3.9628900	9216	3.9645425
9112	3.9596137	9147	3.9612787	9182	3.9629373	9217	3.9645896
9113	3.9596614	9148	3.9613262	9183	3.9629846	9218	3.9646367
9114	3.9597090	9149	3.9613736	9184	3.9630319	9219	3.9646838
9115	3.9597567	9150	3.9614211	9185	3.9630792	9220	3.9647309
9116	3.9598043	9151	3.9614686	9186	3.9631264	9221	3.9647780
9117	3.9598520	9152	3.9615160	9187	3.9631737	9222	3.9648251
9118	3.9598996	9153	3.9615635	9188	3.9632210	9223	3.9648722
9119	3.9599472	9154	3.9616109	9189	3.9632683	9224	3.9649193
9120	3.9599948	9155	3.9616583	9190	3.9633155	9225	3.9649664
9121	3.9600425	9156	3.9617058	9191	3.9633628	9226	3.9650134
9122	3.9600901	9157	3.9617532	9192	3.9634100	9227	3.9650605
9123	3.9601377	9158	3.9618006	9193	3.9634573	9228	3.9651076
9124	3.9601853	9159	3.9618481	9194	3.9635045	9229	3.9651546
9125	3.9602329	9160	3.9618955	9195	3.9635517	9230	3.9652017
9126	3.9602805	9161	3.9619429	9196	3.9635990	9231	3.9652488
9127	3.9603280	9162	3.9619903	9197	3.9636462	9232	3.9652958
9128	3.9603756	9163	3.9620377	9198	3.9636934	9233	3.9653428
9129	3.9604232	9164	3.9620851	9199	3.9637406	9234	3.9653899
9130	3.9604708	9165	3.9621325	9200	3.9637878	9235	3.9654369
9131	3.9605183	9166	3.9621799	9201	3.9638350	9236	3.9654839
9132	3.9605659	9197	3.9622272	9202	3.9638822	9237	3.9655309
9133	3.9606135	9168	3.9622746	9203	3.9639294	9238	3.9655780
9134	3.9606610	9169	3.9623220	9204	3.9639766	9239	3.9656250
9135	3.9607086	9170	3.9623693	9205	3.9640238	9240	3.9656720

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm
9241	3.9657190	9276	3.9673607	9311	3.9689963	9346	3.9706258
9242	3.9657660	9277	3.9674076	9312	3.9690430	9347	3.9706722
9243	3.9658130	9278	3.9674544	9313	3.9690896	9348	3.9707187
9244	3.9658599	9279	3.9675012	9314	3.9691362	9349	3.9707652
9245	3.9659069	9280	3.9675480	9315	3.9691829	9350	3.9708116
9246	3.9659539	9281	3.9675948	9316	3.9692295	9351	3.9708581
9247	3.9660009	9282	3.9676416	9317	3.9692761	9352	3.9709045
9248	3.9660478	9283	3.9676883	9318	3.9693227	9353	3.9709509
9249	3.9660948	9284	3.9677351	9319	3.9693693	9354	3.9709974
9250	3.9661417	9285	3.9677819	9320	3.9694159	9355	3.9710438
9251	3.9661887	9286	3.9678287	9321	3.9694625	9356	3.9710902
9252	3.9662356	9287	3.9678754	9322	3.9695091	9357	3.9711366
9253	3.9662826	9288	3.9679222	9323	3.9695557	9358	3.9711830
9254	3.9663295	9289	3.9679690	9324	3.9696023	9359	3.9712294
9255	3.9663764	9290	3.9680157	9325	3.9696488	9360	3.9712758
9256	3.9664233	9291	3.9680625	9326	3.9696954	9361	3.9713222
9257	3.9664703	9292	3.9681092	9327	3.9697420	9362	3.9713686
9258	3.9665172	9293	3.9681559	9328	3.9697885	9363	3.9714150
9259	3.9665641	9294	3.9682027	9329	3.9698351	9364	3.9714614
9260	3.9666110	9295	3.9682494	9330	3.9698816	9365	3.9715078
9261	3.9666579	9296	3.9682961	9331	3.9699282	9366	3.9715542
9262	3.9667048	9297	3.9683428	9332	3.9699747	9367	3.9716005
9263	3.9667517	9298	3.9683895	9333	3.9700213	9368	3.9716469
9264	3.9667985	9299	3.9684362	9334	3.9700678	9369	3.9716932
9265	3.9668444	9300	3.9684829	9335	3.9701143	9370	3.9717396
6265	3.9668923	9301	3.9685296	9336	3.9701608	9371	3.9717859
6267	3.9669392	9302	3.9685763	9337	3.9702074	9372	3.9718323
6268	3.9669860	9303	3.9686230	9338	3.9702539	9373	3.9718786
6269	3.9670329	9304	3.9686697	9339	3.9703004	9374	3.9719249
6270	3.9670707	9305	3.9687164	9340	3.9703460	9375	3.9719713
9271	3.9671266	9306	3.9687630	9341	3.9703934	9376	3.9720176
9272	3.9671734	9307	3.9688097	9342	3.9704399	9377	3.9720639
9273	3.9672203	9308	3.9688564	9343	3.9704863	9378	3.9721102
9274	3.9672671	9309	3.9689030	9344	3.9705328	9379	3.9721565
9275	3.9673139	9310	3.9689497	9345	3.9705793	9380	3.9722028

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
9381	3.9722491	9416	3.9738665	9451	3.9754778	9486	3.9770831
9382	3.9722954	9417	3.9739126	9452	3.9755237	9487	3.9771289
9383	3.9723417	9418	3.9739587	9453	3.9755697	9488	3.9771747
9384	3.9723880	9419	3.9740048	9454	3.9756156	9489	3.9772204
9385	3.9724343	9420	3.9740509	9455	3.9756615	9490	3.9772662
9386	3.9724805	9421	3.9740970	9456	3.9757075	9491	3.9773120
9387	3.9725268	9422	3.9741431	9457	3.9757534	9492	3.9773578
9388	3.9725731	9423	3.9741892	9458	3.9757993	9493	3.9774035
9389	3.9726193	9424	3.9742353	9459	3.9758452	9494	3.9774492
9390	3.9726656	9425	3.9742814	9460	3.9758911	9495	3.9774950
9391	3.9727118	9426	3.9743274	9461	3.9759370	9496	3.9775407
9392	3.9727581	9427	3.9743735	9462	3.9759829	9497	3.9775864
9393	3.9728043	9428	3.9744196	9463	3.9760288	9498	3.9776322
9394	3.9728506	9429	3.9744656	9464	3.9760747	9499	3.9776779
9395	3.9728968	9430	3.9745117	9465	3.9761206	9500	3.9777236
9396	3.9729430	9431	3.9745577	9466	3.9761665	9501	3.9777693
9397	3.9729892	9432	3.9746038	9467	3.9762124	9502	3.9778150
9398	3.9730354	9433	3.9746498	9468	3.9762582	9503	3.9778607
9399	3.9730816	9434	3.9746959	9469	3.9763041	9504	3.9779064
9400	3.9731278	9435	3.9747419	9470	3.9763500	9505	3.9779521
9401	3.9731741	9436	3.9747870	9471	3.9763958	9506	3.9779978
9402	3.9732202	9437	3.9748340	9472	3.9764417	9507	3.9780435
9403	3.9732664	9438	3.9748800	9473	3.9764875	9508	3.9780892
9404	3.9733126	9439	3.9749260	9474	3.9765334	9509	3.9781348
9405	3.9733588	9440	3.9749720	9475	3.9765792	9510	3.9781805
9406	3.9734050	9441	3.9750180	9476	3.9766251	9511	3.9782262
9407	3.9734511	9442	3.9750640	9477	3.9766709	9512	3.9782718
9408	3.9734973	9443	3.9751100	9478	3.9767167	9513	3.9783175
9409	3.9735435	9444	3.9751560	9479	3.9767625	9514	3.9783631
9410	3.9735896	9445	3.9752020	9480	3.9768083	9515	3.9784088
9411	3.9736358	9446	3.9752479	9481	3.9768541	9516	3.9784544
9412	3.9736819	9447	3.9752939	9482	3.9768999	9517	3.9785001
9413	3.9737281	9448	3.9753399	9483	3.9769457	9518	3.9785457
9414	3.9737742	9449	3.9753858	9484	3.9769915	9519	3.9785913
9415	3.9738203	9450	3.9754318	9485	3.9770373	9520	3.9786369

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
9521	3.9786826	9556	3.9802761	9591	3.9818639	9626	3.9834459
9522	3.9787282	9557	3.9803216	9592	3.9819092	9627	3.9834910
9523	3.9787738	9558	3.9803670	9593	3.9819544	9628	3.9835361
9524	3.9788194	9559	3.9804125	9594	3.9819997	9629	3.9835812
9525	3.9788650	9560	3.9804579	9595	3.9820450	9630	3.9836263
9526	3.9789106	9561	3.9805033	9596	3.9820902	9631	3.9836714
9527	3.9789562	9562	3.9805487	9597	3.9821355	9632	3.9837165
9528	3.9790017	9563	3.9805942	9598	3.9821807	9633	3.9837616
9529	3.9790473	9564	3.9806396	9599	3.9822260	9634	3.9838066
9530	3.9790929	9565	3.9806850	9600	3.9822712	9635	3.9838517
9531	3.9791385	9566	3.9807304	9601	3.9823165	9636	3.9838968
9532	3.9791840	9567	3.9807758	9602	3.9823617	9637	3.9839419
9533	3.9792296	9568	3.9808212	9603	3.9824069	9638	3.9839869
9534	3.9792751	9569	3.9808666	9604	3.9824522	9639	3.9840320
9535	3.9793207	9570	3.9809119	9605	3.9824974	9640	3.9840770
9536	3.9793662	9571	3.9809573	9606	3.9825426	9641	3.9841221
9537	3.9794118	9572	3.9810027	9607	3.9825878	9642	3.9841671
9538	3.9794573	9573	3.9810481	9608	3.9826330	9643	3.9842122
9539	3.9795028	9574	3.9810934	9609	3.9826782	9644	3.9842572
9540	3.9795484	9575	3.9811388	9610	3.9827234	9645	3.9843022
9541	3.9795939	9576	3.9811841	9611	3.9827686	9646	3.9843473
9542	3.9796394	9577	3.9812295	9612	3.9828138	9647	3.9843923
9543	3.9796849	9578	3.9812748	9613	3.9828589	9648	3.9844373
9544	3.9797304	9579	3.9813202	9614	3.9829041	9649	3.9844823
9545	3.9797759	9580	3.9813655	9615	3.9829493	9650	3.9845273
9546	3.9798214	9581	3.9814108	9616	3.9829945	9651	3.9845723
9547	3.9798669	9582	3.9814562	9617	3.9830396	9652	3.9846173
9548	3.9799124	9583	3.9815015	9618	3.9830848	9653	3.9846623
9549	3.9799579	9584	3.9815468	9619	3.9831299	9654	3.9847073
9550	3.9800034	9585	3.9815921	9620	3.9831751	9655	3.9847523
9551	3.9800488	9586	3.9816374	9621	3.9832202	9656	3.9847973
9552	3.9800943	9587	3.9816827	9622	3.9832654	9657	3.9848422
9553	3.9801398	9588	3.9817280	9623	3.9833105	9658	3.9848872
9554	3.9801852	9589	3.9817733	9624	3.9833556	9659	3.9849322
9555	3.9802307	9590	3.9818186	9625	3.9834007	9660	3.9849771

Num.	*Logarithm.*	Num.	*Logarithm.*	Num.	*Logarithm*	Num.	*Logarithm.*
9661	3.9850221	9696	3.9865926	9731	3.9881575	9766	3.9897167
9662	3.9850670	9697	3.9866374	9732	3.9882021	9767	3.9897612
9663	3.9851120	9698	3.9866822	9733	3.9882467	9768	3.9898056
9664	3.9851569	9699	3.9867270	9734	3.9882913	9769	3.9898501
9665	3.9852019	9700	3.9867717	9735	3.9883360	9770	3.9898946
9666	3.9852468	9701	3.9868165	9736	3.9883806	9771	3.9899390
9667	3.9852917	9702	3.9868613	9737	3.9884252	9772	3.9899835
9668	3.9853366	9703	3.9869060	9738	3.9884698	9773	3.9900279
9669	3.9853816	9704	3.9869508	9739	3.9885144	9774	3.9900723
9670	3.9854265	9705	3.9869955	9740	3.9885590	9775	3.9901168
9671	3.9854714	9706	3.9870403	9741	3.9886035	9776	3.9901612
9672	3.9855163	9707	3.9870850	9742	3.9886481	9777	3.9902056
9673	3.9855612	9708	3.9871298	9743	3.9886927	9778	3.9902500
9674	3.9856061	9709	3.9871745	9744	3.9887373	9779	3.9902944
9675	3.9856510	9710	3.9872192	9745	3.9887818	9780	3.9903389
9676	3.9856959	9711	3.9872640	9746	3.9888264	9781	3.9903833
9677	3.9857407	9712	3.9873087	9747	3.9888710	9782	3.9904277
9678	3.9857856	9713	3.9873534	9748	3.9889155	9783	3.9904721
9679	3.9858305	9714	3.9873981	9749	3.9889601	9784	3.9905164
9680	3.9858754	9715	3.9874428	9750	3.9890046	9785	3.9905608
9681	3.9859202	9716	3.9874875	9751	3.9890492	9786	3.9906052
6682	3.9859651	9717	3.9875322	9752	3.9890937	9787	3.9906496
9683	3.9860099	9718	3.9875769	9753	3.9891382	9788	3.9906940
9684	3.9860548	9719	3.9876216	9754	3.9891828	9789	3.9907383
9685	3 9860996	9720	3.9876663	9755	3.9892273	9790	3.9907827
9686	3.9861455	9721	3.9877109	9756	3.9892718	9791	3.9908270
9687	3.9861893	9722	3.9877556	9757	3.9893163	9792	3.9908714
9688	3.9862341	9723	3.9878003	9758	3.9893608	9793	3.9909158
9689	3.9862790	9724	3.9878449	9759	3.9894053	9794	3.9909601
9690	3.9863238	9725	3.9878896	9760	3.9894498	9795	3.9910044
9691	3.9863686	9726	3.9879343	9761	3.9894943	9796	3.9910488
9692	3.9864134	9727	3.9879789	9762	3.9895388	9797	3.9910931
9693	3.9864582	9728	3.9880236	9763	3.9895833	9798	3.9911374
9694	3.9865030	9729	3.9880682	9764	3.9896278	9799	3.9911818
9695	3.9865478	9730	3 9881128	9765	3.9896722	9800	3.9912261

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
9801	3.9912704	9836	3.9928185	9871	3.9943612	9906	3.9958983
9802	3.9913147	9837	3.9928627	9872	3.9944051	9907	3.9959422
9803	3.9913590	9838	3.9929068	9873	3.9944491	9908	3.9959860
9804	3.9914033	9839	3.9929510	9874	3.9944931	9909	3.9960298
9805	3.9914476	9840	3.9929951	9875	3.9945371	9910	3.9960737
9806	3.9914919	9841	3.9930392	9876	3.9945811	9911	3.9961175
9807	3.9915362	9842	3.9930834	9877	3.9946251	9912	3.9961613
9808	3.9915805	9843	3.9931275	9878	3.9946690	9913	3.9962051
9809	3.9916247	9844	3.9931716	9879	3.9947130	9914	3.9962489
9810	3.9916690	9845	3.9932157	9880	3.9947569	9915	3.9962927
9811	3.9917133	9846	3.9932598	9881	3.9948009	9916	3.9963365
9812	3.9917575	9847	3.9933039	9882	3.9948448	9917	3.9963803
9813	3.9918018	9848	3.9933480	9883	3.9948888	9918	3.9964241
9814	3.9918461	9849	3.9933921	9884	3.9949327	9919	3.9964679
9815	3.9918903	9850	3.9934362	9885	3.9949767	9920	3.9965117
9816	3.9919345	9851	3.9934803	9886	3.9950206	9921	3.9965554
9817	3.9919788	9852	3.9935244	9887	3.9950645	9922	3.9965992
9818	3.9920230	9853	3.9935685	9888	3.9951085	9923	3.9966430
9819	3.9920673	9854	3.9936126	9889	3.9951524	9924	3.9966868
9820	3.9921115	9855	3.9936566	9890	3.9951963	9925	3.9967305
9821	3.9921557	9856	3.9937007	9891	3.9952402	9926	3.9967743
9822	3.9921999	9857	3.9937448	9892	3.9952841	9927	3.9968180
9823	3.9922441	9858	3.9937888	9893	3.9953280	9928	3.9968618
9824	3.9922884	9859	3.9938329	9894	3.9953719	9929	3.9969055
9825	3.9923326	9860	3.9938769	9895	3.9954158	9930	3.9969492
9826	3.9923768	9861	3.9939210	9896	3.9954597	9931	3.9969930
9827	3.9924210	9862	3.9939650	9897	3.9955036	9932	3.9970367
9828	3.9924651	9863	3.9940090	9898	3.9955474	9933	3.9970804
9829	3.9925093	9864	3.9940531	9899	3.9955913	9934	3.9971242
9830	3.9925535	9865	3.9940971	9900	3.9956352	9935	3.9971679
9831	3.9925977	9866	3.9941411	9901	3.9956791	9936	3.9972116
9832	3.9926419	9867	3.9941851	9902	3.9957229	9937	3.9972553
9833	3.9926860	9868	3.9942291	9903	3.9957668	9938	3.9972990
9834	3.9927302	9869	3.9942731	9904	3.9958106	9939	3.9973427
9835	3.9927744	9870	3.9943172	9905	3.9958545	9940	3.9973864

Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.	Num.	Logarithm.
9941	3.9974301	9956	3.9980849	9971	3.9987387	9986	3.9993916
9942	3.9974738	9957	3.9981285	9972	3.9987823	9987	3.9994350
9943	3.9975174	9958	3.9981721	9973	3.9988258	9988	3.9994785
9944	3.9975611	9959	3.9982157	9974	3.9988694	9989	3.9995220
9945	3.9976048	9960	3.9982593	9975	3.9989129	9990	3.9995655
9946	3.9976485	9961	3.9983029	9976	3.9889564	9991	3.9996090
9947	3.9976921	9962	3.9983465	9977	3.9990000	9992	3.9996524
9948	3.9977358	9963	3.9983901	9978	3.9990435	9993	3.9996959
9949	3.9977794	9964	3.9984337	9979	3.9990870	9994	3.9997393
9950	3.9978231	9965	3.9984773	9980	3.9991305	9995	3.9997828
9951	3.9978667	9966	3.9985209	9981	3.9991740	9996	3.9998262
9952	3.9979104	9967	3.9985645	9982	3.9992176	9997	3.9998697
9953	3.9979540	9968	3.9986080	9983	3.9992611	9998	3.9999131
9954	3.9979976	9969	3.9986516	9984	3.9993046	9999	3.9999566
9955	3.9980413	9970	3.9986952	9985	3.9993481	10000	4.0000000

FINIS.

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38 „ 10
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